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Optimal Sliding Mode Controllers for Attitude Tracking of Spacecraft

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***Abstract*— This paper studies two optimal sliding mode control laws using integral sliding mode control (ISM) for some spacecraft attitude tracking problems. Integral sliding mode control combining the first order sliding mode and optimal control is applied to quaternion-based spacecraft attitude tracking manoeuvres with external disturbances and an uncertainty inertia matrix. For the optimal control part the state dependent Riccati equation (SDRE) and Control Lyapunov function (CLF) approaches are used to solve the infinite-time nonlinear optimal problem. The second method of Lyapunov is used to show that tracking is achieved globally. An example of multiaxial attitude tracking manoeuvres is presented and simulation results are included to verify the usefulness of these controllers.**

I. Introduction

This paper presents controller designs using the optimal sliding mode to control spacecraft tracking manoeuvres. The optimal sliding mode has been presented in many papers. Young et al. [1] studied the sliding surface design using the linear quadratic regulator (LQR) approach. Some states of the system are considered as the control inputs to the subsystem of the other states and LQ methods can be applied to obtain the optimal control law. The LQR problem for linear time-varying systems has also been investigated in terms of optimal sliding surface design [2]. However the optimal sliding mode of nonlinear systems has been studied rarely. A method for choosing an optimal sliding manifold for a class of nonlinear systems has been presented in [3].

Optimal sliding mode control has been studied by Xu [4]. Because the integral sliding mode is a robust control and the optimal control provides the optimality, one obtains optimality as well as robustness. The controller is developed by adding two control laws together [4]. For the optimal control law design Xu solved the infinite-time nonlinear optimal problem by using the state dependent Riccati equation (SDRE) approach and the control Lyapunov function approach. Early work on the state dependent Riccati equation was studied by Burghart [5] and Wernli [6] The SDRE approach was applied to optimal control and stabilization for nonlinear systems by Banks and Mhana [7]. The explicit control law has been studied for nonlinear system of the form $\dot{x} = A(x)x + B(x)u$. In [8] Cloutier et al. studied nonlinear regulation and nonlinear H_∞ control via the SDRE approach.

On the other hand, the control Lyapunov function (CLF) was introduced for the synthetic problem [9], [10]. In contrast with traditional Lyapunov functions, a CLF can be defined for a system with inputs without specifying a particular feedback function. Sontag [11] has shown that if a CLF is known for a nonlinear system that is affine in the control, then the CLF and the system equations can be used to find controllers that make the system asymptotically stable. Freeman and Kokotovic' [12] have shown that every CLF solves the Hamilton-Jacobi-Bellman (HJB) equation associated with a meaningful cost. In other words, if we have a CLF for a nonlinear system, we can compute the resulting optimal control law without solving the HJB equation. Also Sackmann and Krebs [13] developed a modified optimal control [13] that is an adapted version of the controller [11]. The modified optimal control problem consists of a quadratic performance index and a specific scalar differential equation as a constraint. This method yields an optimal control law in a closed form and achieves global asymptotic stability.

In [4] a specific case of nonlinear systems was studied. There are nonlinear terms only in the final equation of this system, so the method cannot be applied to highly nonlinear systems which have nonlinear terms in all equations of the system (e.g. spacecraft system).

We have developed two controllers for application to spacecraft tracking manoeuvres. The first controller uses the method in [4] and combines this with integral sliding mode control [13] and the SDRE approach [7]. Since the spacecraft systems are highly nonlinear systems, the SDRE approach is rather difficult to apply for the spacecraft systems. The basic concepts in [4] are used for applying the SDRE approach to the spacecraft system. Sackmann and Krebs [13] used the Cayley-Rodrigues parameters for the attitude representation and it was applied to spacecraft rest-to-rest manoeuvres.

The spacecraft tracking system consists of the dynamic equations of the error rate [15] and the kinematics of the attitude error [14], [16]. In this paper we have rewritten the spacecraft system in a form suitable for using the SDRE approach. Once the control design has been completed, we can apply it to spacecraft tracking manoeuvres.

