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Evaluation of different techniques in estimating orientation of crack initiation planes and fatigue lifetime under complex multiaxial loading paths

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Abstract

In the present investigation, the accuracy of two methods, i.e., the Shear Strain Maximum Variance Method (γ-MVM) and the Maximum Damage Method (MDM), in predicting the orientation of the crack initiation plane was checked by considering several results taken from the literature and generated by testing three different metallic materials under complex multiaxial loading. The γ-MVM postulates that the critical plane is that material plane containing the direction experiencing the maximum variance of the resolved shear strain. In contrast, the MDM defines the critical plane as that material plane on which the accumulated damage reaches its maximum value. In the present investigation, the MDM was applied in conjunction with Fatemi-Socie's multiaxial fatigue criterion, Bannantine-Socie's cycle counting method, and Miner's linear rule. The validation exercise being performed demonstrated that both the γ-MVM and the MDM were capable of accurately predicting the orientation of the crack initiation planes in the selected metals. Subsequently, the reliability of three different design methodologies suitable for estimating fatigue lifetime of metals subjected to variable amplitude multiaxial loading was assessed quantitatively by using a number of experimental results taken from the literature. In more detail, Methodology A was based on the MDM applied along with the FS criterion, the BS cycle counting method, and Miner's rule. Methodology B made use of the γ-MVM, the FS criterion, the BS cycle counting method, and Miner's linear rule. Finally, Methodology C involved the γ-MVM, the Modified Manson Coffin Curve Method (MMCCM), the classical Rain-Flow cycle counting method, and Miner's linear rule. According to this systematic validation exercise, the usage of these three design procedures was seen to result in satisfactory predictions, with the estimates falling within an error band of three.

Keywords: multiaxial fatigue; variable amplitude; critical plane; crack initiation plane; nonproportional loading

Nomenclature

$\boldsymbol{a}$	unit vector defining the orientation of axis a
b	axial fatigue strength exponent
b_0	shear fatigue strength exponent
$b(\rho)$	multiaxial fatigue strength exponent depending on ratio ρ
c	axial fatigue ductility exponent
c_0	shear fatigue ductility exponent
$c(\rho)$	multiaxial fatigue ductility exponent depending on ratio ρ
D	damage sum
D_{cr}	critical value of damage sum D
D_{tot}	total value of damage sum D
E	modulus of elasticity
G	shear modulus
k	material constant in the FS parameter
K'	cyclic strength coefficient
K'_{NP}	cyclic strength coefficient under 90 deg out-of-phase loading
$\boldsymbol{n}$	unit vector perpendicular to a generic material plane, Δ
n_i	number of cycles at the i-th loading level
n'	cyclic strain hardening exponent
n'_{NP}	cyclic stain hardening exponent under 90 deg out-of-phase loading
N_b	number of blocks to failure
$N_{b,e}$	estimated number of blocks to failure
N_f	number of cycles to failure
$N_{f,e}$	estimated number of cycles to failure
$N_{f,i}$	number of cycles to failure under i-th loading level constant amplitude loading
q	generic direction on plane Δ

t	time
T	considered time interval
α	angle between direction q and a-axis
$\varepsilon_i(t)$	normal strain components ($i=x, y, z$)
ε'_f	fatigue ductility coefficient
ϕ,θ	polar coordinates defining the orientation of a generic material plane, Δ
γ_a	amplitude of the shear strain
$\gamma_{ij}(t)$	shear strain components ($i, j=x, y, z$)
$\gamma_q(t)$	shear strain resolved along direction q
γ_{MD}	shear strain relative to the critical plane determined by the Maximum Damage Method
γ_{MV}	shear strain resolved along direction, MV, that experiences the maximum variance of the resolved shear strain
γ'_f	shear fatigue ductility coefficient
$\gamma'_f(\rho)$	multiaxial fatigue ductility coefficient depending on ratio ρ
$\Delta\gamma$	shear strain range relative to the critical plane
λ	Stage I crack path angle
λ_e	estimated Stage I crack path angle
ν_e	Poisson's ratio for elastic strain
ν_p	Poisson's ratio for plastic strain
ρ	stress ratio relative to the critical plane ($\rho=\sigma_{n,max}/\tau_a$)
$\sigma_i(t)$	normal stress components ($i=x, y, z$)
σ_n	stress normal to the critical plane
$\sigma_{n,a}$	amplitude of the stress normal to the critical plane
$\sigma_{n,m}$	mean value of the stress normal to the critical plane
$\sigma_{n,max}$	maximum value of the stress normal to the critical plane
σ_y	yield stress
σ'_f	fatigue strength coefficient
τ_a	amplitude of the shear stress relative to the critical plane
$\tau_{ij}(t)$	shear stress components ($i, j=x, y, z$)
τ_m	mean value of the shear stress
τ_{MV}	shear stress resolved along direction, MV, that experiences the maximum variance of the resolved shear strain
τ'_f	shear fatigue strength coefficient
$\tau'_f(\rho)$	multiaxial fatigue strength coefficient depending on ratio ρ

1. Introduction

Since about the middle of the last century, devising sound engineering methodologies suitable for estimating the lifetime of components subjected to variable amplitude (VA) multiaxial loading has been the goal of numerous experimental/theoretical investigations. As far as the design issue is concerned, four key aspects need to be modelled effectively in order to accurately perform the fatigue assessment under

VA multiaxial load histories, i.e.: cyclic stress-strain behaviour, cycle counting, damage and its accumulation [1]. In this complex scenario, examination of the state of the art shows that the highest level of accuracy in estimating multiaxial fatigue lifetime of engineering components and structures is achieved via the so-called critical plane concept [1-5]. Critical plane approaches take as a starting point the idea that fatigue cracks initiate and propagate on certain specific planes. In this context, different strategies have been proposed and validated in order to determine the orientation of such planes. For instance, Findley [6] suggests determining the critical plane by maximizing a linear combination of the shear stress amplitude and the maximum value of the normal stress. In contrast, when the crack initiation process is mainly Mode II governed, Brown and Miller [7] as well as Fatemi and Socie [8] recommend using that material plane experiencing the maximum shear strain amplitude.

