Philosophical Issues from Kripke's 'Semantical Considerations on Modal Logic'

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Abstract.   In 'Semantical Considerations on Modal Logic', Kripke articulates his project in the discourse of "possible worlds". There has been much philosophical discussion of whether endorsement of the Kripke semantics brings ontological commitment to possible worlds. However, that discussion is less than satisfactory because it has been conducted without the necessary investigation of the surrounding philosophical issues that are raised by the Kripke semantics. My aim in this paper is to map out the surrounding territory and to commence that investigation. Among the surrounding issues, and my attitudes to them, are these: (1) the potential of the standard distinction between pure and impure versions of the semantic theory has been under-exploited; (2) there has been under-estimation of what is achieved by the pure semantic theory alone; (3) there is a methodological imperative to co-ordinate a clear conception of the purposes of the impure theory with an equally clear conception of the content the theory; (4) there is a need to support by argument claims about how such a semantic theory, even in an impure state, can fund explanations in the theory of meaning and metaphysics; (5) greater attention needs to be paid to the crucial advance that Kripke makes on the precursors of possible-worlds semantics proper (e.g. Carnap 1947) in clearly distinguishing variation across the worlds within a model of modal space from variation across such models and, finally, (6) the normative nature of the concept of applicability, of the pure semantic theory, is both of crucial importance and largely ignored.

Keywords: Kripke; possible-world semantics; pure and applied semantics; models of modal space; applicability.

1. Introduction

1.1. Overview

The two-fold aim of 'Semantical Considerations on Modal Logic' (Kripke 1963) is to show (in outline): (i) that a family of quantified modal logics can be supplied with a theory of validity of one standard kind (the model-theoretic kind) and, subsequently, (ii) that many such logics have related completeness properties. Kripke (p.64)[1] introduces and narrates his project via an intriguing reference to possible worlds, the intent and ultimate significance of which is unclear. This choice of idiom, naturally, inspires great philosophical interest: but especially so in light of his readers' awareness of the far more sustained and less casual invocation of possible-world talk in his later *philosophical* masterpiece *Naming and Necessity* (Kripke 1980). In this paper, I shall touch on those issues that have dominated the philosophical discussion

arising from Kripke (1963) in the subsequent half-century. But my main purpose is to present and commend a broader perspective on the surrounding philosophical territory within which those well excavated issues are only a part of what is visible and important. Among the salient themes of the broader approach are these: (1) that the potential of the standard distinction between pure and impure versions of the semantic theory has been under-exploited; (2) that there has been under-estimation of what is achieved by the pure semantic theory alone; (3) that there is a methodological imperative to co-ordinate a clear conception of the purposes of the impure theory with an equally clear conception of the content the theory; (4) that there is a need to support by argument claims about how such a semantic theory, even in an impure state, can fund explanations in the theory of meaning and metaphysics; (5) that greater attention needs to be paid to the crucial advance that Kripke makes on the precursors of possible-worlds semantics proper (e.g. Carnap 1947) in clearly distinguishing variation across the worlds within a model of modal space from variation across such models and, finally, (6) that the normative nature of the concept of applicability, of the pure semantic theory, is both of crucial importance and largely ignored.

1.2. The Germ

A possible-worlds semantic theory is a theory that aims to illuminate certain semantic features of expressions and which deploys the concept of a possible world in order to do so. The possible-worlds semantic theory intimated in Kripke (1963) is an interpretation of quantified modal logics and the semantic feature it aims, primarily, to illuminate is validity.[2] Kripke writes:

> To get a semantics for modal logic we introduce the notion of a (normal) modal structure ... [which] ... is an ordered triple (G,K,R) where K is a set, R is a reflexive relation on K and $G \in K$. (p.64)

The semantic theory intimated here is one whose concepts are those of the mathematical theory of models and those concepts do not include that of a possible world. The intimation of a possible-worlds semantic theory arrives with the subsequent comment:

> Intuitively, we look at matters thus: K is the set of all 'possible worlds'; G is the 'real world' and ... H_1RH_2 means intuitively that H_2 is 'possible relative to' H_1 i.e. every proposition *true* in H_2 is *possible* in H_1. (p.64)

But the non-mathematical discourse of possible-worlds is thus introduced here without any clear guidance as to how we are understand the intended significance of this development. Between what appears to be scare quotation ('possible worlds')

and the appeal to intuition ('intuitively') do we have a recommendation to think of matters this way or not? And, recommended or not, is the thought in question that this possible-worlds interpretation is distinguished from others by being the intended interpretation? Or is it the quite different thought that thinking in terms of possible-worlds is a user-friendly, but otherwise strictly insignificant, heuristic. To clarify matters we need the distinction between pure and impure conceptions, and versions, of a semantic theory. That sound advice was first issued by Plantinga (1974, p.126–8) and it appears to have been well taken. For it is now a well established ritual in philosophical discussions that broach possible-worlds semantics that there should be mention of the availability of a pure (formal, mathematical, algebraic) conception of the project. However, subsequent observance of the distinction is often less than diligent and, certainly, I contend, the philosophical value of implementing it rigorously has not been maximised. In this paper, I aim to prosecute the distinction to full effect as the basis of a fruitful re-drawing of the map of the philosophical territory that surrounds Kripkean possible-worlds semantics and which aims to bring neglected regions to the fore.

2. QUIDO Logics and QUIDO Structures

Consider a first-order logic with identity, given purely syntactically by: (a) a grammar on a vocabulary ($\wedge$, $\sim$, $\supset$, $\leftrightarrow$, $\exists$, $\forall$, $=$, $x_1, x_2 \ldots F^1, G^1 \ldots F^2, G^2 \ldots F^3, G^3 \ldots$) and (b) a classical proof-theory. On that basis, we assume that we have fixed and at our disposal the class of theorems of such a logic. We then extend the logical vocabulary to include two sentential monadic operators, box "$\square$" and diamond "$\Diamond$". Each of these has exactly the same syntactic profile as "$\sim$", so substituting either for an occurrence of "$\sim$" preserves well-formedness. "$\square$" is taken as part of the primitive lexicon and "$\Diamond$" is introduced by definition, thus: $\square A =_{\mathrm{df}} \sim\Diamond\sim A$. It merits emphasis that in Kripke (1963) there are no proper names or constant singular terms of any other kind in this vocabulary: not in the primitive lexicon, nor constructed out of it.

The class of logics ("systems") that ultimately concern Kripke (1963), I shall label QUIDO logics. These are so labelled because they are various proof-theoretic enhancements of **QU**antified non-modal logic with identity that are **I**ntensional with respect to the box ("$\square$") of our **D**ual **O**perators.

We can achieve a rough and locally adequate proof-theoretic characterization of *intensionality* as follows: a logic is intensional whenever when it is necessary and sufficient for the unrestricted inter-substitutivity of A and B that $\square(A \leftrightarrow B)$ is a theorem. The specific intensional systems under consideration, then, are the quantified versions of the propositional systems that are labelled (respectively), **M**, **B**, **S4** and **S5**, and which are constructed as follows.

The system **M** has the following axiom schemes and rules:

> A0.   The theorems of the propositional logic[3]
> A1. $\Box A \supset A$
> A2. $\Box(A \supset B) \supset (\Box A \supset \Box B)$
> R1. $A, A \supset B \ / \ B$
> R2. $A \ / \ \Box A$

We obtain the system **S4** by adding to **M** the axiom scheme

> S4. $\Box A \supset \Box\Box A$

We obtain the system **B** by adding to **M** the axiom scheme

> Br. $A \supset \Box\Diamond A$

We obtain the system **S5** by adding to **M** the axiom scheme

> S5. $\Diamond A \supset \Box\Diamond A$

Above and throughout, A, B, C ... are taken for complex formulae that are built from atomic formulae P, Q, R ... by use of the connectives $\wedge$, $\sim$ and $\Box$. All such formula-expressions, atomic or complex, are characterized as 'propositional variables' (p.64).

There ends the syntactic account of QUIDO logics and we turn now to the semantics.

The pure model-theoretic interpretation of classical first-order logic (with identity) is given via a class of models, in each of which there is a domain set d an a assignment function, ψ, that maps: (i) atomic predicates of the language to extensions from the domain of the model and (ii) closed sentences onto semantic values in the range $\{T, F\}$ according to familiar recursive rules governing the first-order logical constants. The pure Kripkean interpretation of the QUIDO logics is given by enriching such an interpretation two-fold.

Firstly, we interpret whichever IDO logic is the propositional core of the QUIDO logic. This calls for an IDO *model-structure* $\langle K, R \rangle$ involving a set K on which there is a binary relation R. A model on such a structure is a binary function, ϕ, from pairs $\langle P, H \rangle$ into the range $\{T, F\}$ where: P varies over atomic formulae, H varies over K-members and $\{T, F\}$ are pure set-theoretic objects that are not further defined or identified. Taking for granted the cases of the constants of ordinary propositional logic, the assignment function is defined for the characteristic dual operators of the IDO logic as follows:

$$\phi\langle\Box A, H\rangle = T \quad \text{iff} \quad \left\{ \begin{array}{l} \forall H'(((H' \in K \wedge R(HH')) \supset (\phi\langle A, H'\rangle = T)); \\ \text{otherwise, } \phi\langle\Box A, H\rangle = F; \end{array} \right.$$

$$\phi\langle\Diamond A, H\rangle = T \quad \text{iff} \quad \begin{cases} \exists H'((H' \in K \wedge R(HH')) \wedge (\phi\langle A, H'\rangle = T)); \\ \text{otherwise, } \phi\langle\Diamond A, H\rangle = F. \end{cases}$$

Secondly, for the interpretation of the full QUIDO logic, that is built around the IDO core, we require a QUIDO model structure, $\langle R, K, D, \psi(H)\rangle$ in which: K and R are as before, D is a domain-set and ψ is a function from K into the power set of D (so each K-member has a local domain $\psi(H)$). A model on a QUIDO model-structure has two components: (i) a function $\xi\langle F, H\rangle$ takes every pair of a primitive predicate and a K-member into an extension, which is a subset of the local domain $\psi(H)$ and (ii) an expanded version of the binary function, ϕ, from pairs $\langle P, H\rangle$ into the range $\{T, F\}$ according to recursive rules for each of the primitive QUIDO constants. The rules for the IDO constants are as per the IDO models and those for the primitive quantifier $\exists$ are as per the orthodox models for first-order logic (p.64–5). In particular, a quantificational formula $\exists x A$ in which x is bound, is evaluated at H by taking the quantifier to range over only the local domain local domain $\psi(H)$ (not over the global domain D).

All of the above, being part of the proof-theory or the model theory for QUIDO logics, is within the jurisdiction of mathematical logic and raises only those philosophical questions that (such) mathematics, in general, raises. This claim is scarcely contestable, and it is endorsed by — inter alia: Plantinga (1974, p.126–7), Quine (1980a, p.173–4), Forbes (1985, p.70–1), Lewis (1986, p.17) and Williamson (2013, p.85). To elaborate in the customary way, the semantic part of this "algebraic" conception of QUIDO logics is a *pure* semantic theory. As such, it says nothing about the existence, nature or extent of possible worlds, of possibilia, or of possibility of any kind. Equally, it says nothing about the existence, nature or extent of the angels, or the cities or anything else (beyond, perhaps, the pure sets) that might — for all we know — instantiate one or other such pure structure. To serve the philosophical purposes of distinguishing rigorously the sort of pure semantic theory just introduced from any further concerns that might arise from its further associations, we ought to speak of such a pure theory, and the class of logics it interprets, only in the austere and non-suggestive terminology that has been used so far. In particular, it is important to resist the temptation, when matters are contained in this sphere to call the logics *modal* logics and to call such an interpretation a *possible-worlds* semantics. It would be no less tendentious, and just as unhelpful, to call these things celestial logics and angelic semantics, or urban logics and metropolitan semantics. To adopt and implement this policy on nomenclature is to obtain very cheap insurance against the confusions, conflations and illicit presuppositions that are apt to take hold if we lapse into speaking otherwise.