For the second controller, we have used the approach in [4] combining this with the integral sliding mode control [14] and the modified optimal control [13] with a CLF. The selected CLF is similar to the function $V(x)$ in [15]. In fact Show and Juang [15] did not prove that this $V(X)$ was a CLF for the spacecraft tracking problems. They selected $V(x)$ to solve the Hamilton-Jacobi partial differential equation and applied the concepts of H_∞ control for their controller design. In this paper we prove that their $V(x)$ is a CLF. Using $V(x)$ and the modified optimal control [13] a new controller has been designed. Numerical simulations of these optimal sliding controllers and the controller developed in [16] are studied in [17] for spacecraft attitude control for rest-to-rest manoeuvres .

This paper is organized as follows. In Section II the dynamic equations of the error rate [15] and the kinematics of attitude error [15], [18] are described. In Section III a new controller combining the method [4], the integral sliding mode [14] and the SDRE approach [7] is presented. The integral sliding mode [14] is applied to the controller and switching function designs, and we use the SDRE approach [3] to solve the nonlinear optimal control problem. In Section IV another controller is designed using the method [4], the integral sliding mode [14] and the modified optimal control [13] with a CLF. Also we prove that a CLF exists and the stability of the spacecraft systems is achieved globally. In Section V an example of spacecraft tracking manoeuvres is presented to make comparisons between the optimal sliding mode controllers using the SDRE and the control Lyapunov function. In Section VI we present conclusions.

II. Mathematical Model of Spacecraft Attitude Tracking Control

A. Dynamic Equations of the Error Rate

A rigid spacecraft rotating under the influence of body-fixed devices is considered. In [18] the dynamic equation is given as

$$J\dot{\omega} = -[\omega\times]J\omega + u + d \tag{1}$$

where $\omega = [\omega_1 \quad \omega_2 \quad \omega_3]^T$ is the angular rate of the spacecraft, $u = [u_1 \quad u_2 \quad u_3]^T$ represents the control vector, $d = [d_1 \quad d_2 \quad d_3]^T$ are bounded disturbances, and J is the inertia matrix. The skew-symmetric matrix $[\omega\times]$ is

$$[\omega\times] = \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix} \tag{2}$$

Denoting $\omega_r = [\omega_{1r} \quad \omega_{2r} \quad \omega_{3r}]^T$ as the desired reference rate and we substitute $\omega_e = \omega - \omega_r$ into (1). We obtain the dynamic equations of the error rate [15]

$$J\dot{\omega}_e = -[\omega_e\times]J\omega_e - [\omega_e\times]J\omega_r - [\omega_r\times]J\omega_e + u_e + d \tag{3}$$

B. Kinematics of the Attitude Error

We explain briefly the attitude error using quaternions. We define here the quaternion $Q = [q^T \quad q_4]^T$ with $q = [q_1 \quad q_2 \quad q_3]^T$ and

$$Q_r = [q_r^T \quad q_{4r}]^T.$$

$q_r = [q_{1r} \quad q_{2r} \quad q_{3r}]^T$ is the desired attitude. Also the attitude error $Q_e = [q_e^T \quad q_{4e}]^T$ with $q_e = [q_{1e} \quad q_{2e} \quad q_{3e}]^T$. Using the quaternion multiplication law, we obtain

$$Q_e = \begin{bmatrix} q_{4r}q - q_4 q_r - [q_r\times]q \\ q_4 q_{4r} + q^T q_r \end{bmatrix} \tag{4}$$

subject to the constraint

$$Q_e^T Q_e = (q^T q + q_4^2)(q_r^T q_r + q_{4r}^2) = 1 \tag{5}$$

The kinematic equation for the attitude error is expressed as [15], [18]

$$\dot{Q}_e = \frac{1}{2}\begin{bmatrix} [q_e\times] + q_{4e}I_{3\times3} \\ -q_e^T \end{bmatrix}\omega_e \tag{6}$$

where I_3 is the 3×3 identity matrix.

