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# Multi anticipative bidirectional macroscopic traffic model considering cooperative driving strategy

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# *Multi anticipative bidirectional macroscopic traffic model considering cooperative driving strategy*

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Recent development of information and communication technologies (ICT) has enabled vehicles to timely communicate with others through wireless technologies, which will form future (intelligent) traffic systems (ITS) consisting of so-called connected vehicles. Cooperative driving with the connected vehicles is regarded as a promising driving pattern to significantly improve transportation efficiency and traffic safety. Such cooperative driving has motivated many studies to model the dynamics of traffic flow under multi anticipative driving strategy (i.e. drivers react to many leading vehicles) or bidirectional strategy (i.e. drivers react to both (single) leading and following vehicles). In the vast literature of traffic flow theory, there are continuum models considering multiple forward anticipative strategy, where the driver reacts to many leaders. To the best of our knowledge, few study effort has been undertaken to include bidirectional driving strategy, where the driver reacts to both direct leader and direct follower, in the continuum traffic flow models. Moreover, the current bidirectional continuum traffic flow model still suffers some drawbacks: considers the behaviour of only a single leading vehicle in the forward looking strategy and neglects the impact of the forward space headway (e.g. the distance between two consecutive vehicles) sensitivity parameter on the (linear) stability of the model. This paper aims to derive a continuum traffic model considering both multiple forward and backward driving strategy. It is shown that the derived model is a generalized version of a current continuum model for ITS and can improve important properties of such bidirectional (continuum) model.

**Keywords:** macroscopic model, bidirectional driving, linear stability, multiple anticipations, anisotropy.

## 1. Introduction

Traffic flow modelling has been well developed since 1950s, when Lighthill and Whitham (1955) and Richards (1956) independently proposed a very simple model to describe the dynamics of traffic flow along the road, and it now still attracts a lot of interests from researchers. In principle, traffic flow modelling can be categorized into three types: microscopic models, mesoscopic models and macroscopic models. The microscopic approach describes traffic flow at a high level of detail such as the movement of individual vehicles (Chadler, Herman, and Montroll 1958; Helly 1959; Bando et al. 1995, 1998; Treiber, Hennecke, and Helbing 2000; Jiang, Wu, and Zhu 2001; Kesting, Treiber, and Helbing 2007, 2010; Laval, Toth, and Zhou 2014), whereas the macroscopic approach represents traffic flow at a low level of detail via aggregate traffic variables such as flow, mean speed and density (Treiber, Hennecke, and Helbing 1999; Gupta and Katiyar 2005, 2006a; Zhang and Wong 2006; Boel and Mihaylova 2006; Laval and Leclercq 2010; Zhang, Wong, and Dai 2011; Helbing et al. 2001; Ngoduy 2012a; Bogdanova et al. 2015). The mesoscopic

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approach, on the other hand, describes traffic flow at a level of detail between microscopic and macroscopic approach through probabilistic terms. An example of mesoscopic models is the gas-kinetic model which is used to derive the macroscopic models based on the method of moments (Helbing 1997; Hoogendoorn 1999; Ngoduy, Hoogendoorn, and van Zuylen 2006; Ngoduy 2008).

In recent years, the development of information and communication technologies (ICT) has enabled vehicles to timely communicate with each other and exchange important information such as the current acceleration or the current speed. These connected vehicles with some common interests can cooperatively drive on road, which may significantly improve the traffic safety and efficiency (van Arem, Van Driel, and Visser 2006; Kesting, Treiber, and Helbing 2010; Arnaout and Bowling 2011; Ngoduy 2012b, 2013a). To account for such changes in the driving strategy, a lot of research has been undertaken to understand how including the information of neighbouring vehicles (both followers and leaders) affects the dynamics of traffic flow. For example, many car-following models have been developed to account for the multiple anticipative driving strategy where the considered vehicle can react to the behaviour of many leading vehicles (Lenz, Wagner, and Sollacher 1999; Ge, Dai, and Dong 2006a; Treiber, Kesting, and Helbing 2006; Hoogendoorn, Ossen, and Schreuder 2006, 2007; Kesting and Treiber 2008; Kesting, Treiber, and Helbing 2010). Such multi anticipative driving strategy generally leads to a better traffic operation such as enhanced capacity and more stable traffic flow with respect to perturbations caused by, for example, sudden deceleration or lane-changes of vehicles (Treiber, Kesting, and Helbing 2006; Kesting, Treiber, and Helbing 2010; Sau et al. 2014; Monteil et al. 2014; Ngoduy 2013b; Ngoduy and Wilson 2014; Ngoduy 2015). A few other research has been conducted to investigate how the information from the following vehicles affects the dynamics of traffic flow (Nakayama, Sugiyama, and Hasebe 2001; Hasebe, Nakayama, and Sugiyama 2003; Ge, Zhu, and Dai 2006b; Sun, Liao, and Peng 2011; Yang et al. 2013; Zheng, Zhong, and Ma 2013; Jin et al. 2014). The effect of the information from the following vehicles on traffic flow dynamics has also been introduced via the honk effect (Zheng, Ma, and Zhong 2011). It has been concluded that a better driving strategy is a balance of indicators describing what is happening behind as well as what you can expect in front (stimulus).

In general, there has been impressive advances in modelling the effect of ICT on traffic dynamics using the microscopic approach (i.e. via car-following models), but very limited has been done to develop the macroscopic (or continuum) model for such intelligent traffic systems. To the best of our knowledge, only a few continuum (macroscopic) models have been developed to consider either multiple forward looking strategy (Wilson et al. 2004; Ngoduy and Wilson 2014) or both single forward looking and backward looking driving strategy (Zheng, Jin, and Huang 2015). It is worth noticing that while the multi anticipative model of Ngoduy and Wilson (2014) is nonlocal (i.e. the vehicles interact with the leader(s) at an advanced distance), the bidirectional model of Zheng, Jin, and Huang (2015) is local but may still reserve the anisotropic property (i.e. the characteristic speeds are always smaller than the average speed of the traffic flow) subject to some conditions, which will be elaborated in the ensuing paper. Note that the anisotropy issue in traffic flow has been well reported in literature (Gupta and Katiyar 2006b; Helbing and Johansson 2009). As shown in our paper, the model of Zheng, Jin, and Huang (2015) only consider one leader and one follower in the driving strategy and its derivation is not exact because of the neglected gradient terms in the expansion, therefore, this model does not include the (forward) headway sensitivity parameter in the linear stability condition if some practical conditions hold. To contribute to the state-of-the-art, this paper puts forward a new continuum model which includes both backward looking and multiple forward looking in the driving strategy. We will show that the proposed model, on the one hand, will improve some important properties of the model of Zheng, Jin, and Huang (2015), and on the other hand, will include the (forward) headway sensitivity parameter in the linear stability condition. The proposed model is more generic than the model of Zheng, Jin, and Huang (2015) to study the effect of ICT on traffic flow dynamics at the macroscopic level (where the information of many neighbouring vehicles contributes to the driving strategy of the driver). In a similar line of Zheng, Jin, and Huang (2015), we first modify the microscopic model of Helly (1959) to include the information of a single follower and multiple leaders in the driving strategy. Then we apply the gradient expansion method to derive a new multi anticipative bidirectional continuum model in which the corrected expansion terms of the space headway are used.

