



UNIVERSITY OF LEEDS

This is a repository copy of *Topology optimization of 3D compliant actuators by a sequential element rejection and admission method*.

White Rose Research Online URL for this paper:  
<http://eprints.whiterose.ac.uk/106532/>

Version: Accepted Version

---

**Proceedings Paper:**

Ansola, R, Vegueria, E, Alonso, C et al. (1 more author) (2016) Topology optimization of 3D compliant actuators by a sequential element rejection and admission method. In: Hristoforou, D and Vlachos, DS, (eds.) IOP Conference Series: Materials Science and Engineering. 5th International Conference on Materials and Applications for Sensors and Transducers (IC-MAST2015), 27-30 Sep 2015, Mykonos, Greece. IOP Publishing: Conference Series .

<https://doi.org/10.1088/1757-899X/108/1/012035>

---

Published under licence in 5th International Conference on Materials and Applications for Sensors and Transducers (ICMAST2015), IOP Publishing Ltd. CC-BY Content from this work may be used under the terms of the Creative Commons Attribution 3.0 licence. Any further distribution of this work must maintain attribution to the author(s) and the title of the work, journal citation and DOI.

**Reuse**

Items deposited in White Rose Research Online are protected by copyright, with all rights reserved unless indicated otherwise. They may be downloaded and/or printed for private study, or other acts as permitted by national copyright laws. The publisher or other rights holders may allow further reproduction and re-use of the full text version. This is indicated by the licence information on the White Rose Research Online record for the item.

**Takedown**

If you consider content in White Rose Research Online to be in breach of UK law, please notify us by emailing [eprints@whiterose.ac.uk](mailto:eprints@whiterose.ac.uk) including the URL of the record and the reason for the withdrawal request.



[eprints@whiterose.ac.uk](mailto:eprints@whiterose.ac.uk)  
<https://eprints.whiterose.ac.uk/>

## Topology optimization of 3D compliant actuators by a sequential element rejection and admission method

This content has been downloaded from IOPscience. Please scroll down to see the full text.

View [the table of contents for this issue](#), or go to the [journal homepage](#) for more

### Download details:

IP Address: 129.11.241.38

This content was downloaded on 24/10/2016 at 16:57

Please note that [terms and conditions apply](#).

You may also be interested in:

### [Fast electrochemical actuator](#)

I V Uvarov, A V Postnikov and V B Svetovoy

### [Influence of throttling of pneumatic actuators at the positioning accuracy](#)

M P Hetmanczyk and P Michalski

### [The design and analysis of a MEMS electrothermal actuator](#)

Wang Suocheng, Hao Yongping and Liu Shuangjie

### [Electromagnetic flat sheet forming by spiral type actuator coil](#)

S. Akbar, M.A. Aleem, M. N. Sarwar et al.

### [Burnishing tool actuators and their influence on the burnishing force components](#)

Yu Chervach, A Kim and D Dorzhiev

### [Wind turbine rotor simulation using the actuator disk and actuator line methods](#)

M Tzimas and J Prospathopoulos

### [Design of FTC structures with PI virtual actuators](#)

Anna Filasová, Dušan Krokavec and Vladimír Serbák

# Topology optimization of 3D compliant actuators by a sequential element rejection and admission method

R Ansola<sup>1,3</sup>, E Veguería<sup>1</sup>, C Alonso<sup>1</sup> and O M Querin<sup>2</sup>

<sup>1</sup> Department of Mechanical Engineering, University of the Basque Country, Alda. Urquijo s/n, 48013 Bilbao, Spain.

<sup>2</sup> School of Mechanical Engineering, University of Leeds, Leeds LS2 9JT, UK.

E-mail: ruben.ansola@ehu.es

**Abstract.** This work presents a sequential element rejection and admission (SERA) method for optimum topology design of three dimensional compliant actuators. The proposed procedure has been successfully applied to several topology optimization problems, but most investigations for compliant devices design have been focused on planar systems. This investigation aims to progress on this line, where a generalization of the method for three dimensional topology optimization is explored. The methodology described in this work is useful for the synthesis of high performance flexure based micro and nano manipulation applications demanding for both sensing and control of motion and force trajectories. In this case the goal of the topology optimization problem is to design an actuator that transfers work from the input point to the output port in a structurally efficient way. Here we will use the classical formulation where the displacement performed on a work piece modelled by a spring is maximized. The technique implemented works with two separate criteria for the rejection and admission of elements to efficiently achieve the optimum design and overcomes problems encountered by other evolutionary methods when dealing with compliant mechanisms design. The use of the algorithm is demonstrated through several numerical examples.

## 1. Introduction

Compliant actuators obtain their mobility from flexibility of their parts as opposed to classic rigid body mechanisms. Therefore they can be built using fewer parts, require fewer assembly processes and need no lubrication. An important application of compliant mechanisms lies in Micro Electro Mechanical Systems (MEMS) design, where due to the small size, hinges and bearings cannot be used. As a result these types of devices must be built and designed as compliant actuators etched out of a single piece of material. The design of compliant mechanisms was initially accomplished by trial and error. However, the idea of introducing more systematic design procedures captured the mind of researchers [1].