III. SDRE Controller

In this section the Xu method [4] and the integral sliding mode [14] are merged to design a new controller. which consists of two parts; the sliding mode and optimal control. The first order sliding mode is used for the sliding mode controller design while the optimal control law is designed using the SDRE approach [7] to solve the infinite-time optimal quadratic problem.

The tracking motion of a rigid spacecraft is considered. For the optimal controller design, the difficulty of using the SDRE approach is how choose the the appropriate matrix $A(x)$. The basic concepts in [13] are difficult to apply. So, we have rewritten the dynamics equations of the error rate in a more suitable form and the appropriate matrix $A(x)$ is then selected. After we obtain the optimal control law, a new optimal sliding mode controller will be designed by combining the optimal control with sliding mode control.

We discuss an optimal control law minimizing the performance index

$$\min_u \int_0^\infty (x^T Q x + u^T R u)dt$$
$$\text{where} \quad \dot{x} = f(x) + G(x)u, \quad x(0) = x_0 \tag{7}$$

and

$$f(x) = \begin{bmatrix} -J^{-1}[\omega_e\times]J\omega_e - J^{-1}[\omega_e\times]J\omega_r - J^{-1}[\omega_r\times]J\omega_e \\ 0.5\left([q_e\times] + q_{4e}I_{3\times3}\right)\omega_e \end{bmatrix} \tag{8}$$

and

$$G(x) = \begin{bmatrix} J^{-1} \\ 0 \end{bmatrix} \tag{9}$$

To apply the SDRE method $f(x)$ must be decomposed as $f(x) = A(x)x$. Obviously it is difficult to obtain the matrix $A(x)$ from the system above. We write $f(x)$ in a more suitable form to choose $A(x)$. Using basic matrix operations, the term $J^{-1}[\omega_e\times]J\omega_r$ in (8) can be written as

$$J^{-1}[\omega_e\times]J\omega_r = -J^{-1}[\alpha\times]\omega_e \tag{10}$$

where

$$[\alpha\times] = \begin{bmatrix} 0 & -\alpha_3 & \alpha_2 \\ \alpha_3 & 0 & -\alpha_1 \\ -\alpha_2 & \alpha_1 & 0 \end{bmatrix}$$

and

$$\begin{aligned} \alpha_1 &= J_{11}\omega_{1r} + J_{12}\omega_{2r} + J_{13}\omega_{3r} \\ \alpha_2 &= J_{21}\omega_{1r} + J_{22}\omega_{2r} + J_{23}\omega_{3r} \\ \alpha_3 &= J_{31}\omega_{1r} + J_{32}\omega_{2r} + J_{33}\omega_{3r} \end{aligned}$$

Now we obtain

$$f(x) = \begin{bmatrix} -J^{-1}[\omega_e\times]J\omega_e + J^{-1}[\alpha\times]\omega_e - J^{-1}[\omega_r\times]J\omega_e \\ 0.5\left([q_e\times] + q_{4e}I_{3\times3}\right)\omega_e \end{bmatrix} \tag{11}$$

and $f(x)$ can be written as

$$f(x) = \begin{bmatrix} -J^{-1}[\omega_e\times]J + J^{-1}[\alpha\times] - J^{-1}[\omega_r\times]J & 0 \\ 0.5\left([q_e\times] + q_{4e}I_{3\times3}\right) & 0 \end{bmatrix} \begin{bmatrix} \omega_e \\ q_e \end{bmatrix} \tag{12}$$

To use the SDRE approach the matrix $A(x)$ is chosen as

$$A(x) = \begin{bmatrix} -J^{-1}[\omega_e\times]J + J^{-1}[\alpha\times] - J^{-1}[\omega_r\times]J & 0 \\ 0.5\left([q_e\times] + q_{4e}I_{3\times3}\right) & 0 \end{bmatrix} \tag{13}$$