3. The Pure Semantics

The Kripkean pure semantic theories described have the potential both to host various sorts of semantically-based elucidations of modality and also to import associated philosophical commitments. That potential, in both respects, is fulfilled when a pure semantic theory is *applied* to a modality and such applications will be discussed extensively in later parts of the paper. However, by focusing on pure QUIDO structures and models on them, we can anticipate the places at which an impure semantic concept of a certain kind and shape can take root and at which associated conceptual and metaphysical commitments are apt to ensue.

3.1. IDO Structures

Recall the IDO model structure, $\langle K, R \rangle$ with binary relation R, set K and models in which an assignment function, ϕ, takes pairs of $\langle$ formulas, K-members $\rangle$ into one of the semantic values $\{T, F\}$. This structure *genus* can be differentiated into species according to the natural mathematical properties of its elements. Since K is a *set*, the first natural difference to consider is that between the empty case and the non-empty case. Since R is a binary relation, the salient differentiating characteristics to consider are those definable out of reflexivity, symmetry and transitivity. For present purposes we need attend only to these differentia of K and R, and henceforth we consider only species of structure in which K is non-empty.[4] Respect for the initial epistemological independence of the semantics from the syntax motivates the introduction of proof-theoretically neutral names for all species of structure. Thus, one species of IDO structure will be labelled the REF-structure, because it is defined by the requirement that the R-relation should be a reflexive relation (on a non-empty set). Similarly, we have the SYMM-structure (R is symmetric), the TRANS-structure (R is transitive) and obvious composites of these terms such as the SYMM-TRANS-structure (R is symmetric and transitive) . . . . and also the EQUIV-structure (as a convenient label for what would canonically be called the REF-SYMM-TRANS structure).

   The strongest *natural* semantic feature of formulas that is suggested by these pure resources, is one that is definable at the level of the genus (that is, on *all* IDO-structures), thus:

(ULTRA-VAL)   ULTRA-VAL (A) iff $\forall \phi'$ on all IDO-structures, and $\forall H \in K$,
                    $\phi'(A, H) = T$.

   The next natural kind of semantic feature suggested is definable on the same pattern but — down a taxonomic level — in terms of species IDO-structures, thus — for example:

(EQUIV-VAL)   EQUIV-VAL (A) iff $\forall \phi'$ on all EQUIV-IDO-structures and $\forall H \in K$, $\phi'(A, H) = T$.

I note three points about Kripke (1963) in relation to the above. Firstly, Kripke never isolates IDO structures: the simplest structures he considers are IDO+ structures (see immediately below). Secondly, Kripke does not define, or even consider, a VAL concept at the level of structure genus: no concept considered is at the level of our ULTRA-VAL, all are at the (species) level of our EQUIV-VAL. Thirdly, Kripke restricts his attention throughout to those IDO-structures (and subsequently QUIDO-structures) that are of the REF species. Since he does so with an application in mind, the restriction is not made in the spirit of purity.[5] Nonetheless, since it simplifies matters to follow Kripke's lead on this point, I shall do so.

3.2. IDO+ Structures

The simplest model-structures considered in Kripke (1963) are not IDO structures but richer IDO+ structures in which there is an additional place for a distinguished K-member, G, — thus: $\langle G, K, R \rangle$. The genus of IDO+ structures is differentiated in exactly the same way as the genus of IDO structures is differentiated: that is on K and on R as indicated above. The primary theoretical facility afforded by the richer structures is (partial) de-relativization of our concept of evaluation. Instead of a value (T or F) attaching to a formula at a K-member and in a model, it can be attached to a formula in a model *tout court* — thus:

(ϕ-MOD) $\phi^*(A) = n =_{\mathrm{df}}$ iff $\phi(A, G) = n$ (for n a variable over $\{T, F\}$).

We can view (ϕ-MOD) as intimating an explicit and reductive definition of its definiendum in terms of the (doubly-relativised) primitive predicate of the theory, $\phi(A, H)$. However, in his consideration of IDO logics and QUIDO logics as logics of standard modalities, Kripke (1963) never puts the IDO+ structures to this work — nor, indeed, to any other.[6,7]

3.3. QUIDO+ Structures

A QUIDO model structure $\langle K, R, D, \psi(H) \rangle$ is an IDO model structure enriched by a domain-set D and a function, ψ, from K into the power set of D (so each K-member has a local domain $\psi(H)$). A QUIDO+ model structure is similarly built on a core IDO+ structure, thus: $\langle G, K, R, D, \psi(H) \rangle$. All of the closed formulas of the QUIDO logic are assigned semantic values by models on these structures: values at a K-Member in the model (QUIDO) or values in the model (QUIDO+).

QUIDO structures afford obvious opportunities to further differentiate species, qua mathematically natural kinds, within the genus. We still, from the IDO case, differentiate according to features of R (and insist on non-empty K). But we may now combine that with differentiation by features of the function $\psi(H)$ and, in particular, by features of the domain, D, of that function. The role of $\psi(H)$ is to assign a local domain to each K-member, and among the *set-theoretically* salient and natural differentiating conditions on domain-variation (for a structure) are these: that each subset of D should be the domain of at least one K-member; that the intersection of distinct K-domains should always be empty; that the value of $\psi(H)$ is constant across K-members and that at every K-member, H, $\psi(H) = D$.[8] I assume, henceforth, that the primary species of QUIDO structures are those defined by such mathematically natural conditions on $\psi(H)$ and by those on R that were introduced previously. As throughout this pure phase of the enquiry, it is an open question whether any such mathematically natural kinds of structure will prove salient in any particular application of the pure semantics.

It is noted that the species of QUIDO structure that are circumscribed in Kripke (1963) are constrained *only* with respect to R: they are entirely unconstrained with respect to $\psi(H)$. Thus various notable species of model-structures (for each R-kind) are *admitted*. These include — notably: those with models whose K-members have intersecting domains and those with models whose K-members have varying domains. For these varying domain models there are admitted two significant sub-cases: (i) downward variation, in which some K-member domains $\psi(H)$ have members that are not members of the distinguished domain $\psi(G)$ and (ii) upward variation, in which members of $\psi(G)$ are absent from some other $\psi(H)$. The species of QUIDO structures that we are asked to consider in Kripke (1963) are less VAL-friendly than those in which more constraints on $\psi(H)$ are imposed — for example, as by Kripke (1959). For absence of constraints breeds variety of models, and variety of models makes for variety of potential counter-models to VAL theses. Famously, the Kripke (1963) deregulation of the domain assignment function has the effect of inVALidating the Barcan Formula (BF), its Converse (CBF) (Barcan 1946) and (double) Necessitism ($\Box\forall\Box\exists$)):

(BF) $\forall x \Box A \supset \Box \forall x A$

(CBF) $\Box \forall x A \supset \forall x \Box A$

($\Box\forall\Box\exists$) $\Box \forall x \Box \exists y (y = x)$[9]

QUIDO+ structures, the other structures that we might develop from the resources they already encompass and models on these structures are further differentiable into species that are both mathematically (fairly) natural and applicable to modality.[10] It is noted that — in the course of his brief remarks on certain other QUIDO models that he does not himself exploit — Kripke expresses his conception

of his project in a telling way — thus:

> For the purposes of modal logic, we hold that [his own models and others] represent alternative conventions. All are tenable. (p.66)

It is difficult to ignore the implicature that there might be purposes other than those of a semantic theory of modal logic, for which establishing the status of a type of model as a tenable convention is not good enough. So we have here a warning about inferring from adequacy to one purpose, to which the semantic theory speaks directly, adequacy to another such purpose, to which it does not speak directly.

3.4. The Pure Work of QUIDO Structures and their Relatives

It is important to identify the work that is done by the pure QUIDO structures, and their relatives, themselves prior to, and independently of, any application to which they might be put.

Certain proof-theoretic results might be established without any appeal to semantic structures. These are intra-syntactic results. Equally, certain results about the relations and orderings among species of structures might be established without appeal to any proof-theoretic considerations. These are intra-semantic results. But by considering how proof-theories relate to QUIDO-structures, in particular, we can further illuminate both and establish inter-categorical results. For the purposes of this paper, there are three kinds of such inter-categorical result that deserve to be highlighted.

Firstly, and crucially, *soundness* and *completeness* are inter-categorical matters par excellence. QUIDO structures allow us to establish lemmas about what is X-VAL over various species of structure X. The X-completeness and X-soundness of a proof theory ϑ is a matter of its capacity to generate as theorems all and (respectively) only such X-Valid formulas. Kripke reports completeness theorems for the quantified versions of each of the systems **M**, **B**, **S4** and **S5** (p.70) referring to their demonstration in Kripke (1959). Once these results are established, it is understandable that the nomenclature for QUIDO structures should be altered to conform to the proof-theories that they perfectly fit — hence the Kripkean terminology of **S4**-structures, **S5**-structures etc. However, it promotes a greater appreciation of the inter-categorical results to set out by using an independent semantic vocabulary and reporting results in an explicitly inter-categorical way. Thus, the report that the system **S4** is sound and complete with respect to REF-TRANS structures is one that conveys more of the achievement than does the nonplussing report the axiom system **S4** is so related to **S4** structures. What needs to be appreciated is that a class of formulas that is natural by one sort of criterion has been discovered to be the very same class of formulas that is natural by a quite different sort of criterion.

Secondly, it is open to us to define various inter-categorical concepts of optimality of QUIDO logics and, building on the soundness and completeness results, to establish pertinent results about these concepts in extension. One such optimality feature is that of being the strongest QUIDO logic (proof theory) that is both sound and complete with respect to the simplest species of model structure. Another such optimality feature is being the weakest proof theory that is both sound and complete with respect to the most demanding kind of model structure.

Thirdly, the soundness and completeness results yield subsequent Safety Results that are sufficient to justify the practice of doing QUIDO logic by proxy in the calculus of first-order logic. The logic of the pure semantic theory, through which pure semantic results are established is a first-order logic. Therefore, we can use first-order logic to set about establishing the VAL-status of any QUIDO formula *A*. But given the crucial inter-categorical results in the background, we are then entitled to draw conclusions about the THEOREM-Status of any QUIDO formula *A*. So a first-order demonstration of X-VAL for a QUIDO formula, *A*, allows us, given completeness, allows us to infer the X-Theoremhood of *A*. What we can also do, and what we cannot (typically) do within the proof theory alone is to demonstrate via first-order methods Non-theoremhood in a QUIDO logic. For a first-order demonstration of X-INVAL allows us, given soundness, to infer Non-(X-Theoremhood). Thus, all that can be established by the use of the proof theory for QUIDO System X — and more! — can be established by the deployment of first-order methods alone.

In sum, even though they rest on semantic considerations it is within the realm of *pure* semantic considerations that all of the following, inter-categorical matters for QUIDO logics — *and any subsequent application of them* — are framed and settled: VAL-ness, SOUND-ness, COMPLETE-ness, OPTIMALITY and SAFETY. And all of this is achieved by QUIDO structures with no need of any enrichment of them.

The formation of QUIDO+ structures, as with the formation of IDO+ structures affords a facility that Kripke (1963) does not exploit: that is, the definition of semantic evaluation relative to a single index (ϕ) as a special case of semantic evaluation relative to an index pair (ϕ, H).

