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Effects of Bearing Clearance on the Chatter Stability of Milling Process

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Abstract: In the present study, the influences of the bearing clearance, which is a common fault for machines, to the chatter stability of milling process are examined by using numerical simulation method. The results reveal that the presence of bearing clearance could make the milling process easier to enter the status of chatter instability and can shift the chatter frequency. In addition, the spectra analysis to vibration signals obtained under the instable milling processes show that the presence of bearing clearance could introduce more frequency components to the vibration responses but, however, under both the stable and instable milling processes, the generated frequency components will not violate the ideal spectra structures of the vibration responses of the milling process, which are usually characterized by the tooth passing frequency and its associated higher harmonics for the stable milling process and by the complex coupling of the tooth passing frequency and the chatter frequency for the instable milling process. This implies that, even under the case with bearing clearance fault, the stability of the milling process can still be determined by viewing the frequency spectra of the vibration responses. Moreover, the phenomena of the chatter frequency shift and the generation of more components provide potential ways to detect the bearing clearance in machines.

1 Introduction

Chatter is a problem of instability occurring in the metal cutting process [1], which is associated with forced and/or self-excited oscillations and usually characterized by violent vibration, loud sound and poor quality of surface finish.

Chatter can cause a life reduction to the tool and affect the productivity by interfering with the normal functioning of the machine process. The problem has been widely investigated by manufacturing community and has been a popular topic for academic and industrial researches. The pioneering chatter instability models were the regeneration theory proposed by Trusty and Polacek [2], and Tobias and Fiswick [3] almost at the same period but independent of each other, which now is referred to by almost every researcher investigating the chatter instability. The regeneration theory has been further developed by many researchers to make it applicable for different metal cutting scenarios including turning and boring [4], drilling [5], milling [6] and grinding [7] operations. Altintas and Weck [8] have contributed a comprehensive review to the fundamental modelling of chatter vibrations and the associated chatter stability lobes for the four metal cutting and grinding processes. Efforts have also been made to improve the prediction accuracy of the chatter stability by taking into account of various factors which are probably encountered in practice, for example the influence of structural interfaces on the dynamic stiffness of the machine [9], the mode-coupling effect [10], the feed speed variation [11] and the effect of noise excitation [12]. In addition, particular attentions have been paid to the vibration response of machine tool because monitoring the vibration frequencies during machining is an efficient way for identifying machine tool chatter and distinguishing between different types of instabilities [13]. During the stable cutting process, the vibration signals are usually dominated by the tooth-passing frequency, and although the higher harmonics may appear in the vibration signals, the motion would still be period-one type [14]. In contrast, during the chatter instable cutting, the machine tool can behave with quasi- period motion [14] or even with chaotic motion [15], and the vibration signals contain multiple frequencies, which usually are dominated by both the tooth-passing frequency and the chatter frequency [14]. The spectra structures of the vibration are well defined, and therefore the stable and instable cutting cases can clearly be distinguished based on the spectra. However, sometimes the nice structure of the chatter frequencies can be influenced or destroyed by several other effects in practice, for example the runout [16]~[18] where the geometric axis of the milling cutter differs from the rotation axis. The bearing clearance is a common fault for the machines, which can lead the machine to complex nonlinear behaviours like quasi-period motion and chaotic motion [19]-[22]. The appearance of bearing clearance fault could considerably deteriorate the performance of machines, and therefore it is of significance to investigate the effects of the bearing clearance to the cutting process and to inspect the spectra structures of the vibration signals in order to accumulate information for the machine condition assessment purpose and also for identifying the machine tool chatter under the bearing clearance case.

In this paper, based on a 2-DOF of milling process model, the effects of the bearing clearance to the cutting process is analyzed by comparing the stability lobe diagrams and the corresponding response spectra for the milling models with and without bearing clearance. The theoretical stability lobe for the milling model without bearing is also calculated using the method proposed by Opitz [23]. The frequency components that arise due to bearing clearance are clearly identified. This provides useful information to determine if the bearing clearance and chatter are present.

2 Model of Milling Process

The 2-DOF model of a workpiece-tool system with bearing clearance is illustrated in Fig 1. The feed direction and spindle rotation are shown for an up-milling operation. The tool is represented by an equivalent two-degree-of-freedom spring-mass-damper system, and the workpiece is assumed to be rigid, and the effect of the bearing clearance is modelled using the discontinuous stiffness model which has been discussed in [20] and [22]. The 2-DOF oscillator with bearing clearance is excited by the cutting force, and the governing equation can be written in the following equation.

$$\left.\begin{aligned} m_x\ddot{x} + c_x\dot{x} + k_{x1}x + \delta(D-\gamma)k_{x2}(D-\gamma)\cos(\varphi) = F_x(t) \\ m_y\ddot{y} + c_y\dot{y} + k_{y1}y + \delta(D-\gamma)k_{y2}(D-\gamma)\sin(\varphi) = F_y(t) \end{aligned}\right\} \tag{1}$$

where x and y are the displacement of the cutter-head in X- and Y- directions respectively, and $D = \sqrt{x^2 + y^2}$, $\cos(\varphi) = x/D$, $\sin(\varphi) = y/D$, and γ is the radial clearance of the bearing, and $\delta(\bullet)$ is a switch function with $\delta(\Delta) = 1$ if $\Delta \geq 0$ and $\delta(\Delta) = 0$ for others. $F_x(t)$ and $F_y(t)$ are the cutting force at time t in X- and Y- directions. The system described by Eq. (1) is a typical non-linear dynamic system with discontinuous stiffness characteristics.

[Fig 1]

For the cutting process study, one key problem is about the modelling of the cutting force. Assuming that the number of teeth in the cutter head is N and the spindle speed is Ω, the tooth passing period τ is equal to

$$\tau = \frac{1}{N\Omega} \tag{2}$$

Referring to Fig 1, the i^{th} tooth is acted upon by an orthogonal force, $F_r(i,t)$ which is the feed force in the radial direction, and $F_t(i,t)$ which is the tangential cutting component in the direction of cutting speed. Usually, the force is taken proportional to the instantaneous chip thickness and the chip thickness direction is perpendicular to the cut surface, as proposed by Koenigsberger et al [24]. Thus

$$\begin{cases} F_r(i,t) = k_n k_t a h(i,t) \\ F_t(i,t) = k_t a h(i,t) \end{cases} \tag{3}$$

where k_t is the specific cutting energy, and k_n is the ratio of radial to tangential cutting forces, *a* is the axial width of cut, and $h(i,t)$ is the chip thickness for tooth *i* of the cutter at time *t*, which consists of a static part due to the rigid body motion of the cutter-head and a dynamic component caused by vibrations of the cutter-head at the present and previous tooth periods and can be determined from

$$h(i,t) = \Delta x(t)\sin\theta(t,i) + \Delta y(t)\cos\theta(t,i) \tag{4}$$

where $\Delta x(t)$ and $\Delta y(t)$ are given by

$$\begin{cases} \Delta x(t) = x(t) - x(t-\tau) + v\tau \\ \Delta y(t) = y(t) - y(t-\tau) \end{cases} \tag{5}$$

and $\theta(t,i)$ is the angular position of tooth *i* at time *t* and is given by

$$\theta(t,i) = 2\pi\Omega t - (i-1)\frac{2\pi}{N} + \theta_0 \tag{6}$$

and θ_0 is the initial angular position of the first tooth.