Under VA multiaxial load histories, defining the orientation of the critical plane is a complex and time-consuming task. To address this intractable problem, several approaches have been formulated and validated which include: the Maximum Damage Method (MDM) first proposed by Bannantine and Socie [9], the Maximum Variance Method developed by Macha [10] and subsequently reformulated by Susmel and co-workers [11, 12], and the weight function method devised by Macha and Capinteri [13-15] as well as by Shang [16].

Turning to the problem of counting cycles under multiaxial VA loading, the Rain-flow cycle counting method [17] is the most commonly used solution to address design problems of practical interest. In particular, amongst those reformulations of the Rain-flow counting method specifically devised to post-process VA multiaxial loading histories, the solution due to Bannantine and Socie [9] as well as due to Wang and Brown [18, 19] deserve to be mentioned explicitly.

Choosing an appropriate damage accumulation model is another tricky problem that needs to be addressed properly in order to accurately take into account the sequence effect under VA multiaxial fatigue loading [20, 21]. In this context, certainly Miner's linear rule still is the simplified model that is most commonly employed in situations of practical interest [22].

In the present investigation, initially the accuracy of the Shear Strain Maximum Variance Method (γ-MVM) as well as of the MDM in predicting the orientation of crack initiation planes is assessed against numerous data taken from the literature and generated under complex loading paths.

Subsequently, three different procedures suitable for estimating multiaxial fatigue lifetime of metallic materials subjected to VA multiaxial load histories are investigated in depth. In more detail, the following three design methodologies will be considered:

(a) Procedure A: Fatemi and Socie's (FS) criterion applied along with the MDM, Bannantine and Socie's (BD) cycle counting method and Miner's linear rule;

(b) Procedure B: the FS criterion applied along with the γ-MVM, the BS cycle counting method and Miner's linear rule;

(c) Procedure C: Modified Manson Coffin Curve Method applied in conjunction with the γ-MVM, Rainflow counting method and Miner's linear rule.

Having implemented these three design procedures in specific numerical codes, their accuracy and reliability will be checked against several experimental results taken from different sources.

2. Determining the orientation of the critical plane under VA multiaxial loading

2.1 The shear strain Maximum Variance Method (γ-MVM)

Based on the research undertaken by Macha [10], recently Susmel [11] has proposed to determine the orientation of the critical plane via the so-called Shear Stress-Maximum Variance Method (τ–MVM). In more detail, as far as fatigue failures in the medium/high-cycle fatigue regime are concerned, this technique estimates the orientation of the critical plane through that direction associated with the maximum variance of the resolved shear stress. Owing to its accuracy and reliability, subsequently, the same idea has been reformulated by Wang and Susmel [12] in terms of strains (the γ–MVM) to estimate the extent of fatigue damage also in the low/medium-cycle fatigue regime. Since this approach will be used extensively in the present investigation, for the sake of clarity, the fundamental concepts on which the γ–MVM is based are briefly reviewed in what follows.

Consider then the component shown in Fig. 1a that is assumed to be subjected to a complex system of forces/moments that lead to tri-axial time-variable stress/strain states at the assumed critical location (point O in Fig. 1), i.e.:

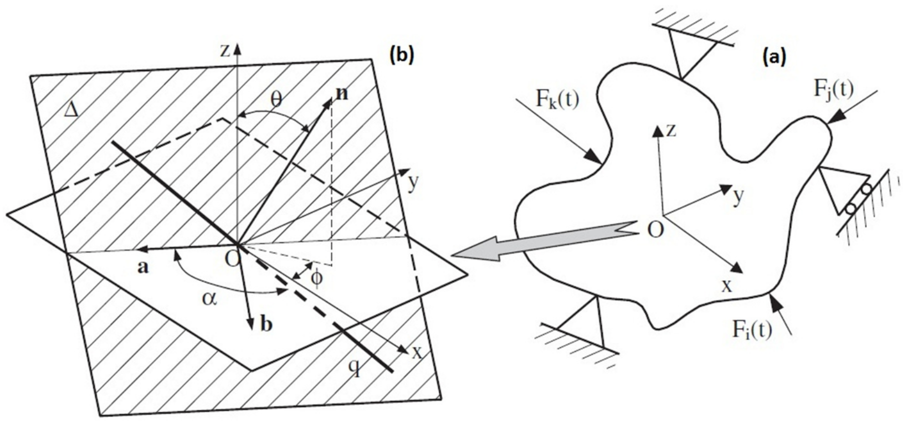


Fig. 1. (a) body subjected to an external system of forces, (b) definition of a generic material plane, Δ, local system of coordinates, and generic direction q on plane Δ.

$$[\sigma(t)] = \begin{bmatrix} \sigma_x(t) & \tau_{xy}(t) & \tau_{xz}(t) \\ \tau_{xy}(t) & \sigma_y(t) & \tau_{yz}(t) \\ \tau_{xz}(t) & \tau_{yz}(t) & \sigma_z(t) \end{bmatrix} \quad (1)$$

$$[\varepsilon(t)] = \begin{bmatrix} \varepsilon_x(t) & \frac{\gamma_{xy}(t)}{2} & \frac{\gamma_{xz}(t)}{2} \\ \frac{\gamma_{xy}(t)}{2} & \varepsilon_y(t) & \frac{\gamma_{yz}(t)}{2} \\ \frac{\gamma_{xz}(t)}{2} & \frac{\gamma_{yz}(t)}{2} & \varepsilon_z(t) \end{bmatrix} \quad (2)$$