The paper is organized as follows. Section 2 presents a car-following model extended to capture multiple forward looking and (single) backward looking strategy. Section 3 derives a multi anticipative bidirectional macroscopic model from the car-following presented in Section 2. In Section 4, we will present some important properties of the new model and compare them against the current bidirectional macroscopic model of Zheng, Jin, and Huang (2015). Section 5 illustrates our model performance via numerical studies. Finally, we conclude the paper in Section 6 .

## Notation

For convenience, the notation in Table below will be used throughout this paper.

| | |
|---|---|
| **Index** | |
| $x_n$ | Location of vehicle $n$ $(m)$ |
| $t$ | Time instant $(s)$ |
| **Microscopic model variables** | |
| $v_n$ | Speed of vehicle $n$ $(m/s)$ |
| $\Delta v_n$ | Relative speed of vehicle $n$ and its leader $n-1$ $(m/s)$ $\Delta v_n \doteq v_{n-1} - v_n$ |
| $s_n$ | Space headway between vehicle $n$ and its leader $n-1$ (m) $s_n \doteq x_{n-1} - x_n$ |
| **Macroscopic model variables** | |
| $V(x,t)$ | Mean speed at location $x$ and time $t$ $(m/s)$ |
| $r(x,t)$ | Density at location $x$ and time $t$ $(vehicle/m)$ |
| **Model parameters** | |
| $i$ | the driving strategy index $(i = 1, 2)$ |
| $m$ | the $m^{th}$ leading vehicle of vehicle $n$ $(m = 1, 2, ..., M)$ |
| $\alpha_i$ | Headway sensitivity parameter for driving strategy $i$ $(1/s^2)$ |
| $\beta_i$ | Relative speed sensitivity parameter for driving strategy $i$ $(1/s)$ |
| $\gamma_i$ | Weight coefficient for driving strategy $i$, $\sum_i \gamma_i = 1$ |
| $a_m$,$b_m$ | Weight factors with for the gaps and relative speeds of vehicle $n$ w.r.t. its $m^{th}$ leader |
| $V_0$ | Free-flow speed $(m/s)$ |
| $V^e(r)$ | Density dependent equilibrium speed $(m/s)$ |

## 2. Multi-anticipative bidirectional microscopic models

Microscopic traffic flow models describe the motion of individual vehicle $n$ in relation to its leading vehicle $n-1$. A generalized microscopic model describing the acceleration of vehicle $n$ considering both leading and following vehicles reads:

$$\frac{dv_n(t)}{dt} = f\left(v_n, s_n, \Delta v_n\right), \tag{1}$$

where $\Delta v_n = \frac{d(x_{n-1}-x_n)}{dt} = \frac{ds_n}{dt} \doteq \dot{s}_n$. In this model, the nonlinear function $f$ can be specifically defined for different models in literature, such as the Intelligent Driver Model (IDM) of Treiber, Kesting, and Helbing (2006), Optimal Velocity Model (OVM) of Bando et al. (1995) or Full Velocity Difference Model (FVDM) of Jiang, Wu, and Zhu (2001), the model of Helly (1959).

In literature, the backward looking effect has been considered using the OVM or FVDM (Nakayama, Sugiyama, and Hasebe 2001; Ge, Zhu, and Dai 2006b; Sun, Liao, and Peng 2011; Yang et al. 2013). Based on the bidirectional looking framework of Jin et al. (2014) which considered the acceleration to consist of two tasks: backward looking and forward looking strategy, Zheng, Jin, and Huang (2015) proposed a bidirectional microscopic model using the Helly car-following model (Helly 1959):

$$\frac{dv_n(t)}{dt} = \underbrace{\left[\gamma_1\alpha_1\left(s_n - S_1(v_n)\right) + \beta_1\Delta v_n\right]}_{\text{forward looking}} - \underbrace{\left[\gamma_2\alpha_2\left(s_n - S_2(v_{n+1})\right) + \beta_2\Delta v_{n+1}\right]}_{\text{backward looking}} \tag{2}$$

where $S_i(.)$ denotes the optimal speed dependent function for forward looking ($i = 1$) and backward looking ($i = 2$), $\gamma_i$ is weight factor for forward looking ($i = 1$) and backward looking ($i = 2$), $\sum_{i=1}^{2}\gamma_i = 1$, $0 \leq \gamma_i \leq 1$, and $\gamma_2 < \gamma_1$ so that the forward-looking driving is more attended.

Nevertheless, the bidirectional model presented in equation (2) does not consider how drivers react to many vehicles ahead (e.g. multiple anticipative driving strategy). Such multiple (forward) anticipations are important in both human driven cars (Lenz, Wagner, and Sollacher 1999; Treiber, Kesting, and Helbing 2006; Hoogendoorn, Ossen, and Schreuder 2006) or cooperative cruise control (van Arem, Van Driel, and Visser 2006; Arnaout and Bowling 2011; Ngoduy 2013b). This paper concerns the extension of the model of Zheng, Jin, and Huang (2015) to capture such multiple (forward) anticipations.

To follow the model in Ngoduy (2015), equation (2) is extended to capture the bidirectional driving strategy considering multiple (forward) anticipations as below:

- Forward looking:

$$f_1 = \alpha_1\left(s_n - \sum_{m=1}^{M} a_m S_1(v_{n-m+1})\right) + \beta_1\sum_{m=1}^{M} b_m\Delta v_{n-m+1} \tag{3}$$

  where $M$ denotes the number of leaders which can affect the behaviour of the considered vehicle $n$. So basically, in the forward looking strategy, the driver of vehicle $n$ will consider the weighted average behaviour of his $M$ leading vehicles. $a_m$ and $b_m$ ($m \in M$) represent, respectively, the weight factors for the gaps and relative speeds which generally satisfy $a_1 > a_2 > ... > a_M$, $b_1 > b_2 > ... > b_M$ and $\sum_m a_m = 1, \sum_m b_m = 1$. Basically, $M$ is related to the traffic conditions as well as the layout of the road. Recent empirical study by Hoogendoorn, Ossen, and Schreuder (2006) based on data collected at a small freeway section in the Netherlands has indicated that $M = 3$ is a reasonable value. However, this is just an indication for a certain freeway section during a certain period of congested traffic. We think there should be more extensive data to be studied in order to get a good conclusion.

- Backward looking:

$$f_2 = -\alpha_2 \left(s_n - S_2(v_{n+1})\right) - \beta_2 \Delta v_{n+1} \tag{4}$$

To combine the multi anticipative forward looking and backward looking strategy, the Helly-type multi anticipative bidirectional model reads:

$$\begin{aligned}\frac{dv_n(t)}{dt} &= \gamma_1 \left[\alpha_1 \sum_{m=1}^{M} a_m \left(s_n - S_1(v_{n-m+1})\right) + \beta_1 \sum_{m=1}^{M} b_m \Delta v_{n-m+1}\right] \\ &- \gamma_2 \left[\alpha_2 \left(s_n - S_2(v_{n+1})\right) + \beta_2 \Delta v_{n+1}\right] \end{aligned} \tag{5}$$

Based on equation (5), we will derive a corresponding multi anticipative bidirectional macroscopic model, which is a generalized model of Zheng, Jin, and Huang (2015). We will show how the multi (forward) anticipations affect the properties of the new model such as the anisotropic and linear stability condition.