From Eq. (3), the cutting force in the feed and normal direction can be solved as follows:

$$\begin{cases} F_x(i,t) = -F(i,t)\cos(\theta(t,i) - \psi) \\ F_y(i,t) = F(i,t)\sin(\theta(t,i) - \psi) \end{cases} \tag{7}$$

where

$$\begin{cases} F(i,t) = a k_t \sqrt{1+k_n^2}\, h(i,t) \\ \psi = \tan^{-1}\dfrac{F_r}{F_t} = \tan^{-1} k_n \end{cases} \tag{8}$$

After carrying out a summation of the forces over the *N* teeth, one can obtain the following expressions for the net force components:

$$\begin{Bmatrix} F_x(t) \\ F_y(t) \end{Bmatrix} = \sum_{i=1}^{N} \begin{Bmatrix} F_x(i,t) \\ F_y(i,t) \end{Bmatrix} = \sum_{i=1}^{N} g(\theta(t,i)) \begin{Bmatrix} -F(i,t)\cos(\theta(t,i)-\psi) \\ F(i,t)\sin(\theta(t,i)-\psi) \end{Bmatrix} \tag{9}$$

The function $g(\theta(t,i))$ is a screen function, and it is equal to 1 if the i^{th} tooth is cutting or 0 when not cutting:

$$g(\theta(t,i)) = \begin{cases} 1 & \text{if } \phi_{\text{ex}} > \theta(t,i) > \phi_{\text{st}} \\ 0 & \text{otherwise} \end{cases} \tag{10}$$

where ϕ_{ex} and ϕ_{st} are the exit and start angles. Substituting Eq.s (4)~(8) into (9) yields

$$\begin{Bmatrix} F_x(t) \\ F_y(t) \end{Bmatrix} = ak_t\sqrt{1+k_n^2}\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}\begin{Bmatrix} \Delta x(t) \\ \Delta y(t) \end{Bmatrix} \tag{11}$$

where

$$\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} = \sum_{i=1}^{N} g(\theta(t,i))\begin{pmatrix} -\sin\theta(t,i)\cos(\theta(t,i)-\psi) & -\cos\theta(t,i)\cos(\theta(t,i)-\psi) \\ \sin\theta(t,i)\sin(\theta(t,i)-\psi) & \cos\theta(t,i)\sin(\theta(t,i)-\psi) \end{pmatrix} \tag{12}$$

As the cutter-head rotates, the directional factors A_{ii} in Eq. (12) vary with time. However, the matrix is period at tooth passing frequency (ω = $N\Omega$) when the workpiece is continuously engaged by the cutter.

3 Chatter Stability Lobes

3.1 Linear Chatter Stability Theory

When the bearing clearance is free for the milling process, then Eq. (1) can be simplified to be

$$\left.\begin{aligned} m_x\ddot{x} + c_x\dot{x} + (k_{x1}+k_{x2})x = F_x(t) \\ m_y\ddot{y} + c_y\dot{y} + (k_{y1}+k_{y2})y = F_y(t) \end{aligned}\right\} \tag{13}$$

and, according to the chatter theory presented in [23], both the vibration components $\Delta x(t)$ and $\Delta y(t)$ are dominated by the chatter vibration frequency ω_c, and so the resultant cutting force can be expressed as $F_x(t) = F_x e^{j\omega_c t}$ and $F_y(t) = F_y e^{j\omega_c t}$. Therefore, the vibrations at present time t and previous tooth $t-\tau$ can be determined as

$$\begin{aligned} x(j\omega_c) = \Phi_X(j\omega_c)F_x(j\omega_c),\ x_d(j\omega_c) = e^{-j\omega_c\tau}\Phi_X(j\omega_c)F_x(j\omega_c) \\ y(j\omega_c) = \Phi_Y(j\omega_c)F_y(j\omega_c),\ y_d(j\omega_c) = e^{-j\omega_c\tau}\Phi_Y(j\omega_c)F_y(j\omega_c) \end{aligned} \tag{14}$$

where Φ_X and Φ_Y are direct frequency response functions in X- and Y- directions respectively, which can be calculated directly from Eq. (13).

$$\left.\begin{aligned} \Phi_X(j\omega_c) = \frac{1}{-m_x\omega_c^2 + jc_x\omega_c + (k_{x1}+k_{x2})} \\ \Phi_Y(j\omega_c) = \frac{1}{-m_y\omega_c^2 + jc_y\omega_c + (k_{y1}+k_{y2})} \end{aligned}\right\} \tag{15}$$

For simplicity, it is assumed that only one tooth is acting on the workpiece, and then the cutting forces at tow directions are determined as

$$\begin{cases} F_x(t) = -F(t)\cos(\theta-\psi) \\ F_y(t) = F(t)\sin(\theta-\psi) \end{cases} \tag{16}$$

The chip thicknesses then can be calculated as

$$\begin{cases} \Delta x(j\omega_c) = x(j\omega_c) - x_d(j\omega_c) \\ \qquad = (e^{-j\omega_c\tau} - 1)\Phi_X(j\omega_c)\cos(\theta-\psi)F(j\omega_c) \\ \Delta y(j\omega_c) = y(j\omega_c) - y_d(j\omega_c) \\ \qquad = -(e^{-j\omega_c\tau} - 1)\Phi_Y(j\omega_c)\sin(\theta-\psi)F(j\omega_c) \end{cases} \tag{17}$$

As the static chip load $v\tau$ does not affect the critical stability of the dynamic machine system and so it can be ignored in the stability analysis, then substituting Eq. (17) into Eq. (4) yields

$$h(t) = (e^{-j\omega_c\tau} - 1)Fe^{j\omega_c t}\Phi_c(j\omega_c) \tag{18}$$

where $\Phi_c(j\omega_c)$ is the oriented transfer function and can be calculated as

$$\Phi_c(j\omega_c) = \left[\Phi_X(j\omega_c)\sin\theta\cos(\theta-\psi) - \Phi_Y(j\omega_c)\cos\theta\sin(\theta-\psi)\right] \tag{19}$$

Generally, the oriented transfer function can be determined for multiple teeth cases by the approach proposed by Opitz [23] as follows

$$\Phi_c(j\omega_c) = V_X\Phi_X(j\omega_c) + V_Y\Phi_Y(j\omega_c) \tag{20}$$

and

$$\begin{cases} V_X = \dfrac{N}{2\pi}\displaystyle\int_{\phi_{\mathrm{st}}}^{\phi_{\mathrm{ex}}} \sin\theta\cos(\theta-\psi)d\theta = \dfrac{N}{4\pi}\left|\theta\sin\psi - \dfrac{1}{2}\cos(2\theta-\psi)\right|_{\phi_{st}}^{\phi_{\mathrm{ex}}} \\ V_Y = \dfrac{N}{2\pi}\displaystyle\int_{\phi_{\mathrm{st}}}^{\phi_{\mathrm{ex}}} -\cos\theta\sin(\theta-\psi)d\theta = \dfrac{N}{4\pi}\left|\theta\sin\psi + \dfrac{1}{2}\cos(2\theta-\psi)\right|_{\phi_{st}}^{\phi_{\mathrm{ex}}} \end{cases} \tag{21}$$