Thus, the optimal control υ^* [7] is given as

$$\upsilon^* = -R^{-1}G^T\Pi(x)x \tag{14}$$

where $\Pi(x)$ is the solution to the generalized SDRE

$$\Pi(x)A(x) + A^T(x)\Pi(x) + Q(x) \\ -\Pi(x)G(x)R^{-1}(x)G^T(x)\Pi(x) = 0 \tag{15}$$

Next we discuss the optimal sliding mode control. Using the Xu method [4] and the integral sliding mode [13], the switching function is designed as

$$s = s_0(x) + \phi \tag{16}$$

Letting $s_0(x) = \omega_e + Kq_e$, (16) becomes

$$s = \omega_e + Kq_e + \phi \tag{17}$$

where K is a 3×3 symmetric positive-definite constant matrix. ϕ is an auxiliary variable that is the solution of the differential equation

$$\dot{\phi} = -\frac{\partial s_0}{\partial x}\left[f(x) + G(x)\upsilon^*\right], \quad \phi(0) = -s_0(x(0)) \tag{18}$$

Here $\frac{\partial s_0}{\partial x} = [I_{3\times3} \quad KI_{3\times3}]^T$ and υ^* is the optimal sliding mode control

$$u = \upsilon^* - M\vartheta_i \tag{19}$$

where M is a 3×3 positive-definite diagonal matrix, and the ith component of ϑ is given by

$$\vartheta_i = \text{sat}(s_i, \varepsilon_i), \quad i = 1, 2, 3. \tag{20}$$

where

$$\text{sat}(s_i, \varepsilon_i) = \begin{cases} 1 & \text{for } \ s_i > \varepsilon_i \\ s_i/\varepsilon_i & \text{for } \ |s_i| \le \varepsilon_i \\ -1 & \text{for } \ s_i < \varepsilon_i \end{cases}$$

Next we show that the control law above is designed such that the reaching and sliding mode conditions are satisfied. The following candidate Lyapunov is selected

$$V = \frac{1}{2}s^T s \tag{21}$$

and we take the time derivative of V with the substitution of $\dot{s}$ and (18). We obtain

$$\dot{V} = s^T\left(\frac{\partial s_0}{\partial x}[f(x) + G(x)\upsilon^*] + \dot{\phi}\right) \tag{22}$$

The control u with external disturbances can be written as

$$u = u_1 + \upsilon^* + \xi \tag{23}$$

Using (7) and (23), the time derivative of V can be written as

$$\begin{aligned} \dot{V} &= s^T(\frac{\partial s_0}{\partial x}[f(x) + G(x)u] \\ &\quad -\frac{\partial s_0}{\partial x}[f(x) + G(x)u - G(x)u_1 - G(x)\xi]) \\ &= s^T(\frac{\partial s_0}{\partial x}G(x)[\xi + u_1]) \end{aligned} \tag{24}$$

Let the discontinuous control input u_1 have the following form

$$u_1 = -M(x)\text{sign}(s) \tag{25}$$

where $M(x) \in \mathcal{R}^{m\times m}$ is a positive definite diagonal matrix. Letting $\Psi = \frac{\partial s_0}{\partial x}G(x)$, we obtain

$$\dot{V} = s^T(\Psi[\xi - M\text{sign}(s)]) \tag{26}$$

We choose s_0 such that Ψ is positive definite and then (26) becomes

$$\dot{V} = |s|(\Psi[\xi\text{sign}(s) - M]) \tag{27}$$

Obviously if $M(x)$ is chosen such that $M(x) > \sup|\xi|$ then $\dot{V} < 0$. This guarantees the reaching and sliding on the manifold.

IV. CLF Controller

This section presents another optimal sliding mode controller design for the spacecraft tracking manoeuvres. A rigid spacecraft rotating under the influence of body-fixed devices is considered. Instead of the SDRE the basic principles in [13] with a CLF is applied to obtain the optimal controller design. We have developed a new control law using the Xu approach [4] combined with integral sliding mode control [14], and the basic concepts in [13] with a CLF. The selected control Lyapunov function is very similar to the function $V(x)$ that Show and Juang [15] selected to solve the Hamilton-Jacobi partial differential equation. We now prove that our chosen function $V(x)$ is a CLF and then use it with the basic principles in [13] to construct a new controller.

