Published paper

Zhang, L., Liu, W., Langley, R.J. (2010) *A class of constant modulus algorithms for uniform linear arrays with a conjugate symmetric constraint*, Signal Processing, 90 (9), pp. 2760-2765
http://dx.doi.org/10.1016/j.sigpro.2010.04.003

A Class of Constant Modulus Algorithms for Uniform Linear Arrays with a Conjugate Symmetric Constraint

Lei Zhang [a] Wei Liu (member, EURASIP) [b] Richard J Langley [c]

[a]*Communications Research Group, Department of Electronic and Electrical Engineering, University of Sheffield, U.K.*

[b]*Communications Research Group, Department of Electronic and Electrical Engineering, University of Sheffield, U.K.*

[c]*Communications Research Group, Department of Electronic and Electrical Engineering, University of Sheffield, U.K.*

Abstract

A class of constant modulus algorithms (CMAs) subject to a conjugate symmetric constraint is proposed for blind beamforming based on the uniform linear array structure. The constraint is derived from the beamformer with an optimum output signal-to-interference-plus-noise ratio (SINR). The effect of the additional constraint is equivalent to adding a second step to the original adaptive algorithms. The proposed approach is general and can be applied to both the traditional CMA and its all kinds of variants, such as the linearly constrained CMA (LCCMA) and the least squares CMA (LSCMA) as two examples. With this constraint, the modified CMAs will always generate a weight vector in the desired form for each update and the number of adaptive variables is effectively reduced by half, leading to a much improved overall performance.

Key words: Constant modulus, uniform linear arrays, conjugate symmetric, blind beamforming

1 Introduction

Constant modulus (CM) based algorithms have been studied extensively in the past for both blind beamforming and equalization [1–5]. Many of the algorithms are de-

Email addresses: `lei.zhang@sheffield.ac.uk` (Lei Zhang), `w.liu@sheffield.ac.uk` (Wei Liu (member, EURASIP)), `r.j.langley@sheffield.ac.uk` (Richard J Langley).

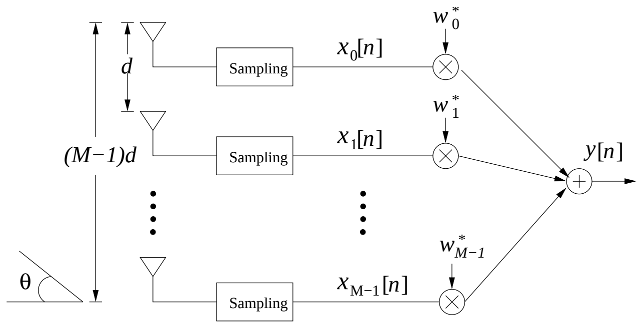

Fig. 1. A narrowband beamforming structure based on a uniform linear array.

rived using the stochastic gradient (SG) method. However, a drawback for this kind of CM algorithms is their sensitivity to stepsizes. A large stepsize leads to a fast convergence speed, but also with a loss of the output signal-to-interference-plus-noise ratio (SINR) and even causes the algorithm unstable. Many variations have been proposed to achieve a faster convergence speed, such as the well-known least squares CMA (LSCMA) [6,7], the one based on recursive least squares (RLS) [8–10] and those with variable stepsizes [11–13]. Given some additional information about the signal, such as the direction of arrival (DOA) angle of the desired signal, linear constraints can be imposed, leading to an improved performance [9,10,12,14].

In general, the more information we add to the algorithm, the less number of snapshots are required for it to arrive at the steady state and more robust the algorithm tends to be. In this paper we will propose a novel approach by incorporating the geometric information of the array system to the original CM cost function based on the commonly used uniform linear array (ULA) structure. For a ULA based CM algorithm to achieve a maximum SINR at its output, we can prove that the weight vector $\mathbf{w}$ should have a conjugate symmetric structure, which is employed to constrain the original CM cost function. By employing the Lagrange multipliers method, a novel adaptive CM algorithm is then derived. The idea is general and can be applied to other CM based algorithms, such as the early mentioned linearly constrained CMA (LCCMA) and the LSCMA, as two representative examples.

This paper is organized as follows. The signal model is described in Section 2; the special structure of the optimum weight vector for maximizing its output SINR is provided and the novel CMA algorithm is derived based on this structure in Section 3. Two extension examples of the proposed approach is given in Section 4. Simulation results are presented in Section 5 and conclusions drawn in Section 6.

2 Signal Model

Consider a ULA with M omnidirectional sensors and an adjacent sensor spacing $d \leq \lambda_0/2$, where λ_0 is the signal wavelength. Suppose there are $L (L \leq M)$ narrowband and uncorrelated CM signals impinging upon the array from the far field with DOA angles $\theta_0, \theta_1, ..., \theta_{L-1}$, respectively. Then the nth snapshot $\mathbf{x}[n]$ of the received array signals can be expressed as

$$\mathbf{x}[n] = \mathbf{A}\mathbf{s}[n] + \mathbf{n}[n] \tag{1}$$

where

$$\begin{aligned} \mathbf{x}[n] &= [x_0[n], x_1[n], \cdots, x_{M-1}[n]]^T \in \mathbb{C}^{M\times 1} \\ \mathbf{s}[n] &= [s_0[n], s_1[n], \cdots, s_{L-1}[n]]^T \in \mathbb{C}^{L\times 1} \end{aligned} \tag{2}$$

are the sensor outputs and the source signals, respectively, and $\mathbf{n}[n]$ is the noise vector. $\{\cdot\}^T$ denotes the transpose operation and $\mathbf{A}$ is the mixing matrix given by

$$\mathbf{A} = [\mathbf{a}(\psi_0), \mathbf{a}(\psi_1), \cdots, \mathbf{a}(\psi_{L-1})] \in \mathbb{C}^{M\times L} \ , \tag{3}$$

where

$$\mathbf{a}(\psi_i) = [1, e^{-j\psi_i}, \cdots, e^{-j(M-1)\psi_i}]^T \tag{4}$$

is the array steering vector with $\psi_i = 2\pi d \sin(\theta_i)/\lambda_0$. The DOA angle θ_i is measured from the broadside direction as indicated in Fig. 1.