By post-processing the above tensors, the instantaneous value of the shear strain, $\gamma_q(t)$, resolved along direction q on plane Δ (see Fig. 1) can be determined directly by calculating the following scalar product [12]:

$$\frac{\gamma_q(t)}{2} = \mathbf{e}(t) \bullet \mathbf{d} \tag{3}$$

where:

$$\mathbf{d} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \\ d_5 \\ d_6 \end{bmatrix} = \begin{bmatrix} \frac{1}{2}\left[\sin(\theta)\sin(2\phi)\cos(\alpha) + \sin(\alpha)\sin(2\theta)\cos(\phi)^2\right] \\ \frac{1}{2}\left[-\sin(\theta)\sin(2\phi)\cos(\alpha) + \sin(\alpha)\sin(2\theta)\sin(\phi)^2\right] \\ -\frac{1}{2}\sin(\alpha)\sin(2\theta) \\ \frac{1}{2}\sin(\alpha)\sin(2\phi)\sin(2\theta) - \cos(\alpha)\cos(2\phi)\sin(\theta) \\ \sin(\alpha)\cos(\phi)\cos(2\theta) + \cos(\alpha)\sin(\phi)\cos(\theta) \\ \sin(\alpha)\sin(\phi)\cos(2\theta) - \cos(\alpha)\cos(\phi)\cos(\theta) \end{bmatrix} \tag{4}$$

$$\mathbf{e}(t) = \begin{bmatrix} \varepsilon_x(t) & \varepsilon_y(t) & \varepsilon_z(t) & \frac{\gamma_{xy}(t)}{2} & \frac{\gamma_{xz}(t)}{2} & \frac{\gamma_{yz}(t)}{2} \end{bmatrix} \tag{5}$$

Using vectors (4) and (5), the variance of the shear strain resolved along generic direction q can then be determined directly as follows:

$$\mathrm{Var}\left[\frac{\gamma_q(t)}{2}\right] = \mathrm{Var}\left[\sum_k d_k e_k(t)\right] = \sum_i \sum_j d_i d_j \mathrm{Cov}\left[e_i(t), e_j(t)\right] \tag{6}$$

As discussed in detail in Refs [11, 12], the key feature of Eq. (6) is that the orientation of the potential critical planes can be determined directly by simply calculating the global maxima of the variance of the resolved shear strain. From a practical point of view, this methodology is suitable for being implemented numerically, with the potential critical planes being selected by simply solving a standard optimization problem. From a computational time viewpoint, the γ-MVM is very efficient, since, as soon as the variance and covariance terms associated with the time-variable strain tensor being post-processed are available, the numerical effort to be made to locate the critical plane is almost independent from the length of the load history under investigation. To conclude, it is worth noticing also that, owing to the fact that the shear strain resolved along the direction of maximum variance is, by definition, a mono-dimensional strain quantity, the γ-MVM allows fatigue cycles under VA multiaxial fatigue loading to be counted unambiguously by making direct use of the conventional uniaxial Rain-Flow counting method.

2.2 The Maximum Damage Method (MDM)

The MDM postulates that the critical plane coincides with that material plane on which the accumulated damage reaches its maximum value. The general procedure to apply those design methodologies based on the MDM in situations of practical interest can be summarised as follows. Initially, the shear and normal stress/strain components relative to a given material plane have to be determined by projecting the assessed loading history on a specific material plane (with this being done by directly manipulating the stress/strain tensors at the assumed critical location). Subsequently, using a specific cycle counting method, the shear and normal stress/strain components relative to the specific plane being investigated are used to identify, count and record the resulting fatigue cycles. As soon as the load spectrum relative to the considered plane is known, the associated damage is estimated cycle-by-cycle according to the adopted multiaxial fatigue criterion, the resulting total damage being calculated by adopting a suitable accumulation rule. According to the MDM's *modus operandi*, the methodology summarised above has to be applied iteratively by considering a number of planes in order to select the one which experiences the maximum extent of damage.

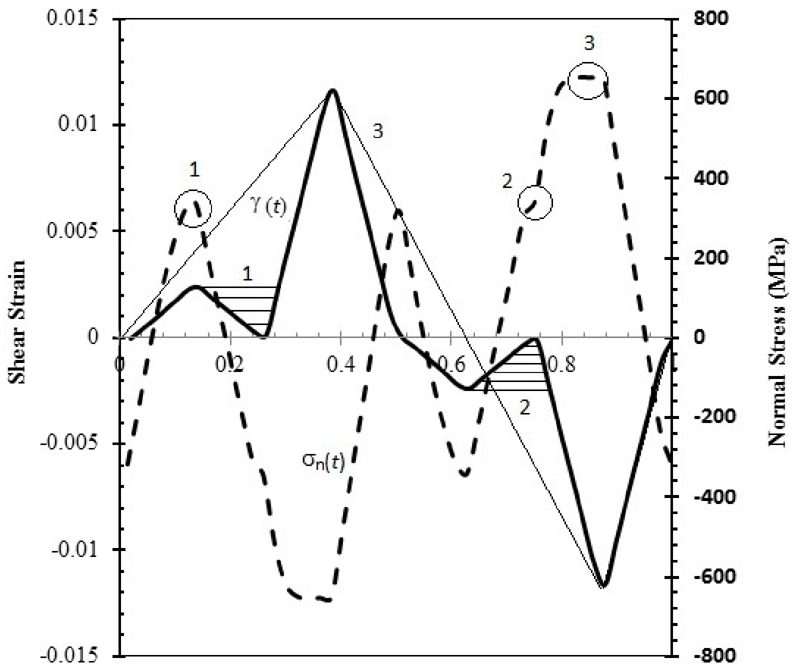
The general procedure summarised above makes it evident that the MDM needs to be applied along with a specific multiaxial fatigue criterion. Further, a suitable cycle counting method as well as an appropriate cumulative damage rule are also required. Accordingly, in the present investigation, the MDM will be used in conjunction with the FS criterion [8], the BS cycle counting method [9] and Miner's linear damage rule [22].

The shear-strain based multiaxial fatigue criterion proposed by Fatemi and Socie [8] can be formalised according to the following well-known relationship:

$$\frac{\Delta\gamma}{2}\left(1+k\,\frac{\sigma_{n,\max}}{\sigma_y}\right)=\frac{\tau'_f}{G}\left(2N_f\right)^{b_0}+\gamma'_f\left(2N_f\right)^{c_0} \tag{7}$$

where, for a given cycle, $\Delta\gamma$ and $\sigma_{n,max}$ are the shear stain range and the maximum normal stress relative to the critical plane, respectively, k is a material constant, and σ_y is the yield stress.