Two different design approaches were considered: lumped and distributed compliant mechanisms. In the first approach [2], rigid body mechanisms were converted into partially compliant mechanisms composed of small flexible pivots and rigid links. In the second approach, distributed compliant mechanisms were obtained with the use of topology optimization techniques, where optimum designs were automatically obtained for prescribed design domains, boundary conditions and functional

<sup>3</sup> To whom any correspondence should be addressed.



specifications. The main advantage of this approach was that there was no need to predetermine the number of links or the location of the flexural joints [3]. Since then, topology optimization has been successfully applied to optimize compliant sensors and actuators in many practical engineering designs with the use of finite element analysis, since this technique enables systematic design directly from the behavioural specifications. The structural topology design problem is formulated as a material distribution problem within a given design domain, where material should be placed and connected to some portions of the boundary with some number of holes inside to optimize an objective function.

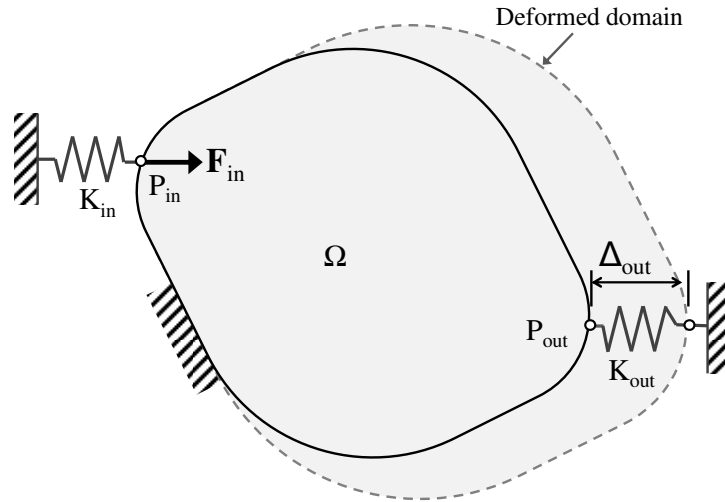
Although the design goals for structures and compliant actuators are different, the same topology optimization methods can be adapted to design both types of elements, introducing some modifications to specifically suit the functional requirement of compliant devices. We should bear in mind that adequate flexibility is deemed essential to afford the required displacement of the actuator at the point of interest. Additionally, a compliant mechanism also needs to be stiff enough to be able to sustain external loads. Therefore, an optimum balance between the two requirements of flexibility and stiffness is essential in the synthesis of compliant actuators.

The pioneering topology optimization method used to design compliant mechanisms was the homogenization method [4]. This technique considered a material model with infinite number of microscopic cells containing varying degrees of solid and void. The optimum topology was obtained by seeking the optimal porosity of the domain. Although this method pioneered the design of compliant mechanisms with topology optimization, the most widely used method for compliant mechanisms is the SIMP parameterization [5]. In this approach, material properties were considered constant within each element and densities were the design variables. The effective property of each element consisted of its density raised to a power and multiplied to the material properties of the solid material. This method was applied to a wide variety of compliant actuators design problems [6]. A number of heuristic or intuition based methods have been also applied to the design of compliant mechanisms (genetic Algorithms [7], Level Set Methods [8] and the Evolutionary Structural Optimization method [9]). More recently the bi-directional evolutionary structural optimization (BESO) method was developed for topological design of compliant mechanisms [10], where solid and void elements are all grouped in one single list and a penalty function is mandatory to reach convergence. The sequential element rejection and addition (SERA) method [11] that the authors propose in this work does not need any artificial material interpolation scheme, and a penalization scheme is not required to find a pure discrete solution.

The Sequential Element Rejection and Admission (SERA) method adds and removes material from the design domain using separate criteria for the update of solid and void elements. This strategy takes into account the current material status and controls how elements change their density, avoiding the algorithm to enter a loop where elements may change randomly from real to virtual and back to the previous material model in each iteration [12]. The optimization problem will be defined as the maximization of the Mutual Potential Energy [13], equivalent to the displacement at the output port. The ratio between input and output stiffness is controlled using a spring model [14], which model the input actuator behaviour and the stiffness of the work piece located at the output port. Examples of these type of actuation principles are electrothermal heating, piezoelectric actuation, shape memory alloys, etc. A well known filtering technique [15] is used to avoid the formation of checkerboard patterns, giving the method the necessarily mesh-independency. The validity of the proposed method for topology optimization of three dimensional actuators is demonstrated with the use of two benchmark problems, where motion is induced by an input force applied directly: an inverter device and a crunching actuator. Future work should consider the application of the SERA method for optimum topology design of more complex thermal and electrothermal devices.

## 2. Topology optimization formulation for three dimensional compliant mechanisms

In order to produce the specific displacement when the load in the input port is applied the compliant mechanism is required to be strong and flexible. Figure 1 shows such a compliant mechanism, occupying a three dimensional domain  $\Omega$ . It is subjected to a force  $F_{in}$  at the input port  $P_{in}$  and is supposed to produce an output displacement  $\Delta_{out}$  at the output port  $P_{out}$ .



**Figure 1.** Problem definition

Obtaining the optimum design that converts the input work into an output displacement in a predefined direction is the aim of the topology optimization procedure implemented here for 3D compliant actuators design. The mathematical formulation of the required output displacement is expressed as the Mutual Potential Energy (MPE). Then the problem can be written as the maximization of the Mutual Potential Energy (MPE) subjected to a target volume fraction  $V^*$ .

$$\max \text{MPE} \quad (1)$$

$$\text{s.t. } \sum_{e=1}^N \rho_e \leq V^*, \quad \rho_e = \{\rho_{\min}, 1\}, \quad e=1, \dots, N \quad (2)$$

where  $\rho_e$  is the density of the  $e^{\text{th}}$  finite element,  $N$  is the number of finite elements,  $V_e$  is the volume of the  $e^{\text{th}}$  element,  $V_{\text{Tot}}$  is the total volume for the domain and  $\rho_{\min}$  is the minimum density considered, a typical value of which is  $10^{-4}$ .

