Proceedings Paper:

Kosiba, E.A., Liu, G., Shtessel, Y.B. et al. (1 more author) (2006) Output tracking via sliding modes in causal systems with time delay modeled by higher order pade approximations. In: Proceedings of the International Workshop on Variable Structure Systems. International Workshop on Variable Structure Systems 2006 , VSS'06 , 05-07 Jun 2006, Alghero, Italy. Institute of Electrical and Electronics Engineers , pp. 250-255. ISBN 1-4244-0208-5

https://doi.org/10.1109/VSS.2006.1644526

Proceedings of the 2006 International Workshop on Variable Structure Systems
Alghero, Italy, June 5-7, 2006

Output Tracking via Sliding Modes in Causal Systems with Time Delay Modeled by Higher Order Padé Approximations

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Abstract— **Output tracking in a SISO causal uncertain nonlinear system with an output subject to a time delay is considered using sliding mode control. A higher order Padé approximation to a delay function with a known time delay is used to construct a model of a transformed system without a time delayed output and is employed in a feedback sliding mode control. This model functions as a predictor of the plant states and the plant output, but is of nonminimum phase due to the application of the Padé approximation. The method of the *stable system center* is used to stabilize the internal dynamics of this plant model, and a control is developed using a sliding surface to allow the plant to track a arbitrary reference profile given by an exogenous system with a known characteristic equation. Simulations of the system are performed for the plant model using a first, second and third order Padé approximations and a controller in plant feedback mode. Numerical examples for Padé approximations of increasing order are considered and compared.**

I. Introduction

OUTPUT tracking is important in many control systems, including electric power converters, robot manipulators and aerospace control systems [1]. The development of control algorithms, including sliding mode control, to allow output tracking of a reference profile for a system with an output time delay has been considered previously [1]-[6]. Time delay compensation techniques, developed in [4,5] mostly for systems with input delay, have been proposed to design the control input based on predicted values of the state variables.

The output tracking of a real-time reference profile in nonlinear uncertain systems with output delay by sliding mode control is considered in [7],[8]. The sliding mode control algorithm is designed for the approximate nonminimum phase model of a system with output delay. In these papers the plant state and the plant output predictor is based on the Padé approximation of the first order. The transformed system model with the Padé approximation replacing the delay element is of a nonminimum phase [9]. The developed output tracking sliding mode control algorithm [7],[8] suitable for the output tracking in causal nonminimum phase systems employs the method of stable system center [10],[11]. This technique is based on the transformation of nonminimum phase output tracking in causal systems to state variable tracking. The bounded state reference profiles are generated using custom-designed equations of the system center [10],[11].

The Padé approximations yield the time delay function model of a reasonable accuracy within a limited bandwidth [12]. It means that the agility of the system dynamics and of the reference output profile must be within this bandwidth in order the controller to provide an accurate output tracking. Since sliding mode control generates the switching control function of a very high frequency, inaccuracy in the output time delay approximation can lead to a significant control chattering [2],[3],[7],[8]. A contribution of this paper is in extending sliding mode output tracking control in causal nonlinear uncertain systems with output time delay modeled by the first order Padé approximation that is developed in [7],[8] to the case with higher order Padé approximation. Numerical computer simulations compare the sliding mode control performance in systems with the output time delay using the first, second and third order Padé approximations. It is shown that the higher order Padé approximation yields the better tracking accuracy of an arbitrary output reference profile given by a linear exogenous system with a known characteristic equation. In particular, the higher frequency of control switching and lower amplitude of oscillations in the output tracking error are achieved.

II. Problem Formulation

A. Mathematical Model

Consider a controllable fully feedback linearizable nonlinear SISO dynamic system without time delay

$$\begin{aligned} \dot{x} &= \phi(x,t) + g(x,t)u \\ y &= h(x) \end{aligned} \tag{1}$$

where $x(t) \in \Re^n$ is a state vector, $y(t) \in \Re^1$ a controlled output and $u(t) \in \Re^1$ is a control input. The output command (tracking) profile $y_c(t)$ is given in real time for the output $y(t)$ to be tracked asymptotically: $y(t) \to y_c(t)$.