The solution proposed by Bannantine and Socie [9] to count fatigue cycles on a specific material plane takes full advantage of the Rain-Flow counting method. This technique makes use of a master channel and some auxiliary channels. For those materials in which the crack initiation process is Mode II governed, the shear strain is recommended as being used as the master channel. In contrast, for those materials characterised by a cracking behaviour that is mainly Mode I dominated, the normal strain is employed instead as reference strain information. After selecting the appropriate master channel, the Rain-Flow method is then used to defined and count the fatigue cycles. If the FS criterion is employed, then the shear strain has to be used as master channel, with the normal stress signal becoming the auxiliary channel. A schematic chart showing the way the BS cycle counting method works when it is applied along with the FS criterion is shown in Fig.2.



Cycles	γ_a	$\sigma_{n,\max}$(Mpa)
1	0.0011292	339
1	0.0011292	347
1	0.0116354	654

Fig. 2. Example showing the use of the BS cycle counting method applied along with the FS criterion.

After determining all the cycles associated with the material plane being investigated, the resulting total damage is then calculated according to Palmgren-Miner's linear rule as follows:

$$D_{\text{tot}} = \sum_{i=1}^{j} \frac{n_i}{N_{f,i}} \tag{8}$$

where, for the i-th loading level, n_i is the number of counted cycles, $N_{f,i}$ is the associated number of cycles to failure (under constant amplitude loading), and D_{tot} is the total value of the damage sum. Finally, according to the MDM's *modus operandi*, amongst all the material planes being explored, the critical plane is defined as the one on which D_{tot} reaches its maximum value.

3. Selected methodologies to estimate fatigue lifetime under VA multiaxial loading

The present paper aims to assess also the accuracy of three different critical plane based design methodologies by considering a number of experimental results generated under VA multiaxial fatigue loading. For the sake of clarity, in what follows the three design methodologies being investigated will be reviewed briefly by considering the following three key aspects: (i) determination of the critical plane, (ii) quantification of the damage extent, and (iii) estimation of fatigue lifetime.

As shown in the flowchart of Fig. 3, Methodology A is based on the use of the MDM applied along with the FS criterion, the BS counting method, and Miner's linear rule. In more detail, according to the procedure reviewed under 2.2, for a given material plane, the corresponding stress/strain components are used to count the fatigue cycles according to the BS method. Subsequently, the damage on the plane being investigated is calculated according to the FS criterion, with the cumulated damage being estimated by using Miner's linear rule. Finally, after determining the critical plane as that experiencing the maximum damage, the number of cycles to failure, $N_{f,e}$, is directly estimated from the corresponding value of D_{tot}, Eq. (8), according to the following standard relationship:

$$N_{f,e} = \frac{D_{cr}}{D_{tot}} \sum_{i=1}^{j} n_i \tag{9}$$

where D_{cr} is the critical value of the damage sum. According to Miner's approach, D_{cr} should be taken invariably equal to unity [22]. However, much experimental evidence suggests that D_{cr} varies in the range 0.02-5, with its average value being approximately equal to 3 [23].

Turning to the second design technique being considered in the present investigation (i.e., Methodology B), this procedure is based instead on the combined use of the γ-MVM, the FS criterion, the BS cycle counting method, and Miner's rule. In more detail, after determining the orientation of the critical plane through the γ-MVM, the damage on this specific plane is assessed by applying the FS criterion along with the BS cycle counting method and the damage accumulation rule due to Miner – see Eq. (8). Finally, fatigue lifetime is estimated from the calculated value for D_{tot} via Eq. (9). The procedure for the in-field usage of Methodology B is schematically shown in Fig. 3b.

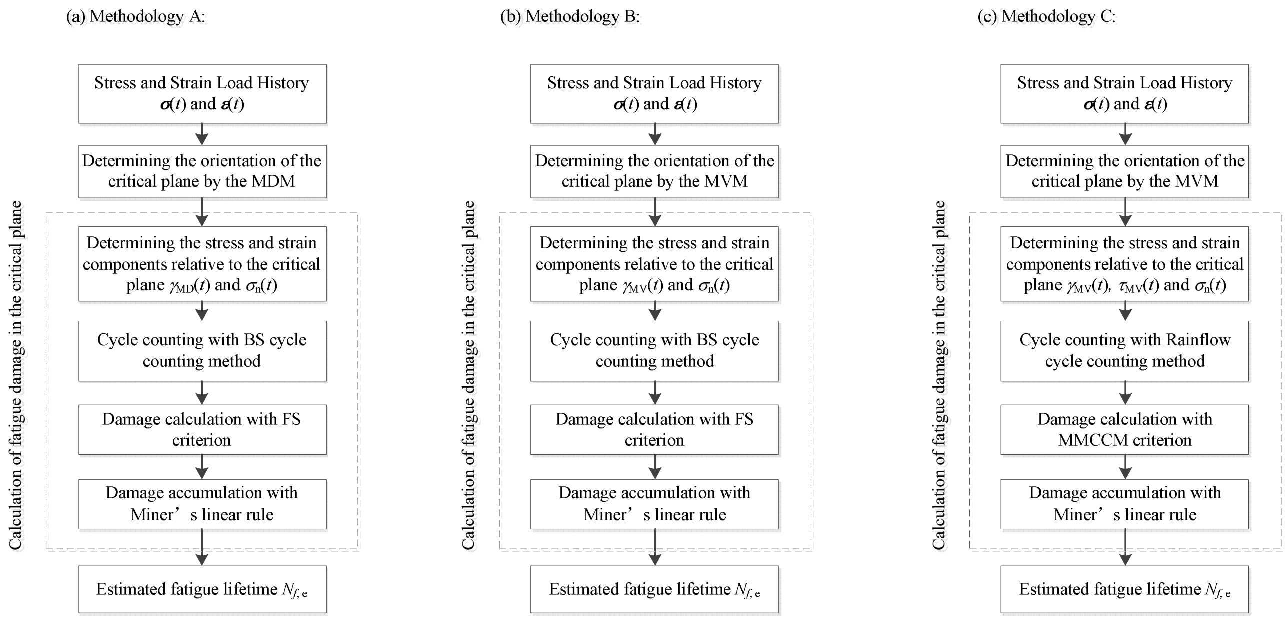


Fig. 3. Summary of the investigated procedures suitable for estimating lifetime under VA multiaxial fatigue loading

Methodology C involves the γ–MVM, the Modified Manson Coffin Curve Method (MMCCM) [24], the classical Rain-Flow cycle counting method, and Miner's linear rule, with such a procedure being summarised in Fig. 3c. Since the MMCCM' *modus operandi* has already been discussed elsewhere in great detail [12, 24], in what follows its key features will be recalled briefly, the reader being referred to the original sources for a detailed description of this strain based critical plane approach.