## 3. Multi anticipative bidirectional continuum model

We adopt a gradient expansion technique to derive our multi anticipative bidirectional macroscopic model. First, the macroscopic model variables are obtained from the microscopic models by the following linear transformation:

$$s_n(t) \to h(x,t) = \frac{1}{r(x,t)} - \frac{1}{2r^3}\frac{\partial r}{\partial x} - \frac{1}{6r^4}\frac{\partial^2 r}{\partial x^2} + ... \tag{6}$$

$$v_n(t) \to V(x,t), \quad \Delta v_{n-m+1}(t) \to mh\frac{\partial V}{\partial x}, \quad \Delta v_{n+1} = v_n(t) - v_{n+1}(t) \to h\frac{\partial V}{\partial x} \tag{7}$$

$$S_1(v_{n-m+1}(t)) \to \frac{1}{R_1^e\left[V(x+(m-1)h,t)\right]}, \quad S_2\left(v_{n+1}(t)\right) \to \frac{1}{R_2^e\left[V(x-h,t)\right]} \tag{8}$$

where $R_i^e(.)$ denotes the equilibrium speed dependent density with respect to the forward looking ($i=1$) or backward looking ($i=2$) strategy. Note that Zheng, Jin, and Huang (2015) have neglected the gradient terms in the expansion of the space headway in equation (6), which consequently led to complete different properties of the resulting macroscopic equations as shown in the ensuing paper.

Applying a first order Taylor expansion for the following quantities:

$$\begin{aligned} &h(x,t) \approx \frac{1}{r} - \frac{1}{2r^3}\frac{\partial r}{\partial x} \\ &\frac{1}{R_1^e\left[(V(x+(m-1)h,t)\right]} \approx \frac{1}{R_1^e(V)} - \frac{m-1}{r\left[R_1^e(V)\right]^2}\frac{dR_1^e(V)}{dV}\frac{\partial V}{\partial x}, \\ &\frac{1}{R_2^e\left[V(x-h,t)\right]} \approx \frac{1}{R_2^e(V)} + \frac{1}{r\left[R_2^e(V)\right]^2}\frac{dR_2^e(V)}{dV}\frac{\partial V}{\partial x} \end{aligned}$$

leads to the following multiple anticipative bidirectional model:

$$\frac{dV(x,t)}{dt} = \frac{\partial V}{\partial t} + V\frac{\partial V}{\partial x} = \gamma_1\alpha_1\left(\frac{1}{r} - \frac{1}{R_1^e(V)}\right) - \gamma_2\alpha_2\left(\frac{1}{r} - \frac{1}{R_2^e(V)}\right) - \frac{\gamma_1\alpha_1 - \gamma_2\alpha_2}{2r^3}\frac{\partial r}{\partial x}$$
$$+ \frac{1}{r}\left[\gamma_1\beta_1\sum_{m=1}^{M} b_m m - \gamma_2\beta_2 + \frac{\gamma_1\alpha_1 R_{1,V}^e}{[R_1^e(V)]^2}\sum_{m=1}^{M} a_m(m-1) + \frac{\gamma_2\alpha_2 R_{2,V}^e}{[R_2^e(V)]^2}\right]\frac{\partial V}{\partial x}$$

which can be rewritten as:

$$\frac{\partial V}{\partial t} + (V - c_0)\frac{\partial V}{\partial x} + c\frac{\partial r}{\partial x} = \gamma_1\alpha_1\left(\frac{1}{r} - \frac{1}{R_1^e(V)}\right) - \gamma_2\alpha_2\left(\frac{1}{r} - \frac{1}{R_2^e(V)}\right) \tag{9}$$

where $R_{i,V}^e = \frac{dR_i^e(V)}{dV}$ and

$$c = \frac{\gamma_1\alpha_1 - \gamma_2\alpha_2}{2r^3} \tag{10}$$

$$c_0 = \frac{1}{r}\left[\gamma_1\beta_1\sum_{m=1}^{M} b_m m - \gamma_2\beta_2 + \frac{\gamma_1\alpha_1 R_{1,V}^e}{[R_1^e(V)]^2}\sum_{m=1}^{M} a_m(m-1) + \frac{\gamma_2\alpha_2 R_{2,V}^e}{[R_2^e(V)]^2}\right] \tag{11}$$

It is obvious that if $M = 1$ and $c = 0$ we will obtain the model of Zheng, Jin, and Huang (2015). It is straightforward to test our model with different combinations of equilibrium speeds for forward (i.e. $R_1^e(V)$) and backward (i.e. $R_2^e(V)$) looking as in Zheng, Jin, and Huang (2015). However, for the sake of simplicity, this paper uses the same equilibrium relations for both forward and backward looking, for example:

$$\text{Density dependent speed}: V^e(r) = \frac{V_0}{2}\left[\tanh\left(\frac{r^{-1} - s_0}{l} - \theta\right) + \tanh\theta\right], \tag{12}$$

which can be rewritten as:

$$\text{Speed dependent density}: R^e(V) = \left[s_0\left(\frac{1}{2}log\left(\frac{1+W}{1-W}\right) + \theta\right) + l\right]^{-1} \tag{13}$$

where $W = \frac{2V}{V_0} - \tanh\theta$, the free speed $V_0 = 30$m/s, critical headway $s_0 = 40$m, average vehicle length $l = 4$m, constant parameter $\theta = 1.5$.

## 4. Model properties

This section is devoted to study the differences between our model and the model of Zheng, Jin, and Huang (2015), that is how the multiple forward looking affects some important model properties.

### 4.1. *Characteristic speeds*

Equation (9) is combined the conservation law equation:

$$\frac{\partial r}{\partial t} + \frac{\partial (rV)}{\partial x} = 0 \tag{14}$$

to form a system of strictly hyperbolic partial differential equations:

$$\frac{\partial U}{\partial t} + \mathcal{J}\left(\mathcal{U}\right)\frac{\partial U}{\partial x} = S \tag{15}$$

where

$$U = \begin{bmatrix} r \\ V \end{bmatrix}, \quad \mathcal{J}\left(\mathcal{U}\right) = \begin{bmatrix} V & r \\ c & V - c_0 \end{bmatrix}, \quad S = \begin{bmatrix} 0 \\ \gamma_1\alpha_1\left(\frac{1}{r} - \frac{1}{R_1^e(V)}\right) - \gamma_2\alpha_2\left(\frac{1}{r} - \frac{1}{R_2^e(V)}\right) \end{bmatrix}$$

It has been well known that the model equation (15) is strictly hyperbolic with two distinct eigenvalues corresponding to two separate characteristic speeds:$\lambda_1 = V + 0.5\left(\sqrt{c_0^2 + 4rc} - c_0\right)$ and $\lambda_2 = V - 0.5\left(\sqrt{c_0^2 + 4rc} + c_0\right)$. Therefore, the model is said to be strictly anisotropic if $\lambda_1 \leq V$ which leads to $c_0 \geq 0$ and $c \leq 0$. That is, in addition to the anisotropic condition in Zheng, Jin, and Huang (2015), i.e. $c_0 \geq 0$, our model needs an extra condition: $c \leq 0$, which can be written as: $\gamma_1\alpha_1 \leq \gamma_2\alpha_2$. As the driver is practically assumed to pay more attention to the leading vehicle(s) than the following one (i.e. $\gamma_1 \geq \gamma_2$), the condition $c \leq 0$ leads to:

$$c \leq 0 \leq\Leftrightarrow 1 \geq \frac{\gamma_2}{\gamma_1} \geq \frac{\alpha_1}{\alpha_2} \tag{16}$$