B. Coordinate Transformation

The system (1) can be easily transformed [9] to

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$$y^{(n)} = \varphi(\xi,t) + b(\xi,t)u \quad (2)$$

where $\xi = [y, \dot{y}, ..., y^{(n-1)}]^T \in \Re^n$, and n is the relative degree. Following the approach in [13], we define a coordinate transformation for $\xi = [y, \dot{y}, ..., y^{(n-1)}]^T \in \Re^n$ to transform the system (1) to a form with relative degree equal to one. The new coordinate basis is

$$\begin{pmatrix} z \\ q \end{pmatrix} = \begin{bmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \vdots & & \ddots & 0 \\ a_0 & a_1 & \dots a_{n-2} & 1 \end{bmatrix} \cdot \xi, \quad (3)$$

$z(t) \in \Re^{n-1}$, $q(t) \in \Re$

and the coordinate

$$q(t) = y^{(n-1)}(t) + a_{n-2}y^{(n-2)}(t) + ... + a_1\dot{y}(t) + a_0 y(t) \quad (4)$$

is a new output. A new output-tracking profile is introduced

$$q_c(t) = y_c^{(n-1)}(t) + a_{n-2}y_c^{(n-2)}(t) + ... + a_1\dot{y}_c(t) + a_0 y_c(t) \quad (5)$$

so that $q(t) \to q_c(t)$ implies that $y(t) \to y_c(t)$ asymptotically with the eigenvalues defined by the polynomial

$$\Delta_n(\lambda) = \lambda^{n-1} + a_{n-2}\lambda^{n-2} + ... + a_1\lambda + a_0 . \quad (6)$$

where $a_{n-1}, ..., a_1, a_0$ are specified nonnegative constants. Eigenvalues of the polynomial (6) are to be located in the left hand side of the complex plane or in the imaginary axis.

The system (1) is rewritten in the new basis (3)

$$\begin{cases} \dot{z}_1 = z_2, \dot{z}_2 = z_3 \\ \dots\dots\dots\dots \\ \dot{z}_{n-1} = q - a_{n-2}z_{n-1} - ... - a_1 z_2 - a_0 z_1 \\ \dot{q} = \hat{\varphi}(z,q,t) + \hat{b}(z,q,t)u \end{cases} \quad (7)$$

where

$$\hat{\varphi}(z,q,t) = -a_0 a_{n-2} z_1 - (a_1 a_{n-2} - a_0) z_2 - ... - (a_{n-2}^2 - a_{n-3}) z_{n-1} + a_{n-2} q + \varphi(z, q - a_0 z_1 - ... - a_{n-2} z_{n-1}, t),$$

$$\hat{b}(z,q,t) = b(q - a_0 z_1 - ... - a_{n-2} z_{n-1}, t).$$

The internal dynamics of the system (7) are stable and can be disregarded when solving the output tracking problem.

C. Output Tracking Time-delayed Problem

Output tracking in system (1) can be transformed by (3) to output tracking in a scalar system

$$\dot{q} = \hat{\varphi}(z,q,t) + \hat{b}(z,q,t)u \quad (8)$$

where $q \in \Re^1$, $u \in \Re^1$; $\hat{b}(\cdot) = \hat{b}_0(\cdot)(1 + \delta(\cdot))$, $|\delta(\cdot)| \le \beta_1 < 1$ $\hat{\varphi}(\cdot) < \alpha_1$, $\alpha_1 > 0$. The function $\hat{b}_0(\cdot) > 0$ is assumed known. We now assume that the system output of (8) is accessible with time delay

$$\hat{y}(t) = q(t - \tau) . \quad (9)$$

The problem is to design sliding mode control $u(t)$ that forces the output variable $\hat{y}(t)$ to track asymptotically the command profile $q_c(t)$ described by an exogenous system with known stable characteristic polynomial

$$P_k(\lambda) = \lambda^k + p_{k-1}\lambda^{k-1} + ... + p_1\lambda + p_0 \quad (10)$$

where k is the order of the exogenous system, and $p_{k-1}, ..., p_1, p_0$ are specified nonnegative constants. Note that we require

$$\lim_{t\to\infty} |q_c(t) - \hat{y}(t)| = 0. \quad (11)$$

The problem can be reformulated by replacing the time-delay function (9) by the Pade′ approximations [12].