The Mutual Potential Energy in equation (3) was defined as the deformation at a prescribed output port in a specified direction. To obtain the MPE, two load cases are calculated:

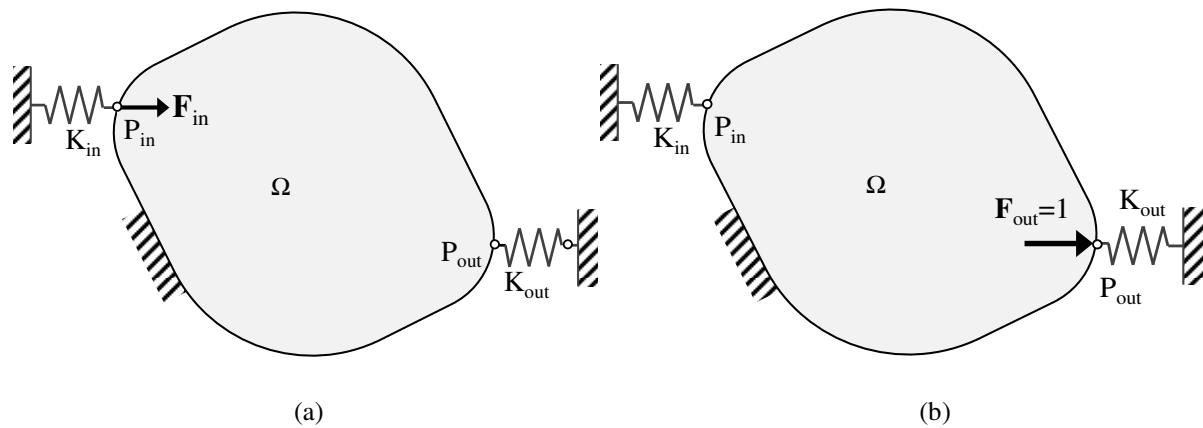
- 1) The Input Force Case, where the input force  $F_{in}$  is applied to the input port  $P_{in}$ , named with the subscript 1 in equations (3) and (4) and figure 2a;
- 2) The Unit Force Case, where a unit force is applied at the output port  $P_{out}$  in the direction of the desired displacement, named with the subscript 2 in equations (3) and (5) and Figure 2b.

$$\text{MPE} = \mathbf{U}_2^T \cdot \mathbf{K} \cdot \mathbf{U}_1 \quad (3)$$

$$\mathbf{K} \cdot \mathbf{U}_1 = \mathbf{F}_1 \quad (4)$$

$$\mathbf{K} \cdot \mathbf{U}_2 = \mathbf{F}_2 \quad (5)$$

where  $\mathbf{K}$  is the global stiffness matrix of the structure;  $\mathbf{F}_1$  is the nodal force vector which contains the input force  $F_{in}$ ;  $\mathbf{F}_2$  is the nodal force vector which contains the unit output force  $F_{out}$ ; and  $\mathbf{U}_1$  and  $\mathbf{U}_2$  are the displacement fields due to each load case.



**Figure 2.** Representation of two load cases

In order to control the displacement amplification it is possible to specify different values of the output spring,  $K_{out}$ , that simulates the resistance in the output displacement. The spring model of figure 1 is used to define the stiffness ratio between the input and output ports. The artificial input spring  $K_{in}$  together with a spring force  $F_{in}$  simulates the input work of the actuator.

### 3. Sensitivity number computation

A sensitivity analysis is carried out as part of the optimization process to provide information on how sensitive the objective function is to small changes in the design variables. This sensitivity number in each element determines which elements are removed or added so that the objective function is maximized.

$$\alpha_e = -U_{1e}^T \cdot \Delta K_e \cdot U_{2e} \quad (6)$$

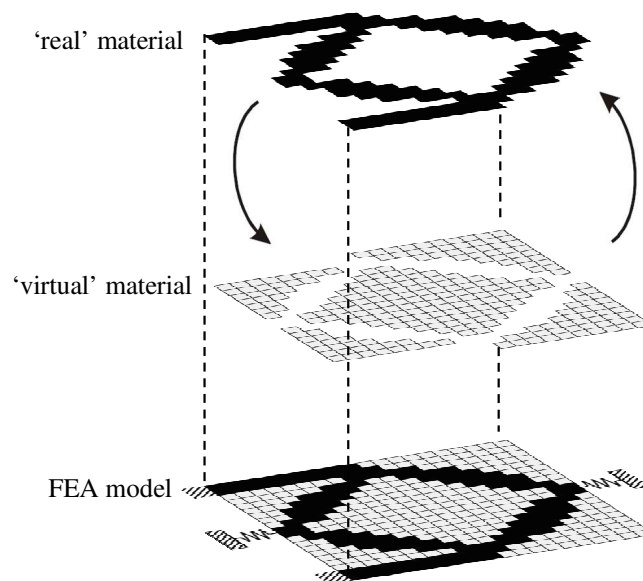
where  $U_{1e}$  is the displacement vector of element  $e$  due to the applied load  $F_1$ ;  $U_{2e}$  is the displacement vector of element  $e$  due to the output load vector  $F_2$ ; and  $\Delta K_e$  is the variation of the elemental stiffness matrix.