Another relative of a QUIDO structure is its distinguished model. I mark this feature explicitly in the vocabulary of the pure semantic theory by a singular-term-forming operator "!", so that "Π!" means *the distinguished model of structure species (or genus)* Π. The facility offered by this new distinguishing move, as with its predecessor is (further) de-relativization: now, the definition of semantic evaluation simpliciter (*) as a special case of semantic evaluation relative to a single index (a model index) (ϕ). To have a value simpliciter or absolutely is to have that value at the distinguished *K*-member in the distinguished model:

(ABS) $*(A) = n$ iff $(\phi(A, G) = n) \wedge \phi = \Pi!$

We do not need distinguished models or distinguished elements of models whenever the definitions and results require only that we consider what is the case for all models (for an arbitrary model) or only what is the case for every K-member in every model (for an arbitrary K-member). In that event, QUIDO structures (or IDO structures) suffice.

It is to be noted, then, that Kripke (1963) in his treatment of standard modality does not make this new distinguishing move (among models) either. There is no play with distinguished models (of any structure) and there is no attempt to define — by that route or otherwise — a formula's having simpliciter the semantic values T, F.[11]

3.5. Philosophical Commitment Points in QUIDO Structures and their Relatives

I presume Quinean (orthodox and conservative) sufficient conditions of ontological commitment — thus: one who endorses a semantic theory is ontologically committed — at least pro tem — to the sets over which it appears to quantify and to whatever things one is committed to taking to be the members of those sets.[12] Accordingly, I shall presume that one who endorses a pure Kripkean semantic theory for QUIDO logics is committed (pro tem) to the existence of those sets, and their members, that are intimated by the models of the relevant structures. The distinctive ontological commitments that arise from an applied version of such a pure semantic theory then arise, potentially, from two structure points in the models. Firstly, there is intimated in all relevant structures, models that contain a non-empty K-set. The bare minimum (pro tem) commitment associated with any application of any pure QUIDO semantic theory is commitment to at least one such model, to one such set and to one such set-member. However, and although it is very often overlooked, the *potential* for ontological commitment to K-members significantly outstrips commitment to any K-set that is an element of *any* single model. For, in all foreseeable cases, the theories asserted will be many-model theories, and with such theories comes commitment to an uberset, $K+$, which is the union of all the K-sets from across the models.[13] So the ontological K-hierarchy has uberset $K+$ at the top, then sets K and then K-members. Secondly, in all QUIDO structures — and sparing repetition of the reasoning from the K-case — there is also potential commitment at every level of the ontological D-hierarchy, which has uberset $D+$ at the top, then model-sets D and then D-members.

Various points of potential conceptual or ideological commitment are also identifiable in and around QUIDO structures and their relatives. I shall not, here, open the question of the differences between conceptual and ideological commitments and the different conditions under which these different kinds of commitment ensue. Suffice to say that we ought to be sensitive to those meaningful but non-quantificational facets or aspects of the theory that at least raise the question of such non-ontological

commitments. And there are two salient cases of this sort. Firstly, within both IDO+ and QUIDO+ structures there is position, definitive of those structures, *G*, intimating a feature of *K-member-distinction*. Secondly, assignable to any model structure term, there is the ! label intimating a feature of Π-*model-distinction*.

This concludes the account of the pure semantics for IDO and QUIDO logics that can be distilled from Kripke (1963).

4. The Application of the Pure Semantics to Modalities

4.1. Applications in General and Modal Applications in Particular

The impure semantic riches suggested by such pure Kripkean semantic theories have proved not so much a philosophers' paradise as a philosophers' playground. For, perhaps impatient to grab what appears to lie in wait, we philosophers have been inclined to rush in without displaying due care at those crucial points at which it is our job as philosophers to be careful. From the very start, the "application" is, in this context, dangerously ambiguous between a semantic theory with a certain impure *content* and a *purpose* to which that theory is put. So a more rigorous approach is required.

As I shall frame matters, an impure semantic theory is a version, or instance, of a pure semantic theory that: (i) has as its object a given pre-identified subject matter; (ii) has the form of a given pure semantic theory and (iii) involves commitment to specific content at some (at least some) of the philosophical commitment points identified points in the pure theory.[14] Moreover, such an impure theory ought to be associated with a purpose where the notion of purpose is two-dimensional, involving: (iv) a certain particular range of semantic features that are intended for elucidation and (v) a conception of the kind of elucidation intended.

On subject matter, I shall be concerned only with impure semantic theories that are intended to be about a pre-identified modality. Various subject matters are susceptible to logical analysis in terms of QUIDO-systems and QUIDO-structures: the propositional attitudes, temporality, games, quantum mechanics etc.[15] But I shall not be concerned with impure semantic theories whose intended subject matter is so palpably non-*modal*. I cast my subject matter as modality of a *pre-identified kind* in the conviction that we can recognize and name kinds of modality after reflection on our practical concerns and (non-semantic) theoretical projects. Thus, illustrating in the mode of possibility we have: technological possibility, nomological possibility, epistemic possibility, legal possibility, logical possibility, metaphysical possibility etc. It is a matter to be settled further down the line, if at all, how these kinds are precisely individuated or how they are related to each other. The relevant adjectives and adverbs (technological, metaphysically etc.) are what are supposed to fit in the

place marked by "X" in various specifications of the further elements of the QUIDO structure (X-possible worlds, X-possibilia etc.). But I shall suppress the relativizing term, "X" when operating within a context that allows that. Furthermore, I shall presume that we are dealing with aspects of modal subject matter that are comfortably representable in regimentation by the dual operators of QUIDO logics: but I will not broach here the issue of which aspects those are.

QUIDO logics become *modal* logics as a matter of *intention* and QUIDO models become part of a possible-worlds semantic theory as a matter of their *content*. For philosophical clarity, QUIDO logics should be called *modal* logics (when and) only when they are deployed with the intention or purpose of representing the inferential properties of modal expressions: expressions which our usage, or perhaps stipulation, has already settled as being modal. Associated Kripkean semantic interpretations of such modal logics should be called *possible-worlds* semantic theories only if they are intended to represent given semantic features of the modal expressions, via appropriate models, in terms of quantification over possible-worlds (and where the proponent of the theory takes the entities in question to be deserving of the "possible-worlds" name). Thus, we are now concerned with logics for a pre-identified modality in which □ and ◊ are dual operators, of necessity and possibility and with possible-worlds semantic theories of these logics.[16]

4.2. Possible-Worlds Semantic Theories: Minimal Content and Conditional Modal Results

Pure Kripke semantics becomes an impure possible-worlds semantic theory (for a quantified logic of a given modality) with the selection of a species of QUIDO structure and the postulation of special, impure, possible-worlds models that conform to that structure. If understanding is to advance, we must think of this postulated possible-worldly content as involving both a minimal (shallow and horizontal) development and also, potentially a (deep and vertical) development.

The minimal development involves a specification of the elements that amounts only to owning the names that they have in a possible-worlds semantic theory — committing to a conception of those names as non-arbitrary and appropriate. Thus, to take the comprehensive case of a distinguished QUIDO model, $\langle G, K, R, D, \psi(H) \rangle!$, we have ownership of the following vocabulary: the actualized world, (G); the set of all the X-possible worlds, (K); the relation of relative X-possibility (aka X-accessibility), (R); the set of all X-possibilia, (D); the set of possibilia that exist in possible-world H, $\psi(H) \rangle$ and the intended space of possible worlds, given by $\langle G, K, R, D, \psi(H) \rangle!$. And below this level of description we do well also to at least allow for ownership of an "at-ness" locution of representation by a model. This is available as a locution that potentially unifies what is represented by a world about its domain (existing at) and what is the case there (true at).

So, to illustrate intent, what is left open at this point is whether or not possible-worlds are to be taken, further, as things of any of the kinds that they might (in this context) reasonably be supposed to be — maximal sums, sets or conjunctions of propositions, states of affairs, properties, pictures or spatiotemporally-located individuals. How exactly we proceed within that limitation is a question of deep content to which we shall return. But what is supposed to be ruled out now, in this context and at this stage, is that the possible worlds of the models for a modal logic might be taken to be the drawers in my desk, the nations of the earth or anything else that is not of that intended (maximal, worldly) ilk that we can recognize when we see it. With that minimal specification of possible-worldly content in place we can fill out the impure semantic picture by owning as versions of the semantic predicates the more widely deployable: True, (T); False, (F) and Valid, (VAL). Three important points can be introduced following these clarifications and each will subsequently be elaborated.

Firstly, we can now substitute all relevant impure terms in the appropriate pure definitions and claim the ensuing results. Secondly, it is no longer an open question whether such results are really claims about the semantic features of a given modality rather than about something else: given the facts about the subject matter intended and the content assigned that *is* what they are doing. Thirdly, however, there is an important sense in which the relevant results — which are undoubtedly theses about the semantic features of a given modality — are still only conditionally established.

We can confidently introduce the impure semantic terms into pure definitions and ensuing results because: (a) the results of the pure semantic theory are established with respect to *all* models that are instances of given structures and, therefore, (b) they devolve to any impure semantic theory as we understand that here, since (c) each is but a sub-theory in which the models are a special restricted range of all of those that are instances of the structure. Of course, if you refuse point blank to accept the existence of possible worlds then you will not accept that there are models that contain them and so you will not accept that there are such things to *be* among the totality of models. But otherwise, you have in prospect impure results about — say — the Validity of the B schema for nomological necessity, the Completeness of an **S5**-logic of metaphysical modality and the Safety of using first-order methods to determine that the S5-schema is not a theorem of **S4**. With the pure results established unconditionally, the ensuing impure results are in prospect. But wherever we are concerned with a given pre-identified modality (nomological, metaphysical, etc.) the results are, emphatically, in prospect *only* and remain conditional. The condition that pertains, and whose satisfaction remains to be established, is that the given modality *merits* being modelled in the relevant species of pure structure. We return to that all-important issue in 5 below.

4.3. Modal Spaces and their Hyper-Spaces: Variation Across Worlds versus Variation Across Models

It is appropriate, as a matter of minimal commitment to a possible-worlds semantics for a modality, to say that each model represents a version of X-modal space — a version of the space of metaphysically possible worlds, of the technologically possible-worlds, etc. A possible-worlds semantic theory of a modality, then, commits its proponent to the existence of whatever such models are required to represent an appropriate variety of such modal spaces. We must then be vigilant in observing the semantically crucial distinction between the familiar variation across possible worlds within a model and this recently noted variation across modal spaces within the logical hyperspace of that modality. Once we register this distinction various tempting and casual characterizations of relevant pluralities are revealed as invitations to confusion that ought to be refused.