The condition $c_0 \geq 0$ can be written as below:

$$c_1 \geq 0 \tag{17}$$

and

$$\gamma_2 \leq \frac{1}{1 + \frac{c_2}{c_1}} \tag{18}$$

where

$$\begin{aligned} c_1 &= \beta_1 \sum_{m=1}^{M} b_m m + \frac{\alpha_1 R_{1,V}^e}{\left[R_1^e(V)\right]^2} \sum_{m=1}^{M} a_m(m-1) \\ c_2 &= \beta_2 - \frac{\alpha_2 R_{2,V}^e}{\left[R_2^e(V)\right]^2} \end{aligned}$$

Coupling condition (18) with condition (16) results in:

$$\frac{\gamma_1\alpha_1}{\alpha_2} \leq \gamma_2 \leq \min\left(\gamma_1, \frac{1}{1 + \frac{c_2}{c_1}}\right) \tag{19}$$

If we neglect the contribution of the density gradient term in the space headway expansion (which is not exact), condition (19) is reduced to the model of Zheng, Jin, and Huang (2015) if $M = 1$.

### 4.2. *Linear stability conditions*

The linear stability method considers how small perturbations around the homogeneous and stationary solutions influence the stability of traffic flow. Nevertheless, we are aware that the conditions that are stable according to this linear analysis might actually still show nonlinear or short-wave lengths instabilities. However, in general the linear analysis gives sound insights in the general behaviour of the model used. We are also aware that investigation of nonlinear and short-wavelength instabilities will be important to explain complex transitions of congested traffic states. Such nonlinear analysis will be left in our future work.

*At microscopic level* (i.e. for car-following models), the homogeneous and stationary solutions are $[v^e, s^e, 0]$ where $v^e = V(s^e)$ is the speed at the stationary solution. To follow the derivation in Ngoduy (2015) for the anticipative bidirectional Helly-type model, we can obtain the general (linear) stability condition below

$$(\gamma_1\alpha_1 - \gamma_2\alpha_2) + (\gamma_1\beta_1 - \gamma_2\beta_2)\left(\gamma_1\alpha_1 S^e_{1,v} - \gamma_2\alpha_2 S^e_{2,v}\right) - 0.5\left(\gamma_1\alpha_1 S^e_{1,v} - \gamma_2\alpha_2 S^e_{2,v}\right)^2 \geq 0 \tag{20}$$

where $S^e_{1,2,v} = \frac{dS^e_{1,2}}{dv} > 0$. This stability condition reduces to:

- If $\gamma_2 = 0$ (i.e. forward looking driving strategy only):

$$1 + \beta_1 S^e_{1,v} - 0.5\alpha_1\left(S^e_{1,v}\right)^2 \geq 0 \tag{21}$$

- If $S^e_1(v) = S^e_2(v) = S^e(v)$:

$$1 + (\gamma_1\beta_1 - \gamma_2\beta_2)\, S_e - 0.5\,(\gamma_1\alpha_1 - \gamma_2\alpha_2)\,(S_e)^2 \geq 0 \tag{22}$$

It is clear that both headway sensitivity and relative speed sensitivity parameters affect the linear stability condition of the bidirectional Helly-type model.

*At macroscopic level*, the homogeneous and stationary solutions are $[r^e, V^e]$, where $V^e = V^e(r^e)$, $V^e(r^e)$ is the speed at the stationary solution. The linear stability condition of the bidirectional continuum model in Zheng, Jin, and Huang (2015) reads:

$$\frac{1}{r^e}\left[\gamma_1\beta_1 - \gamma_2\beta_2 + \frac{\gamma_2\alpha_2 R^e_{2,V}}{[R^e_2(V)]^2}\right] + \frac{\gamma_1\alpha_1 - \gamma_2\alpha_2}{r^e}\left[\frac{\gamma_1\alpha_1 R^e_{1,V}}{[R^e_1(V)]^2} - \frac{\gamma_2\alpha_2 R^e_{2,V}}{[R^e_2(V)]^2}\right]^{-1} \geq 0 \tag{23}$$

Note that the linear stability condition (23) completely neglects the contribution of the density gradient term and only considers one vehicle in the forward-looking strategy. Generally speaking, both the headway sensitivity parameters (i.e. $\alpha_1$, $\alpha_2$) and the relative speed sensitivity parameters (i.e. $\beta_1$, $\beta_2$) contribute to the linear stability condition of the bidirectional continuum traffic model. Since $R^e_{i,V} \leq 0$, increased $\gamma_2$ leads to the reduced left hand side of equation (23), which intuitively results in the destabilized traffic flow. The above linear stability condition reduces to the following conditions:

- If $\gamma_2 = 0$ (i.e. forward looking driving strategy only):

$$\gamma_1\beta_1 + \left[\frac{R^e_{1,V}}{[R^e_1(V)]^2}\right]^{-1} \geq 0 \tag{24}$$

- If $R_1^e(V) = R_2^e(V) = R(V)$

$$\gamma_1\beta_1 - \gamma_2\beta_2 + \frac{\gamma_2\alpha_2 R_V}{R^2} + \left[\frac{R_V}{R^2}\right]^{-1} \geq 0 \tag{25}$$

In either case, the forward headway sensitivity parameter (i.e. $\alpha_1$) does not contribute to the linear stability condition. This property makes the continuum model of Zheng, Jin, and Huang (2015) inconsistent with the car-following model, from which it is derived (i.e. the Helly-type model). We will show in the ensuing section that such headway sensitivity parameter does play an important role in the linear stability condition of the newly derived continuum model.

Let's rewrite our model in the following general form:

$$\frac{\partial V}{\partial t} + V\frac{\partial V}{\partial x} = A\left(r, V, r_x, V_x\right) \tag{26}$$

and perform the linear analysis as detailed in Treiber and Kesting (2013) we obtain the following linear stability condition:

$$-\left(r^e\frac{A_r}{A_V}\right)^2 + r^e\left(\frac{A_r}{A_V}A_{Vx} - A_{rx}\right) \geq 0 \tag{27}$$

where

$$A_r = \frac{\partial A}{\partial r}, \quad A_V = \frac{\partial A}{\partial V}, \quad A_{rx} = \frac{\partial A}{\partial r_x}, \quad r_x = \frac{\partial r}{\partial x}, \quad A_{Vx} = \frac{\partial A}{\partial V_x}, \quad V_x = \frac{\partial V}{\partial x}$$

From our model, we can derive the stability condition as follows:

$$\begin{aligned} &A_r = -\frac{\gamma_1\alpha_1 - \gamma_2\alpha_2}{(r^e)^2}, \quad A_V = \frac{\gamma_1\alpha_1 R^e_{1,V}}{(R^e_1)^2} - \frac{\gamma_2\alpha_2 R^e_{2,V}}{(R^e_2)^2} \\ &A_{rx} = c, \quad A_{Vx} = c_0 \end{aligned}$$