The MMCCM [24] postulates that the degree of multiaxiality and non-proportionality of the stress state at the critical locations can be quantified through the following stress ratio:

$$\rho = \frac{\sigma_{n,m} + \sigma_{n,a}}{\tau_a} = \frac{\sigma_{n,max}}{\tau_a} \tag{10}$$

where τ_a is used to denote the shear stress amplitude relative to the critical plane, whereas $\sigma_{n,m}$, $\sigma_{n,a}$, and $\sigma_{n,max}$ are the mean value, the amplitude, and the maximum value of the stress normal to the critical plane, respectively.

As far as VA multiaxial load histories are concerned, τ_a and $\sigma_{n,a}$ in the Eq. (10) are the equivalent amplitudes of the shear and normal stress, respectively. These two stress quantities are determined according to the following definitions [12]:

$$\tau_a = \sqrt{2 \cdot \mathrm{Var}[\tau_{MV}(t)]} \tag{11}$$

$$\sigma_{n,a} = \sqrt{2 \cdot \mathrm{Var}[\sigma_n(t)]} \tag{12}$$

where

$$\mathrm{Var}[\tau_{MV}(t)] = \frac{1}{T}\int_0^T [\tau_{MV}(t) - \tau_m]^2 \cdot dt \tag{13}$$

$$\mathrm{Var}[\sigma_n(t)] = \frac{1}{T}\int_0^T [\sigma_n(t) - \sigma_{n,m}]^2 \cdot dt \tag{14}$$

In Eq. (11) $\tau_{MV}(t)$ denotes the instantaneous value of the shear stress resolved along the direction of maximum variance, whereas $\sigma_n(t)$ is the instantaneous value of the stress perpendicular to the critical plane.

For a given value of ρ, the profile of the corresponding modified Manson–Coffin curve can be estimated by using the following general relationship:

$$\gamma_a = \frac{\tau'_f(\rho)}{G}(2N_f)^{b(\rho)} + \gamma'_f(\rho) \cdot (2N_f)^{c(\rho)} \tag{15}$$

where $\tau'_f(\rho)$, $b(\rho)$, $\gamma'_f(\rho)$, and $c(\rho)$ are fatigue constants that can be extrapolated from the fully-reversed uniaxial and torsional fatigue curves via the following relationships [12, 24]:

$$\frac{\tau'_f(\rho)}{G} = \rho \cdot (1+\nu_e)\frac{\sigma'_f}{E} + (1-\rho)\frac{\tau'_f}{G}$$

$$\gamma'_f(\rho) = \rho \cdot (1+\nu_p)\varepsilon'_f + (1-\rho)\gamma'_f$$

$$b(\rho) = \frac{b \cdot b_0}{(b_0 - b)\rho + b}$$

$$c(\rho) = \frac{c \cdot c_0}{(c_0 - c)\rho + c}$$

4. Validation by experimental data

A number of suitable experimental data were selected from the technical literature in order to check the accuracy of the γ-MVM and the MDM in predicting the orientation of the crack initiation planes as well as of design procedures A, B, and C in estimating fatigue lifetime under VA multiaxial fatigue loading. In more detail, we considered the experimental results generated by Kim et al. [25] by testing specimens of S45C steel under short variable amplitude multiaxial loading and by Shamsaei et al. [26] by testing samples of 1050QT steel and 304L steel under discriminating strain paths.

The summary of the static and fatigue properties of the materials being investigated are reported in Tables 1 and 2. As to the values listed in these tables, it has to be pointed out that, when the required fatigue properties were not available in the original articles, they were estimated by using the following engineering rules [1]:

$$\tau'_f = \frac{\sigma'_f}{\sqrt{3}}\ ;\ \ \gamma'_f = \sqrt{3}\varepsilon'_f\ ;\ \ b_0 = b\ ;\ \ c_0 = c$$

The investigated loading paths are shown in Fig. 4, with such load histories being given, in general, in terms of strain components measured at the assumed crack initiation locations. Accordingly, when the associated stress components were not available in the original sources, they were estimated from the provided strain paths by using the model devised by Jiang and Sehitoglu [27, 28]. In this context, the hardening effect under non-proportional loading was taken into account by correcting the reference stabilised stress/strain curves by making the following assumption [1]:

$$K'_{\mathrm{NP}} = 1.25 \cdot K'\ ;\ \ n'_{NP} = n'$$

where K'_{NP} and n'_{NP} are the cyclic strength coefficient and cyclic strain hardening exponent under 90° out-of-phase loading, respectively.

Table 1 Static properties of the investigated materials

Material	Ref.	E (GPa)	G (GPa)	σ_y (MPa)	k in FS
S45C	[25]	186	70.6	496	1
1050 QT steel	[26]	203	81	1009	0.6
304L stainless steel	[26]	195	77	208	0.15

Table 2 Fatigue properties of the investigated materials

Material	Ref.	K' (MPa)	n'	ε'_f	σ'_f (MPa)	b	c	γ'_f	τ'_f (MPa)	b_0	c_0
S45C	[25]	1215	0.217	0.359	923	-0.099	-0.519	0.198	685	-0.12	-0.36
1050 QT steel	[26]	1558	0.123	2.01	1346	-0.062	-0.725	**3.48**	**777**	**-0.062**	**-0.725**
304L stainless steel	[26]	2841	0.371	0.122	1287	-0.145	-0.394	**0.211**	**743**	**-0.145**	**-0.394**