The average of dynamic chip thickness leads to mean dynamic resultant cutting force as follows

$$Fe^{j\omega_c t} = ak_t\sqrt{1+k_n^2}(e^{-j\omega_c\tau} - 1)Fe^{j\omega_c t}\Phi_c(j\omega_c) \tag{22}$$

From Eq. (22), the chatter stability lobes of the milling process can be estimated by solving the following characteristic equation

$$1 + (1 - e^{-j\omega_c T})a_{\lim}k_t\sqrt{1+k_n^2}\Phi_c(j\omega_c) = 0 \tag{23}$$

where $a_{\lim}$ is the maximum axial width of cut for chatter vibration free machining. From the characteristic Eq. (23), the stability lobes are then solved as

$$\left.\begin{aligned} a_{\lim} &= \frac{-1}{k_t\sqrt{1+k_n^2}\,\mathrm{Re}(\Phi_c(j\omega_c))} \\ T &= \frac{1}{\omega_c}(\varepsilon + 2k\pi) \rightarrow n = \frac{60}{NT} \\ \varepsilon &= \pi + 2\tan^{-1}\left(\frac{\mathrm{Im}(\Phi_c(j\omega_c))}{\mathrm{Re}(\Phi_c(j\omega_c))}\right) \end{aligned}\right\} \tag{24}$$

where T is the tooth passing period, n [rev/min] is the spindle speed.

3.2 Nonlinear Effects in Milling Process

Although the above linear stability theory could predict most of important phenomena in cutting dynamics, in recent years there has been a resurgence of interesting in modelling milling process as nonlinear dynamics for a better insight to the complex dynamics in cutting materials. Generally speaking, the major nonlinear effects on milling dynamics include [25]:

1) material constitutive relations (stress versus strain, strain rate and temperature),
2) tool-structure nonlinearities,
3) friction at the tool-chip interface,
4) loss of tool-workpiece contact,
5) influences of machine drive unit.

Most research efforts have usually been made to investigate the first four kinds of nonlinear effects [14][25]-[28], and results indicated that such nonlinear cutting dynamics could exhibit a global sub-critical Hopf bifurcation (initially super-critical and then turning sub-critical at higher vibration amplitudes); on the other hand, few researches have been done to the fifth one. The nonlinear cutting process is usually investigated by using the following dimensionless dynamic equation

$$\ddot{\eta} + 2\xi\dot{\eta} + p^2(\eta + \beta_2\eta^2 + \beta_3\eta^3 + \cdots) = p^2 w(\Delta f + \alpha_2\Delta f^2 + \alpha_3\Delta f^3 + \cdots) \quad (25)$$

This model incorporates both structural $(\beta_2, \beta_3, \cdots)$ and material nonlinearities $(\alpha_2, \alpha_3, \cdots)$. As a common fault for machine drive units, the appearance of the bearing clearance could cause discontinuously nonlinear stiffness characteristics to entire cutting system and so as to deteriorate the machine performance. Essentially, the milling process with bearing clearance can be regarded as a specific case of model (25) because in mathematics the Weierstrass approximation theorem [29] guarantees that any continuous function on a closed and bounded interval can be uniformly approximated on that interval by a polynomial to any degree of accuracy. Therefore, the restoring force of the spring with discontinuously nonlinear stiffness caused by bearing clearance can be approximated as a polynomial function as shown in Fig 2.

[Fig 2]

When nonlinearities are considered in the milling process, the linear chatter stability theory is no longer valid in determining the chatter stability lobe, and so to analyze the nonlinear cutting dynamics researchers often have had to resort to the perturbation method and numerical simulation method. In addition, in the linear chatter stability study, numerical simulation methods have also been widely used to verify the accuracy of various frequency domain stability lobe prediction models like the model expressed by Eq. (24). In this study, the effects of bearing clearance on the cutting dynamics are investigated by using numerical simulation method.

3.3 Effects of Bearing Clearance on the Chatter Stability

The investigation of the effects of bearing clearance on the chatter stability, numerical simulations is conducted using the fourth order *Runge-Kutta* method. In the numerical simulations, the stability limits are obtained by gradually increasing the axial width of cut while holding all the other parameters constant until instability occurred. This procedure is repeated at different spindle speed to construct the chatter stability lobe. In this study, a milling operation that was originally investigated numerically by Balachandran [14] is considered here. The cutter-head is assumed to have six teeth, and the entry and exit angles ϕ_{ex}, ϕ_{st} are 67 ^{o} and 139^{o} respectively. The specific cutting energy is taken as k_t =900 Mpa, and the proportionality factor k_n is 0.3, and the constant feed speed *v* is 0.5×10^{-5} m / teeth. The modal parameters about the cutter-head are given in Table 1.

[Fig 3]

[Fig 4]

Table 1. Modal parameters of the cutter-head

	Mass (kg)	Stiffness (k_1+k_2) (N/m)	Damping Ratio (%)
X	2.75×10^{-2}	8.79×10^{5}	1.39
Y	2.96×10^{-2}	9.71×10^{5}	1.38

In Fig 3, the results show the comparison between the stability lobes which are obtained by the method presented as Eq. (24) and by the time domain simulation respectively. Basically, the comparisons between the two results are good, especially in the regions between the peaks. As there is no well-established method available to theoretically analyze the chatter stability of the system described as Eq. (1) where the effects of the bearing clearance are taken into account, only the time domain simulation method is used to predict the stability lobe. When the bearing clearance is considered, the stiffness variety due to the clearance is taken as $k_{x2} = 0.5k_{x1}$ and $k_{y2} = 0.5k_{y1}$, the radial clearance γ is 5μm and it is assumed that the presence of clearance will not affect the system damping. Fig 4 shows the stability lobes, which are predicted by the time domain simulation for with clearance and without clearance. The comparisons between the two stability lobes especially at the region around the lobe where k = 1 clearly indicate that the presence of the bearing clearance has reduced the dominant chatter frequency and the limit of the axial width of cut, and an obvious leftward shift of the lobes can be observed. The reducing of the chatter frequency and the cutting width limit can be easily explained as it is known that the presence of bearing clearance in a machine could introduce the stiffness decrease to the whole machine system, and consequently reduce the natural frequencies of the machine system. The stiffness decrease would make it easier for the machine to enter the status of chatter instability.

4 Steady-State Response Analysis

The steady-state vibration response analysis has always played very important role in monitoring the condition of the milling process to prevent the machine from the damage due to the chatter instability. It has been known that, in the case of bearing clearance free, if there is no chatter instability happening [14], the principal period is equal to the tooth passing period τ, and the vibrations are periodic-one type and the ideal vibration frequencies can be determined as

$$f = \frac{kN\Omega}{60} \text{ [Hz]} \qquad k=0; \pm 1; \pm 2, \dots \quad (26)$$

When the chatter instability happens, the type of stability loss will correspond to the secondary Hopf bifurcation of periodic systems, and quasi-periodic vibrations [14] will arise, and the corresponding vibration frequencies are

$$f = \pm\frac{\omega_c}{2\pi} + \frac{kN\Omega}{60} \qquad k=0; \pm 1; \pm 2, \dots \quad (27)$$

The nice spectra structures of the vibration provide convenient methods to identify the stable and instable cutting cases. In this section, the numerical simulation method will be used to verify the spectra structures. Moreover, the influence of bearing clearance to the vibration spectra structures of the milling process is also investigated. The milling processes at two different spindle speeds 3720 rpm and 9000 rpm are considered for different scenarios, i.e. the cutting process is stable or instable, and the bearing clearance is free or presents.