Firstly, we are well advised to avoid using the term *logical space* as a blanket term for whatever is represented by an arbitrary model, for this is doubly treacherous. For one thing, the semantical interpretation of the *logic* of a modality is something that is executed at the level of a modal hyperspace and not at the level of a modal space: matters of logical validity are matters concerning all of the admissible models of the totality of (say) the metaphysically possible worlds. So one would more accurately say that the *logical space for the metaphysical modality* is a logical hyperspace and not any one space of metaphysically possible worlds. For another thing, if we can neutralize that first threat of confusion and wish to persevere with calling some modal *space(s)* "logical", we should do that only if it is the logical modality, identified as such, that is the intended subject matter of the semantic theory. *If* it is the logical modality, pre-identified as such, that is our subject matter, then it is appropriate to say of the impure models we invoke that each represents a version of logically modal space — a space of logically possible worlds. And if one is prepared to commit to there being an intended model in that case, calling *it* logical space (tout court) might be put down to harmless ellipsis — ellipsis on "the space of logically possible worlds in the intended model". But if you begin by presenting your intended subject matter as anything other than "logical" modality then substantial work, and explanatory responsibility, must be undertaken in order to earn the right to apply the "logical" epithet to the modality subsequently in the course of semantic, or metaphysical, theorizing. The point is most telling when we are entertaining the prospect of there being modalities that are absolute but characterizable other than as logical. For it should be a substantial hypothesis, and not a matter for casual presumption, that the intended model of the metaphysically possible worlds is — even in extension — an accurate representation of the intended space of logical possibility. It is a different and no less substantial hypothesis that the logical hyperspace for logical modality is

identical to the logical hyperspace for metaphysical modality. Equally, it is unjustified to set out by helping ourselves to the "logical" epithet in this context as a synonym for maximal or absolute. There are substantive and difficult questions about whether any kind of alethically modal space is absolute, or even maximal, in its inclusion of all alethic modalities — either in the intended case or otherwise. And these questions ought not to be spoiled by the un-necessary and presumptious characterization of (de dicto) whatever models or spaces that have these features as "logical".[17]

Secondly, we are also well advised to avoid transferring casually to the semantics of modal logic the argot surounding the semantics of non-modal logic in calling alternate models representations of "different possibilities" or "different ways that an X-modal space *could* be". To show that caution, is required I offer an example of a line of thought that has some prima facie plausibility. Take as hypotheses: (a) that nothing is both red all over and green all over and (b) it is absolutely necessarily so. In the case of non-modal logic, (a), and modal logic, (b), we might expect unintended models to represent the falsehood of the hypothesis in question. But in that case what is represented in those unintended models is not properly called a *possibility*. For what are truly possibilities have already been exhausted by what is represented (as true) in the intended model and the negation of hypothesis (a) is *not* represented in (any possible world in) that model. So what is represented in the worlds in the alternates to the intended model are not invariably and obviously (other) possibilities.[18] There is however another subtle distinction to be observed here.

Even if one were entirely convinced that the alternate models in the semantics of a modality do not invariably represent possibilities, there is something else that might be granted. By hypothesis, then, what the unintended models do not do in such a case is to represent something that is in fact possible. But it is a different matter whether what is represented by (a world in) a model of a modal logic is thereby represented *as* possible. This is the sense of *represented* that includes *misrepresented* as a special case. In the case of the interpretation of non-modal logic we might easily acquiesce in the idea that the unintended models misrepresent by representing as non-modal fact that which often is contrary to non-modal fact. That is what the totality of such unintended models is for: it is to represent all cases of being contrary to (non-modal) fact that is permitted by the definition of a model for the non-modal logic. In the case of the interpretation of modal logic, then we might equally acquiesce in the idea that the unintended models misrepresent by representing as modal fact that which often is contrary to modal fact. That is what the totality of such unintended models is for: it is to represent all cases of being contrary to modal fact that is permitted by the definition of a model for the modal logic. So the moral of the story is that the difference between variation within and variation across modal spaces is one that necessitates our distinguishing carefully the claim that what is represented *is* a possibility from the claim that it is being represented *as* a possibility.

It is noted, finally, that this distinction suggests a way of understanding Kripke's use of quotation in the infamous passage cited above at the outset — thus:

> Intuitively, we look at matters thus: K is the set of all 'possible worlds'; G is the 'real world' and ... (p.63)

For perhaps this is not *scare* quotation at all but intimation of the following idea: that for all possible-world-models of the given structure, by having a world w as a member of such a K the model thereby *calls w* a "possible world". This is invariably representation of a world *as* possible and but certainly not invariably accurate representation of a world that *is* possible. Representation, by a model, of a world as possible is the analog of the more familiar notion of representation of a possible world, by itself, as actualized: the latter is supposed to be achieved just by a possible world's being a possible world and in all but one case it is a feat of self-misrepresentation.

4.4. Identifying The Commitment-Drivers in Possible-Worlds Semantic Theories

There are three points (shared by the pure models of IDO+ structures and of QUIDO+ structures) at which impure ideological or conceptual commitments for a possible-worlds semantic theory are foreshadowed. The points are: (i) the AT, or representation, feature found with claims that something exists at or is the case at a possible world (in a model); (ii) the distinguishing feature, within a model, of a world as actualized and (iii) the distinguishing, within the space of models, of a model as intended. Any possible-world semantic theory that incorporates models at any level of structure requires commitments involving some version of the AT feature. If such a theory is concerned only with Validity and not with Truth its distinctive non-ontological commitments end there. Any possible-world semantic theory that is concerned with Truth simpliciter requires, in addition, commitments involving the intendedness feature and the actualization feature. We have seen that the pure theory distilled from Kripke (1963) is of the former kind: accordingly, its impure instances are limited in respect of non-ontological commitment to an AT feature (or features) alone. To anticipate deep matters, momentarily, in the cause of illustration, ATness might be taken as: homogeneous or heterogeneous over exists AT and true AT; it might in either case be taken as primitive, or as (in some sense) theorizable in terms of set-membership, or part-hood or some sort of consequence relation.

It is, to say the least, a rather important feature of the Kripke (1963) theory that its range — and taking the author at his word — is limited to the semantic aspect of the logical concept of validity. Granted, it is not easy to see why we would have any philosophical interest in the notion of Validity other than in virtue of its connections to Truth. But a semantic theory of Validity for a modality which, for its philosophical

significance, *presupposes* Truth (à la Kripke (1963)) is quite a different animal from a semantic theory that seeks to encompass within its range an elucidation of Truth.

The starting point for the identification of drivers of ontological commitments in possible-worlds semantic theories is, of course, consideration of the case of commitment to the existence of possible worlds. In that case, our Archimedean point is as follows. Any possible-world semantic theory that incorporates models at any level of structure requires commitment to the existence of at least one possible world.[19] And when we are concerned with the territory of quantified modal logics there is also commitment to the existence of the possibilia individuals that exist in that world at least. However, it is strictly consistent with the minimal characterization of a possible-worlds semantic theory that it should be ontologically committed only to one, actualized, possible world and only to the actualized possibilia that exist at it.[20] Ontological commitments to non-actualized possible worlds and to non-actualized possible individuals are driven by certain specific commitments about the extent of modal truth: specifically, by (certain) denials of necessity. It is to such negative claims that the theory assigns truth-conditions that have the logical form of existential quantifications. For positive claims of necessity have the logical form of, non-existentially-committing, universal generalizations. What follows is a swift characterization of the necessities whose denial is effective in the respective cases.

Firstly, in a one-world model we will have, in all relevant structures, modal truths but also modal collapse. Whatever is true at the actualized world is, of course, thereby true at some world and thus we have truths of possibility. But in a one-world model, whatever is true at the actualized world is also thereby true at every world and thus we have truths of necessity.[21] That also settles that there will be truths of impossibility — as long as it is not the case that absolutely everything is true at the actualized world. For what is false at the actualized world, in such a model, is thereby false at all possible worlds and, thus, an impossibility. It is a requirement of there being modal truths of non-necessity in a model — that is, truths of contingency or of possibility-not — that it is a many-world model. For given that P is true in the actualized world, possibly-not-P will also be true at that world only if there is an accessible world at which not-P. Under the supposition that that P is true in the actualized world (and with ongoing classical restrictions) the actualized world itself cannot be such a world. And so, the truth of possibly-not-P requires the truth of another world and, given that there is but one actualized world, that other world will be a non-actualized world. All of that concerns what goes on in a model — an arbitrary model — and raises the question of whether a world that is non-actualized in a model is a world that is non-actualized simpliciter. The answer to that question, on all pertinent and reasonable presumptions must be negative. For in a perfectly legitimate contrary-to-modal-fact representation in a unintended model, a world other than the (really) actualized world would be *represented as* the actualized world of the model. So, fi-

nally and with the intended level of precision, the precise source of commitment to non-actualized possible worlds in commitment to there being at least some modal truths (simpliciter) of the form possibly not-*P*. For it is these truths (simpliciter) that require the existence in the intended model of possible worlds that are represented as non-actualized: and it is those worlds that really are the non-actualized worlds *tout court*. For convenience, I shall refer to this precise source of commitment to the existence of non-actualized worlds somewhat less precisely as commitment to contingent truth (or to contingency).[22]

Secondly, as is familiar, the question of the existence of possibilia (other than possible worlds) arises only with quantified modal logics via their QUIDO structures and the element D. Consider, then a QUIDO model in which — cutting to the case we know to be crucial — the intended model is such that it contains only the one, actualized, possible world and the domain of possibilia that exist at it. Without metaphysical prejudice we label these the actualized possibilia. It will be true in such a model that necessarily each of these exists and necessarily that all of these exist. For in this model all (actualized) individuals exist at every possible world. Expansion of such a model with respect to the number of worlds will have no impact on which modalized *existence* claims are true unless we have variation in domains across those worlds. As long as there is constancy of domain, changes in extension of many other predicates will occur as we move from world to world (the representatives of x is blue, x is taller than y ...). But the extension of the predicate (that represents) y *being such that x is identical to it* will remain identical to D, and to $d'(w*)$. Thus each and every actualized individual (indeed each and every possible individual in the model) will exist at every possible world. Taking the hint from the earlier case of propositional modal logic, we seek the generator of non-actualized possible individuals in the realm of contingency. But in this case, not just any relevant contingency will do. What we need is the contingency born of upward domain variation: not that born of downward domain variation. The semantic conditions of truth (simpliciter) of a claim *that some x that exists might not have done* are these: there should be a non-actualized world at which x does not exist and that, clearly, is not a requirement of the existence of any possible individual. The semantic conditions of truth (simpliciter) of a claim *that some x might have existed but does not* are these: there should be a world at which there exists some y and that y is not identical to any x that exists in the actualized world. That requires that the y in question is not in the domain of the actualized world and that is to say it is a possible individual that is not an actualized individual which is to say — finally — a non-actualized possible individual. For convenience, I shall refer to this precise driver of commitment to the existence of non-actualized possibilia as commitment to truths of possible extra existence.[23,24] I do not — of course — claim that those commitments to contingency that drive commitment to the non-actualized entities is, in either, case inescapable.

One could choose to avoid commitments to non-actualized possible worlds by declining to commit to any contingent truth and one could choose to avoid commitments to non-actualized possible worlds by declining to commit to truths of possible extra existence.[25]

Thirdly, there is the subtle, difficult and neglected question of whether commitment to impossible worlds is driven by the proponent of a Kripkean possible-worlds semantics for modal logics who has orthodox commitments about the extent of Validities. The point here is not one about the driving of a commitment to the existence of impossibilia by any commitment in the realm of modal *truth*.[26] It is a point about the logical hyperspace of the modality rather than about the intended modal space. The suspicion of such a commitment to the existence of impossible worlds is rooted in an intuition about what models (ought to) do.

The relevant intuition about the unintended models for a logic is that their point is to represent variation in non-logical fact. In the case of modal logics, then, the point of unintended models would be to represent as possible, inter alia, what is contrary to modal fact (but not contrary to modal-logical fact). And that, indeed, suggests that there should be impossible worlds in unintended models — as long as one is in the market for a modality that is absolute but (in some relevant sense) inequivalent to the strictly logical modality. So what might thus be envisaged as being represented as true in the worlds of unintended models would be analytic impossibilities, such as something's being both red all over and green all over, or metaphysical impossibilities such as something's having water in it but no Hydrogen. But whatever else is going on here, it will be noted that the picture appears to require constancy of meaning of non-logical predicates across models and so across the variations that changes of model permits. For say that the intended model represents the modal fact of the necessity of water containing Hydrogen by a certain formula being true at every possible world in the model: $\forall x(Wx \supset Hx)$. Then if it is the case that another model represents, contrary to modal fact, that it is not necessary that water contains hydrogen, and it does so by containing a world at which that formula is false, then that requires that the formula in question means what it meant before in the other model. For only then can we presume that the modal fact represented as holding in one model is the very modal fact that is represented as failing to hold in the other. It is not obvious that this picture is coherent. But if it is coherent it appears to impose an interesting and significant limitation on the power of the possible-world semantic theory. For if meaning is something that can persist across models it cannot be something that is even co-ordinate with any situation that obtains within any one model. The semantic theory of Kripke (1963), of course, expresses no aspiration to take the semantic feature of meaning within its scope. But the point is worth noting for the benefit of those who would so extend it.