D. Padé Approximations

The Padé approximations for a time delay τ can be represented as a ratio of two polynomial functions of the Laplace variable s with real coefficients. This is [12]

$$\frac{\hat{y}(s)}{q(s)} = e^{-s\tau} \approx \frac{\sum_{i=0}^{j} N_i(-s)^i}{\sum_{i=0}^{j} D_i s^i} = \frac{\sum_{i=0}^{j} (-1)^i \frac{(2j-i)!}{(j-i)!i!}(\tau s)^i}{\sum_{i=0}^{j} \frac{(2j-i)!}{(j-i)!i!}(\tau s)^i} \quad (12)$$

where s is the Laplace variable and j is the order of the Padé approximation. Let us introduce a new output variable $\tilde{y}$ assuming that equality (12) becomes exact. In particular, we obtained the following state variable Padé approximation:

$$\dot{\eta} = 2\left(\eta/\tau - 2/\tau\right)\tilde{y}, \quad j = 1$$

$$\begin{bmatrix} \dot{\eta}_1 \\ \dot{\eta}_2 \end{bmatrix} = \begin{bmatrix} 0 & -12/\tau^2 \\ 1 & 6/\tau \end{bmatrix} \begin{bmatrix} \eta_1 \\ \eta_2 \end{bmatrix} - 2\begin{bmatrix} 0 \\ 6/\tau \end{bmatrix}\tilde{y}, \quad j = 2$$

......

$$\begin{bmatrix} \dot{\eta}_1 \\ \dot{\eta}_2 \\ \dot{\eta}_3 \\ \dot{\eta}_4 \\ \vdots \\ \dot{\eta}_j \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 & \pm D_0 \\ 1 & 0 & 0 & 0 & 0 & \vdots \\ 0 & 1 & 0 & 0 & \ddots & -D_{j-4} \\ 0 & 0 & 1 & 0 & \ddots & D_{j-3} \\ 0 & \ddots & \ddots & \ddots & \ddots & -D_{j-2} \\ 0 & 0 & 0 & 0 & 1 & D_{j-1} \end{bmatrix} \begin{bmatrix} \eta_1 \\ \eta_2 \\ \eta_3 \\ \eta_4 \\ \vdots \\ \eta_j \end{bmatrix} - 2\begin{bmatrix} D_0 \text{ or } 0 \\ \vdots \\ 0 \\ D_{j-3} \\ 0 \\ D_{j-1} \end{bmatrix}\tilde{y} \quad (13)$$

Taking into account (12) and (13) the system (8) and (9) can be approximately replaced by a state variable model. In particular, for the first order Padé approximation the system (8), (9) can be modeled by [7],[8]

$$\begin{cases} \dot{\eta} = \frac{2}{\tau}\eta - \frac{4}{\tau}\tilde{y} \\ \dot{\tilde{y}} = \overline{\varphi}(z,q,\eta,\tilde{y}) - \hat{b}(z,q,t)u \end{cases} \quad (14a)$$

where $\overline{\varphi}(z,q,\eta,\tilde{y}) = -\hat{\varphi}(z,q,t) + \frac{2}{\tau}\eta - \frac{4}{\tau}\tilde{y}$, and the new output $\tilde{y}$ is an approximation for the original output $\hat{y}$.

Similarly, using a second order Padé approximation the system (8) and (9) can be approximately replaced by

$$\dot{\eta}_1 = -\frac{12}{\tau^2}\eta_2, \quad \dot{\eta}_2 = \eta_1 + \frac{6}{\tau}\eta_2 - \frac{12}{\tau}\tilde{y}$$
$$\dot{\tilde{y}} = -\eta_1 - \frac{6}{\tau}\eta_2 - \frac{12}{\tau}\tilde{y} + \hat{\varphi}(z,q,t) + \hat{b}(z,q,t)u \tag{14b}$$

The systems in eqs. (14a) and (14b) and are of nonminimum phase with unstable forced zero dynamics. They are also systems without time delay, where the output and states are predicted using the Padé approximations of any selected order. So, the original tracking control problem with output time delay has been approximately transformed into a nonminimum phase output-tracking problem without delay. The nonminimum phase output tracking problem (11), (14) with $\tilde{y}$ standing for $\hat{y}$ is addressed using the method of the stable system center that transforms the output tracking in a nonminimum phase system to the state variable tracking.