By applying a set of coefficients w_i, $i = 0, \ldots, M-1$ to the received array signals $\mathbf{x}[n]$, we obtain the beamformer output $y[n] = \mathbf{w}^H\mathbf{x}[n]$, where $\{\cdot\}^H$ denotes the Hermitian transpose operation and $\mathbf{w}$ is the weight vector given by

$$\mathbf{w} = [w_0, w_1, \cdots, w_{M-1}]^T \ . \tag{5}$$

3 The Proposed Algorithm

The proposed idea is general and not limited to a specific constant modulus algorithm. As a starting point, we consider the CM algorithm minimizing the following cost function [1,5]

$$J_{CM} = \frac{1}{4}E\{(|y[n]|^2 - 1)^2\} \ . \tag{6}$$

where $E\{\cdot\}$ is the expectation operation. A recursive update equation for $\mathbf{w}$ can be obtained as follows,

$$\mathbf{w}[n+1] = \mathbf{w}[n] - \mu(|y[n]|^2 - 1)y^*[n]\mathbf{x}[n] \tag{7}$$

where μ is the stepsize. The above algorithm can be applied to any array structure for blind beamforming and in this paper we only consider the ULA case.

3.1 The Weight Vector for Optimum SINR

It can be proved that if we constrain the CMA to maximize its output SINR, then $\mathbf{w}$ will have the following general symmetric structure

$$\mathbf{w}_{opt} = \mathbf{J}\mathbf{w}_{opt}^* \tag{8}$$

where $\mathbf{J} \in \mathbb{C}^{M\times M}$ is the exchange matrix defined as

$$\mathbf{J} = \begin{bmatrix} 0 & \dots & 0 & 1 \\ 0 & \dots & 1 & 0 \\ \vdots & \ddots & \vdots & \vdots \\ 1 & \dots & 0 & 0 \end{bmatrix}. \tag{9}$$

Proof:

Without loss of generality, assume the first signal s_0 is the desired one. Then the optimum $\mathbf{w}$ for maximizing the output SINR can be expressed as [3]

$$\mathbf{w} = \mu_0 \mathbf{R}_{xx}^{-1}\mathbf{a}(\psi_0) \tag{10}$$

where μ_0 is an arbitrary nonzero complex-valued constant and $\mathbf{R}_{xx} = E\{\mathbf{x}[n]\mathbf{x}^H[n]\}$ is the correlation matrix of $\mathbf{x}[n]$.

Since $\mathbf{R}_{xx}$ is a Hermitian Toeplitz matrix, we have $\mathbf{R}_{xx} = \mathbf{J}\mathbf{R}_{xx}^*\mathbf{J}$. Note that $\mathbf{J} = \mathbf{J}^{-1}$, then we have

$$\mathbf{R}_{xx}^{-1} = (\mathbf{J}\mathbf{R}_{xx}^*\mathbf{J})^{-1} = \mathbf{J}(\mathbf{R}_{xx}^{-1})^*\mathbf{J}. \tag{11}$$

Moreover, for a ULA, its steering vector has the following property:

$$\mathbf{a}(\psi_0) = e^{-j(M-1)\psi_0}\mathbf{J}\mathbf{a}^*(\psi_0). \tag{12}$$

Substituting (11) and (12) into (10), and noticing that $\mathbf{J}\mathbf{J} = \mathbf{I}$, we have

$$\begin{aligned} \mathbf{w} &= \mu_0 \mathbf{R}_{xx}^{-1}\mathbf{a}(\psi_0) \\ &= \mu_0 \mathbf{J}(\mathbf{R}_{xx}^{-1})^*\mathbf{J}e^{-j(M-1)\psi_0}\mathbf{J}\mathbf{a}^*(\psi_0) \\ &= \frac{\mu_0}{\mu_0^*}e^{-j(M-1)\psi_0}\mathbf{J}\mu_0^*(\mathbf{R}_{xx}^{-1})^*\mathbf{J}\mathbf{J}\mathbf{a}^*(\psi_0) \\ &= \frac{\mu_0}{\mu_0^*}e^{-j(M-1)\psi_0}\mathbf{J}\mathbf{w}^*. \end{aligned} \tag{13}$$

By defining

$$e^{j\phi} = \frac{\mu_0}{\mu_0^*}e^{-j(M-1)\psi_0}. \tag{14}$$

we can draw a conclusion that $\mathbf{w}_{opt}$ has a general conjugate symmetric property given by

$$\mathbf{w} = e^{j\phi}\mathbf{J}\mathbf{w}^* \ . \tag{15}$$

Since the optimum SINR solution is ambiguous to the phase response, we can combine $e^{-j\phi/2}$ with the original $\mathbf{w}$ to form a new $\mathbf{w}$, which will not change the output SINR.