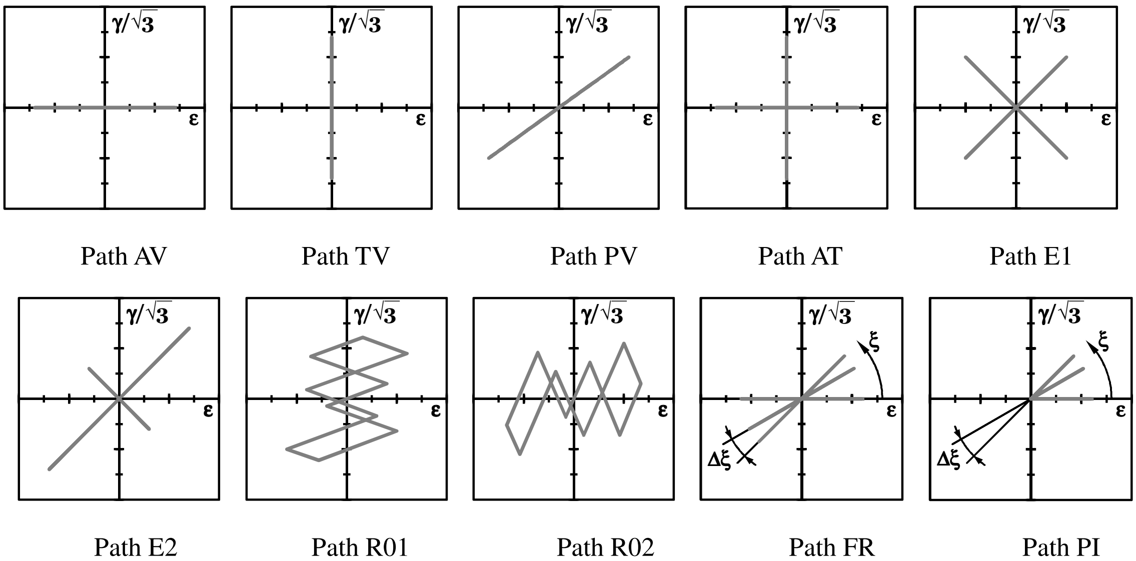


Fig. 4. Investigated loading paths

4.1 Validation of critical plane orientation

As far as metallic materials subjected to fatigue loading are concerned, it is commonly accepted [1, 29-31] that Stage I cracks initiate on those materials planes experiencing the maximum shear, with the subsequent propagation occurring on those planes that are perpendicular to the normal stress (Stage II).

According to this classic schematisation, both the γ-MVM and the MDM (as described under 2.2) can be used to estimate the orientation of Stage I planes.

Figs 5 and 6 show the comparison between predicted and experimental orientation of the crack initiation planes for the materials being considered, with the definition of the angle giving the Stage I plane orientation being shown in the same figures. These two diagrams confirm that both the γ-MVM and the MDM are capable of predicting the orientation of the crack initiation plane quite accurately. In particular, 95% of the predictions made by the γ-MVM are seen to fall within an error band of ±20%, with 98% of the estimates falling within an error interval of ±30%. Turning to the MDM, Figs 5 and 6 make it evident that the systematic usage of this methodology resulted in 88% of the estimates falling within and error interval of ±20% and 95% within an error interval of 30%.

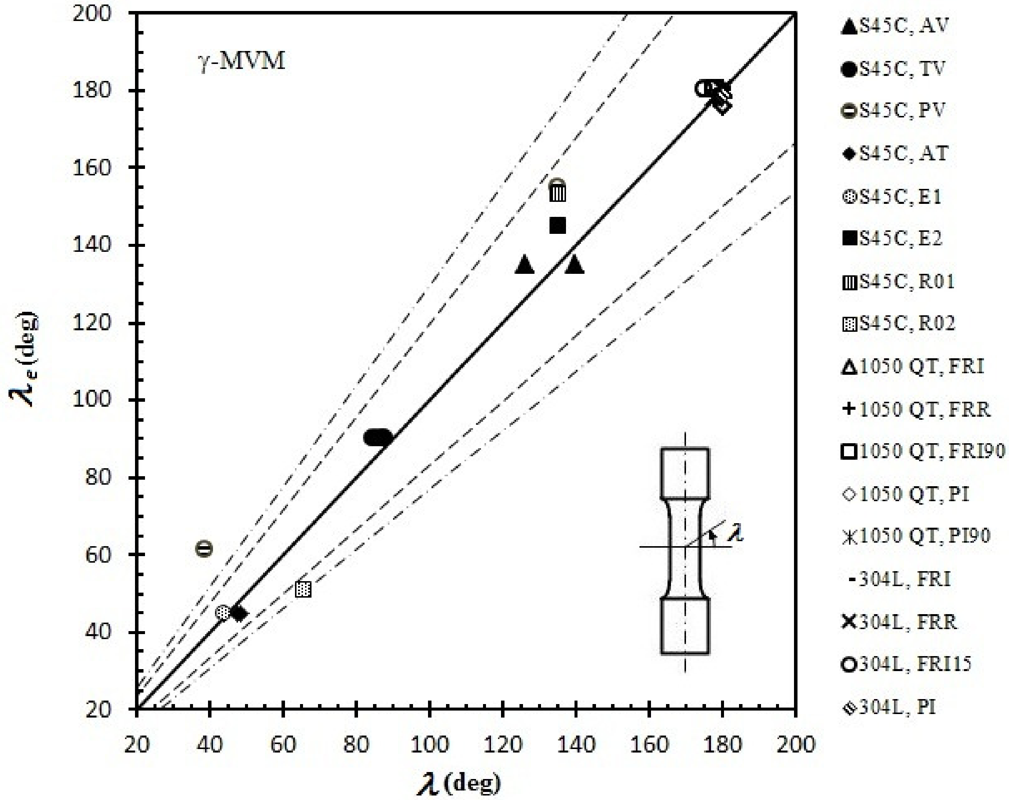


Fig. 5. Accuracy of the γ-MVM in estimating the orientation of Stage I planes.