Now we discuss the optimal control law design. For a CLF $V(x)$ an optimal control law can be designed as [13]

$$u_{opt} = -R^{-1}(x)(L_GV)^T\lambda \tag{28}$$

where

$$\lambda = \frac{L_fV - \sqrt{(L_fV)^2 + x^TQxL_GVR^{-1}(x)(L_GV)^T}}{L_GVR^{-1}(x)(L_GV)^T}$$

In order to use this controller for solving the infinite-time optimal problem we have to guarantee that the candidate function $V(x)$ is a CLF.

A CLF $V(x)$ is a $\mathcal{C}^1$, positive definite, radially unbounded function satisfying

$$L_GV = 0 \Longrightarrow L_fV < 0 \quad \forall x \ne 0 \tag{29}$$

Next we prove that our chosen function $V(x)$ is a CLF. It is similar to the function $V(x)$ in [14]. A CLF candidate is selected as

$$V(x) = \frac{1}{2}\left[a\omega_e^T J\omega_e + 2b\omega_e^T Jq_e + cq_e^T q_e\right] \tag{30}$$

where a, b and c are nonnegative constants. Since J is symmetric and positive definite, $V(x)$ can be written as

$$V(x) = \frac{1}{2}\begin{bmatrix}\omega_e^T & q_e^T\end{bmatrix}\begin{bmatrix}aJ & bJ\\ bJ & c\end{bmatrix}\begin{bmatrix}\omega_e\\ q_e\end{bmatrix} \tag{31}$$

The conditions for the $V(x)$ to be positive defined are

$$c > 0, \quad acJ > b^2J^2 \tag{32}$$

Using (31), we obtain

$$\frac{\partial V(x)}{\partial x} = \begin{bmatrix}aJ\omega_e + bJq_e\\ bJ\omega_e + cq_e\end{bmatrix} = \begin{bmatrix}aJ & bJ\\ bJ & c\end{bmatrix}\begin{bmatrix}\omega_e\\ q_e\end{bmatrix} \tag{33}$$

and

$$\begin{aligned}\left[\frac{\partial V(x)}{\partial x}\right]^T &= \begin{bmatrix}\omega_e^T & q_e^T\end{bmatrix}\begin{bmatrix}aJ & bJ\\ bJ & c\end{bmatrix}\\ &= \begin{bmatrix}\omega_e^T aJ + q_e^T bJ & \omega_e^T bJ + q_e^T c\end{bmatrix}\end{aligned} \tag{34}$$

Thus

$$\begin{aligned}L_G V &= \begin{bmatrix}\omega_e^T aJ + q_e^T bJ & \omega_e^T bJ + q_e^T c\end{bmatrix}\begin{bmatrix}J^{-1}\\ 0\end{bmatrix}\\ &= \omega_e^T a + q_e^T b\end{aligned} \tag{35}$$

Therefore, if $L_G V = 0$, then we have

$$\omega_e^T = -\frac{b}{a}q_e^T \tag{36}$$

Next we show that if $L_G V = 0$, then $L_f V < 0$ for all $x \neq 0$. Letting $\Gamma(\omega_e, \omega_r) = -J^{-1}[\omega_e\times]J\omega_e - J^{-1}[\alpha\times]\omega_e - J^{-1}[\omega_r\times]J\omega_e$, $f(x)$ can be written as

$$f(x) = \begin{bmatrix}\Gamma(\omega_e, \omega_r)\\ 0.5\left([q_e\times] + q_{4e}I_{3\times3}\right)\omega_e\end{bmatrix} \tag{37}$$

and

$$\begin{aligned}L_f V &= (\omega_e^T aJ + q_e^T bJ)\Gamma(\omega_e, \omega_r) + (\omega_e^T bJ +\\ &\qquad q_e^T c)(0.5\left([q_e\times] + q_{4e}I_{3\times3}\right)\omega_e)\end{aligned} \tag{38}$$