Defining

$$\mathbf{w}_{opt} = (e^{-j\phi/2}\mathbf{w}) \tag{16}$$

and substituting it into the general form (15) leads to the result of (8). This completes the proof.

One note is that as long as the linear array is center symmetric, the steering vector $\mathbf{a}(\psi_0)$ will have the property described by (12) and as a result the optimum $\mathbf{w}$ will have the conjugate symmetric structure. However, in a multipath environment, the combined steering vector for each of the original source signals will lose the structure given in Equations (4) and (12) and the conjugate symmetric property will be invalid, which presents an important limitation to its effective application.

It is noticed that [15] has given the same conclusion of (8) by assuming that the phase origin of the steering vector is at the geometric center of the array and μ_0 is a real constant. We can see from the proof that the conclusion is actually valid even without these assumptions. Furthermore, we have also provided a more general result in Equation (15), which will be used in the constrained LCCMA example studied in Section 4.1.

3.2 The Algorithm

The aim of a CM algorithm is to extract the desired signal with a constant modulus. In a noise-free environment and under the condition of $M \geq L$, the CMA will be able to extract the desired signal completely without any interfering signals left at its output, which will give a maximum output SINR of infinity. In this case, the optimum solution of the CMA coincides with that of the maximum SINR beamformer [16]. In the presence of noise, although the optimum solution of a CM algorithm will not necessarily give the maximum output SINR, it has been shown that at least the optimum CMA solution is very close to the one giving the maximum SINR [17].

Based on this, we can regulate the CM algorithms in the direction of maximizing its output SINR by constraining them to the derived conjugate symmetric structure in (8). Taking the CM cost function J_{CM} in (6) as an example, we can combine it with the structure of (8) in the following way

$$\min_{\mathbf{w}} J_{CM} = \frac{1}{4} E\{(|y[n]|^2 - 1)^2\}$$
$$\text{subject to } \mathbf{w}[n] - \mathbf{J}\mathbf{w}^*[n] = \mathbf{0} \tag{17}$$

or equivalently

$$\min_{\mathbf{w}} J_{CM} = \frac{1}{4} E\{(|y[n]|^2 - 1)^2\}$$
$$\text{subject to } (\mathbf{w}[n] - \mathbf{J}\mathbf{w}^*[n])^H (\mathbf{w}[n] - \mathbf{J}\mathbf{w}^*[n]) = 0. \tag{18}$$

Notice that $(\mathbf{w}[n] - \mathbf{J}\mathbf{w}^*[n])^H (\mathbf{w}[n] - \mathbf{J}\mathbf{w}^*[n])$ is real-valued. We can form the following Lagrangian cost function with a real-valued undetermined Lagrange multiplier λ

$$Q_{CM} = \frac{1}{4}(|y[n]|^2 - 1)^2 + \lambda(\mathbf{w}[n] - \mathbf{J}\mathbf{w}^*[n])^H (\mathbf{w}[n] - \mathbf{J}\mathbf{w}^*[n]). \tag{19}$$

Taking the gradient of (19) with respect to $\mathbf{w}^*$, we have

$$\nabla_{\mathbf{w}^*} Q_{CM} = (|y[n]|^2 - 1) y^*[n] \mathbf{x}[n] + 2\lambda(\mathbf{w}[n] - \mathbf{J}\mathbf{w}^*[n]) \; . \tag{20}$$

Changing the weight vector in the negative direction of the gradient in (20), scaled by a constant stepsize μ, we have the following update equation for the weight vector $\mathbf{w}$

$$\mathbf{w}[n+1] = \mathbf{w}[n] - \mu \mathbf{r}[n] - 2\mu\lambda(\mathbf{w}[n] - \mathbf{J}\mathbf{w}^*[n]) \tag{21}$$

where

$$\mathbf{r}[n] = (|y[n]|^2 - 1) y^*[n] \mathbf{x}[n]. \tag{22}$$

Note $\mathbf{w}[n+1]$ should satisfy the constraint equation (8), i.e.

$$\mathbf{w}[n+1] = \mathbf{J}\mathbf{w}^*[n+1] \; . \tag{23}$$

Substituting (21) into (23) and after simplification, we have

$$2\mu\lambda(\mathbf{w}[n] - \mathbf{J}\mathbf{w}^*[n]) = \frac{(\mathbf{w}[n] - \mathbf{J}\mathbf{w}^*[n]) - \mu(\mathbf{r}[n] - \mathbf{J}\mathbf{r}^*[n])}{2} \; . \tag{24}$$

Substitute (24) into (21) to eliminate λ, leading to the following solution

$$\mathbf{w}[n+1] = \frac{\mathbf{w}[n] + \mathbf{J}\mathbf{w}^*[n]}{2} - \frac{\mu(\mathbf{r}[n] + \mathbf{J}\mathbf{r}^*[n])}{2} \; . \tag{25}$$

We can re-arrange (25) to the following form

$$\mathbf{w}[n+1] = \frac{\mathbf{w}[n] - \mu \mathbf{r}[n]}{2} + \frac{\mathbf{J}\mathbf{w}^*[n] - \mu \mathbf{J}\mathbf{r}^*[n]}{2}. \tag{26}$$