III. The Method of Stable System Center in Nonminimum Phase Output Tracking

The results are formulated for the following nonminimum phase system:

$$\begin{cases} \dot{\psi} = Q_1\psi + Q_2 y + f(t) \\ \dot{y} = \varphi(y,\psi,t) + \omega(y,\psi)u \end{cases} \tag{15}$$

where $\psi(t) \in R^{n-m}$ is the internal state, $y(t) \in \Re^m$ the controlled vector-output, $u(t) \in \Re^m$ the control function; $\varphi(\cdot) \in \Re^m$ is a bounded vector field and $\omega(\cdot) \in \Re^{m\times m}$ is a nonsingular matrix; $Q_1 \in \Re^{(n-m)\times(n-m)}$ is a known nonsingular non-Hurwitz matrix, $Q_2 \in \Re^{(n-m)\times m}$ is a known matrix, the pair (Q_1, Q_2) is completely controllable; $f(t)$ is an unmatched [1,2] external disturbance.

Remark. The system (15) is nonminimum phase, since $Q_1 \in \Re^{(n-m)\times(n-m)}$ is a non-Hurwitz matrix that yields instability of the zero dynamics.

The problem is to provide the tracking of a causal smooth reference (command) profile, $y \to y_c(t)$, in the presence of an unmatched bounded external disturbance $f(t)$.

A. Replacing output tracking by state tracking

Now, we have the output tracking problem for the nonminimum phase system (15) which is in the normal canonical form [9]. We seek state tracking control in order to apply traditional state variable sliding mode control techniques [2,3]. Rewriting system (15) in errors we obtain

$$\begin{cases} \dot{e}_\psi = Q_1 e_\psi + Q_2 e_y + (\dot{\psi}_c - Q_1\psi_c - Q_2 y_c - f(t)) \\ \dot{e}_y = -\varphi(y_c - e_y, \psi_c - e_\psi, t) - \omega(y_c - e_y, \psi_c - e_\psi)u + \dot{y}_c \end{cases} \tag{16}$$

where $e_y = y_c - y$, $e_\psi = \psi_c - \psi$.

Equations of system center (ideal internal dynamics) that define a command (tracking) profile $\psi_c(t)$ for the internal state vector $\psi(t)$, can be written as [8],[9]:

$$\dot{\psi}_c = Q_1\psi_c + Q_2 y_c + f(t). \tag{17}$$

The disturbance $f(t)$ can be estimated using an exact second order sliding mode differentiator [14]. This is

$$\hat{f}(t) = \hat{\dot{\psi}} - Q_1\psi - Q_2 y \tag{18}$$

Once ψ_c is identified by integrating eq. (17), the problem to provide state tracking in the system (15) can be solved using standard sliding mode control [2],[3]. The system's (15) state tracking error dynamics are described by

$$\begin{cases} \dot{e}_\psi = Q_1 e_\psi + Q_2 e_y \\ \dot{e}_y = -\varphi(y_c - e_y, \psi_c - e_\psi, t) - \omega(y_c - e_y, \psi_c - e_\psi)u + \dot{y}_c(t) \end{cases} \tag{19}$$

Define the sliding surface $\sigma \in \Re^m$ as

$$\sigma = e_y + K e_\psi = 0, \; K \in \Re^{m\times(n-m)}, \tag{20}$$

Eqs. (19), (20) reduce in the sliding mode to

$$\begin{cases} \dot{e}_\psi = (Q_1 - Q_2 K)e_\psi \\ e_y = -K e_\psi \end{cases} \tag{21}$$

Since the pair (Q_1, Q_2) is completely controllable, we select K so that the eigenvalues of $(Q_1 - Q_2 K)$ lie sufficiently deep in the left half plane. Then the system (22) is locally asymptotically stable. We achieve asymptotic output tracking in the sliding mode, i.e. $e_y \to 0$.

The standard sliding mode controller [2,3], providing existence of the sliding mode (21), can be easily designed as

$$u = \omega(\cdot)^{-1}\left[\hat{\dot{y}}_c - \hat{\varphi}(\cdot) + K(Q_1 e_\psi + Q_2 e_y) - \frac{\rho}{2} SIGN(\sigma)\right] \tag{22}$$

with $\hat{\dot{y}}_c$ and $\hat{\varphi}(\cdot)$ estimates of $\dot{y}_c$ and $\varphi(\cdot)$ respectively, and $SIGN(\sigma) = [sign(\sigma_1), sign(\sigma_2), ..., sign(\sigma_m)]^T$, $\rho > \max_{i=1,m} |\hat{\varphi}_i(.) - \varphi_i(.)|$.