Substituting (36) in (38) we obtain

$$L_f V = (-\frac{b^2}{a}q_e^T J + q_e^T c)\left(-\frac{b}{2a}[q_e\times]q_e - \frac{b}{2a}q_{4e}I_{3\times3}q_e\right) \tag{39}$$

Since $[q_e\times]q_e = 0$, (39) becomes

$$\begin{aligned}L_f V &= (-\frac{b^2}{a}q_e^T J + q_e^T c)\left(-\frac{b}{2a}q_{4e}I_{3\times3}q_e\right)\\ &= (\frac{b^3}{2a^2}q_e^T J - \frac{b}{2a}q_e^T c)(q_{4e}I_{3\times3}q_e)\end{aligned} \tag{40}$$

The term $\frac{b^3}{2a^2}q_e^T J$ in (40) can be written as

$$\begin{aligned}\frac{b^3}{2a^2}q_e^T J &= \frac{b^3}{2a^2}q_e^T J^2 J^{-1}\\ &= \frac{b}{2a^2}q_e^T b^2 J^2 J^{-1}\end{aligned} \tag{41}$$

Using condition $acJ > b^2J^2$, we obtain

$$\begin{aligned}\frac{b^3}{2a^2}q_e^T J &< \frac{b}{2a^2}q_e^T(acJ)J^{-1}\\ &< \frac{b}{2a}q_e^T c\end{aligned} \tag{42}$$

So if $L_G V = 0$ then $L_f V < 0$ for all $x \neq 0$. This guarantees that the candidate $V(x)$ is a CLF for system (2).

Next we show that modified optimal control [13] with this CLF $V(x)$ yields global asymptotic stability. Using the basic concepts [13] the time derivative of a CLF $V(x)$ is

$$\begin{aligned}\dot{V} &= \frac{\partial V}{\partial x}(f(x) + Gu)\\ &= L_f V + L_G V u\end{aligned} \tag{43}$$

Substituting (28) into (43) yields

$$\dot{V}(x) = -\sqrt{(L_f V)^2 + x^T Q x L_G V R^{-1}(x)(L_G V)^T} \tag{44}$$

Clearly $\dot{V}(x)$ is negative definite and global asymptotic stability has been proved.

Letting $v^* = u_{opt}$ and substituting (28) into (19), a new controller design has been obtained.

V. Simulation Results

An example of a rigid-body micro satellite [19] is presented with numerical simulations to validate and compare both controllers. The spacecraft is assumed to have the inertia matrix

$$J = \begin{bmatrix}10 & 1.0 & 0.7\\ 1.0 & 10 & 0.4\\ 0.7 & 0.4 & 8\end{bmatrix} \quad \mathrm{kg}\cdot\mathrm{m}^2$$

The weighting matrices are chosen to be $Q = \mathrm{diag}(1,1,1,5,5,5)$ and $R = \mathrm{diag}(5,5,5)$. The initial conditions are $q_e(0) = [0.3 \quad -0.2 \quad -0.3 \quad 0.8832]^T$ and $\omega_e(0) = [0.06 \quad -0.04 \quad 0.05]^T$ rad/s. For the SDRE controller the control vector is designed using (19) with an optimal control (14) while for the CLF controller we use (19) with optimal control (28). For the switching function design (17) we choose the same constant matrix K. $K = \lambda I_{3\times3}$ with $\lambda = 0.2$. To obtain $s(0) = 0$ the initial ϕ is chosen to be $\phi(0) = -(\omega_e(0) + Kq_e(0))$. Suppose that the desired angular velocities are

$$\omega_r(t) = \begin{bmatrix}0.05\sin(\frac{\pi t}{100})\\ 0.05\sin(\frac{2\pi t}{100})\\ 0.05\sin(\frac{3\pi t}{100})\end{bmatrix} \quad \mathrm{rad/s.}$$