It is obvious that the numerator of the first part of (26) corresponds to the original CMA described in (7); while for the numerator of the second part, we have

$$\mathbf{J}\mathbf{w}^*[n] - \mu\mathbf{J}\mathbf{r}^*[n] = \mathbf{J}(\mathbf{w}[n] - \mu\mathbf{r}[n])^* = \mathbf{J}\hat{\mathbf{w}}^*[n+1] \tag{27}$$

where $\hat{\mathbf{w}}[n+1] = \mathbf{w}[n] - \mu\mathbf{r}[n]$ is the original CMA. Therefore the update equation can be separated into two steps as follows:

$$\begin{cases} \hat{\mathbf{w}}[n+1] = \mathbf{w}[n] - \mu\mathbf{r}[n] \ ; \\ \mathbf{w}[n+1] = \frac{1}{2}(\hat{\mathbf{w}}[n+1] + \mathbf{J}(\hat{\mathbf{w}}[n+1])^*) \ . \end{cases} \tag{28}$$

As the solution to the constrained minimization problem in (17), it subjects a normal CMA to a $\mathbf{w}$ with a conjugate symmetric property. For even M, If we know $w_1, w_2 \cdots w_{M/2}$, another half of the coefficients would have been decided according to the constraint equation (8). For an odd M, the coefficient $w_{(M+1)/2}$ at the middle must be real to meet the constraint equation, so we have the same conclusion as in the even M case. As a result, the number of variables in the adaptation has effectively been reduced by half by the proposed constraint and therefore we can expect an increased convergence speed and the output SINR will be better than the original CMA since the constraint guarantees a $\mathbf{w}$ in the structure corresponding to a maximum output SINR.

3.3 Analysis of the Proposed Algorithm

We first define the weight error vector $\mathbf{e}_w[n+1] = \mathbf{w}[n+1] - \mathbf{w}_{opt}$. Using (8) and (28), we arrive at

$$\begin{aligned} \mathbf{e}_w[n+1] &= \frac{1}{2}[\hat{\mathbf{w}}[n+1] + \mathbf{J}(\hat{\mathbf{w}}[n+1])^*] - \mathbf{w}_{opt} \\ &= \frac{1}{2}[\hat{\mathbf{w}}[n+1] - \mathbf{w}_{opt} + \mathbf{J}(\hat{\mathbf{w}}[n+1] - \mathbf{w}_{opt})^*] \ . \end{aligned} \tag{29}$$

Defining the matrix $\mathbf{R}_{cx} = \mathbf{I} - \mu(|y[n]|^2 - 1)\mathbf{x}[n]\mathbf{x}^H[n]$ and using (28), we have

$$\begin{aligned} \hat{\mathbf{w}}[n+1] - \mathbf{w}_{opt} &= \mathbf{R}_{cx}\mathbf{w}[n] - \mathbf{w}_{opt} \\ &= \mathbf{R}_{cx}(\mathbf{e}_w[n] + \mathbf{w}_{opt}) - \mathbf{w}_{opt} \\ &= \mathbf{R}_{cx}\mathbf{e}_w[n] - \mu(|y[n]|^2 - 1)\mathbf{x}[n]\mathbf{x}^H[n]\mathbf{w}_{opt}. \end{aligned} \tag{30}$$

Note that for the optimum $\mathbf{w}$, $\nabla_{\mathbf{w}^*} Q_{CM}$ in Equation (20) will be zero. Then we have $\mu(|y[n]|^2 - 1)\mathbf{x}[n]\mathbf{x}^H[n]\mathbf{w}_{opt} = 0$. Substituting (30) into (29) and then taking the expectation operation, we have

$$E\{\mathbf{e}_w[n+1]\} = \frac{1}{2}[E\{\mathbf{R}_{cx}\mathbf{e}_w[n]\} + E\{\mathbf{J}\mathbf{R}_{cx}^*\mathbf{J}\mathbf{J}\mathbf{e}_w^*[n]\}]$$
$$= \frac{1}{2}E\{\mathbf{R}_{cx}\}E\{\mathbf{e}_w[n] + \mathbf{J}\mathbf{e}_w^*[n]\} = E\{\mathbf{R}_{cx}\}E\{\mathbf{e}_w[n]\} \quad (31)$$

where we have used the fact $E\{\mathbf{R}_{cx}\} = E\{\mathbf{J}\mathbf{R}_{cx}^*\mathbf{J}\}$ and $\mathbf{e}_w[n] = \mathbf{J}\mathbf{e}_w^*[n]$. The stable condition of $\lim_{n\to\infty} E\{\mathbf{w}[n+1]\} = \mathbf{w}_{opt}$ or $\lim_{n\to\infty} E\{\mathbf{e}_w[n+1]\} = \mathbf{0}$ is equivalent to $E\{\Pi_{n=1}^{\infty}\mathbf{R}_{cx}\} = \mathbf{0}$ [18]. A sufficient condition for stability is that the stepsize is limited to the following range [18]

$$0 \le \mu \le \min_k \frac{2}{\lambda_k^{\mathbf{R}_{vx}}} \quad (32)$$

where $\lambda_k^{\mathbf{R}_{vx}}$ is the $k-$th eigenvalue of $\mathbf{R}_{vx}$ and

$$\mathbf{R}_{vx} = E\{(|y[n]|^2 - 1)\mathbf{x}[n]\mathbf{x}^H[n]\}\ . \quad (33)$$

The steady-state excess MSE (EMSE) analysis is an effective method to evaluate the performance of an adaptive algorithm [19]. One can follow the general steps presented in [20] and a recent paper with a more complete analysis to the CM based algorithms [13], and it is omitted here.

4 Extensions of the Proposed Approach

The proposed constrained approach is general and can be applied to other types of CMAs directly. In the following we will consider two representative examples.