The only problem still to be resolved is obtaining a stable solution ψ_c (the system center) to the unstable equations of the system center (17).

B. Stable system center

Consider the exosystems for $\theta_c = Q_2 y_c + \hat{f}$. Let the "cumulative" characteristic polynomial for this exosystem be in the format

$$P_k(\lambda) = \lambda^k + p_{k-1}\lambda^{k-1} + ... + p_1\lambda + p_0, \tag{23}$$

where k is the order of this exosystem, and $p_{k-1}, ..., p_1, p_0$ are specified numbers.

The stable system center $\tilde{\psi}_c(t)$ can be computed using the following theorem.

Theorem 1. *Given the nonminimum phase system (15) with the measurable state vector* (ψ, y) *and the following set of conditions:*

1. the matrix $\tilde{Q}_1$ *in eq. (17) is nonsingular.*

2. the output reference profile $y_c(t)$ and the unmatched disturbance $f(t)$ can be piecewise represented by known linear exosystems with a characteristic polynomial (4).

Then

1. the output tracking in real time of a bounded reference profile, $y_c(t) \in \Re^m$, can be replaced by tracking the state reference profile $(\psi_c y_c)^T \in \Re^n$, such that $(\psi, y)^T \to (\psi_c, y_c)^T$ asymptotically with given eigenvalues;

2. the internal state reference profile $\widetilde{\psi}_c \in \Re^{n-m}$ is generated by the matrix differential equation

$$\begin{aligned}&\widetilde{\psi}_c^{(k)} + c_{k-1} I \cdot \widetilde{\psi}_c^{(k-1)} + ... + c_1 \dot{\widetilde{\psi}}_c + c_0 \widetilde{\psi}_c = \\ &-\left(P_{k-1}\theta_c^{(k-1)} + ... + P_1 \dot{\theta}_c + P_0 \theta_c\right)\end{aligned} \quad (24)$$

where the constants $c_{k-1},...,c_1,c_0$ are chosen to provide any desired eigenvalues for $\widetilde{\psi}_c \to \psi_c$, and matrices $P_{k-1},...,P_1,P_0 \in \Re^{(n-m)\times(n-m)}$ are given by

$$\begin{aligned}&P_{k-1} = \\ &\left(I + c_{k-1}Q_1^{-1} + ... + c_0 Q_1^{-k}\right)\cdot\left(I + p_{k-1}Q_1^{-1} + ... + p_0 Q_1^{-k}\right)^{-1} - I \\ &P_{k-2} = c_{k-2}Q_1^{-1} + ... + c_0 Q_1^{-(k-1)} - \\ &(P_{k-1} + I)\cdot\left(p_{k-2}Q_1^{-1} + ... + p_0 Q_1^{-(k-1)}\right) \\ &\vdots \\ &P_1 = c_1 Q_1^{-1} + c_0 Q_1^{-2} - (P_{k-1} + I)\cdot\left(p_1 Q_1^{-1} + p_0 Q_1^{-2}\right) \\ &P_0 = c_0 Q_1^{-1} - (P_{k-1} + I)\cdot p_0 Q_1^{-1}\end{aligned} \quad (25)$$

Proof: See [11].

Thus, (24) and (25) determine the stable system centre $\widetilde{\psi}_c(t)$ for the system (15), which asymptotically converges to the ideal internal dynamics (unstable) in eq. (17). Now we can use the stable system center $\widetilde{\psi}_c(t)$ for $\psi_c(t)$ when computing the sliding surface (21), bearing in mind that $\widetilde{\psi}_c(t) \to \psi_c(t)$ asymptotically.