The tracking problem is considered in the presence of external disturbance $d(t)$. The disturbance model [19] is

$$\begin{aligned}d(t) &= 0.01\times\begin{bmatrix}2\sin(\frac{\pi t}{100}) + 2\sin(\frac{2\pi t}{100}) - \cos(\frac{3\pi t}{100})\\ -2\sin(\frac{\pi t}{100}) - 2\sin(\frac{2\pi t}{100}) + \cos(\frac{3\pi t}{100})\\ 2\sin(\frac{\pi t}{100}) + 2\cos(\frac{2\pi t}{100}) + \sin(\frac{3\pi t}{100})\end{bmatrix}\\ &\quad + 0.2\times\begin{bmatrix}\delta(70,2)\\ \delta(80,2)\\ \delta(90,2)\end{bmatrix} + \begin{bmatrix}0.005\\ 0.005\\ 0.005\end{bmatrix} \quad \mathrm{Nm.}\end{aligned} \tag{45}$$

where $\delta(t_i, \Delta t_i)$ denotes an impulsive disturbance with magnitude 1 and width Δt_i seconds, activated at the time instant

t_i.

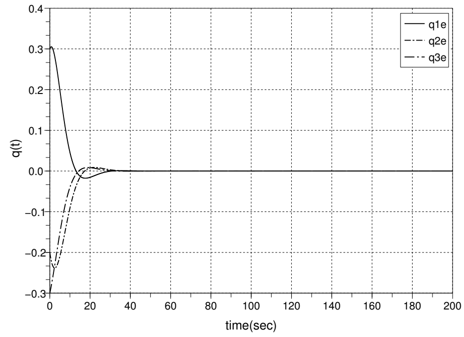

Fig. 1. Quaternion error using controller S (SDRE).

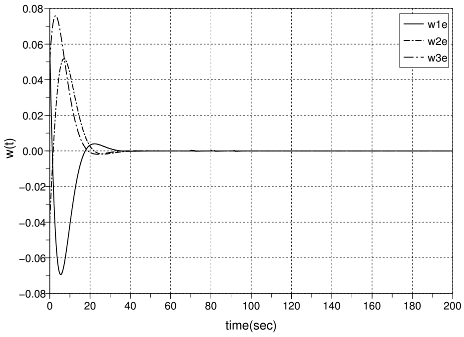

Fig. 2. Relative rate error using controller S (SDRE).

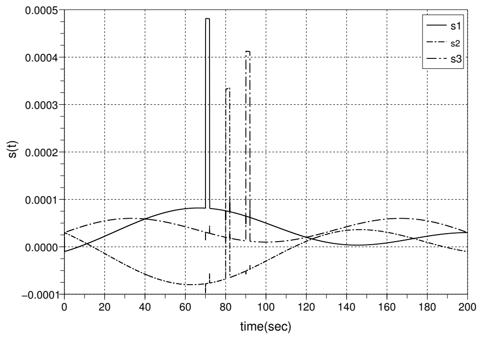

Fig. 3. Sliding functions using controller S (SDRE).

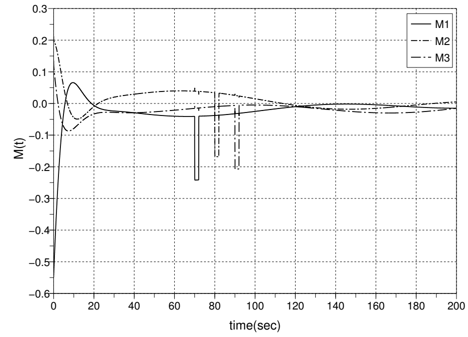

Fig. 4. Control torques using controller S (SDRE).

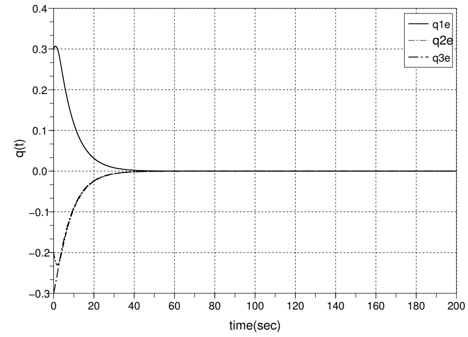

Fig. 5. Quaternion error using controller C (CLF).