4.1 Constrained LCCMA

The performance of a CMA can be improved significantly with additional signal information in the form of linear constraints [11,14], such as the DOA angle of the desired signal [12]. For this class of linearly constrained CMAs, we can also add the conjugate symmetric constraint and derive the corresponding constrained LCCMAs.

Suppose the response of the beamformer to the desired signal is constrained to 1, i.e., $\mathbf{a}^H(\psi_0)\mathbf{w} = 1$. With (10), we have $\mathbf{a}^H(\psi_0)(\mu_0\mathbf{R}_{xx}^{-1}\mathbf{a}(\psi_0)) = 1$. Then μ_0 has to be real-valued and $\mu_0/\mu_0^* = 1$, since $\mathbf{a}^H(\psi_0)\mathbf{R}^{-1}\mathbf{a}(\psi_0) \in \mathbb{R}$. From (14) we then have $\phi = -(M-1)\psi_0$.

Now we can formulate the constrained LCCM problem as

$$\min_{\mathbf{w}} J_{CM} = \frac{1}{4} E\{(|y[n]|^2 - 1)^2\}$$
$$\text{subject to } \mathbf{a}^H(\psi_0)\mathbf{w} = 1 \quad \text{and } \mathbf{w} - e^{j\phi}\mathbf{J}\mathbf{w}^* = \mathbf{0} \,. \tag{34}$$

A two-step constrained LCCMA can be derived using the Lagrange multipliers method in a similar way as in Section 3.2, given by

$$\begin{cases} \hat{\mathbf{w}}[n+1] = \mathbf{w}[n] - \mu[\mathbf{I} - \frac{1}{M}\mathbf{a}(\psi_0)\mathbf{a}^H(\psi_0)]\mathbf{r}[n] \,; \\ \mathbf{w}[n+1] = \frac{1}{2}[\hat{\mathbf{w}}[n+1] + e^{j\phi}\mathbf{J}(\hat{\mathbf{w}}[n+1])^*] \,. \end{cases} \tag{35}$$

The first step in (35) is exactly the same as the original LCCMA derived without the proposed conjugate symmetric constraint [20]. Therefore the effect of the weight constraint is again equivalent to adding a second step to the original algorithm. Notice that the general conjugate symmetric property in Equation (15) is used here instead of the phase-shifted version in (8).

4.2 Constrained LSCMA

In the second example, we consider the LSCMA case with the proposed conjugate symmetric constraint. The coefficients vector of an LSCMA can be updated block by block or sample by sample and here we only consider the sample by sample case, i.e. the update occurs every time a new signal sample is received.

The proposed constrained adaptive LSCMA algorithm is based on the following formulation

$$\min_{\mathbf{w}} J_{LS} = \sum_{i=1}^{K} ||\mathbf{w}^H[n]\mathbf{x}[nK+i]| - 1|^2$$
$$\text{subject to } \mathbf{w} - \mathbf{J}\mathbf{w}^* = \mathbf{0} \tag{36}$$

where K is the number of samples considered. A new cost function with an undetermined Lagrange multipliers λ can be formed as

$$\begin{aligned} Q_{LS} &= \sum_{i=1}^{K} ||\mathbf{w}^H[n]\mathbf{x}[nK+i]| - 1|^2 + \lambda(\mathbf{w}[n] - \mathbf{J}\mathbf{w}^*[n])^H(\mathbf{w}[n] - \mathbf{J}\mathbf{w}^*[n]) \\ &= \|\mathbf{q}(\mathbf{w})\|_2^2 + \lambda(\mathbf{w}[n] - \mathbf{J}\mathbf{w}^*[n])^H(\mathbf{w}[n] - \mathbf{J}\mathbf{w}^*[n]) \end{aligned} \tag{37}$$

where $\mathbf{q}(\mathbf{w}) = [q_1(w), q_2(w), \cdots, q_K(w)]^T$ and $q_i(w) = |\mathbf{w}^H[n]\mathbf{x}[nK+i]| - 1$. Q_{LS} has a Taylor-series expansion with the following form

$$
\begin{aligned}
Q_{LS}(\mathbf{w}+\boldsymbol{\delta}) = & \|\mathbf{q}(\mathbf{w})+\mathbf{D}(\mathbf{w})^H\boldsymbol{\delta}\|_2^2+\lambda(\mathbf{w}[n]-\mathbf{J}\mathbf{w}^*[n])^H\boldsymbol{\delta} \\
& +\lambda(\mathbf{w}[n]-\mathbf{J}\mathbf{w}^*[n])^H(\mathbf{w}[n]-\mathbf{J}\mathbf{w}^*[n])
\end{aligned} \tag{38}
$$

where $\mathbf{D}(\mathbf{w})$ is the partial derivative of $\mathbf{q}(\mathbf{w})$ and $\boldsymbol{\delta}$ the offset vector. Taking the gradient with respect to $\boldsymbol{\delta}$ and setting it to zero, we obtain

$$
\begin{aligned}
\boldsymbol{\delta} &= -[\mathbf{D}(\mathbf{w})\mathbf{D}^{\mathbf{H}}(\mathbf{w})]^{-1}[\mathbf{D}(\mathbf{w})\mathbf{q}(\mathbf{w})+\lambda(\mathbf{w}[n]-\mathbf{J}\mathbf{w}^*[n])] \\
&= -(\tilde{\mathbf{R}}[n])^{-1}[\tilde{\mathbf{x}}[n]\mathbf{y}_e[n]+\lambda(\mathbf{w}[n]-\mathbf{J}\mathbf{w}^*[n])]
\end{aligned} \tag{39}
$$