IV. Example: Sliding mode tracking controller design for the system with output time delay

Given a 2nd order plant [7],[8]

$$\dot{x}_1 = x_2, \quad \dot{x}_2 = -x_2 + u, \quad y = x_1 \quad (26)$$

The desired command output profile is given in the form

$$y_c = \overline{A} + \overline{B}\sin\omega_n t. \quad (27)$$

The relative degree of the plant is equal to two. Transforming the plant to the form (7) with relative degree equal to one, introduce a new state vector in accordance with eq. (3)

$$\begin{bmatrix} z_1 \\ q \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ a_0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}. \quad (28)$$

The system (26) is rewritten in the new basis (28) as

$$\begin{cases} \dot{z}_1 = -a_0 z_1 + q \\ \dot{q} = -(a_0 - 1)a_0 z_1 + (a_0 - 1)q + u \end{cases} \quad (29)$$

The new output q is related to the original output y by

$$q = \dot{y} + a_0 y \Rightarrow q = \dot{x}_1 + a_0 x_1. \quad (30)$$

The command profile for the new output is computed to be

$$q_c = \dot{y}_c + a_0 y_c \Rightarrow q_c = A + B\sin\omega_n t + C\cos\omega_n t \quad (31)$$

where $A = a_0\overline{A},\ B = a_0\overline{B},\ C = \omega_n\overline{B}$. The signal in (31) can be described by a linear exogenous system of differential equations with the characteristic equation (23):

$$P_3(\lambda) = \lambda^3 + 0\cdot\lambda^2 + \omega_n^2\cdot\lambda + 0 \quad (31a)$$

The system (28) is of relative degree one with stable zero dynamics. The output $y(t)$ reaches $y_c(t)$ asymptotically with eigenvalue $\lambda = -a_0$ once $q(t)$ (30) reaches $q_c(t)$. We assume that the system output (29) is accessible with a time delay $\hat{y} = q(t-\tau)$. The problem is to design sliding mode control u that provides asymptotic tracking $\hat{y} \to q_c$.

A. The first-order Padé approximation

Replacing the time-delay function by the first-order Padé approximation as in (13) the system (29) is approximately represented by a nonminimum phase system without delay [7],[8]

$$\begin{aligned}&\dot{z}_1 = -a_0 z_1 + \eta - \widetilde{y}, \quad \dot{\eta} = \frac{2}{\tau}\eta - \frac{4}{\tau}\widetilde{y} \\ &\dot{\widetilde{y}} = \left(\frac{2}{\tau} - a_0 + 1\right)\eta + \left(a_0 - 1 - \frac{4}{\tau}\right)\widetilde{y} + (a_0 - 1)a_0 z_1 - u\end{aligned} \quad (32)$$

where the output $\widetilde{y}$ is an approximation to the output $\hat{y}$.

Remark. The zero dynamics of the system are

$$\dot{z}_1 = -a_0 z_1 + \eta, \quad \dot{\eta} = \frac{2}{\tau}\eta \quad (33)$$

The equation for z_1 is stable and generates a bounded profile given bounded input η. The problem is to stabilize the equation for η while providing asymptotic output tracking $\widetilde{y} \to q_c$.

Let the equations of the stable system center (24) and (25) have $c_0 = 1000,\ c_1 = 300,\ c_2 = 30,\ \tau = 0.2,\ \omega_n = 2,\ a_0 = 20$, the parameters P_0, P_1, P_2 are computed as $P_0 = 100,\quad P_1 = 36.9,\quad P_2 = 6.7$, and $\theta_c = -20\left(20\overline{A} + 20\overline{B}\sin 2t + 2\overline{B}\cos 2t\right)$.

The system center can be written as the transfer function

$$\widetilde{\eta}_c(s) = -\frac{6.7s^2 + 36.9s + 100}{s^3 + 30s^2 + 300s + 1000}\theta_c(s) \quad (34)$$

which is employed when implementing or simulating the system (26). The sliding surface $\sigma = e_q - 0.75\widetilde{e}_\eta$ yields the sliding mode asymptotic tracking dynamics (assuming $\widetilde{e}_\eta \to e_\eta$) obtained in the format (21)

$$\dot{e}_\eta = -10e_\eta,\ e_q = -C_1 e_\eta \tag{35}$$

Finally $e_\eta \to 0$ and $e_q \to 0$ asymptotically, yielding $\tilde{y} \to y_c$ in the sliding mode. The corresponding sliding mode controller has been designed using the simplest relay format $u = -25sign(\sigma)$.