Another intuition about models, however, begins with the opposite thought to

that which emerges from the first. This intuition about the models (for modal logics) is that their point is to represent meaning-variation: to display all the variations of meanings of expressions that the value-assignments of the model theory permits. We might add, although this is not strictly an essential part of the thought, that the model theory for the logic is fixing the meaning of what it treats as the logical expressions (in this case, expressions including the modal operators) while displaying all the variations of meanings of the non-logical expressions. In any case, what we lose on this view is the idea that what unintended models are doing is representing as possible a content (or a fact) that the intended model represents as impossible. What the unintended models are doing, more like, is representing variations in the interpretation of interpretable expressions that are consistent with the trans-model interpretation of the logical constants. What the evaluation of the model theory does is to fix the truth-value-relevant behaviour of the logical constants at every world in every model. And to investigate the consequences of this, and this alone, for validity of formulas containing these expressions it allows the meanings of all other expressions to vary from model to model. In that case, if the intended model represents the necessity of water containing Hydrogen by having the formula, $\forall x(Wx \supset Hx)$, true at every possible world in the model, it cannot be read off from the falsehood of the same formula in a world in another model that what is represented is the non-necessity of water containing Hydrogen. So there is no commitment afoot in the latter representation to anything that deserves to be called an *impossible world*: for that would be a world one at which what is true is a content representing that which is contrary to modal fact.

However, having registered and developed these intuitions about the role of models, the effective question is that of the variation across models which is required by the semantic theory at hand. And, I shall argue, those requirements align with the second intuition in not generating commitment to impossible worlds out of natural commitments about the extent of validity.[27]

In this matter, it is the understanding of the syntax of the semantic theory itself, rather than that of its object-language, that is crucial.

The kind of formula that is properly called Valid is an axiom scheme and is a formula of a semantic meta-language. Take, then, the case of the scheme:

S4. $\Box A \supset \Box\Box A$

Here, Kripke calls 'A' a propositional variable and that gets the point across that the schema is not a thing that is true or false but at most a thing that has true or false instances. The true and false instances of formulas involving 'A' are closed sentences of the object-language. But just as no such meta-language schema is properly called True, no object-language sentence is properly called Valid. Now, take as a hypothesis

that the following sentence of English, with its usual meaning, expresses an (absolutely) necessary truth: " All water molecules contain Hydrogen". So the following is true: " It is necessary that all water molecules contain Hydrogen". And, with meaning preserved, let us represent that former sentence in our object-language in the notation of propositional modal logic alone and then in the notation of quantified modal logic as follows:

(1) $\Box P$
(2) $\Box \forall x (Wx \supset Hx)$

In both cases we have truth-evaluable sentences at the level of the object-language. The legitimate connection that these sentences have to the meta-linguistic concept of Validity is via the following question. Does either sentence display a *schematic form* that is Valid? That is to ask if either of the following schemes of the meta-language is valid:

(1*) $\Box A$
(2*) $\Box \forall x (\vartheta x \supset \Pi x)$

And, of course, neither scheme is Valid, given that box, as is now fixed, is an operator of alethic necessity. For that would be to say that every closed sentence of the relevant form would be true in every model and thereby in the intended model. And thereby we would have either, (1*) the necessary truth of every closed sentence — and hence a reduction, in the case of any *A* and Not-*A* — or, not much better, (2*) of at least every universal generalization. So we are bound to accept such orthodox commitments to Invalidity and to postulate appropriate worlds. But the counter-worlds in question need be only worlds at which some instance of the relevant schema is false. For recall, the only candidate around for being true or false is such a schema instance. Accordingly, all that is required to invalidate the schema is the falsehood in a model and a possible world of a (truth-evaluable, object language) sentence that shares a form (1*) with (1) or the form (2*) with (2). And that, of course is NOT to require the falsehood in a model and a possible world of (1*) or (2*) nor of the (ex hypothesi) necessary truths P or $\forall x (Wx \supset Hx)$. In other words to distance claims of necessity from claims of validity in a modal logic we need not even go near sentences that have the same non-modal content (for example, about water and about hydrogen) as those that in fact express the necessities in question. We need only consider closed sentences that have the same modal-logical form and, moreover, to find the right sentences we may not even have to look beyond the intended model. Thus, I conclude that orthodox commitments about Invalidity have *not* been shown to impose on the proponent of our possible-world semantic theory a commitment to the existence of impossible worlds.

4.5. The What and The Why of Non-Minimal Content

Non-minimal, or deep, content is added to a minimally impure possible-worlds se-
mantic theory by an increase in philosophical commitments about the entities and
concepts involved. Primarily, deep content is a matter of metaphysical or conceptual
elaboration — for example: a metaphysical account of the kind of thing that a pos-
sible world is; a conceptual or ideological account of the predicates of accessibility
or actualization or a characterization of the domain over which the quantifiers of the
theory range. Such an account may be offered is the spirit of reductionism: that is by
offering an informative definition, analysis, re-description or identification. Such an
account may, equally, be offered is the spirit of primitivism: that is by asserting unde-
finability, unanalysability or insusceptibility to theoretically significant re-description
or identification. No doubt, further and more sophisticated theoretical attitudes are
available. But for present purposes I shall take just these two kinds of non-quietist,
non-neutral, non-agnostic and positive theoretical stances, on each relevant question,
to be exhaustive.

 The vast majority of philosophical discussion around possible worlds has been
taken up with such issues of deep content. Should we best think of the possible worlds
as (maximal) sums of spatiotemporally located individuals, as propositions, as states
of affairs, as properties or as sets of any of the above? Should we take the terms
"possible world" and "actualization" as semantic primitives or as definable? Should
we distinguish, ideologically, the actual from the actualized? If so, should we take the
non-actualized possibilia to be identical to things that actually exist, and in which we
already believe, or is our commitment to them to be viewed as a warranted expansion
of ontology? I have pursued these questions extensively elsewhere (Divers 2002) and
my aim here is not to re-open them. Rather it is to make a methodological point
about the appropriate approach to such questions following reflection on an impure
Kripkean possible-worlds semantic theory for quantified modal logics.

 My methodological contention is that taking a position on deep content is sen-
sible only if it is well motivated, and that is to say purpose-driven. The pre-eminent
rationale for a deep content proposal should conform to the following pattern. Given
that one takes the purpose(s) of the semantic theory to be this (these), one's op-
tions for thinking about the concepts and entities involved are those, and among
these options, theoretical considerations give such reason as we have for thinking
about the concepts and entities involved in this way rather than in that way. This
much is rational. But it is not obviously rational to insist in advance of requirements
of purpose that possible worlds must be abstract entities, or must be characterizable
adequately only via the involvement of primitively modal concepts etc.[28] In any case,
as I understand it, the project of developing a possible-worlds semantic theory, from
the minimal stage is to take a position on content that will support adequately the

identification of an intended range of models of modal space for *a given modality* and *for given purposes*. Indeed, it is tempting to treat as pseudo-questions those of what possible worlds or possibilia *really* are, unless there is a context of purpose onto which we can project the question as one about best fit to a theoretical role.

4.6. **Purpose: Semantic Range and Elucidation Type**

The conception of purpose that we need is two-dimensional: one dimension is that of semantic range, the other is that of the type of elucidation intended of what lies in that range.

The dimension of semantic range can, in the present context, extend maximally over a theory of truth, a theory of validity and a metalogic. Here, a metalogic is just a facility to define and address the extension of the already encountered, and validity-dependent, concepts of soundness, completeness, safety and optimality. Kripke (1963) offers a semantic theory whose range is a theory of validity and a metalogic. To repeat an important point, that the interest of such a theory should in some sense presuppose truth is granted but that is not the same as the theory encompassing truth within the range of semantic concepts that ait aims to elucidate. In any case, to cover all relevant ground, I shall continue to consider a Kripkean semantic theory that is of maximal relevant range and so encompasses a theory of truth within its range.

The dimension of elucidation type covers the various kinds of improvement of perspective on modal truth and validity that we might think that we acquire as a result of the theory. For present purposes, I shall distinguish five prima facie types of elucidation.

The first and least ambitious level of claim that might be made is that the elucidation afforded is (merely) *heuristic*. This is the name that I choose for a category that is needed in order to file all versions of the following idea. That what we are given by Kripke's (half-hearted) suggestions about how we may think of the elements (as possible worlds etc.) is a way of thinking and speaking that is attractive, but not in itself of any logical or philosophical cash value. It is, all will agree, in many ways easy and natural for us to imagine, think of and speak of a space of objects (cf Quine 1969). And when we speak of possible worlds as forming a modal space (as they do in each model) iterated modal expressions, in particular, become tractable in away that they would not be otherwise. Indeed, when attempting to come to terms with, say, the impossibility of contingent necessity say, it is all but irresistible to think terms of a space of worlds, and a journey around it, once we are aware of that option. However, the heuristic thought goes, if we do gain any serious insight by indulging in the idiom it is collateral and indirect: it is like doing so by way of getting a joke, or a bump on the head or grasping a metaphor (cf Davidson 1978). It might even be argued that such heuristic elucidation is a pervasive, critical and essential part of

our beginning to theorize any subject matter (cf Quine 1980). So heuristic need not carry the suggestion of intellectual insignificance or triviality. There may well be *deferred* philosophical value in the heuristic since it may suggest to us how to go about, or how not to go about, those projects in which the achievement of philosophical benefit immediately and directly *is* the point. But in pleading (mere) heuristic, one unloads both immediate costs and immediate benefits. We have no obligation to give any deep account of what we are saying, but nor are we entitled simply to repeat what we are saying — even if so little is demanded of us than to tell literal truths about the semantic features of the modal idioms.

The second level of claim that might be made for the elucidatory capacity of the theory is that it is, or yields, a useful *instrument*. The defining idea here is that the primary philosophical value of the theory lies with its providing techniques or methods that we are entitled to use in order to reach conclusions about validity: about which modal conclusions do and which do not, follow from which modal premises. So the instrumental conception of the theory is validity-focused and we put aside whether it allows us any facility will modal truth as a by-product. The central instrumentalist value in prospect is that (appeal to) possible-worlds semantic theory allows us to do modal logic by proxy through only first-order methods. The glaring challenge that any such instrumentalist must confront is to say something about why the first-order methods are reliable. And then a dilemma looms. On one hand, the would-be instrumentalist might then be drawn into making deeper claims about the impure semantics, in which case she is no longer just an instrumentalist about the impure theory. On the other hand, the would-be instrumentalist might point out the following truth: the completeness of various logics, as guaranteed by the pure semantics, is available to her as a guarantor of the safety of the first-order methods. But in that case, we need have here nothing that requires the play with impure semantic theory. What we may have is no more than the combination of a user-friendly idiom whose philosophical cash value does not derive from its content, even minimally construed. This dilemma fuels the suspicion that that such instrumentalism about a genuinely impure possible-worlds semantic theory of validity may be an untenable position that is set to collapse under minimal scrutiny.