The first scenario considered is the stable milling processes at the spindle speed 3720 rpm. The axial width of cut a is taken as 0.08 mm. From the stability lobes shown in Fig 4, it can be known that the milling processes are stable for all the cases with and without bearing clearance. The simulation results including the sampled vibration signals at both the X- and Y- directions, the cutter-head orbit constructed from the vibration signals and the FFT spectra of the vibration signals are all presented. In addition, in order to clearly show the period one and quasi-periodic vibrations, the Poincaré sections are also plotted together with the orbits. The

Poincaré sections can be obtained by plotting the stroboscopically sampled data in the x-y plane.

[Fig 5]

Fig 5 shows the results obtained under the case without bearing clearance for the first scenario. It is a stable milling process where the dominant peak in the FFT spectra is at the tooth passing frequency $f_{\Omega} = N\times\Omega/60 = 372$ Hz. Although the FFT spectra have higher harmonics, the cutter-head orbit and the Poincaré section clearly show a period-one motion. Obviously, the spectra are also consistent with the frequency structure determined by Eq. (26).

[Fig 6]

The results shown in Fig 6 are obtained under the case with bearing clearance. By comparing the results shown Fig 5, it can be found that there is no significant difference caused by the bearing clearance when the milling process is stable. The cutter-orbit and the Poincaré section clearly show the motion is still period-one type. The spectra also indicate that the dominant peak is still the component at the tooth passing frequency. As many researches have revealed, the discontinuous nonlinearity like the stiffness change due to bearing clearance can introduce higher harmonics to the force responses for the systems subjected to harmonic forces. Therefore, the spectrum structure of the bearing clearance fault is coincident with the frequency structure of the stable milling process and, therefore, in this case, it is difficult to distinguish between the cases with bearing clearance and without bearing clearance from the frequency spectra under the stable cutting. Having a closer comparison between the vibration signals under the cases with and without bearing clearance, it can be seen that the amplitudes of the vibration signals with bearing clearance are relatively bigger than those without bearing clearance, and the associated second harmonics also become larger. The increasing vibration intensity caused by the presence of bearing clearance can certainly reduce the quality of the finished surface and, however, these tiny differences might be able to provide a way to detect the bearing clearance fault in machines, but which would be not very effective.

[Fig 7]

[Fig 8]

The scenarios shown in Fig 7 and Fig 8 are corresponding to chatter instable milling processes without and with bearing clearance respectively. The cutter-head orbits and the associated Poincaré sections, the shapes of which are close ring, all clearly indicate that the cutter-heads are acting with quasi-periodic motions. For the case without bearing clearance, besides the component at the tooth passing frequency f_{Ω} and it associated higher harmonics, there are extra peaks appearing in the response spectra. In the spectrum of the vibration signal in X-direction, the dominant component is 915Hz, which actually is the chatter frequency $f_c = \omega_c/2\pi$. There are even richer frequency components appearing in the spectrum of the vibration signal in Y-direction, i.e. $f_c - f_\Omega = 543$Hz and $f_c - 2f_\Omega = 171$Hz. It can be known that the frequency structure of vibration signals satisfy that determined by Eq. (27). In the results shown in Fig 8, the chatter frequency f_c is determined as 940Hz, which is bigger than the chatter frequency under the case without bearing clearance. Moreover, it can be seen that, comparing to the spectra of the vibration signals obtained under the case without bearing clearance, besides for the components $f_c - f_\Omega = 568$Hz and $f_c - 2f_\Omega = 196$Hz, more frequency components arise in the spectra of the vibration signals with bearing clearance, i.e. the frequency components $3f_\Omega - f_c = 176$Hz, $4f_\Omega - f_c = 548$Hz and $5f_\Omega - f_c = 920$Hz. It is quite clearly that, under the instable milling process, the presence of the bearing clearance can not only shift the chatter frequency but also excite more frequency components. The shift of the chatter frequency and the generation of more frequency components caused by the bearing clearance to the instable milling process provide a more effective method than inspecting the amplitude of the second harmonic to detect the presence of bearing clearance in machines. It is worth noting here that, comparing the frequency components of the vibration signals for the bearing clearance case with the spectra structure defined by Eq. (27), it can be easily seen that the extra frequency components introduced by the bearing clearance are still contained in the spectra structure determined by Eq. (27). This implies the presence of bearing clearance will not make the vibration signal under the instable milling process to violate the well defined spectra structures, and therefore the stable and instable cutting processes can still be distinguished based on the vibration spectra.

[Fig 9]

[Fig 10]

The results shown in Fig 9 and Fig 10 are also obtained under the chatter instable milling processes without and with bearing clearance respectively, but the spindle speed is 9000rpm. Again, the cutter-head orbits and the ring-type Poincaré sections all indicate that the cutter-heads are acting with quasi-periodic motions. From the spectra shown in Fig 9 (C), it can be seen that the chatter frequency component $f_c = 526$Hz only appears in the vibration signal in Y-direction, and the vibration signal in X-direction is a pure sinusoidal signal whose frequency is determined as 900 Hz, which is the tooth passing frequency. This implies that although the entire milling process is chatter instable but, strictly speaking, the instable motion only happens in Y-direction. The motion difference between X- and Y-direction is due to the asymmetric of the cutter-head as the modal parameters shown in Table 1 indicate. In addition, it can also be seen that the spectrum structure of the vibration signal in Y-direction is consistent with the spectra structure defined by Eq. (27). The results shown in Fig 10 are obtained under the case with bearing clearance. Obviously, the chatter frequency components arise in all the spectra of the vibration signals in both directions, which mean, in this case, the motions in both directions are chatter instable. Comparing with the instable motion only in X-direction under the bearing clearance free case, in certain sense, the instable motions in both directions indicate that the presence of bearing clearance could make the milling process easier to enter the instable status. From the spectra in Fig 10(C), the chatter frequency in the case with bearing clearance is determined as 524Hz, which is different from the 526Hz component in the case without bearing clearance. Moreover, obviously, more frequency components have arisen in the spectra, i.e. $2f_c - f_\Omega$ = 148Hz, $3f_c - f_\Omega$ = 672Hz, $f_c + f_\Omega$ = 1424Hz and $2(f_\Omega - f_c)$ = 752Hz. Clearly, the spectra structures under the case with bearing clearance still satisfy the well defined spectra structures given by Eq. (27). Once again, the spectra structures of the vibration signals obtained under spindle speed = 9000 rpm indicate that, even under the bearing clearance case, the stable and instable cutting processes can still be distinguished based on the vibration spectra**.**

5 Conclusions

Bearing clearance is a common fault for machines, which can deteriorate the performance of machines. In the present study, by using a discontinuous stiffness model to represent the effects of the bearing clearance, the influences of the bearing clearance to the chatter stability of milling process have been examined though numerical simulation method. The chatter lobes determined by numerical simulations reveal that the bearing clearance can reduce the dominant chatter frequency and the limit of the axial width of cut so that the milling process with bearing clearance would more easily enter the instable cutting than that without bearing clearance. Moreover, when the chatter instability happens during the milling process, the presence of the bearing clearance can shift the chatter frequency. In addition, the spectra structures of vibration signals obtained under both the stable and instable milling process have been inspected, and the results show that the presence of bearing clearance could introduce more frequency components to the vibration responses. However, under both the stable and instable milling processes, the bearing clearance will not make the frequency structures of the vibration signals violate the ideal spectra structures of the vibration responses of the usual milling process. This means that, even under the case with bearing clearance fault, the stability of the milling process can still be determined by viewing the frequency spectra of the vibration responses. Moreover, the phenomena of the chatter frequency shift and the generation of more components can be regarded as the fault features of the bearing clearance.