It is straightforward to show that if $M = 1$ and $c = 0$, we obtain the linear stability condition of the model of Zheng, Jin, and Huang (2015). For the sake of simplicity, we assume that $R_1^e(V) = R_2^e(V) = R(V)$, the linear stability of our model reads:

$$\begin{aligned} &\frac{R^4}{(R_V)^2} + \frac{R^2}{R_V}\left(\gamma_1\beta_1\sum_{m=1}^{M} b_m m - \gamma_2\beta_2\right) + \gamma_1\alpha_1\left(\sum_{m=1}^{M} a_m(m-1) + 1/2\right) + \frac{\gamma_2\alpha_2}{2} \leq 0 \\ \Leftrightarrow &\frac{R^4}{(R_V)^2} + \frac{R^2}{R_V}\left(\gamma_1\beta_1\sum_{m=1}^{M} b_m m - \gamma_2\beta_2\right) + \gamma_1\alpha_1\left(\sum_{m=1}^{M} a_m m - 1/2\right) + \frac{\gamma_2\alpha_2}{2} \leq 0 \end{aligned} \tag{28}$$

It can be seen that if $\gamma_2 = 0$, our model does include the impact of $\alpha_1$ on the linear stability condition via the contribution of the multi anticipation term $\gamma_1\alpha_1\sum_{m=1}^{M} a_m(m-1)$ and the density gradient term $\frac{\gamma_1\alpha_1}{2}$. As $R_V \leq 0$, $\gamma_2$ contributes positively to the left hand side of equation (28) which consequently destabilizes traffic flow, which is in contradiction to the findings in literature (Nakayama, Sugiyama, and Hasebe 2001; Ge, Zhu, and Dai 2006b; Sun, Liao, and Peng 2011; Yang et al. 2013), where it was found that backward looking strategy improves the linear stability condition. These results will be illustrated numerically in Section 5 for different sets of model parameters.

## 5. Numerical studies

### 5.1. *Numerical solution*

The time and space are divided into time step $\Delta t$ and cell length $\Delta x$ so that $\Delta x \geq \Delta t V_0$. To follow Zheng, Jin, and Huang (2015), we adopt the numerical method below for every cell $i$ and time instant $k$:

(1) Density update

$$r_i(k+1) = r_i(k) + \frac{\Delta t}{\Delta x} r_i(k) \left(V_i(k) - V_{i+1}(k)\right) + \frac{\Delta t}{\Delta x} V_i(k) \left(r_{i-1}(k) - r_i(k)\right) \quad (29)$$

(2) Speed update

- If $V_i(k) \geq c_{i,0}(k)$

$$\begin{aligned} V_i(k+1) = V_i(k) &+ \frac{\Delta t}{\Delta x} \left(c_{i,0}(k) - V_i(k)\right) \left(V_i(k) - V_{i-1}(k)\right) \\ &+ \frac{\Delta t}{\Delta x} c_i(k) \left(r_i(k) - r_{i-1}(k)\right) + \Delta t \text{ RHS} \end{aligned} \quad (30)$$

- If $V_i(k) < c_{i,0}(k)$

$$\begin{aligned} V_i(k+1) = V_i(k) &+ \frac{\Delta t}{\Delta x} \left(c_{i,0}(k) - V_i(k)\right) \left(V_{i+1}(k) - V_i(k)\right) \\ &+ \frac{\Delta t}{\Delta x} c_i(k) \left(r_{i+1}(k) - r_i(k)\right) + \Delta t \text{ RHS} \end{aligned} \quad (31)$$

where RHS denotes the right hand side of equation (9).