The third level of claim that might be made for the elucidatory capacity of the theory is that it is, in at lest some aspects, a literally accurate representation of the semantic facts about the relevant modal expressions and the sentences in which they figure. I shall take this as a conception of the theory as one that tells the (literal) truth about semantic features of modal expressions.[29] However the truth, although perfectly literal, may yet be qualified. When we pay attention to the theory we may find that some of the truths that it affords are conditional statements or truths that rely for their truth on the presence of some crucial relativizing, index. But, for all that, the claim is that there is literal truth to be had. My principle claim about the

hierarchy of elucidatory ambition is that it is *here*, with commitment to literal truth, that metaphysical and conceptual responsibilities ensue and questions of deep content kick in. At any lower level of ambition what is being claimed does not demand a position on deep content: but commitment to this type of elucidation does. Now that we have truth: what is primitive, what is not and how so?[30]

The penultimate level of claim in the hierarchy of elucidatory ambition is that the impure semantic theory is true and *semantically explanatory*. Thus, the explanations discerned will be of those semantic facts about validity for modal inferences and perhaps truth for modal sentences that the theory is explicitly equipped to generate and display.

The ultimate level of claim, then, is that the impure possible-worlds semantic theory is (true and) *supra-semantically explanatory*. Various philosophical conceptions of how we should go about the theory of meaning, or metaphysics or (perhaps even) epistemology, will encompass a conception of truth-values and, especially, *truth-conditions* for relevant sentences: truth-conditions that are generated in a certain way or specified in a certain form. The prospect then arises that our possible-world semantic theory treats the truth-conditions of modal sentences in a way that the proponents of these conceptions will recognize, or at least allow, as philosophically propitious for their supra-semantic purposes. Ultimately, then, a possible-worlds semantic theory, may be held to contribute to such explanations as those of the nature of possibility (and necessity), the meaning of modal sentences or the source of modal knowledge. Arrival at this ultimate level of claim of *supra-semantically explanatory* power for the semantic theory calls for an interlude.

4.7. Truth-Roles in a Semantical Theory of Logics, and Beyond

The Kripkean semantics for quantified modal logics in its pure state already allows us to make an important structural connection between sentences and inferences. We can see that whenever a complex sentence of a certain form is Val there is a corresponding inference that is T-preserving for every degree of relativization of T: T-in-a-*K*-member-in a model; T-in-a model and T-simpliciter. A class of corresponding impure (and partial) semantic theories is introduced via uniform substitution of 'T' occurrences by a predicate of truth (and of 'F' by a predicate of falsehood). On one way of looking at the matter, we then have here, a *Truth-role* that is defined in terms of other semantic concepts. In the first part, the definition is explicit since Truth is defined as Truth-at-the-actualized-world-in-the-intended-model. But the definition of the un-relativized Truth-role bottoms out in an implicit definition of the primitive and relativized Truth-role: one which ensures that the un-relativized Truth-role can be played by any property that is distributed and preserved over sentences as the evaluation rules of the semantics require. In sum, what we get from the impure se-

mantics is a partial definition of a non-relativized truth-predicate, Θ^1, in terms of the impure and relativized truth-predicate, Θ^3 and (what are identified as) the logical constants of the language $(\wedge, \ldots, \square)$:

(TΘ) Θ^1 is the feature such that: (i) $\Theta^1(A)$ iff $\Theta^3(A$, actualized world, intended model) where (ii) $\Theta^3(A)$ is the feature that is distributed and preserved thus: $\Theta^3(B \wedge C, H, M)$ iff $\Theta^3(B, H, M)$ and $\Theta^3(C, H, M)$ ... and ... $\Theta^3(\square B, H, M)$ iff for all $H*$ accessible from H, $\Theta^3(B, H*, M)$.

However — and rightly or wrongly — potential interest in the impure semantics of quantified modal logics greatly outstrips our interest in logic alone. For the semantics: (a) circumscribes a truth-role that applies to modal sentences and thereby (b) promises to illuminate modality via connections between that truth-role and other truth-roles that are partially (and in the first instance, very loosely) defined in terms of high-level theoretical commitments in the theory of meaning, in metaphysics and in epistemology — thus:

(TΨ) The meaning of a sentence, S, is given by its Ψ–conditions.
(TΦ) The ontological commitments associated with asserting a sentence S are to those things that make that sentence Φ.
(TΩ) A belief that S is knowledge that S when S is warranted and S is Ω.[31]

In real cases, we should expect to see expressions of finer-grained commitments with correspondingly finer-grained roles specified in further terms from the given source of interest — for example, in the case of meaning:

(TΨ*) The meaning of a sentence, S, is given when its Ψ–conditions are presented in a certain way $\wedge$ To understand a sentence is to know what its Ψ–conditions (so presented) are.

And any such refinement of grain only serves to sharpen the obvious challenge to any proponent of the semantic theory. That is to take a position on: (a) what kind of supra-semantic significance, if any, the impure Kripkean metalogical theory of validity for quantified modal logic is supposed to have and (b) the basis on which any such connection is claimed (working hypothesis, argument, etc.). The maximal position in the former respect would be that there is (at least) one alethic property that unifies all of the conceptually disparate truth-roles by being relatable to each in appropriate ways. And a multitude of intermediate combinations are available for consideration. Perhaps it will be held that some/any alethic property that is apt to play the validity role is also apt to play the meaning-role, but not to play the metaphysical-role. But then philosophical satisfaction demands that we are also given a story about the grounds or basis on which such a position is being taken. Were there

no such connections to be made or at least worthy of exploration, it is difficult to see why there would be any interest in an *impure* version of the meta-logical theory of validity. However, I note that there is no claim of supra-semantic significance made for the semantic theory in Kripke (1963). Indeed, it would be of significance to establish any source at all in which a connection between the possible-worlds semantics for quantified modal logics and any supra-semantic concerns is supported by anything more substantial than tacit presumption. Having made the point, I shall be concerned now only with the consequences of hypotheses of connections between the semantic and the supra-semantic: not the grounds on which they are claimed.

4.8. Co-Ordination Obligations

The primary obligations of the philosopher who deals in Kripkean possible-worlds semantics are: (a) to make clear the envisaged purposes of the theory (range and elucidation-type intended) and (b) to co-ordinate commitments of minimal and non-minimal content to those purposes. Such co-ordination requires respecting at every turn the questions of whether given resources, that are clearly identified, are sufficient or, especially, *required* for those purposes.

Take a philosopher who accepts (at least) that a possible-worlds semantic theory for quantified modal logics genuinely represents facts (literal truths) about the semantic features of X-modal expressions. In particular, the range of concern extends to truth and so the commitment is that the theory genuinely represents the facts about truth — simpliciter — conditions for X-modal sentences. This establishes properly purpose. We assume further that our philosopher has minimal orthodox views about the extent of X-contingent truth and X-possible extra existence (i.e. that neither category is empty). Following the discussion of minimal commitments and their sources, we know that such a philosopher is — at least pro tem — committed to the existence of: non-actualized possible worlds and non-actualized world possible individuals. What such a philosopher can or must say at the level of deep content is constrained by what further theoretical ambitions are afoot.

Here is a first example. If the semantic theory is intended as offering a theory of meaning and in a way that incorporates a meaning-reductive analysis of modal terms, that requires that it should present an account of truth-conditions for modal statements in non-modal terms. Accordingly, the ontology (and more) that is thus far presented in the key *modal* theoretical terms "possible-world", "possible individual", "relative possibility", etc. must be shown to be presentable in non-modal terms. It is not an option, given the stated further purpose. to take the modal terms of the semantic theory as semantically unanalysed primitives. Approaching from the other side, if the project (never mind why) begins with the insistence that the key *modal* theoretical terms retain their initial status as primitives (that are not eliminable by

explicit definition) then co-ordination requires abandonment of the project of providing a meaning-reductive analysis.

Here is a second example. On one conception of a theory of understanding we have to credit competent speakers of a language with having an internally representation (even implicit knowledge) of an appropriate compositional semantics that attributes truth-conditions, in a certain way, to the sentences of the language. Now, among the many obstacles that lie in front of that conception is the objection that it cannot appropriate a semantic theory of the sort that defines truth as truth in the intended model. Accordingly, she who would endorse the Kripkean semantic theory we have described as one that tells the truth about truth for modal sentences has a co-ordination problem. Naturally, such a theorist might simply abandon or have no interest in the proposition that her semantic theory can serve a theory of understanding. That would be a co-ordination success. But if serving a theory of understanding is maintained as a prospective supra-semantic role, co-ordination must be achieved otherwise.

Here is a third example. If the semantic theory is intended as explaining, or perhaps even only as accurately displaying that, metaphysically, in virtue of which true modal sentences are true then it must take at least many of its key prima facie metaphysical commitments as primitive. For example, and putting aside whether it is a very promising project, if the idea is to back the theory as displaying that truths of possibility are true in virtue of the existence of possible worlds and/or possible individuals, then there can be no question — given the character of such an explanation — of taking back or weaselling away the commitment to the existence of the possibilia. The possibilia might subsequently be characterized or identified in further terms but that is a different matter to their existence being disavowed. This latter point has an important generalization that is worth making explicit.

4.9. Co-Ordination Obligations and Deflationary Re-Interpretation of the Theory

It is suggested that we can, by a certain method, have "the benefits" associated with deploying a possible-worlds semantic theory without paying the ontological cost in commitment to non-actualized possible worlds or possibilia. The underlying method is that of offering a re-interpreting translation of the theory in a new medium from which the prima facie ontological commitments are absent. Moreover, in the most prominent cases, the effective device in the new medium is usually supposed to be a non-factive operator, inside whose scope existential quantifiers are freed of the ontological burden that they carry when unprotected. Thus, we have, in prospect, fictional-prefix re-interpretations of the possible-worlds semantic theory: for prefix fictionalism see Rosen (1990). And we also have, explicitly, various modal-operator

re-interpretations of the possible-worlds semantic theory: for such modalism see Prior & Fine (1978), especially Fine (1978), and Forbes (1985, 1989). For the proponents of such re-interpretation projects, the recognition of obligations to co-ordinate purpose with deep content will bring serious challenges from many sources, including the following observations.

Firstly, and quite generally, what we need to see in support of such projects are: (a) clear and definite conceptions of purpose and (b) reasons for believing that those purposes can be served — unamended — under the proposed re-interpretation. In the absence of those supporting elements, it is of no use to claim that we can have "the benefits" without the ontological costs. For what we have, at most, then, is a defence of the second conjunct by pointing to the non-factivity of an operator. And that alone, without a defence of the first conjunct, cuts no ice.[32] Secondly, it simply is not the case that all of the elucidatory ambitions associated with the deployment of a possible-worlds semantic theory will survive the discharging of ontological commitments. We have already seen counterexample, with the hypothesis of possibilia as truth-makers for possibility claims, and others are around the corner.[33] Thirdly, the very idea of constructing a theory for impure semantic purposes, in particular, and then re-interpreting it must surely start off under suspicion of incoherence. What is the literal truth about the semantic properties of modal expressions I am supposed to grasp by looking at the sentences of a theory that turn out to mean something other than what they appear to mean? The theory appears to say that the conditions under which claims of contingency are true involve the existence of non-actualized possible worlds. But it cannot be that truth I am grasping for, it turns out on re-interpretation, that the theory does not say that at all. Indeed, it cannot be saying that under re-interpretation for if it were we would not have re-interpretation at all, deflationary or otherwise.