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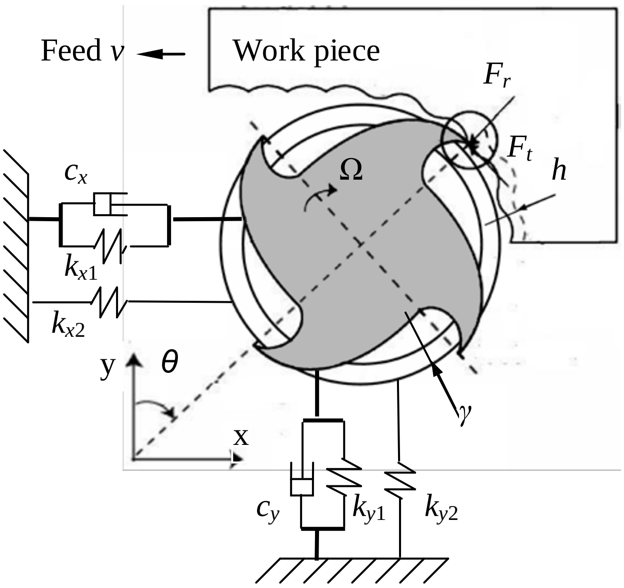

Fig 1 The model of milling process with bearing clearance

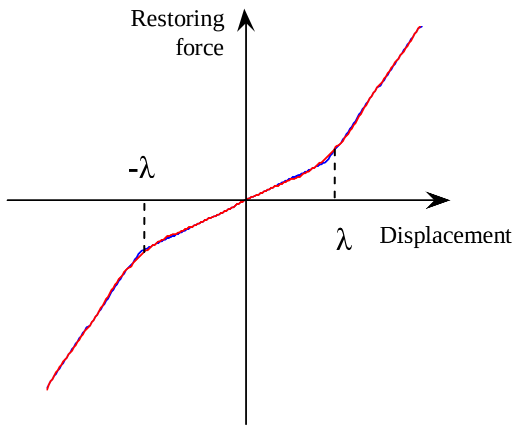

Fig 2, the polynomial approximation of the restoring force of a spring with discontinuous stiffness

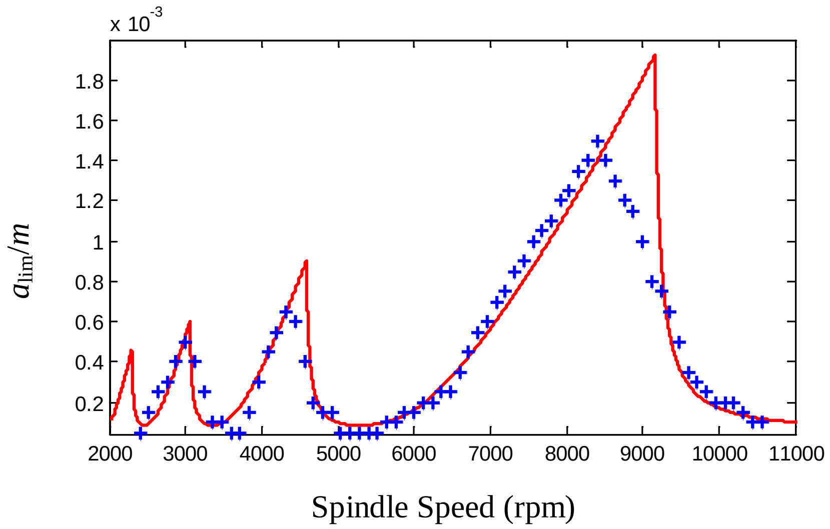

Fig 3, The stability lobes [– : predicated by Eq. (19); + : predicated by time domain simulation]

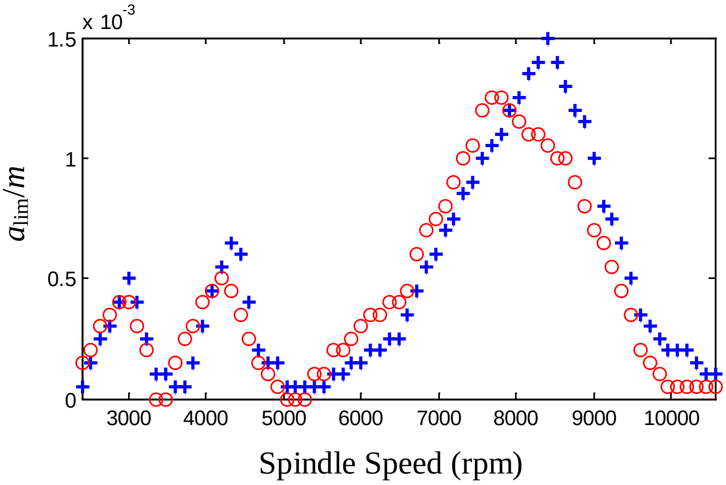

Fig 4, The stability lobes [+: without bearing clearance; ○: with bearing clearance]