### 4. Sequential Element Rejection and Admission method for three dimensional compliant mechanisms

The method considers two separate material models, ‘Real’ and ‘Virtual’ material and two separate criteria of rejection and admission of elements allow material to be introduced and removed from the design domain by changing its status from ‘virtual’ to ‘real’ and vice versa (figure 3). The ‘real’ material present at the end of the optimization will be the optimized topology. The twelve steps that drive the SERA method for compliant mechanisms are given below:

- Define the design problem. The maximum design domain must be defined and meshed with finite elements. All boundary constraints, loads and the target volume fraction  $V^*$  must also be specified.
- Assign ‘real’ and ‘virtual’ material properties to the initial design domain.
- Calculate the variation of the volume fraction in the  $i^{th}$  iteration which consists of the volume fraction to be added  $\Delta V_{Add}(i)$  and removed  $\Delta V_{Remove}(i)$ .
- Carry out a Finite Element Analysis of the two load cases to produce the displacement vectors  $U_1$  and  $U_2$ . The elemental and global stiffness matrixes,  $K_e$  and  $K$ , are also calculated as part of the FEA.
- Calculate the elemental sensitivity numbers.

- Apply a mesh independent filtering to the sensitivity numbers.
- Separate the sensitivity numbers into ‘real’ and ‘virtual’ materials,  $\alpha_R$  and  $\alpha_V$ .
- Define the threshold values for ‘real’ and ‘virtual’ material,  $\alpha_R^{th}$  and  $\alpha_V^{th}$ .
- Remove and add elements.
- Calculate the volume of the ‘real’ material in the domain.
- Calculate the convergence criterion  $\varepsilon_i$ .
- Repeat steps until the target volume is reached and the optimization converges. The final topology is represented by the ‘real’ material in the design domain.



**Figure 3.** The SERA material models

#### 4.1. Definition of the starting design domain

The SERA method can start from a full design domain (all elements are ‘real’ material), from a void design domain (all elements are ‘virtual’ material), and also with any amount of material present in the domain. In this last case, a combination of ‘real’ and ‘virtual’ material is the starting point. For any of these cases, the material present in the domain is assigned the ‘real’ material properties and material not present in the domain is assigned the ‘virtual’ material properties. The method converges toward the optimum topology regardless of the initial design domain. However, the initial design domain determines the number of iterations needed to achieve that optimum. The examples shown at the end of the paper were all solved starting from a full design domain. Nevertheless, expressions for both cases are included in the following subsections for the sake of completeness.

#### 4.2. Calculation of volumes to add $\Delta V_{Add}(i)$ and remove $\Delta V_{Remove}(i)$

In the first stage of the process different amounts of material are added and removed in each iteration until the target volume fraction  $V^*$  is reached. In the second stage, once the target volume fraction is reached, material re-distribution takes place by both adding and removing the same amount of material until the problem converges.

4.2.1. First stage: real material volume fraction and target volume fraction are different. Two starting domain cases exist:

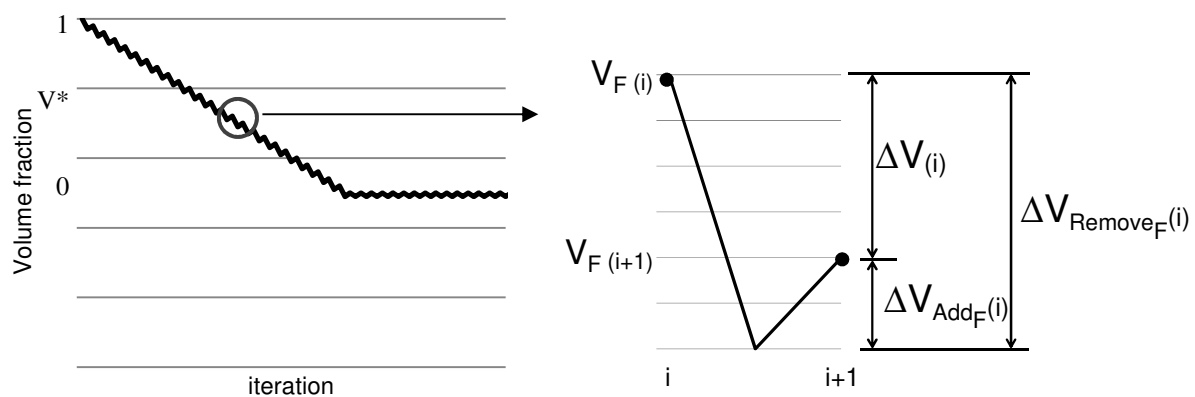
a) When the design starts with a volume fraction higher than the target fraction or a full domain  $V_F(0)$  is the volume in the design domain at the beginning of the process. The volumes in the following iterations will be calculated using the equation (7). In each iteration the variation of the volume will be calculated with equation (8) and then the amount of material to add and remove will be calculated in equations (9) and (10), where: PR is the Progression Rate, with typical values ranging between 0.01 and 0.05; SR is the Smoothing Ratio, with typical values in the range between 1.2 and 1.4.

$$V_F(i) = \max(V_F(i-1) \cdot (1-PR), V^*) \quad (7)$$

$$V(i) = |V_F(i) - V_F(i-1)| \quad (8)$$

$$\Delta V_{Add_F}(i) = \Delta V(i) \cdot (SR-1) \quad (9)$$

$$\Delta V_{Remove_F}(i) = \Delta V(i) \cdot SR \quad (10)$$



**Figure 4.** Removal and addition of material from a full domain

b) When the design starts with a volume fraction lower than the target fraction or a void domain  $V_V(0)$  is the volume in the design domain at the beginning of the process. The volumes in the following iterations will be calculated using the expression (11). In each iteration the variation of the volume will be calculated with equation (12) and then the amount of material to add and remove will be calculated using equations (13) and (14).