B. The higher order Padé approximation

This procedure was repeated for Padé approximations of orders 2 and 3, using eq. (14a) for $j = 2$ and $j = 3$. In these cases the number of internal states of the model for the system (28) without output delay increases by one for each increase in the order of the Padé approximation, retaining the original relative degree of the system (28), which is equal to one. The parameters P_0, P_1, P_2 in (24, (25) become matrices, and the gain K in (20) becomes a vector, i.e. $P_i \in \Re^{jxj}$, $K \in \Re^{jx1}$. The parameters P_i are generated using eq. (25). The equation of the system center (24) becomes a matrix differential equation of the third order, given the third order characteristic equation of the exogenous input (31a). The system center transfer functions, as in (34), are formed from the elements of P_i by combining terms to form a matrix transfer function

$$\begin{bmatrix} s^2 P_{11} + sP_{11} + P_{11} & \cdots & s^2 P_{1j} + sP_{1j} + P_{1j} \\ \vdots & \ddots & \vdots \\ s^2 P_{j1} + sP_{j1} + P_{j1} & \cdots & s^2 P_{jj} + sP_{jj} + P_{jj} \end{bmatrix} \begin{bmatrix} \theta_{1c} \\ \vdots \\ \theta_{jc} \end{bmatrix}$$
$$= \begin{bmatrix} s^3 + c_2 s^2 + c_1 s + c_0 \\ \vdots \\ s^3 + c_2 s^2 + c_1 s + c_0 \end{bmatrix} \begin{bmatrix} \tilde{\eta}_{1c} \\ \vdots \\ \tilde{\eta}_{jc} \end{bmatrix}$$

where j is the order of the Padé approximation used in the model. The gain vector K in (20) is identified by using the state error equation in the sliding mode

$$\dot{e}_\eta = (Q_1 - Q_2 K)e_\eta, e_y = -Ke_\eta \tag{36}$$

and solving for the characteristic polynomial of the result. For all Padé approximation cases $j = 1,2,3$ used as examples herein, the equations were evaluated for chosen eigenvalue(s) $\lambda_i = -10$ and the gains K_j were identified accordingly. The corresponding sliding mode controller has been designed using the simplest relay format $u = 25sign(\sigma)$ for the 2nd order Padé approximation and $u = -25sign(\sigma)$ for the 3d order

The results of simulations are shown in Figures 1-3 for a reference profile satisfying the exogenous system with a given characteristic equation (31a)

$$q_c(t) = 1(t) - 1.5(t-15) + \{0.5[1(t-4) - 1(t-10)] + 0.25[1(t-10) - 1(t-15)]\}\sin 2t \tag{37}$$

Figures 1,2, and 3 compare the plant output command and output for a Padé model of order 1,2, and 3 respectively. Figures 4, 5, and 6 are zoomed views of figures 1,2, and 3, demonstrating lower amplitude but higher frequency oscillations (chattering) as the order of the Padé model increase from 1 to 3.