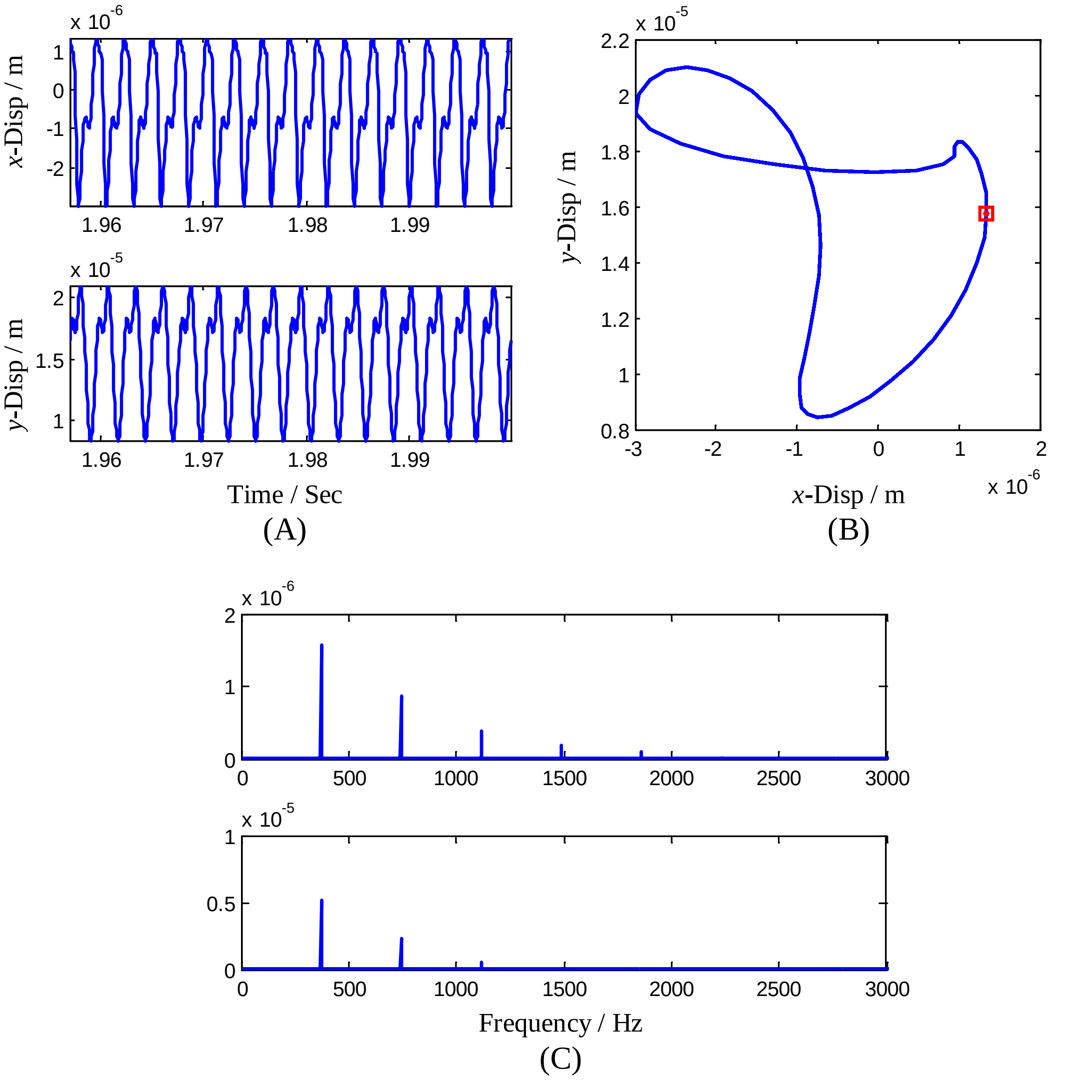

Fig 5, the results for the stable scenario at speed 3720 rpm and without bearing clearance [the sampled vibration signals (A), and the cutter-head orbit along with Poincaré section (B), and the FFT spectra (C)]

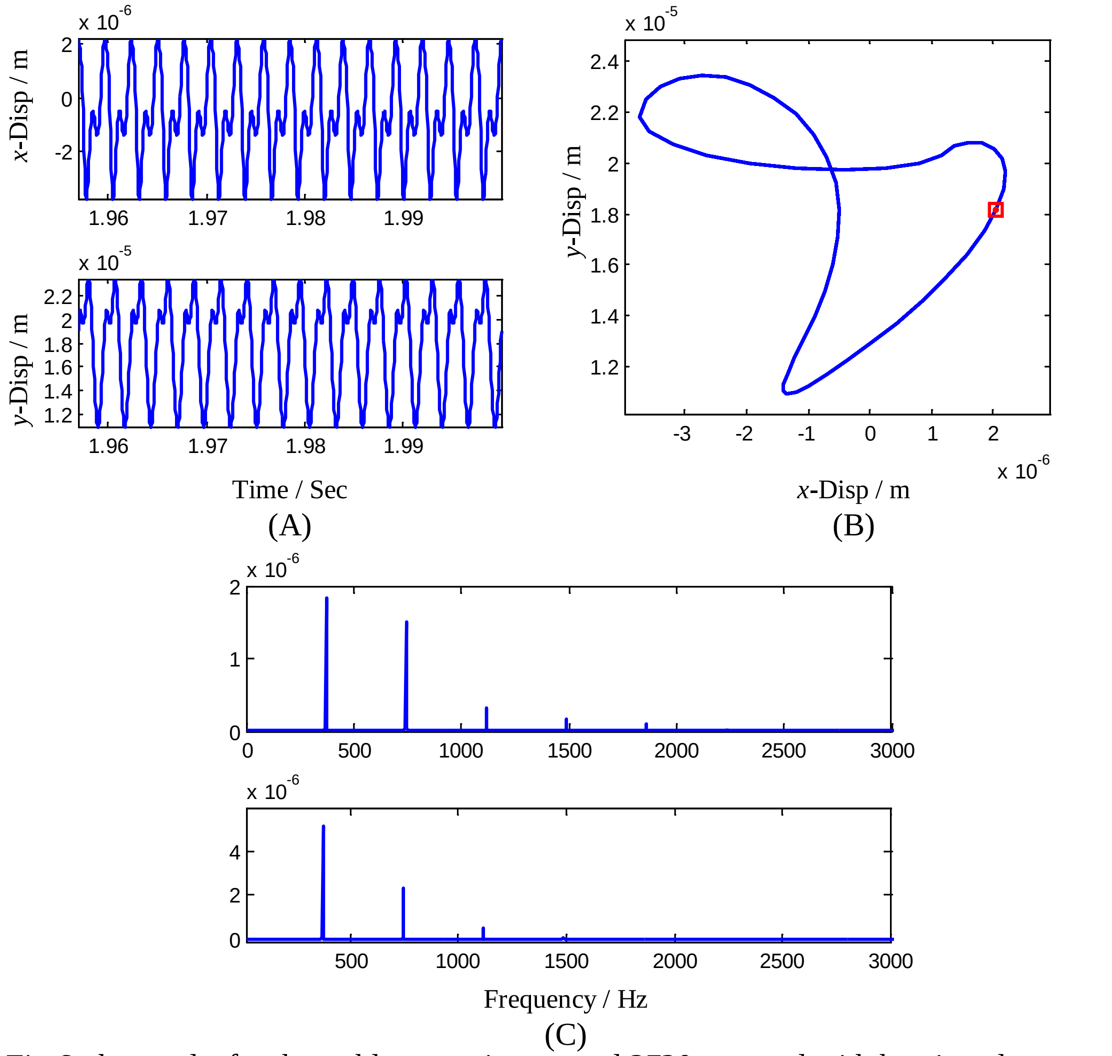

Fig 6, the results for the stable scenario at speed 3720 rpm and with bearing clearance [the sampled vibration signals (A), and the cutter-head orbit along with Poincaré section (B), and the FFT spectra (C)]