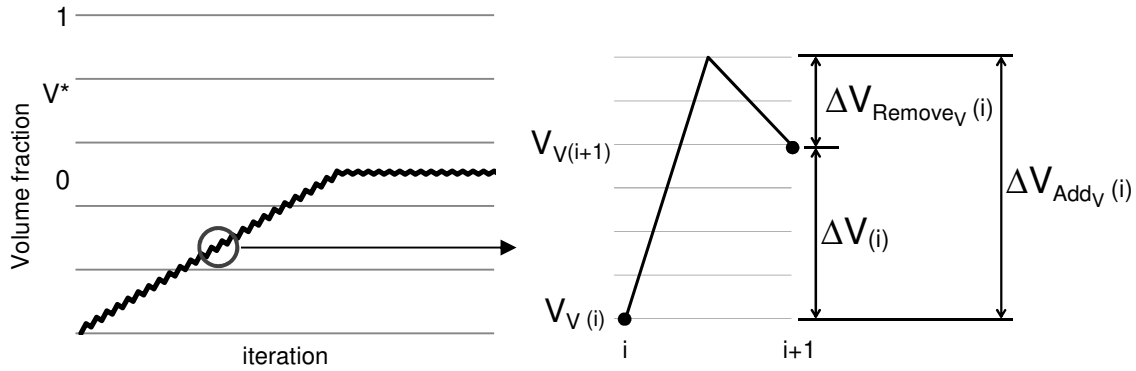
$$V_V(i) = \min(V_V(i-1) + PR, V^*) \quad (11)$$

$$V(i) = |V_V(i) - V_V(i-1)| \quad (12)$$

$$\Delta V_{Add_V}(i) = \Delta V(i) \cdot SR \quad (13)$$

$$\Delta V_{Remove_V}(i) = \Delta V(i) \cdot (SR-1) \quad (14)$$





**Figure 5.** Removal and addition of material from a void domain

A graphical representation of the removal and addition of elements in each iteration for case (a) is given in figure 5 and for case (b) in figure 6. In both cases, each iteration consists of two sub-steps which add and remove material from the design domain. The difference depends on the amount of material to be added or removed so that the volume fraction in that iteration decreases for case (a) or increases for case (b).

*4.2.2. Second stage: real material volume fraction and target volume fraction are similar.* The process of material re-distribution takes place: this consists of both adding and removing the same amount of material from the design domain:

$$\Delta V_{Add_{F \text{ or } V}}(i) = \Delta V_{Remove_{F \text{ or } V}}(i) = \beta \cdot V^* \quad (15)$$

where  $\beta$  is the material re-distribution fraction, with typical values ranging between 0.001 and 0.005.

#### 4.3. Removal and addition of elements

The sensitivity number for the  $e$ th finite element  $\alpha_e$  (6) is a function of the variation between two iterations in the stiffness matrix of that element  $\Delta \mathbf{K}_e$  (16).

$$\Delta \mathbf{K}_e = \mathbf{K}_e(i) - \mathbf{K}_e(i-1) \quad (16)$$

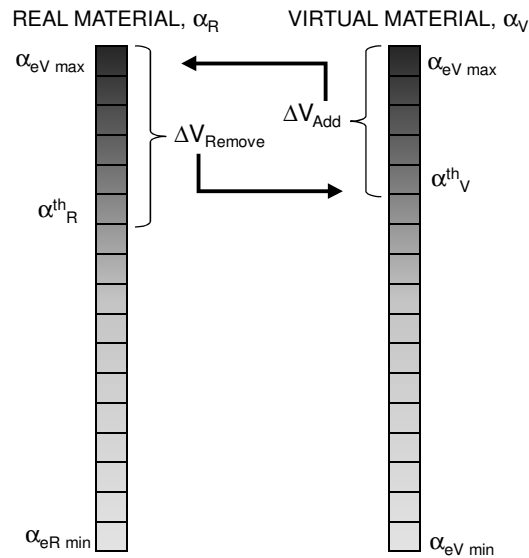
where  $\mathbf{K}_e(i)$  is the stiffness matrix in the  $i$ th iteration for the  $e$ <sup>th</sup> finite element; and  $\mathbf{K}_e(i-1)$  is the stiffness matrix in the  $(i-1)$ <sup>th</sup> iteration for the same finite element. If an element is added,  $\mathbf{K}_e(i) = \mathbf{K}_e$  and  $\mathbf{K}_e(i-1) \approx 0$ , so the variation of the elemental stiffness matrix is  $\Delta \mathbf{K}_e = \mathbf{K}_e$ . But if an element is removed,  $\mathbf{K}_e(i) \approx 0$  and  $\mathbf{K}_e(i-1) = \mathbf{K}_e$ , and  $\Delta \mathbf{K}_e = -\mathbf{K}_e$ .

Then, the elemental sensitivity numbers for the ‘real’ and ‘virtual’ material are given by equations (17) and (18), respectively.

$$\alpha_{eR} = \mathbf{U}_{1e}^T \cdot \mathbf{K}_e \cdot \mathbf{U}_{2e} \quad (17)$$

$$\alpha_{eV} = -\mathbf{U}_{1e}^T \cdot \mathbf{K}_e \cdot \mathbf{U}_{2e} \quad (18)$$

As the objective is to maximize the MPE, the elements with the higher values of sensitivity number are the ones to be added and removed (figure 6). Before the addition or removal of elements according to this sensitivity values, these values will be smoothed with a mesh independency filter. The threshold values  $\alpha_{th}^R$  and  $\alpha_{th}^V$  are the sensitivity values that remove or add the amount of volume  $\Delta V_{Remove}(i)$  and  $\Delta V_{Add}(i)$  defined for each iteration are given in figure 6.