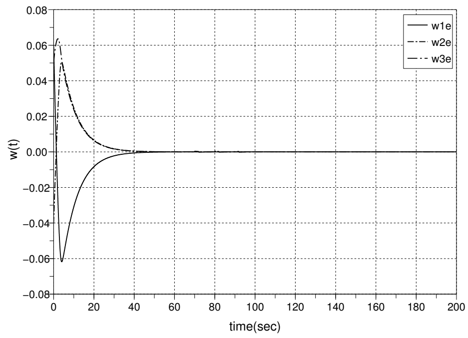

Fig. 6. Relative rate error using controller C (CLF).

The simulation results of the SDRE and CLF controllers are compared. As shown in Figs. 1, 2, 5 and 6 for the SDRE controller the quaternion and angular velocity error reach zero after 35 seconds while for the CLF controller is attained after 50 seconds. For both controllers, the sliding vectors (Figs. 3 and 7) are on the sliding manifold at time zero and very close to zero thereafter. Since integral sliding mode control is applied, there is no reaching time. The effect of external disturbances is apparent.

As shown in Figs. 1 and 5 both controllers stabilize the closed loop system of the rigid spacecraft. The effect of external disturbances on the tracking outputs is reduced. In Fig. 8 the trajectories of control torques for the CLF approach show a faster rate of change during the first 10 seconds when compared with the SDRE approach. In view of these simulation results the SDRE controller is considered more suitable for practical spacecraft manoeuvres of this specific type. Other model and tracking manoeuvres may yield different behaviour. However, the success of the SDRE approach depends on a good choice of the matrix $A(x)$. It is difficult to obtain global stability because of the limitations

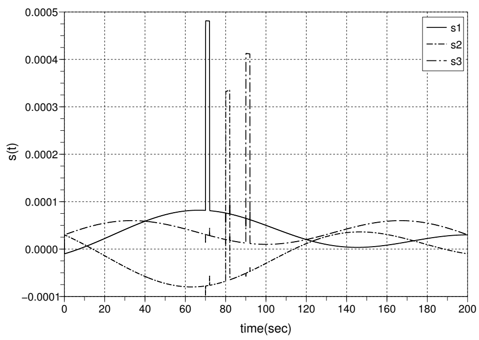

Fig. 7. Sliding functions using controller C (CLF).

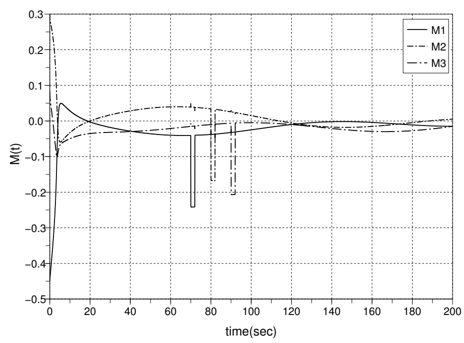

Fig. 8. Control torques using controller C (CLF).

of this technique. On the other hand the CLF approach has an explicit formula which yields global asymptotic stability. Its implementation for an optimal controller design is probably better when compared with the SDRE method.

VI. Conclusion

We have studied two controller designs using the integral sliding mode to control some spacecraft tracking manoeuvres. Our new optimal sliding mode control laws have been successfully applied to the spacecraft tracking manoeuvres. To obtain these controller designs integral sliding mode control combined with first order sliding mode and optimal control has been applied to quaternion-based spacecraft attitude tracking manoeuvres with external disturbances and an uncertain inertia matrix. The state dependent Riccati equation (SDRE) and the control Lyapunov function (CLF) are used to solve the infinite-time nonlinear optimal problem. The second method of Lyapunov theory is used to show that tracking is achieved globally. An example of multiaxial attitude tracking manoeuvres is presented and simulation results are included to verify the usefulness of these controllers.

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