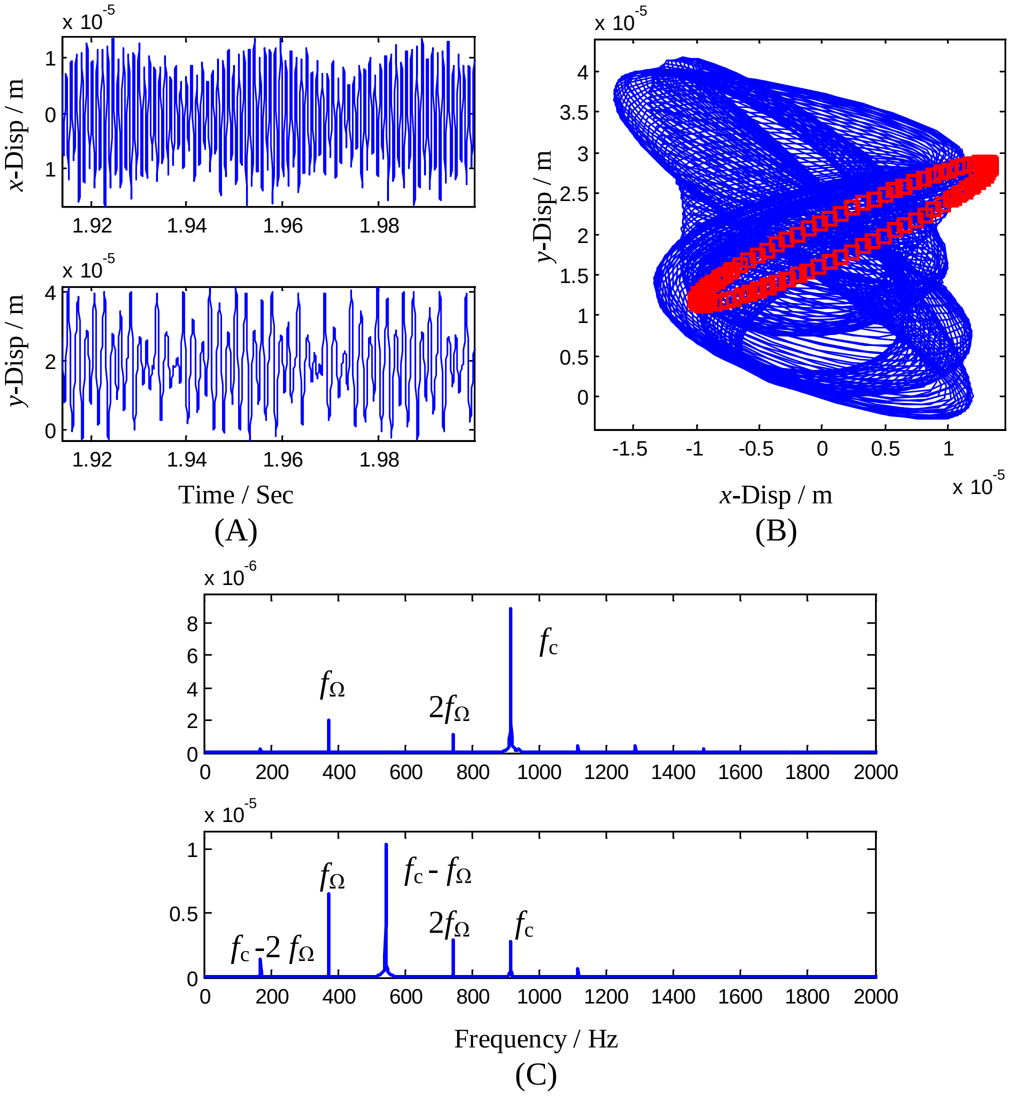

Fig 7, the results for the instable scenario at speed 3720 rpm and without bearing clearance [the sampled vibration signals (A), and the cutter-head orbit along with Poincaré section (B), and the FFT spectra (C)]

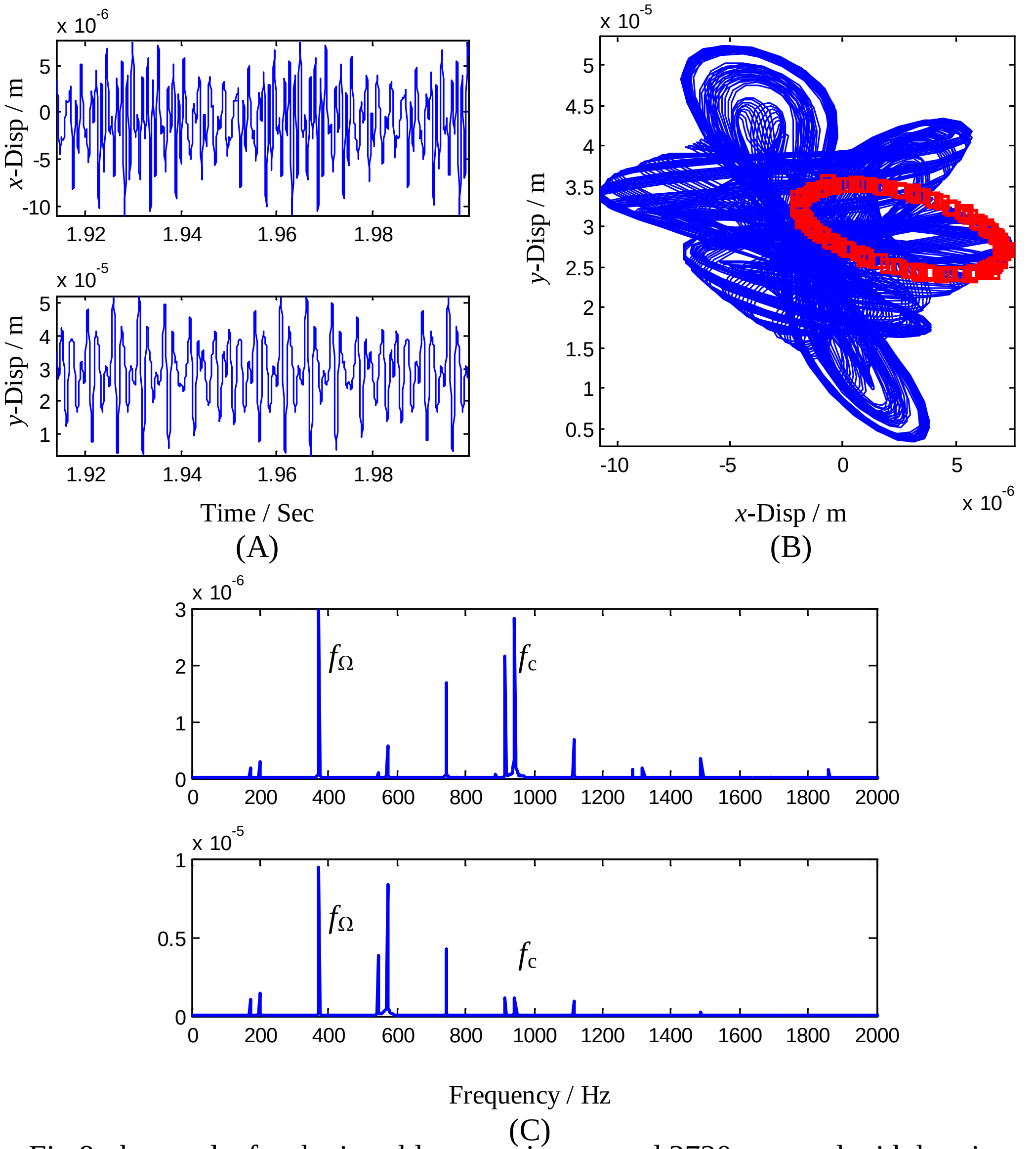

Fig 8, the results for the instable scenario at speed 3720 rpm and with bearing clearance [the sampled vibration signals (A), and the cutter-head orbit along with Poincaré section (B), and the FFT spectra (C)]