**Figure 6.** Lists of real and virtual material and the volumes to be removed and added

#### 4.4. Convergence criterion

The convergence criterion is defined as the change in the objective function in the last 10 iterations (19), which is considered an adequate number of iterations for the convergence study. It implies that the process will have a minimum of 10 iterations as the convergence criterion is not applied until the iteration number has reached this minimum number of iterations.

$$\varepsilon_i = \frac{|\sum_{i=9}^{i=5} MPE_i - \sum_{i=4}^i MPE_i|}{\sum_{i=4}^i MPE_i} \quad (19)$$

where  $\varepsilon_i$  is the convergence criterion, with typical values ranging between 0.001 and 0.01.

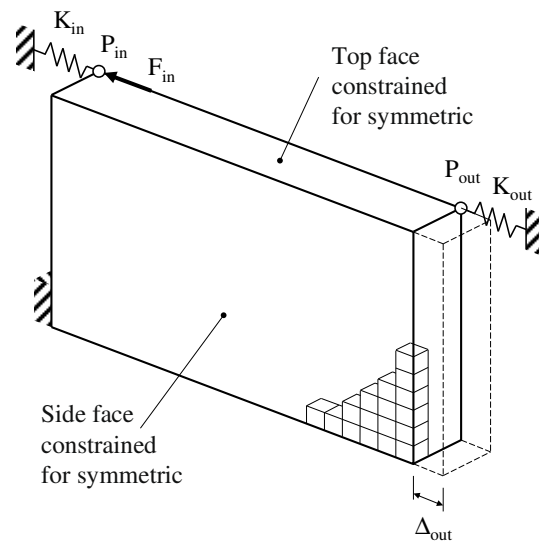
## 5. Numerical examples

Three dimensional inverter and crunching benchmark actuators are presented to demonstrate the efficiency of the proposed method. Many examples of compliant mechanisms, such as inverters and grippers, can be found in the literature. Here, some of them have been re-designed using the proposed method and illustrating the ability of the design method for 3D applications. The material properties used in all examples are the following: Young's modulus is taken to be  $E = 200$  GPa and the Poisson's ratio is  $\nu = 0.3$ . The density of the virtual material is taken to be  $\rho_{\min} = 10^{-4}$ , which is equivalent to 0.01 % of the stiffness of a real material. The algorithm has been applied for simple compliant actuators optimization under directly applied input loads. The topologies presented in this section were modelled using geometrical nonlinear elastic analysis, because otherwise the results may be useless in some cases as large displacement mechanisms, even if obtained results do not show any locking or related problems.

### 5.1. Inverter mechanism

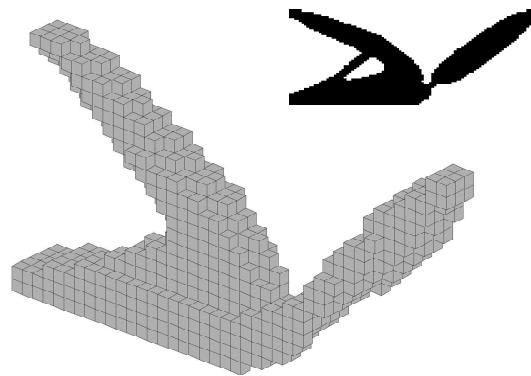
The design domain for the inverter mechanism is shown in figure 7. The aim of this topology optimization problem is to design an actuator that converts an input displacement on the left edge to a displacement in the opposite direction on the right edge. The domain consists of a parallelepiped of size  $40 \times 40 \times 5$  mm discretized using cubic eight node finite elements. The boundary is fixed at the upper and bottom left corners and a standard input force  $F_{\text{in}} = 1$  kN is applied. A stiffness ratio of  $K_{\text{out}}/K_{\text{in}} = 1$  is defined for the springs that model the input actuator and the output workpiece. The

target volume fraction  $V^*$  is 0.4 and the filter radius used to prevent chequerboard is  $r_{\min} = 1.5$  mm. Both the top face and the side displacement are imposed taking advantage of the symmetry; i.e., nodes can only move within the planes shown in the figure. Therefore the results shown in the following subsections correspond to the quarter part of the full design domain.