Finally in this regard, let us re-assert the kind of semantic theory and the kind of elucidation that puts commitment to possible worlds and possibillia in prospect. In the negative, if we are taking the possible worlds semantic theory as a pure theory or if we are taking it as impure but only of heuristic value then such commitment is not in prospect. In those cases, we also have to co-ordinate accordingly what purposes might be served. But what is not required is deflationary re-interpretation for the theories, so understood, are not inflated (in the relevant sense) in the first place. The re-interpretation project is well motivated and apposite only if we have a possible-worlds semantic theory taken impurely and which is supposed to achieve the accurate and literal representation of something or other. But when one re-interprets the sentences in which such a theory is couched, to the suggested deflationary effect, it is no longer an impure possible-worlds semantic theory. So, in sum, it is hard to resist the idea that little of these re-interpretation projects will survive the scrutiny that is born of the elementary requirement to co-ordinate purpose with deep content.[34]

5. The Normativity of Applicability and Logic-Selection

5.1. The Appropriateness of a Model Structure for a Modality

The applicability of a particular pure semantic theory to the subject matter of a given modality essentially involves the association of that subject matter with a certain species of model structure. Since the selection of a structure determines which systems are sound and/or complete it is natural to say that the application of an impure semantic theory necessarily involves the attribution of a given logic as the logic of that modality. The subject of this final section is the normative conditions of applicability to a modal subject matter and (consequently) of logic-selection.

It is within a pure semantic theory that metalogical concepts are defined, and that results about their extensions established. The results in question offer, at least, benefits to our inferential practices. For example, Completeness lets us know that we need no further rules (all validites are already at our disposal in the preferred proof theory), it guarantees the safety of the practice of carrying out our modal business in the medium of first-order logic when is suits us to do so, and it allows us to show what cannot be derived from given rules. The availability of such enhancements to our modal reasoning marks an advance. How significant an advance is debatable. But, in any event, its availability is conditional. For what remains to be established in order that, all or any of, the semantic machinery should properly be at our disposal in modal reasoning in a given case is that we have identified an appropriate semantic structure — one that is *eligible* to model the particular subject matter in question. This is a good time at which to remind ourselves that definitions of Val-concepts that we find in the pure semantics of Kripke (1963) are all relativized to a given kind of model structure (**M**, **B**, **S4** or **S5**) and that this relativity transmits to the concepts of Soundness and Completeness. The resources are there to define a concept of ULTRA-Validity that would put no restrictions on the structure of models but it is telling that no philosophical reason has emerged to criticize Kripke for having passed on that option. The resources are there also to define Validity (completeness and soundness) simpliciter for a given modality X according to a (now) familiar pattern of appealing to a distinguished case:

> For Modality X, $Val^*(A)$ iff A is @-Val and @ is the right structure for X

But I cannot see how our philosophical interest could survive the removal of the residual relativization of rightness of a structure for a given modality. So it is all about which structure is right for a given modality. This is a normative matter: the structure that we identify for our (impure) semantic theory has to be appropriate for the given modality and we have to have something to say about our entitlement or justification in treating it as such. But how are we to address it?

The customary *non-method* is one of occasional ad hoc observations in a method-ological vacuum, and my aim is to improve on this position. My aim is not so ambitious as to establish that a given modal logic is in any case the right one: nor even to establish a protocol for selecting the right modal logic in general. More modestly, I hope to set out and order the considerations from which a reflective and systematic approach to these matters might be developed.

5.2. Some Basic Propositions of Methodology

The first item of methodological business is to embed awareness that we should not expect — or at least we are not entitled to presume — that just any considerations of modal truth will have a proper role in the selection of a validity-determining structure. Once we are in the business of subject matter-specific logics — modal logics, temporal logics, epistemic logics — the default expectation must be that there are generalizations about these subject matters that merit representation as true in the intended model but not, thereby, in all models. I shall not attempt to argue here that there are such cases in for any given modal subject matter — either in general or by example. Moreover, it may be that there is something about at least certain modal subject matters that allow us to argue, a priori, that they are not typical in this respect and that they afford no such cases. But once we are aware of the spectre of highly general but logically insignificant truths, the matter deserves attention and proper consideration before appeal to an arbitrary general truth of modality can be admitted as holding sway in the selection, via structures throughout a hyperspace of models, of a logic of modality.

If we are looking to compile an inventory of *all* the pre-semantic-theoretical features of a modality that might be relevant to the selection of a structure as eligible, and to weight them appropriately, the second item of methodological business is to register (a spectrum of) relevantly different kinds of practice in which our various modalizings are rooted. Some modalities are given to us in ordinary usage: typically idioms of practical modality of what can or can't be so or be done in a conversationally circumscribed context. Other modalities are given to us by technical, specialist or perhaps even scientific — but still non-philosophical — practice: typically the can's and cannot's of technology, of biology, of physics or even mathematics or logic. Then yet other modalities are given to us by philosophy itself, and not just the kind of philosophy that involves discerning or explicating what we find in ordinary or specialist philosophical practice: thus, for example, analytic possibility and metaphysical possibility. It might be that these different cases call for different *weightings* of potentially relevant considerations.

The third methodological proposition is that the case for modeling a particular modality in a particular structure cannot be made with reference only to pure terms.

Certainly, the pure semantics and the proof-theory will allow us to establish the locations at which certain qualities are optimal and are likely, for that reason, to prove the most desirable in relevant respects. Thus we can identify the systems that are proof-theoretically strongest (**S5**), or weakest (**M**), and those structures that are semantically least demanding on accessibility (REF) or most demanding (EQUIV) and — as earlier noted — we can then define and locate variously maximal or optimal inter-categorical combinations of these features. Nor, of course, is it out of the question that we could reverse-engineer a conception of a modality as just that modality that is salient in this or that logical respect. But no such considerations are sufficient for our stated purpose. For one thing, it is perfectly reasonable to ask for an account of why maximizing in some such respect is (optimally) desirable if it is to be presented as such. Moreover, and crucially, it is essential that an entitlement be established to adopt whatever logical package is supposed to be most desirable in the case of a specific modal subject matter. A pure study can tell us that the package that gives the strongest proof theory and the simplest structure of accessibility in extension is **S5**. But if that is the optimally desirable feature, we need to have a discriminating justification for deploying it as the logic of some modalities but not others. For, presumably, there is no question of a blanket justification for deploying the strongest logic of alethic modality in all cases just because it is the strongest.

The justification for selecting a modal logic for a given modal subject- must always involve appeal to considerations about that subject matter that relate it to independently desirable features of the logic.

5.3. The Consequences for Modal Logic of Adequately Identifying a Subject matter for Application

One candidate for having modal-logic-selecting significance is the battery of constraints that is exerted by standards of competent usage. If argument is wanted here it would best proceed by establishing a link to the methodological requirement that a prospective impure semantic theory needs to have a sufficiently stable and pre-theoretically identifiable subject matter. Criteria of competent usage will concern which (basic) assertions and (basic) patterns of inference are, by standards of competence, permissible, mandated or forbidden. And, the thought is, (some core of) these provide a body of non-negotiable, pre-theoretically given, subject matter-fixing data. Needless to say, there will be questions about whether any facts established through usage have this status and how we can recognize those that do. And many will be especially hostile to the candidacy if we label the intended constraints as what is *analytic* of the modal operators in question. But even Quine (1953) recognizes (in the non-modal case, of course, of identity) that subject-matter-fixing considerations are something of a sine qua non, and our aim here is to register all reasonable candidates.

I shall now comment briefly on the likely extent and effect of such considerations in the modal cases.

I venture that such considerations (always) settle the question of whether a modality under study is — and so ought to be treated in the semantic theory as — alethic: that is, as supporting necessity elimination and possibility introduction. For, I suggest, if alethic status is not so settled (prior to semantic theorizing) we do not have data of sufficient quality to circumscribe an appropriate subject matter. Certainly, if alethic status is not settled by criteria of competent usage, it is quite unclear how such modal idioms could be sufficiently disciplined to be viable, never mind useful. So there are two semantic-theoretically tractable cases to consider. If relevant data of usage circumscribe a non-alethic modality X, the selection of certain Kripke-structures as eligible and the subsequent application of a modal logic is not ruled out. But that case leads into a realm of Kripke structures where we will not follow. If relevant data of usage circumscribe an alethic modality, completeness requires a structure in which accessibility is, at least, reflexive: a REF-structure or above. However, it is natural to think that valuable although completeness may be, what is non-negotiable for the selection of a logic for a given modal subject matter is that it should be sound. Indeed, it is often asserted that a logic that is known to be incomplete is the right logic for a subject matter even when it is also known that it has "rivals" that are known to be complete.[35] What we need to do to settle soundness is to settle the question of whether, for a particular alethic modal subject matter, *more than* reflexivity is attributable to the accessibility relation. To that end, relevant considerations will naturally be sought by searching further under the heading of criteria of competent usage. But is not within my present remit to pursue that search.

5.4. The Consequences for Modal Logic of Modally Significant Relations

The question of the consequences for logic selection of subject matter fixing is one that, clearly, applies quite generally and not just to modal logic and modal subject matter. By contrast the next potential source of logic selection is not so general. It is not so particular as to be a feature of only the modal case: for it is a (pure) feature of all IDO and QUIOD modal structures and that means that it will figure in all applications of those structures with modalities being one such case. The source in question is facts about relevant relations.

A familiar pattern of argument for the selection of a modal logic, and one that is more at home at the theoretical end of our concerns with modality, proceeds as follows: (A) identify a relation that is relevant to the given modal subject matter; (B) establish the profile of that relation with respect to reflexivity, symmetry and transitivity; (C) identify the Kripke structure that embeds that profile and (D) con-

clude that the right modal logic for the subject-mater in question is that which is sound and complete with respect to that model structure. Thus one might seek to conclude that the right logic for nomological modality is **S5**, because the key relation is w has exactly the same laws as v, that expresses an equivalence relation and that profile defines the **S5** structure. Or, one might seek to conclude that the right logic of metaphysical modality is **S4** because: (further lemma) metaphysical modality is combinatorial; the key relation is *v is generable from the fundamental elements of w*; that relation is reflexive and transitive but not symmetric and that profile defines the **S4** structure.[36] To reiterate intent, my immediate remit is not to scrutinize cases but rather to comment on the rules of the game. And I have two such comments.

Firstly, then, it is evident that more than one relation will be relevant to the given subject matter. In the case of nomological modality, for example, we also have *no law of w is violated at v*. Presumably we can rely on the factivity of lawhood to make this reflexive, but it is certainly not symmetric or transitive and so it does not have a profile that selects the **S5** structure (but rather selects M). Perhaps the only sensible thing to say in this light is that we have a suite of nomological modalities — generated by different relations that are eligible for the nomological accessibility role — rather than embrace the project of pinning down the one true nomological modality. But the primary methodological significance of the phenomenon is that we have reason to be sceptical in general of the inevitability of finding an applied semantics (and an ensuing logic) that correctly and uniquely does justice to all the concerns and interests of our pre-semantic modalzing in and around a given territory, such as the nomological.

Secondly, one must consider whether any prospective accessibility relation really is a proper candidate for holding sway throughout all models. Lest the point be mis-construed, the IDO and QUIDO structures are, of course, set-up so that *whatever is put in the R-role* has that breadth of significance: it must be apt to extend over all the possible worlds in a model and its holding in all models is what defines the structure. But, at first glance at least, it is not obvious that there is good reason to take it that such just any structural fact about the relatedness of possible worlds within the intended model is eligible to have the significance that an accessibility relation must have: that is in constraining admissibility on all models. What the intended model might well settle, and in any case let us grant that it does, is that it is not a contingent matter that a given relation (say the combinatorial relation mentioned above) is symmetric. But equally it will be settled by the intended model that (say) Water's containing Hydrogen is a non-contingent matter. However, the necessity of Water's containing Hydrogen is not something that it would be apt to represent as holding in every admissible model of metaphysical necessity. This leads us back towards the difficult territory of what our expectations of models are and what their role is in an impure Kripke semantics for modal logic. But I trust that enough has been offered to

show that there is good cause to pause for thought here. And among related ways of stimulating that thought may be to pose this related questions. Ought we be distinguishing here between what is necessarily true of relations and what is true of relation-expressing predicates in virtue of the meanings that they have? And ought we be inferring from these considerations that, depending on their "source" some necessities are projectible across the hyperspace of models while others are not. In any case, eligibility for the role of accessibility relation requires such projectibility and the proponent of any candidate for the role ought to be prepared to back the kind of argument outlined above by a defence of such projectibility.