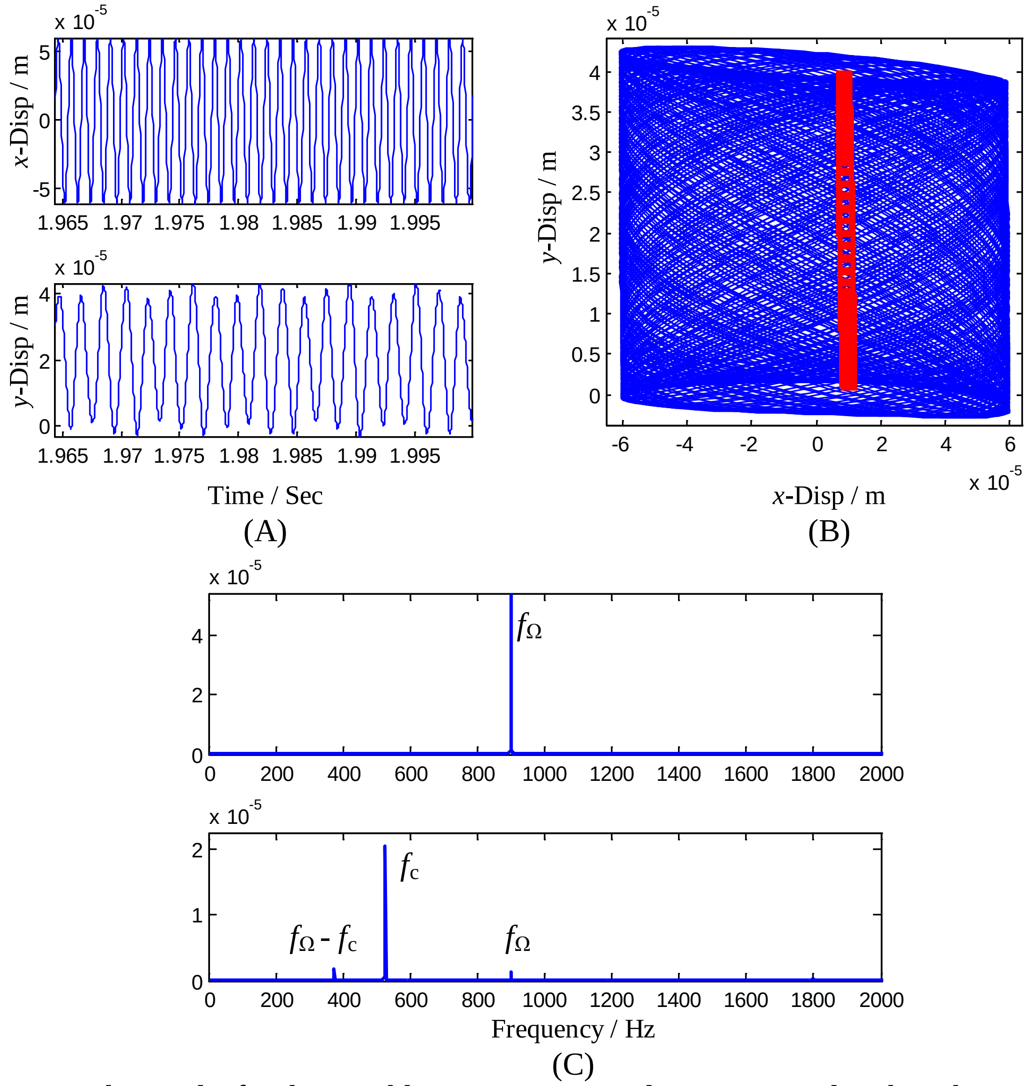

Fig 9, the results for the instable scenario at speed 9000 rpm and without bearing clearance [the sampled vibration signals (A), and the cutter-head orbit along with Poincaré section (B), and the FFT spectra (C)]

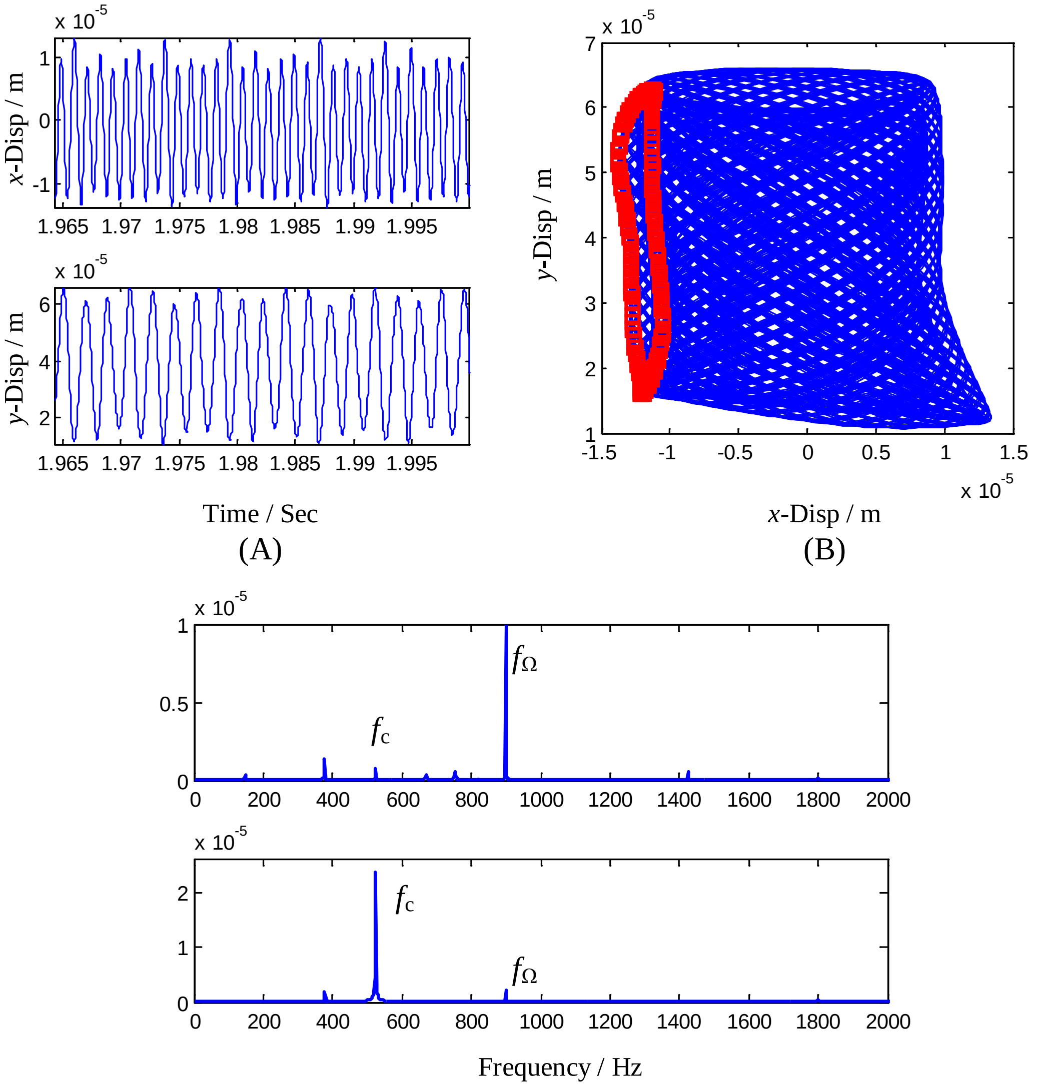

Fig 10, the results for the instable scenario at speed 9000 rpm and with bearing clearance [the sampled vibration signals (A), and the cutter-head orbit along with Poincaré section (B), and the FFT spectra (C)]