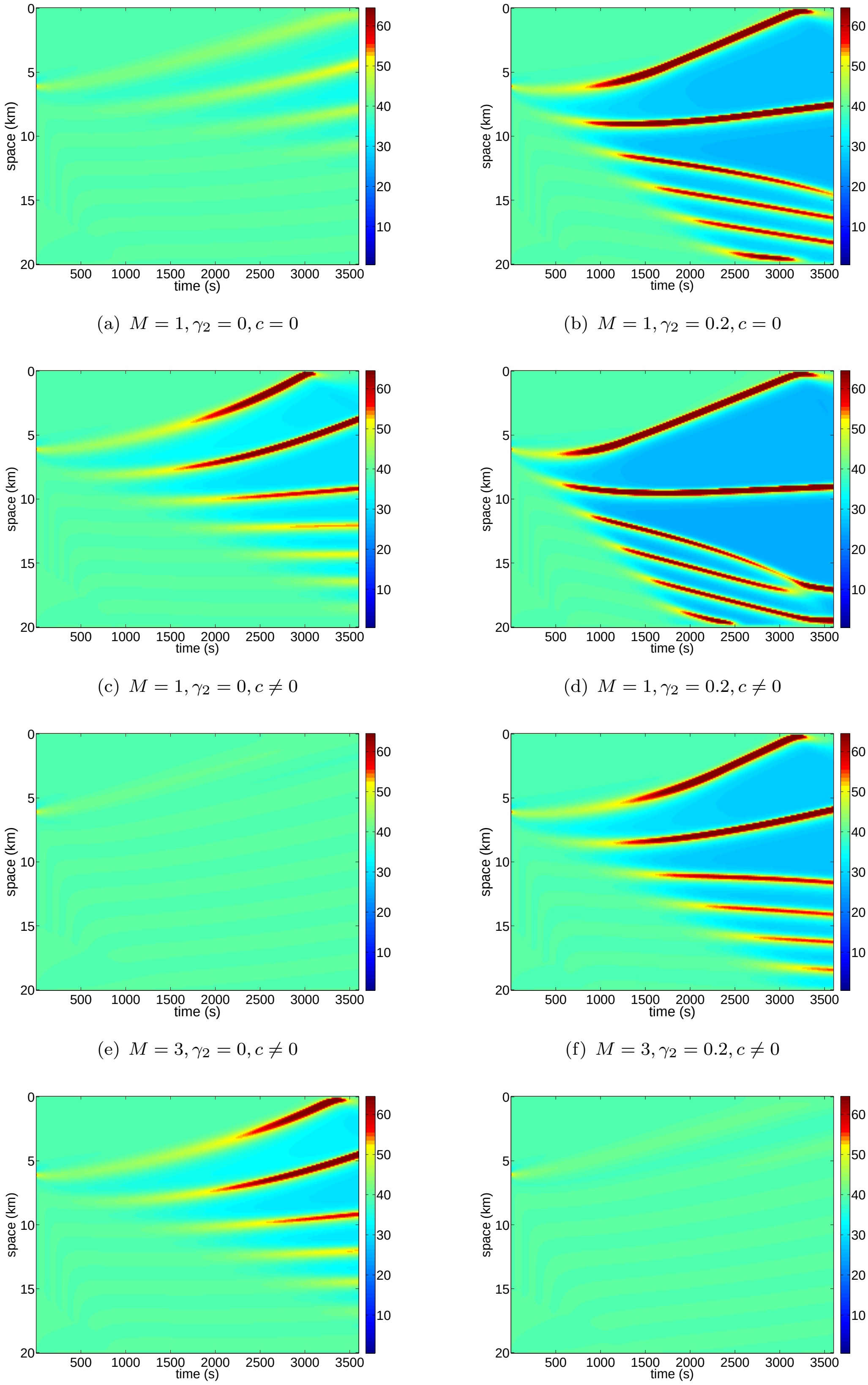
### 5.2. *Traffic instabilities*

| Model parameters | Stability function value | Traffic state | Figure |
|---|---|---|---|
| $M = 1, \gamma_2 = 0, c = 0$ | -0.04 | stable | Figure 1(a) |
| $M = 1, \gamma_2 = 0.2, c = 0$ | 0.11 | unstable | Figure 1(b) |
| $M = 1, \gamma_2 = 0, c \neq 0$ | 0.123 | unstable | Figure 1(c) |
| $M = 1, \gamma_2 = 0.2, c \neq 0$ | 0.148 | unstable | Figure 1(d) |
| $M = 3, \gamma_2 = 0, c \neq 0$ | -0.133 | stable | Figure 1(e) |
| $M = 3, \gamma_2 = 0.2, c \neq 0$ | 0.031 | unstable | Figure 1(f) |
| $M = 1, \gamma_2 = 0.1, c \neq 0$ | 0.08 | unstable | Figure 1(g) |
| $M = 3, \gamma_2 = 0.1, c \neq 0$ | -0.05 | stable | Figure 1(h) |

Table 1.: Traffic instabilities under different sets of model parameters

We will investigate how the proposed model replicates the well-known traffic instabilities triggered by a small perturbation. Hence, we will adopt the periodic boundary condition with the following initial conditions:

$$r(x,0) = r_0 + \delta r_0 \left[\cosh^{-2}\left(\frac{160}{L}\left(x - \frac{5L}{16}\right)\right) - 0.25 \cosh^{-2}\left(\frac{40}{L}\left(x - \frac{11L}{36}\right)\right)\right] \quad (32)$$




(a) $M = 1, \gamma_2 = 0, c = 0$

(b) $M = 1, \gamma_2 = 0.2, c = 0$

(c) $M = 1, \gamma_2 = 0, c \neq 0$

(d) $M = 1, \gamma_2 = 0.2, c \neq 0$

(e) $M = 3, \gamma_2 = 0, c \neq 0$

(f) $M = 3, \gamma_2 = 0.2, c \neq 0$

(g) $M = 1, \gamma_2 = 0.1, c \neq 0$

(h) $M = 3, \gamma_2 = 0.1, c \neq 0$

Figure 1.: Propagation of traffic instabilities due to a small initial perturbation.

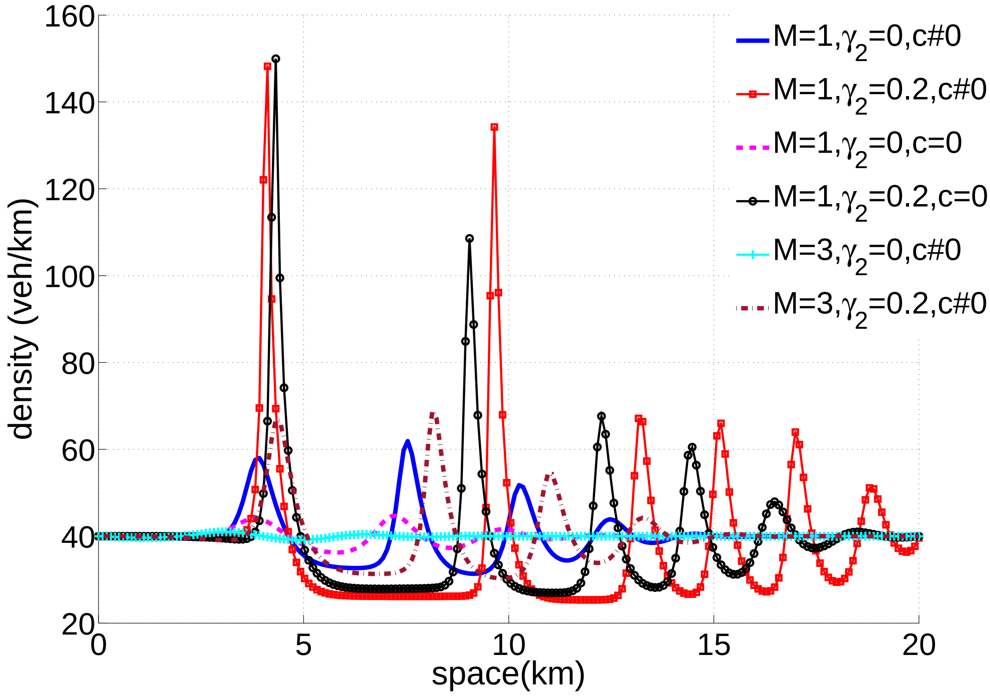


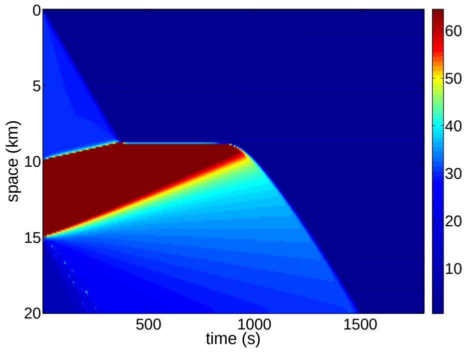

Figure 2.: Density profiles in different models at $t = 1200$s

where $r_0 = 40$veh/km, $\delta r_0 = 30$veh/km, $L = 20000$m. In this test, we choose $\Delta x = 100$m and $\Delta t = 2$s. Other model parameters which will be used throughout this paper are: $\alpha_1 = 0.1s^{-2}$, $\alpha_1 = 0.01s^{-2}$, $\beta_1 = 0.2s^{-1}$, $\beta_1 = 0.02s^{-1}$. The values of the stability function calculated by the left hand side of equation (28) are given in Table 1, which lead to different stable regimes for different model parameters. According to equation (28), traffic becomes stable if the stability function is smaller than zero and unstable otherwise. The initial speed is to follow an equilibrium relationship: $V(x, 0) = V^e(r(x, 0))$ defined above. The dynamics of traffic flow reproduced by our model for such initial and boundary conditions are shown in Figure 1.

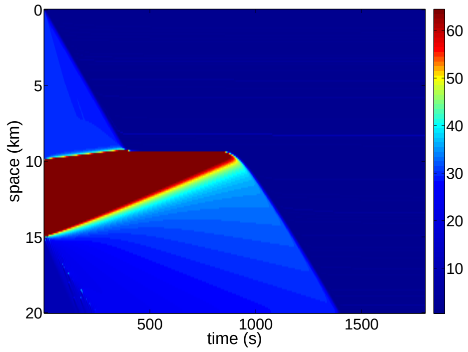
It can be seen from Figure 1 that, for both models, increasing reaction to the following vehicle (more attention to the backward looking) will destabilize traffic flow with respect to the initial perturbation, triggering stop-and-go waves. Figure 1 also confirms that multi anticipative driving strategy stabilizes traffic flow. Figure 1 and Table 1 support numerically the linear stability results above. Figure 2 describes that increased backward looking strategy amplifies the magnitude of the density oscillation while multi anticipative strategy reduces such magnitude. Therefore, it is concluded that backward looking contributes negatively to the stability of traffic flow.