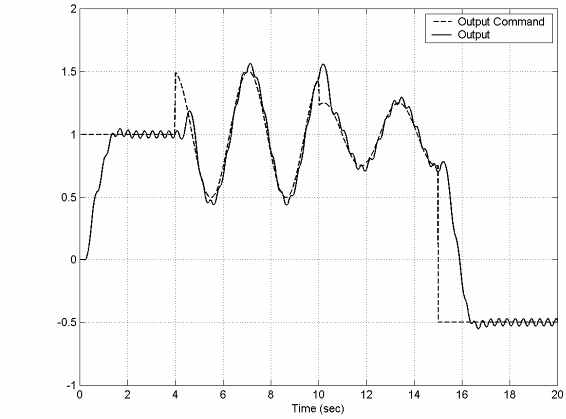

Figure 1: Plant with 1st Order Padé Model/Controller

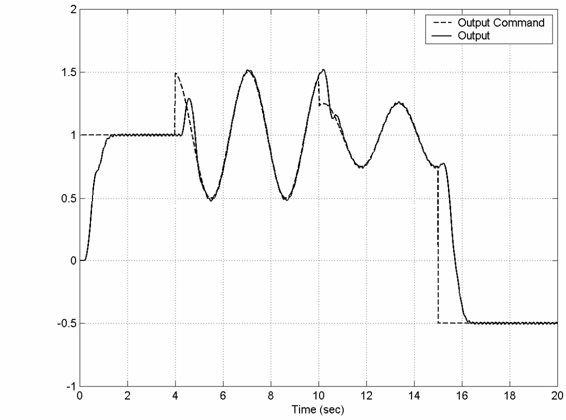

Figure 2: Plant with 2nd Order Padé Model/Controller

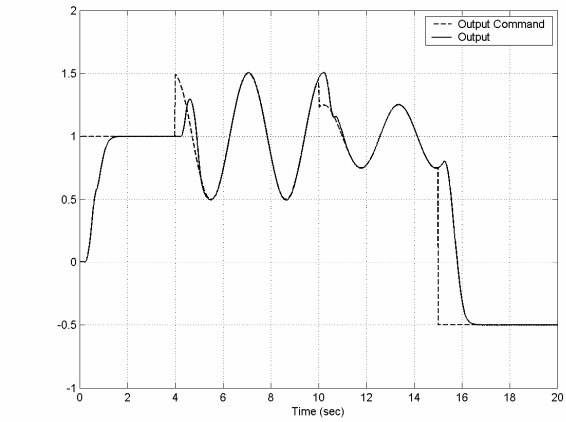

Figure 3: Plant with 3rd Order Padé Model/Controller

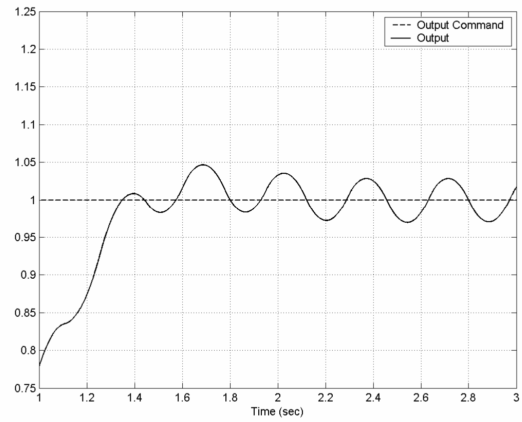

Figure 4: Detail View of 1st Order Padé Example

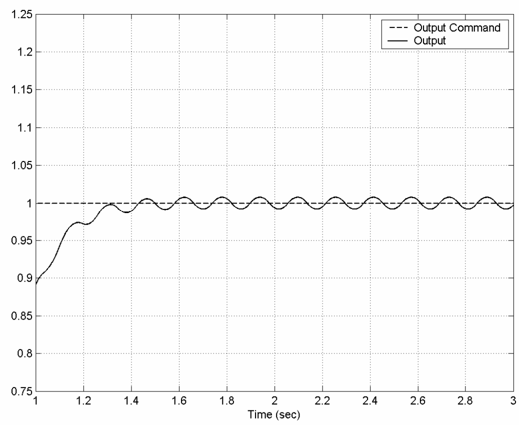

Figure 5: Detail View of 2nd Order Padé Example

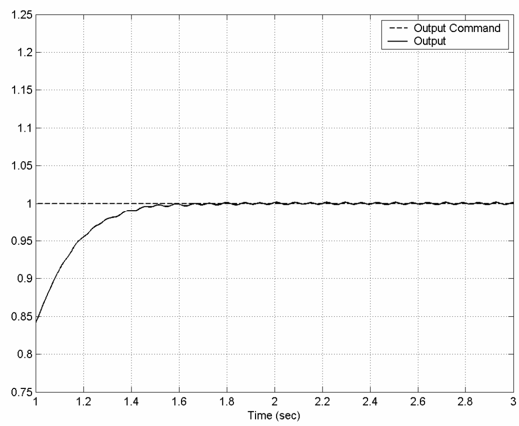

Figure 6: Detail View of 3rd Order Padé Example

V. Conclusions

Output tracking in causal nonlinear systems with an output delay via sliding mode control is considered. The higher order Padé approximation for the time delay function is used. Bounded state tracking profiles are generated by equations of the stable system center, which performs a stable dynamic inverse of a dynamically extended model of the plant incorporating stable exogenous models for plant inputs and a nonminimum-phase model for the delay. It is shown that the higher order Padé approximation yield the better tracking accuracy of an arbitrary output reference profile given by a linear exogenous system with a known characteristic equation. In particular, the higher frequency of control switching and lower amplitude of oscillations in the output tracking error are achieved for the higher order Padé approximations.

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