with $\tilde{\mathbf{x}}[n]=[\mathbf{x}[nK+1],\cdots,\mathbf{x}[nK+K]]$, $\mathbf{y}_e[n]=\mathbf{y}[n]-\tilde{\mathbf{y}}_e[n]$ and

$$
\begin{aligned}
\mathbf{y}[n] &= [y[nK+1],\cdots,y[nK+K]]^H \\
\tilde{\mathbf{y}}_e[n] &= [\frac{y[nK+1]}{|y[nK+1]|},\cdots,\frac{y[nK+K]}{|y[nK+K]|}]^H\ ,
\end{aligned} \tag{40}
$$

where we have used the fact [6]:

$$
\mathbf{D}(\mathbf{w})\mathbf{D}^{\mathbf{H}}(\mathbf{w})=\tilde{\mathbf{R}}[n]=\sum_{i=1}^{K}\mathbf{x}[nK+i]\mathbf{x}^H[nK+i] \tag{41}
$$

and

$$
\mathbf{D}(\mathbf{w})\mathbf{q}(\mathbf{w})=\tilde{\mathbf{x}}[n]\mathbf{y}_e[n]. \tag{42}
$$

Noting that $\mathbf{w}[n]=(\tilde{\mathbf{R}})^{-1}[n]\tilde{\mathbf{x}}[n]\mathbf{y}[n]$, we then arrive at the following update equation for the constrained LSCMA

$$
\begin{aligned}
\mathbf{w}[n+1] &= \mathbf{w}[n]+\boldsymbol{\delta}[n] \\
&= \mathbf{w}[n]-\tilde{\mathbf{R}}^{-1}[\tilde{\mathbf{x}}[n]\mathbf{y}_e[n]+\lambda(\mathbf{w}[n]-\mathbf{J}\mathbf{w}^*[n])] \\
&= \tilde{\mathbf{R}}^{-1}[n][\tilde{\mathbf{x}}[n]\tilde{\mathbf{y}}_e[n]+\lambda(\mathbf{w}[n]-\mathbf{J}\mathbf{w}^*[n])] \\
&= \hat{\mathbf{w}}[n+1]+\tilde{\mathbf{R}}^{-1}[n]\lambda(\mathbf{w}[n]-\mathbf{J}\mathbf{w}^*[n])
\end{aligned} \tag{43}
$$

where $\hat{\mathbf{w}}[n+1]=(\tilde{\mathbf{R}}[n])^{-1}\tilde{\mathbf{x}}[n]\tilde{\mathbf{y}}_e[n]$ is the original LSCMA update equation. Substituting (43) into the constraint equation to eliminate λ leads to the following constrained LSCMA

$$
\begin{aligned}
\mathbf{w}[n+1] = & \hat{\mathbf{w}}[n+1]-\tilde{\mathbf{R}}^{-1}[n][\tilde{\mathbf{R}}^{-1}[n]+\mathbf{J}(\tilde{\mathbf{R}}^*[n])^{-1}\mathbf{J}]^{-1} \\
& \cdot(\hat{\mathbf{w}}[n+1]-\mathbf{J}\hat{\mathbf{w}}^*[n+1])\ .
\end{aligned} \tag{44}
$$

When $K\to\infty$, we have $\tilde{\mathbf{R}}^{-1}[n]=\mathbf{J}(\tilde{\mathbf{R}}^*[n])^{-1}\mathbf{J}$. Then for a large K, we can have the approximation $\tilde{\mathbf{R}}^{-1}[n]\approx\mathbf{J}(\tilde{\mathbf{R}}^*[n])^{-1}\mathbf{J}$, which gives

$$
\tilde{\mathbf{R}}^{-1}[n][\tilde{\mathbf{R}}^{-1}[n]+\mathbf{J}(\tilde{\mathbf{R}}^*[n])^{-1}\mathbf{J}]^{-1}\approx\frac{1}{2}\ . \tag{45}
$$

Then (44) can be further simplified as

$$\mathbf{w}[n+1] \approx \frac{1}{2}(\hat{\mathbf{w}}[n+1] + \mathbf{J}\hat{\mathbf{w}}^*[n+1]) \ . \tag{46}$$

So the derived constrained LSCMA is still following the same general solution form as described in (28).

5 Simulations

Simulations have been performed based on a ULA with $M = 8$ sensors and a spacing of $d = \lambda_0/2$. We assume that all of the CM signals have the same power with a signal-to-noise ratio (SNR) of 20dB unless otherwise specified and all are generated by the QPSK modulation scheme.

Simulation 1: There are one desired signal and three interfering signals, arriving from the DOA angles $10°$, $-30°$, $50°$ and $-70°$, respectively. Fig. 2 shows the learning curves of the traditional CMA (T-CMA) ($\mu = 0.0003$), the constrained CMA (C-CMA) ($\mu = 0.001$), the traditional LSCMA (T-LSCMA) ($K = 30$), the constrained LSCMA (C-LSCMA) ($K = 30$), the traditional LCCMA (T-LCCMA) ($\mu = 0.0003$) and the constrained LCCMA (C-LCCMA) ($\mu = 0.0005$). The stepsizes are chosen empirically for each pair of algorithms to reach approximately the same steady state output SINR. We can see that due to the additional DOA information of the desired signal, the T-LCCMA and the C-LCCMA have achieved a much better performance than the basic T-CMA and C-CMA, respectively. However, the proposed algorithms (C-CMA and C-LCCMA) have a much faster convergence rate than the corresponding traditional ones. For the LSCMA pair (T-LSCMA and C-LSCMA), their convergence speed is not controlled by any stepsize and it seems that they have a similar convergence speed. However, the proposed C-LSCMA has achieved a higher output SINR given the same block size.