### 5.3. *Shock-wave propagation patterns*

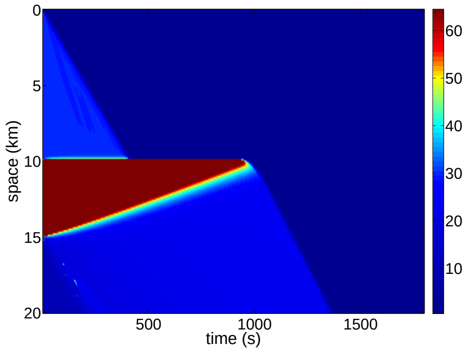
In this section, we will exhibit our model performance with respect to the shock-wave propagation. The proposed model is simulated using the open boundary condition with



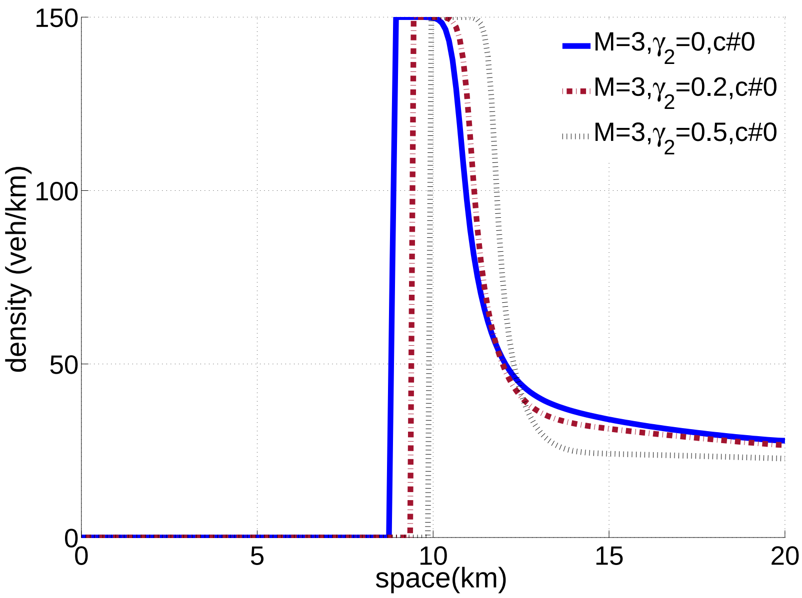

(a) $M = 3, \gamma_2 = 0, c \neq 0$




(b) $M = 3, \gamma_2 = 0.2, c \neq 0$




(c) $M = 3, \gamma_2 = 0.5, c \neq 0$




(d) Density profile at $t = 600s$

Figure 3.: Shock-wave propagation with different levels of backward looking.

the following initial condition:

$$r(x,0) = \begin{cases} 30veh/km, & \text{if } \ 0 \leq x \leq 10000m \\ 150veh/km, & \text{if } \ 10000m \leq x \leq 15000m \\ 10veh/km, & \text{otherwise} \end{cases} \tag{33}$$

Figure 3 shows that increased backward looking level will reduce the shock-wave speed. At a low level of backward looking, the front shock is still moving upstream from the bottleneck location ($x = 10000$m), indicating that the congestion affects traffic upstream of the bottleneck, as seen in Figures 3(a)-3(b). That is traffic dynamics are affected by the downstream condition. However, at a higher level of backward looking, the front shock is standstill at the bottleneck location (i.e. Figure 3(c)) indicating that the congestion affects traffic downstream of the bottleneck: pushing effect from the upstream vehicles. Such pushing effect can be seen clearly in Figure 3(d) where the location of the front shock is moving downstream with increased level of backward attention.

## 6. Concluding remarks

This paper has proposed a macroscopic traffic model which can capture both the backward looking and multiple forward looking strategy. Such proposed driving strategy can be realized through cooperative driving strategy of connected vehicles in our future traffic systems. Our proposed model has been derived from the same car-following model, namely the model of Helly (1959), as in Zheng, Jin, and Huang (2015) using the gradient expansion method. We have shown that the proposed model is a generalized version of the model of Zheng, Jin, and Huang (2015), which is more consistent with the original car-following model by, on the one hand, correcting the space headway expansion via the inclusion of the density gradient term, and on the other hand, considering the forward multi-anticipative driving strategy. Numerical examples with both periodic and open boundary conditions have supported our analytical findings, indicating that while the multiple looking ahead strategy improves traffic flow, the backward looking strategy has negative impact on traffic flow: it destabilizes traffic flow with respect to small perturbations.

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## References

Arnaout, G., and S. Bowling. 2011. "Towards reducing traffic congestion using cooperative adaptive cruise control on a freeway with a ramp." *Journal of Industrial Engineering and Management* 4: 699–717.

Bando, M., K. Hasebe, K. Nakanishi, and A. Nakayama. 1998. "Analysis of Optimal Velocity Model with Explicit Delay." *Physical Review E* 58: 5429–5435.

Bando, M., K. Hasebe, A. Nakayama, A. Shibata, and Y. Sugiyama. 1995. "Dynamical Model of Traffic Congestion and Numerical Simulation." *Physical Review E* 51: 1035–1042.

Boel, R., and L. Mihaylova. 2006. "A compositional stochastic model for real time freeway traffic simulation." *Transportation Research Part B* 40: 319–334.

Bogdanova, A., M.N. Smirnova, Z. Zhu, and N. N. Smirnov. 2015. "Exploring peculiarities of traffic flows with a viscoelastic model." *Transportmetrica A: Transport Science* in press: doi:10.1080/23249935.2015.1030472.

Chadler, R. E., R. Herman, and E. W. Montroll. 1958. "Traffic Dynamics: Studies in Car Following." *Operations Research* (6): 165–184.

Ge, H. X., S. Q. Dai, and L. Y. Dong. 2006a. "An extended car-following model based on intelligent transporation system application." *Physica A* 365: 543–548.

Ge, H. X., H. B. Zhu, and S. Q. Dai. 2006b. "Effect of looking backward on traffic flow in a cooperative driving car following model." *The European Physical Journal B* 54: 503–507.

Gupta, A.K., and V.K. Katiyar. 2005. " Analyses of shock waves and traffic jams." *Journal of Physics A* 38: 4069–4083.

Gupta, A.K., and V.K. Katiyar. 2006a. " Phase transition of traffic states with an on-ramp." *Physica A* 371: 674–682.

Gupta, A.K., and V.K. Katiyar. 2006b. " A new anisotropic continuum model for traffic flow." *Physica A* 368: 551–559.

Hasebe, K., A. Nakayama, and Y. Sugiyama. 2003. "Dynamical Model of a Cooperative Driving System for Freeway Traffic." *Physical Review E* 68: 0261021–0261026.

Helbing, D. 1997. *Verkehrsdynamik*. Berlin: Springer-Verlag.

Helbing, D., A. Hennecke, V. Shvetsov, and M. Treiber. 2001. "MASTER: Macroscopic Traffic Simulation Based on a Gas-kinetic Non-local Traffic Model." *Transportation Research B* 35 (2): 183–211.

Helbing, D., and A. F. Johansson. 2009. "On the Controversy around Daganzo's Requiem for and Aw-Rascle's Resurrection of Second-Order Traffic Flow Models." *The European Physical Journal B* 69: 549–562.