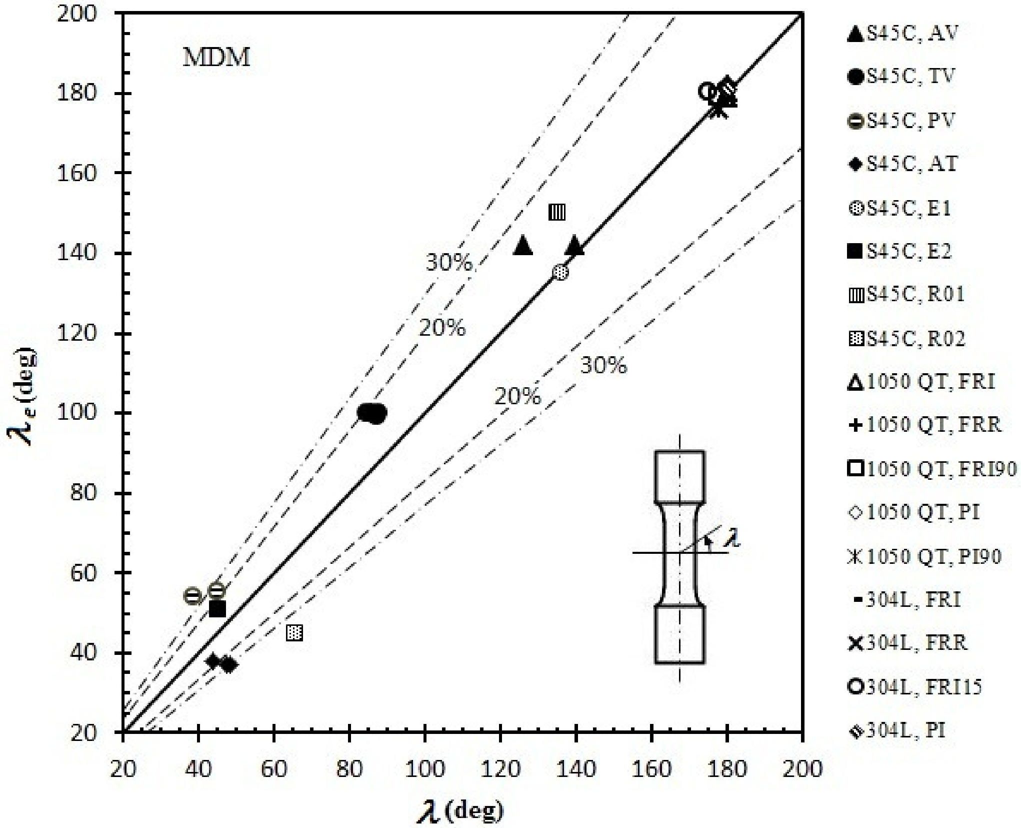


Fig. 6. Accuracy of the MDM in estimating the orientation of Stage I planes

4.2 Accuracy of the considered design methodologies in estimating fatigue lifetime

The predicted versus experimental fatigue lifetime diagrams built by adopting design procedures A, B and C are reported in Figs 7 to 9. As it can be seen from these Figures, the majority of the predictions made by these three methodologies fall within an error factor of about 3. This further confirms that these three design procedure can be used safely in situations of practical interest to design real components against VA multiaxial fatigue loading.

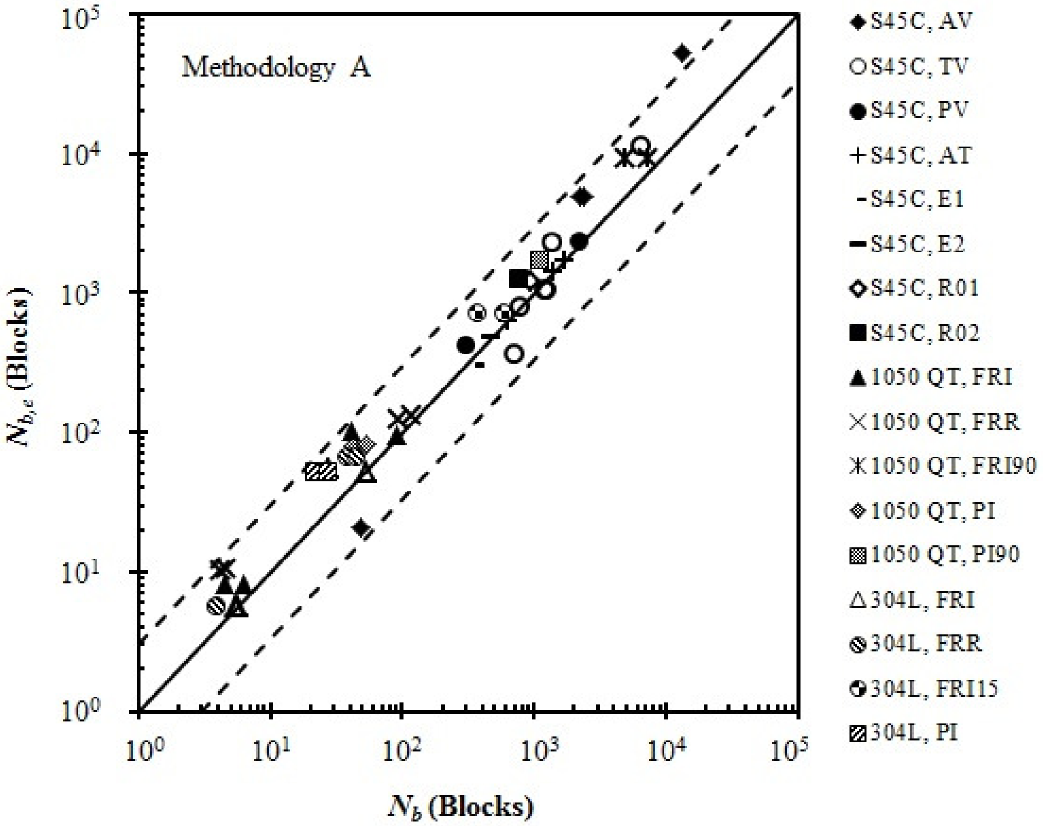


Fig. 7. Methodology A: comparison between experimental and predicted fatigue lifetime