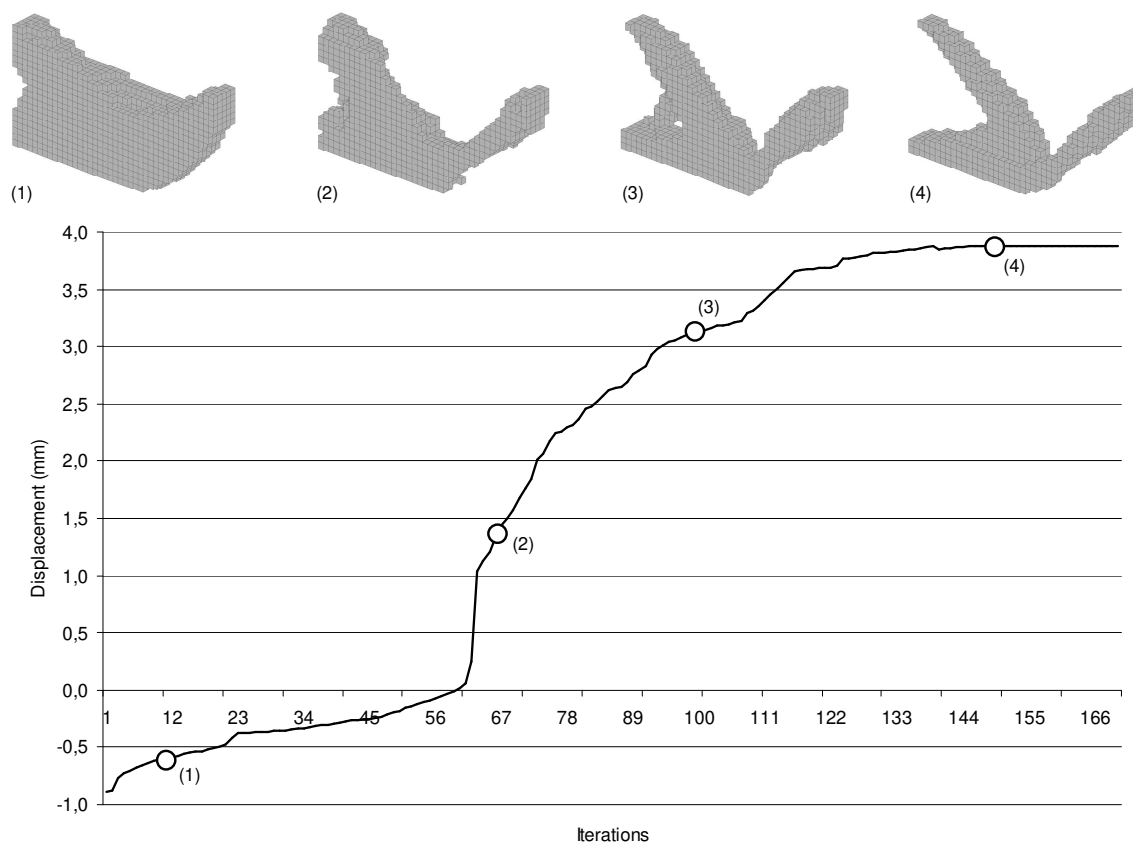
**Figure 7.** Design domain and boundary conditions for the inverter mechanism

The optimized topology was obtained after approximately 150 iterations. Figure 8 shows the optimum topology of the analyzed quarter part of the domain. It can be observed that the obtained material distribution and the connections of the supports with the input and output ports are basically similar to the classical planar inverter mechanism with slight differences. This example agrees well also with the solution offered by the SIMP parametrization strategy. One can notice that the resulting mechanism is not truly compliant because there is a moment free one-node connected hinge. This hinge should be avoided performing a post processing of the resulting topology, where it can be substituted with a long slender compliant hinge. An alternative solution is to applied diverse methods proposed in literature to alleviate the problem from the very beginning of the optimum design problem formulation.



**Figure 8.** Optimum topology for the inverter mechanism

In figure 9 the output displacement evolution is plotted as a function of the iteration number and snapshots of the actuator topologies at the different stages are shown. As it can be observed in the evolution history, the objective function does not have positive values until a well defined mechanism is constructed with material joining the input and output ports. This is because the objective function represents the displacement in the output port and therefore positive values can only be obtained when the material distribution promotes the displacement transmission through the hinges created during the optimization process. Initial displacement at the output port was 0.9603 mm inwards, while final displacement is 3.742 mm outwards, this time along the desired direction. Figure 9 describes the gradual formation of the optimum design and illustrates how the material rejection and addition takes place, using an image sequence captured during the process. The optimal topology is the same regardless of the path of the optimization, even if it can take different number of iterations until the topology efficiently grows towards the optimum, depending on the initial material distribution and the final volume fraction.



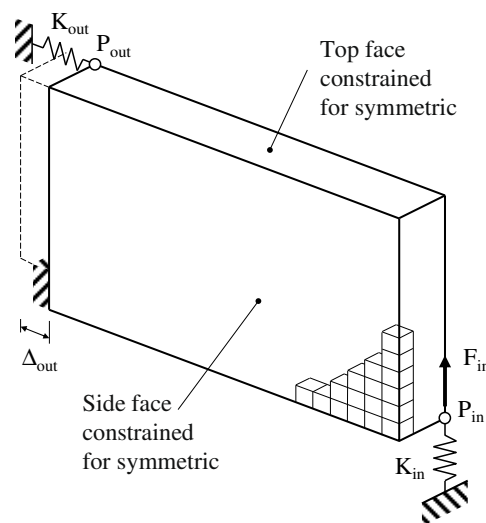
**Figure 9.** Evolution history for the inverter mechanism.

### 5.2. Crunching mechanism

The design domain for a crunching mechanism is shown in figure 10. Again it is a square of size  $40 \times 40$  mm with a thickness of 5 mm and subdivided using 1 mm cubic eight node finite elements. The mechanism is supported at the left side and is subject to a vertical input load  $F_{in} = 1$  kN at the upper and lower right corners, where due to the symmetry of the problem only a quarter of the full design domain is modelled. The objective is to maximize the horizontal displacement on the output port in the outward directions by distributing material in the design domain area. The allowable amount of material for this example is taken to be 40% of the full design domain. The spring based formulation

used in this work allows the displacement amplification to be controlled by specifying different values of the input and output springs as part of the definition of the problem. As in the previous example a ratio of  $K_{out}/K_{in} = 1$  has been adopted.