Helly, W. 1959. "Simulation of bottlenecks in a single lane traffic flow." *International Symposium on the Theory of Traffic Flow* 207–238.

Hoogendoorn, S. P. 1999. "Multiclass Continuum Modelling of Multilane Traffic Flow." PhD thesis. Delft University of Technology, Netherlands.

Hoogendoorn, S. P., S. Ossen, and M. Schreuder. 2006. "Empirics of Multianticipative Car-Following Behavior." *Transportation Research Record* 1965: 112–120.

Hoogendoorn, S. P., S. J. L. Ossen, and M. Schreuder. 2007. "Properties of a microscopic heterogeneous multi-anticipative traffic flow model." In *Proceedings of the 17th International Symposium on Transportation and Traffic Theory*, 583–606.

Jiang, R., Q. Wu, and Z. Zhu. 2001. "Full velocity difference model for a car-following theory." *Physical Review E* 64: 017101–017104.

Jin, P. J., D. Yang, B. Ran, M. Cebelak, and C. M. Walton. 2014. "Bidirectional Control Characteristics of General Motors and Optimal Velocity Car-following Models." *Transportation Research Record* 2381: 110–119.

Kesting, A., and M. Treiber. 2008. "How Reaction Time, Update Time, and Adaptation Time Influence the Stability of Traffic Flow." *Computer-Aided Civil and Infrastructure Engineering* 23: 125–137.

Kesting, A., M. Treiber, and D. Helbing. 2007. "General lane-changing model MOBIL for car-following models." *Transportation Research Record* 1999: 86–94.

Kesting, A., M. Treiber, and D. Helbing. 2010. "Enhanced Intelligent Driver Model to Access the Impact of Driving Strategies on Traffic Capacity." *Philosophical Transactions of the Royal Society A* 368: 4585–4605.

Laval, J., C. S. Toth, and Y. Zhou. 2014. "A parsimonious model for the formation of oscillations in car-following models." *Transportation Research Part B* 70: 228–238.

Laval, J. A., and L. Leclercq. 2010. "Continuum Approximation for Congestion Dynamics Along Freeway Corridors." *Transportation Science* 44: 87–97.

Lenz, H., C.K. Wagner, and R. Sollacher. 1999. "Multi-anticipative car-following model." *The European Physical Journal B* 7: 331–335.

Lighthill, M. H., and G. B. Whitham. 1955. "On Kinematic Waves 2: A Theory of Traffic Flow on Long, Crowded Roads." *Proceedings of the Royal Society of London* 229: 317–345.

Monteil, J., R. Billot, J. Sau, and N.-E. El Faouzi. 2014. "Linear and Weakly Nonlinear Stability Analyses of Cooperative Car-Following Models." *IEEE Transactions on Intelligent Transporta-*

*tion Systems* 15: 2001–2013.
Nakayama, A., Y. Sugiyama, and K. Hasebe. 2001. "Effect of looking at the car that follows in an optimal velocity model of traffic flow." *Physical Review E* 65: 0161121–0161126.
Ngoduy, D. 2008. "Operational effects of acceleration lane on main traffic flow at discontinuities." *Transportmetrica* 4: 195–2007.
Ngoduy, D. 2012a. "Effect of driver behaviours on the formation and dissipation of traffic flow instabilities." *Nonlinear Dynamics* 69: 969–975.
Ngoduy, D. 2012b. "Application of gas-kinetic theory to modelling mixed traffic of manual and adaptive cruise control vehicles." *Transportmetrica A: Transport Science* 8: 43–60.
Ngoduy, D. 2013a. "Platoon-based macroscopic model for intelligent traffic flow.." *Transportmetrica B: Transport Dynamics* 1: 153–169.
Ngoduy, D. 2013b. "Instabilities of cooperative adaptive cruise control traffic flow: a macroscopic approach.." *Communications in Nonlinear Science and Numerical Simulation* 18: 2838–2851.
Ngoduy, D. 2015. "Linear stability of a generalized multi-anticipative car following model with time delays.." *Communications in Nonlinear Science and Numerical Simulation* 22: 420?–426.
Ngoduy, D., S. P. Hoogendoorn, and H. J. van Zuylen. 2006. "New Continuum Traffic Model for Freeway with On- and Off-ramp to Explain Different Traffic Congested States." *Transportation Research Records* 1965: 91–102.
Ngoduy, D., and R. E. Wilson. 2014. "Multi-anticipative nonlocal second order traffic model.." *Computer-Aided Civil and Infrastructure Engineering* 29: 248–263.
Richards, P. I. 1956. "Shock Waves on the Highway." *Operations Research* 4: 42–51.
Sau, J., J. Monteil, R. Billot, and N.-E. El Faouzi. 2014. "The root locus method: application to linear stability analysis and design of cooperative car-following models." *Transportmetrica B: Transport Dynamics* 2: 60–82.
Sun, D. H., X. Y. Liao, and G. H. Peng. 2011. "Effect of looking backward on traffic flow in an extended multiple car-following model." *Physica A* 390: 631–635.
Treiber, M., A. Hennecke, and D. Helbing. 1999. "Derivation, Properties and Simulation of a Gas-kinetic-based, Non-local Traffic Model." *Physical Review E* 59: 239–253.
Treiber, M., A. Hennecke, and D. Helbing. 2000. "Congested Traffic States in Empirical Observations and Microscopic Simulations." *Physical Review E* 62: 1805–1824.
Treiber, M., and A. Kesting. 2013. *Traffic Flow Dynamics*. Germany: Springer.
Treiber, M., A. Kesting, and D. Helbing. 2006. "Delays, inaccuracies and anticipation in microscopic traffic model." *Physica A* 360: 71–88.
van Arem, B., C. J. G. Van Driel, and R. Visser. 2006. "The impact of cooperative adaptive cruise control on traffic flow characteristics." *IEEE Transactions on intelligent transporation systems* 7: 429–436.
Wilson, R. E., P. Berg, S. Hooper, and G. Lunt. 2004. "Many neighbour interaction and nonlocality in traffic models." *The European Physical Journal B* 39: 397–408.
Yang, S., W. Liu, D. Sun, and C. Li. 2013. "A New Extended Multiple Car-Following Model Considering the Backward-Looking Effect on Traffic Flow." *Journal of Computational and Nonlinear Dynamics* 8: 011016–011021.
Zhang, P., and S. C. Wong. 2006. "Essence of conservation forms in the travelling waves solutions of higher order traffic models." *Physical Review E* 74: 0261091–02610910.
Zhang, P., S. C. Wong, and S. Q. Dai. 2011. "A conserved higher-order anisotropic traffic flow model: Description of equilibrium and non-equilibrium flows." *Transportation Research Part B* 43: 562–574.
Zheng, L., P. J. Jin, and H. Huang. 2015. "An anisotroic continuum model considering bidirectional information impact." *Transportation Research Part B* 75: 36–57.
Zheng, L., S. F. Ma, and S. Q. Zhong. 2011. "Analysis of honk effect on the traffic flow in a cellular automaton model." *Physica A* 390: 1072–1084.
Zheng, L., S. Q. Zhong, and S. F. Ma. 2013. "Towards the bi-directional cellular automaton model with perception ranges." *Physica A* 392: 3028–3038.