Simulation 2: The tracking ability of the proposed algorithms is studied in Fig. 3. There are two stages in the simulations: at the first stage (snapshot number n from 1 to 2000), the settings are the same as in simulation 1; at the second stage (snapshot number n from 2001 onwards), an additional CM interfering signal arrives at the array from the direction $-20°$. From the figure we can see that the proposed CM algorithms have adapted to the new environment very quickly and even achieved a better output SINR compared to the traditional ones with the given stepsizes.

Simulation 3: In the third set of simulations, we study the BER (bit error rate) performance of the proposed algorithms, with the result shown in Fig. 4. The settings are the same as in Simulation 1. However, the input SNR varied and we have chosen the same stepsize (or block size) for the corresponding pairs of algorithms. For the CMA pair, the stepsize is $\mu = 0.002$ and for the LCCMA pair, it is $\mu = 0.005$.

The weight vector we used for calculating the BER was obtained after 2000 iterations and averaged over 100 rounds of simulations. For both the CMA and the LSCMA cases, the BER result is improved significantly; while for the LCCMA case, the original algorithm has already achieved an extremely low BER result due to the effective incorporation of the DOA information of the desired signal and the improvement by the proposed algorithm is relatively small.

Simulation 4: We shall now examine the sensitivity of the proposed algorithm (C-CMA) to its stepsize. Consider a simplified scenario with 20dB input SNR, based on a $3-$element ULA, with only one desired CM signal and one interfering CM signal, arriving from $\theta_0 = 10°$ and $\theta_1 = -30°$, respectively. We change the stepsize of both the proposed and the traditional CMA from 0.002, 0.005, 0.01 to 0.02. The results are shown in Fig. 5, where the convergence speed increases for both algorithms when the stepsize becomes larger and larger. However, the output SINR of the T-CMA drops significantly for the case of $\mu = 0.02$, while the C-CMA almost keeps the same steady state error and reaches almost the same output SINR. For the LCCMA pair, both of them converge very quickly due to the imposed linear constraint and the sensitivity study is therefore omitted here.

6 Conclusions

A new class of constrained CMAs has been proposed based on the ULA structure. It is derived by constraining the weight vector to a conjugate symmetric form, which corresponds to a beamforming solution with the maximum output SINR. There are essentially two steps in the update equation, with the first one being the traditional update equation and the second one imposing the desired structure. With this constraint, the number of variables in the update equation has effectively been reduced by half. As a result, a higher convergence rate has been achieved. Moreover, since the imposed structure is derived from a maximum SINR beamformer, the proposed algorithm also leads to a higher output SINR given the same stepsize or block size.

References

[1] R. Gooch and J. Lundell, “CM array: an adaptive beamformer for constant modulus signals,” in *Proc. IEEE International Conference on Acoustics, Speech, and Signal Processing*, New York, NY, 1986, pp. 2523–2526.

[2] H. Krim and M. Viberg, “Two decades of array signal processing research: the parametric approach,” *IEEE Signal Processing Magazine*, vol. 13, no. 4, pp. 67–94, July 1996.

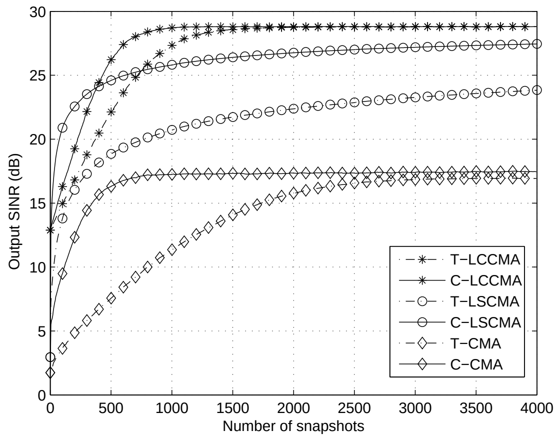

Fig. 2. Output SINR versus the number of snapshots.

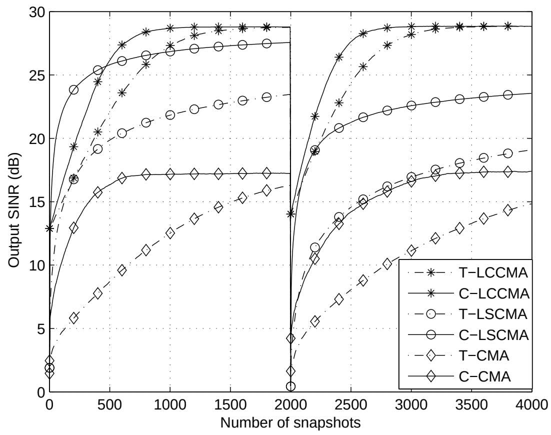

Fig. 3. Output SINR versus the number of snapshots, with $L = 4$ interfering signals for the first 2000 snapshots and $L = 5$ after that.