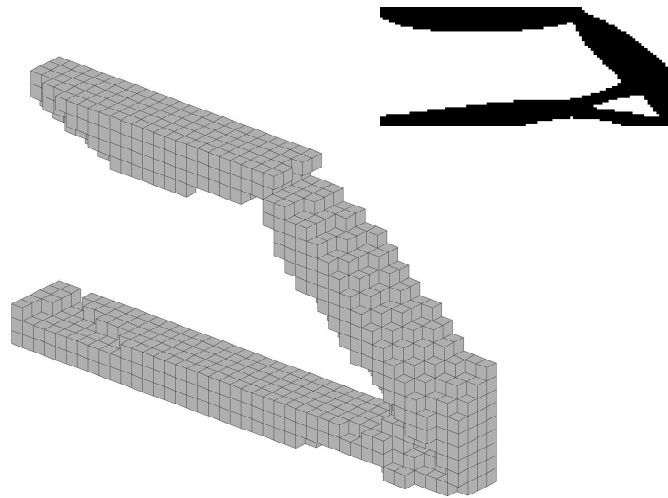
Figure 11 shows the obtained optimum topology for the crunching mechanism, which resembles the solutions obtained by other authors using different material models and optimization algorithms for two dimensional problems such as SIMP. The same figure includes the optimized topology for the planar actuator optimum design problem. The resulting three dimensional topology is comparable with the 2D solution, obtained with the SERA procedure described in this work. Although topologies are not exactly the same, the most important features of the design are present in both cases. It must be noted as well that the angle between the bars and the location of holes depends heavily on the smoothing of sensitivities and stiffness of the structure and springs, which reasonably are different in both cases.



**Figure 10.** Design domain and boundary conditions for the crunching mechanism

## 6. Conclusions

An extension of the sequential element rejection and admission (SERA) method for topology optimization of three dimensional compliant actuators has been presented in this work. This method overcomes the issues noticed evolutionary methods are used for the design of compliant mechanisms. The main difference of this method with respect to other bi-directional methods which add and remove elements from the design domain is that solid and void materials are treated separately so that the addition and removal of elements have separate criteria. The problem of designing compliant mechanisms is defined here as the maximization of the mutual potential energy and a spring model is used to control the input and output stiffness. This formulation meets the flexibility and stiffness requirements necessary to design compliant devices that satisfy the kinematic requirements and at the same time withstand the applied loads. The solved numerical examples agree well with solutions offered by the SIMP interpolation scheme and demonstrate that the SERA method is both an efficient method of designing 3D compliant actuators. Optimum topologies resemble the solutions obtained with the same method for two dimensional problems. It should be mentioned that both SIMP and SERA methods show the same issues concerning one-node connected hinges, so techniques to avoid them should be explored and implemented. The generalization of the method for more complex electrothermal actuators in three dimensions is left for future work.



**Figure 11.** Optimum topology for the crunching mechanism.

## 7. References

- [1] Howell L 2001 *Compliant mechanisms* (New York: Wiley)
- [2] Howell, L L and Midha A 1994 A Method for the Design of Compliant Mechanisms With Small-Length Flexural Pivots *ASME J. Mech. Des.* **116** 280–90
- [3] Bendsoe M P and Sigmund O 2003 *Topology optimization: theory, method and application* (Berlin: Springer)
- [4] Ananthasuresh G K, Kota S and Gianchandani Y 1993 Systematic synthesis of micro compliant mechanisms: preliminary results *Proc. 3rd national conf of appl mech and robot (Cincinnati)* vol 2 p 82
- [5] Bendsoe M P 1994 Optimal shape design as a material distribution problem *Struct. Optim.* **1** 193–202
- [6] Sigmund O 2001 Design of multiphysics actuators using topology optimization. Part I: One-material structures *Comput. Meth. Appl. Eng.* **190** 6577–604
- [7] Parsons R and Canfield S 2002 Developing genetic programming techniques for the design of compliant mechanisms *Struct. Multidisc. Optim.* **24** 78–86
- [8] Yulin M and Xiaoming W 2004 A level set method for structural topology optimization and its applications *Adv. Eng. Softw.* **35** 415–441
- [9] Ansola R, Vegueria E, Canales J and Tárrago J 2007 A simple evolutionary topology optimization procedure for compliant mechanism design *Finite Elem. Anal. Des.* **44** 53–62
- [10] Huang X, Li Y, Zhou S W, Xie Y M 2014 Topology optimization of compliant mechanisms with desired structural stiffness *Engineering Structures.* **79** 13–21
- [11] Rozvany G I N and Querin O M 2002 Theoretical foundations of Sequential Element Rejections and Admissions (SERA) methods and their computational implementations in topology optimisation *Proc. 9th AIAA/ISSMO symposium on multidisciplinary analysis and optimization (Georgia)* p 5521
- [12] Alonso C, Querin O M and Ansola R 2013 A Sequential Element Rejection and Admission (SERA) method for compliant mechanisms design *Struct. Multidisc. Optim.* **47** 795–807
- [13] Shield R and Prager W 1970 Optimal structural design for given deflection *J. Appl. Math. Phys.* **21** 513–23
- [14] Ananthasuresh G K, Kota S and Gianchandani Y 1994 A methodical approach to the design of compliant micromechanisms *Proc. Solid state sensor and actuator workshop* p 189-192

- [15] Sigmund O and Petersson J 1998 Numerical instabilities in topology optimization: a survey on procedures dealing with checkerboards, mesh-dependencies and local minima *Struct. Multidisc. Optim.* **16** 68–75.

### **Acknowledgments**

This work was financially supported by the Ministry of Education, Culture and Sport in Spain through the project DPI2012-36600 and the Department of Education, Universities and Research of the Basque Government through the project IT691-13. Their support is greatly appreciated.