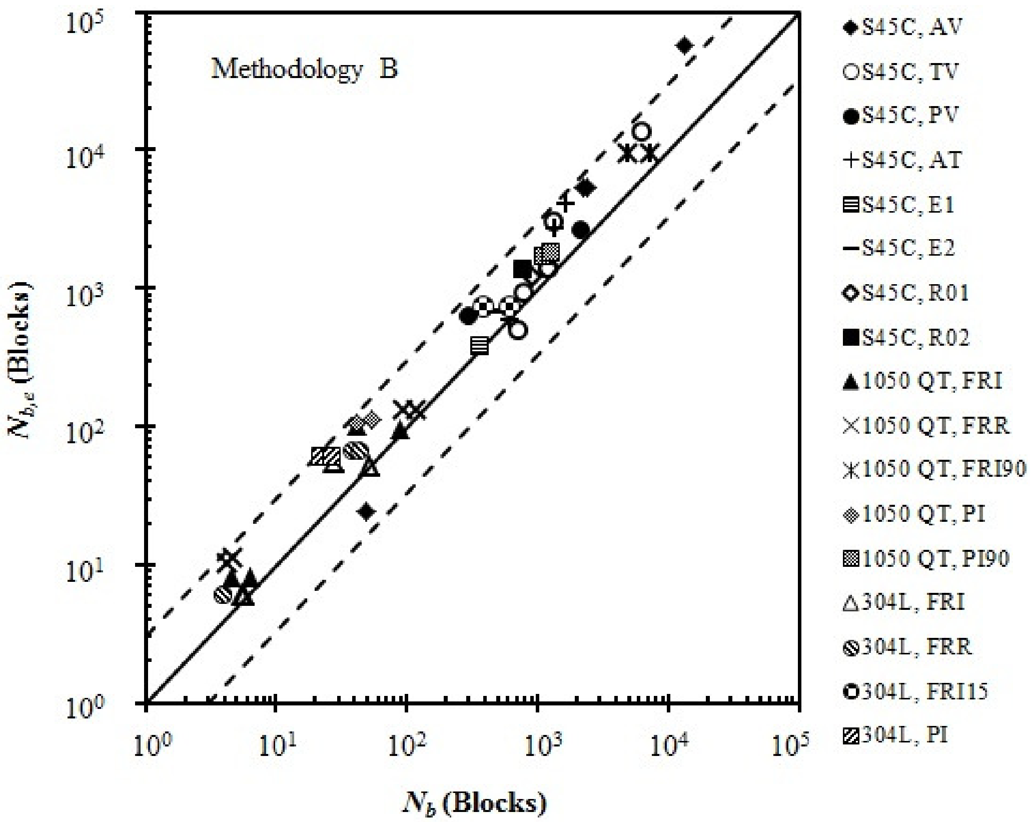


Fig. 8. Methodology B: comparison between experimental and predicted fatigue lifetime.

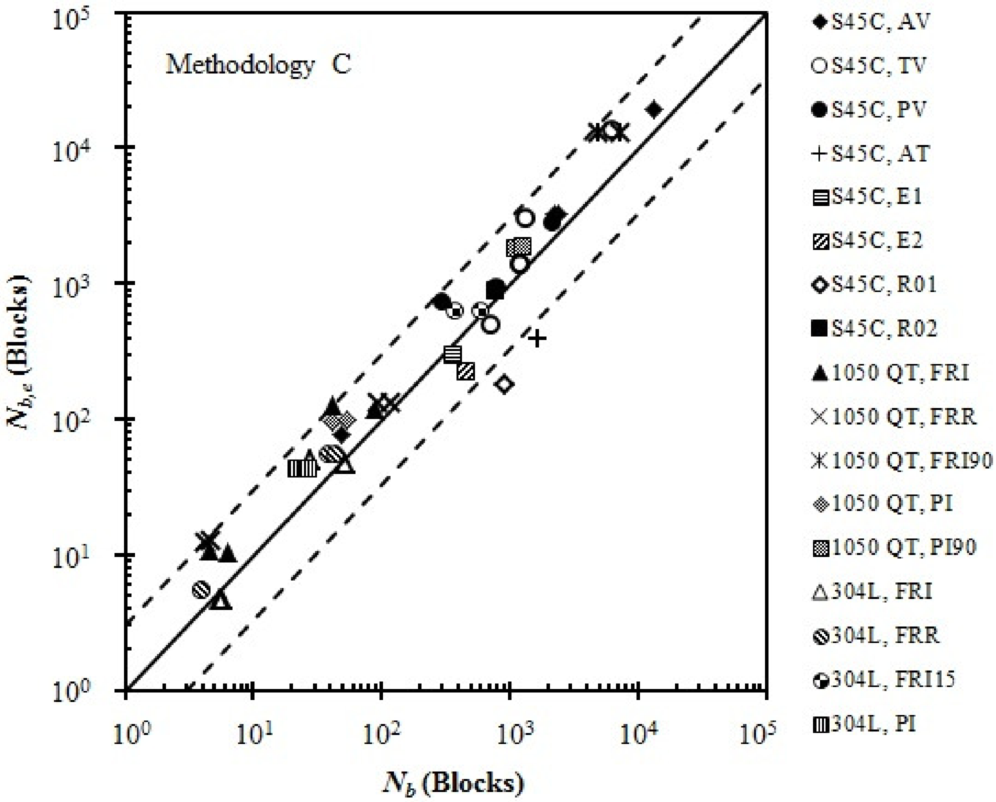


Fig. 9. Methodology C: comparison between experimental and predicted fatigue lifetime.

5. Conclusions

- The result of the validation exercise discussed in the present paper suggests that the usage of both the γ-MVM and the MDM resulted in a satisfactory level of accuracy when these two different methodologies were used to predict the orientation of the crack initiation plane in the metallic materials being considered.
- This investigation further confirms that the three VA multiaxial fatigue life assessment methodologies being investigated allowed fatigue lifetime under VA/complex multiaxial load histories to be estimated by always reaching an adequate level of accuracy.
- More work needs to be done in this area to verify the accuracy and reliability of the considered techniques to perform the VA multiaxial fatigue assessment in the presence of local stress/strain concentration phenomena.

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