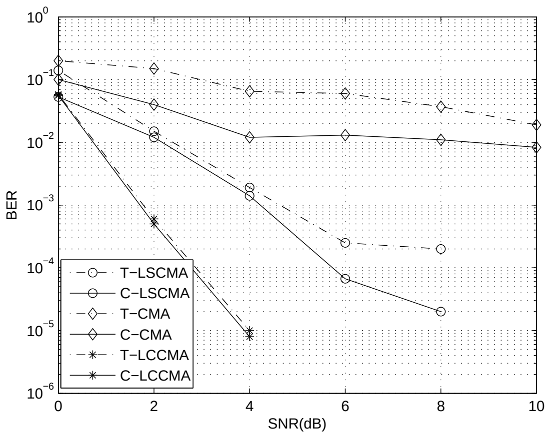

Fig. 4. BER performance versus the input signal to noise ratio (SNR).

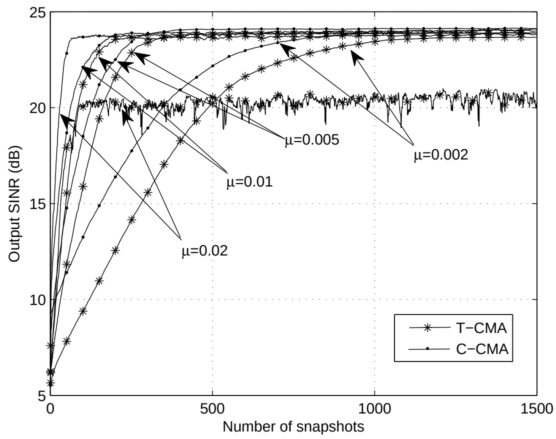

Fig. 5. Output SINR versus the number of snapshots for different stepsizes.

[3] L. C. Godara, "Application of antenna arrays to mobile communications, Part II: Beam-forming and direction-of-arrival estimation," *Proceedings of the IEEE*, vol. 85, pp. 1195–1245, 1997.

[4] C. R. Johnson, P. Schniter, T. J. Endres, J. D. Behm, D. R. Brown, and R. A. Casas, "Blind equalization using the constant modulus criterion: A review," *Proceedings of the IEEE*, vol. 86, no. 10, pp. 1927–1950, October 1998.

[5] S. Chen, A. Wolfgang, and L. Hanzo, "Constant modulus algorithm aided soft decision directed scheme for blind space-time equalisation of simo channels," *Signal Processing*, vol. 87, pp. 2587–2599, 2007.

[6] B. G. Agee, "Least-squares CMA: A new technique for rapid correction of constant modulus signals," in *ICASSP*, 1986.

[7] T. E. Biedka, W. H. Tranter, and J. H. Reed, "Convergence Analysis of the Least Squares Constant Modulus Algorithm in Interference Cancellation Applications," *IEEE Transactions on Communications*, vol. 48, no. 3, pp. 491–501, March 2000.

[8] Y. Chen, T. Le-Ngoc, B. Champagne, and X. Changjiang, "Recursive least squares constant modulus algorithm for blind adaptive array," *IEEE Transactions on Signal Processing*, vol. 52, pp. 1452 – 1456, 2004.

[9] R. C. de Lamare and R. Sampaio-Neto, "Blind adaptive code-constrained constant modulus algorithms for CDMA interference suppression in multipath channels," *IEEE Communications Letters*, vol. 9, pp. 334 – 336, 2005.

[10] R. C. de Lamare, M. Haardt, and R. Sampaio-Neto, "Blind adaptive constrained reduced-rank parameter estimation based on constant modulus design for CDMA interference suppression," *IEEE Transactions on Signal Processing*, vol. 52, pp. 2470 – 2482, 2008.

[11] R. C. de Lamare and R. Sampaio-Neto, "Low-complexity variable step-size mechanisms for stochastic gradient algorithms in minimum variance CDMA receivers," *IEEE Transactions on Signal Processing*, vol. 54, pp. 2302–2317, 2006.

[12] L. Wang, Y. Cai, and R. C. de Lamare, “Low-complexity adaptive step size constrained constant modulus SG-based algorithms for blind adaptive beamforming,” in *IEEE Proceedings ICASSP*, Las Vegas, NV, April 2008.

[13] Y. Cai and R. C. de Lamare, “Low-complexity variable step-size mechanism for code-constrained constant modulus stochastic gradient algorithms applied to CDMA interference suppression,” *IEEE Transactions on Signal Processing*, vol. 57, pp. 313–323, 2009.

[14] J. Miguez and L. Castedo, “Linearly constrained constant modulus approach to blind adaptive multiuser interference suppression,” *Communications Letters*, vol. 2, pp. 217–219, 1998.

[15] K. C. Huarng and C. C. Yeh, “Adaptive beamforming with conjugate symmetric weights,” *IEEE Transactions on Antennas and Propagation*, vol. 39, no. 7, pp. 926–932, 1991.

[16] J. Li and P. Stoica, *Robust adaptive beamforming*. New Jersey, U.S.A.: John Wiley & Sons, Inc., 2005.

[17] A. Leshem and A. J. van der Veen, “Blind source separation: The location of local minima in the case of finitely many samples,” *IEEE Trans. Signal Processing*, vol. 56, pp. 4340–4353, 2008.

[18] M. Honig, U. Madhow, and S. Verdu, “Blind adaptive multiuser detection,” *IEEE Transactions on Information Theory*, vol. 41, pp. 944–960, 1995.

[19] A. H. Sayed, *Adaptive Filters*. New Jersey, U.S.A.: John Wiley & Sons, Inc., 2008.

[20] J. B. Whitehead and F. Takawira, “Performance analysis of the linearly constrained constant-modulus algorithm-based multiuser detector,” *IEEE Transactions on Signal Processing*, vol. 53, pp. 643–653, 2005.