5.5. Modal Logic Without a Description of the Accessibility Relation?

Given the kinds of descriptions of relations on possible worlds that we are used to it is, at least typically, not difficult to determine status with respect to symmetry and transitivity. And accordingly, whenever an IDO structure or QUIDO structure is determined a modal logic thereby determined too. But there are at least two ways in which we might find ourselves in a position that falls short of meeting the requirement to *describe* the accessibility relation for the structure. And it is not obviously irrational to hold out hope of access to an effective impure semantic theory for the modality while consciously occupying that position.

Firstly, one might wish to select by their consequences unmediated hypotheses of symmetry, transitivity or both. This might be born of either despair or agnosticism about pinning down the right description of the right relation. But, in any event, the initial idea would be to hypothesize (say) transitivity without describing a relation and showing that the relation so described is transitive. The follow-up would be to endorse one such hypothesis in light of its "pay-off".

Secondly, on a subtly different note, one might — again out of despair or agnosticism — think that the best we can say is this: even if there is a fact of the matter about the nature of the accessibility relation for this alethic modality, we may never be in a position to assert confidently what that fact is *beyond asserting that it is reflexive* (that much being settled by non-negotiable facts about the identity of the subject matter). In that case one would be in a position to deploy the **M**-rules in confidence of their soundness, but not of their completeness. And therefore, one would not be in a position to make a claim about which is the right logic for that modality.

There is a rich seam to be explored here and — again — it is not my present purpose to explore it. But I shall take a liberty of partiality in elaborating the position of those who are inclined to take **M** as the default logic in use for all alethic modalities.

5.6. **In Brief Support of the Default Use of M**

The salient consequence of having at our disposal only such a weak logic as **M** is that we have to recognize the distinctness, *in a certain sense*, of infinitely many complex modes of the given modality. **M** allows us, always, to intervene in a chain of modal operators according to the alethic laws, by introducing an outer ◊ (possibility introduction) or eliminating an outer □ (necessity elimination). But there is no such *general* facility to eliminate an outer possibility operator (from ◊□ to □, or ◊◊ to ◊) or to introduce an outer necessity operator (from ◊ to □◊, or □ to □□). So the overall effect is that various chains of modal operators are not logically equivalent to any shorter or simpler chain. Thus, given only M, logically distinct modes of X-modality are represented by each of the infinitely many syntactically distinct chains: ◊*A*, ◊◊*A* ... □*A*, □□*A*, ... ◊*A*, ◊□*A* ... ◊*A*, □◊*A* ... □*A*, ◊□*A* ... □*A*, □◊*A* ...). This may, then, seem a significant disadvantage "of economy" when compared with the stronger modal logics in the field and especially (taking the strongest) **S5** where equivalence results allow reduction to only a few logically distinct modes: *A*, ∼*A*, ◊*A*, ◊∼*A*, □∼*A*, □*A*.[37] But let us be clear about what is common ground between the **M**-hypothesis and the **S5**-hypothesis and about why any residual discrepancy should be judged significant.

The first common consequence is that all well-formed formulas (of arbitrary length) will have an interpretation in accordance with the fully compositional assignment procedures that the impure semantic theory inherits from the pure. So all relevant formulas are (declared) meaningful on both hypotheses. The second common consequence is metaphysical: no ideological commitment to complex modal states (of the kind intimated by iterated modal operators) ensues. On the **S5**-hypothesis the question, we might say, does not arise, since semantic analysis weeds out iteration. The **M**-hypothesis does not resolve the matter in that way and so, certainly, leaves semantically open the question of such a commitment. But what does not follow from the **M**-sanction of various chains of operators as semantically primitive is commitment to assert any closed sentence in which such chains have wide scope. In particular, the **M**-hypothesis does not involve a commitment to assert sentences of any syntactic type other than those that are semantically primitive on the **S5**-hypothesis. We should not, of course expect of a (hypothesis of) commitment to a *logic* to have any such substantive assertoric commitments. But we are in a region where the methodology has to be made explicit, and a reminder also steers us away from error.

There is however, a discrepancy of practical potential. For **S5** offers greater inferential facility with iterated modalities than does **M**: the former allows us to make inferences from premises and to conclusions over a greater range of iterated modal forms. But in the face of that (undisputed) observation it is inevitable that the de-

fender of **M** will attempt to put pressure on the idea that we can expect much, if anything, in the way of fulfilment of that potential. And she will naturally attempt to do so by emphasizing the narrowness and remoteness of the range of circumstances in which such iterations are assertible or in which anything turns on their assertibility.

To begin with the hardest hitting of observations there is no shortage of cases in which it is doubtful that competent usage even permits iteration. Can one properly assert that it is analytically necessary that it is analytically necessary that *P*? For that looks like claiming that "it is analytically necessary that *P*" is true in virtue of the meanings of the expressions *it* contains. No less dubious is the assertion that it is physically necessary that it is physically necessary that *P*. For that looks like claiming that "it is physically necessary that *P*" has the same status in respect of physical modality as does "Nothing travels faster than light". But how could that be? The claim here is emphatically *not* that such iterations are meaningless. And remember that under the **M**-hypothesis we are precisely committed to an impure semantic theory that assigns them (non-arbitrary, compositionally generated) truth-conditions. If further characterization of the flaw is wanted, perhaps it is that such iterations are (meaningful) category mistakes. But the bottom line is that there is no foreseeable circumstance in which one would be motivated to make such an assertion nor, accordingly, to take it as a premise when the aim was to get at the modal truth by executing inference that is taken to be sound. Granted, in the case of other modalities in which we have a practical or theoretical interest there is no such compelling basis to allege that a category mistake is afoot. And the most compelling case in that respect may well be that of metaphysical modality. But being acquitted of the charge of category error is not to be cleared of all charges of lacking clear conditions of assertibility. The defender of **M** will prosecute an interest in the question of whether, for example, it is metaphysically necessary that Water contains Hydrogen. For that will be co-ordinate with her preparedness to appeal to the non-modal proposition necessitated without reservation in reasoning from any suppositions. But it will strike her as perverse that her skepticism about necessity in such a case, and her inferential dispositions altered, might properly be dissolved on the strength of an attempt to sell her the oddity which is the proposition that it is possibly possible that it is necessary that Water contains Hydrogen. In the central case of epistemologically significant inference we proceed from what we take ourselves to have the greater justification for asserting and that does not promise a great career for iterated modalities in being fed into strong modal logics to generate basic modal conclusions. In sum, skepticism about modal logics that are stronger than **M** has a decent working basis in default skepticism about the assertibility conditions of the discrepant iterative modes themselves.

This, I hope to have shown, is but one of many important and philosophical issues that are raised by Kripke's classic paper and that remain ripe for exploration half a century on.

Acknowledgments

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Notes

[1] When references take the form of names of page numbers alone, they are to Kripke (1963).

[2] Contrast the possible-worlds semantic theory of Lewis (1968) whose aim is to provide insights into the meanings of modal expressions without buying into the project of interpreting, or even recognizing, any special *logic* of modality. It is made abundantly clear in the opening sentences of Lewis (1968, p.113) that such a contrast is intended. Certainly, the resources of counterpart theory that are deployed in Lewis (1968) might be deployed by others who have in mind the Kripkean project of interpreting quantified modal logics: thus see Hughes & Cresswell (1996, p.353–8). But the Kripkean logical project and Lewisian non-logical project are different projects, any connections between which need to be established by argument.

[3] Kripke (p.63) writes, "Truth-functional tautologies" and while we know that the two characterizations are equivalent in extension, I prefer to avoid all semantic vocabulary in the syntactic phase of development.

[4] Another natural differentiator of sets is, of course, (further) measures of cardinality: but there is no cause to consider those here.

[5] The effect of considering only REF structures is to ensure the VALidity in all structures of those principles that are conjointly characteristic of the alethic modalities — thus: $A \supset \Diamond A$ and $\Box A \supset A$. So when it comes to modal applications, in due course, the restriction to alethic modalities is the default presumption.

[6] By contrast, in relation to the logic of provability, Kripke does exploit the full IDO+ structures to make the de-relativizing move that I have described (p.71). In that, far less discussed, phase of the paper, and displaying his mathematical sensibilities, Kripke is happy to consider IDO+ structures in which the elements of the K-set are further models!

[7] Another kind of semantic facility is afforded by the IDO+ expansion if we add certain further operators to the language. In that event, the distinguished *K*-member, *G*, is apt to play an essential role in the evaluation clause for such an operator. An application-neutral term for such operators is *fixed-point* operators. But peeking again behind the veil of purity, in modal applications such operators are Actuality operators of the rigidifying, or return-key, type (see Hodes 1984). In this paper, and again remaining faithful to the remit of Kripke (1963), there will be no discussion of these more exotic modal logics nor of the semantic issues they add.

[8] There is no value in proliferating species nomenclature at this stage.

[9] The shift to admitting variant-domain structures was motivated by a reconception, in Kripke (p.65), of the proof theory for QUIDO logics. Kripke (1959) took Prior (1956) to have provided a proof in good standing of the Barcan Formula (BF) and realized that this formula would further be *validated* only if downward domain variation were inadmissible. (Validation of CBF requires ruling out upward variation). The later view that Prior's attempt is not a proof in good-standing deprives BF of its status as a theorem and, so, of its power to require, in the name of completeness, VALidity-ensuring restrictions on models.

[10] To indicate the flexibility of the resources offered by Kripke structures I shall mention some cases that have been exploited in various developments of the applied semantics of modality. Since pure characterizations threaten to be lengthy and cumbersome I shall resort to impure characterizations, noting that the impure characterizations are dispensable throughout. Thus, consider each of the following generalizations of which the Kripkean species is a special sub-species: (i) models containing non-classical assignment functions in which closed formulas are taken at each *K*-member onto one subset of $\{T, F\}$, allowing for value gaps and value gluts; (ii) models containing counterpart-theoretic evaluation of open formulas at a world, where evaluation via transworld identity is a special case of such evaluation that is permitted when structures tolerate intersecting *K*-domains; (iii) models in which the fundamental form of evaluation is of a sequence in which the first element is itself a sequence thus — evaluation at a *K*-member in the model is a special case of evaluation of a centred *K*-member, such as $\langle H, o \rangle$, in the model where $o \in \psi(H)$ and (iv) model structures in which the set of possible worlds, *K*, is a proper subset of the set of all worlds in the space, *W*, that broader set containing also impossible worlds. Kripke remarks explicitly, and then only briefly, on case (i), contrasting his own thoroughly bivalent approach with the alternative that he takes to be suggested by Frege (1892) and Strawson (1950) and implemented by Hintikka (1961) and Prior (1957). This is the occasion of his "convention" remark quoted above. I read Kripke as alluding to case (ii), and the prospect of counterpart-theoretic development, in his remarks about the prospect of a broader of identity (p.70).

[11] The natural path of construction is from QUIDO structures to QUIDO+ structures and then to distinguished QUIDO+ models. But it is formally available to distinguish in either one of these ways without distinguishing in the other. That is, we can take an IDO or QUIDO structure in which there is no distinguished W-member and distinguish a model in that structure. Or we can, as Kripke (1963) does, take IDO+ and QUIDO+ structures in which there is a distinguished W-member but decline to invoke a distinguished model of the structure. That is formally unobjectionable but not obviously well motivated even from the pure standpoint never mind the impure modal standpoint.

[12] The "pro tem" qualification is there so as to not rule out at this stage, and without argument, the prospect of an attempted ontological deflation by re-interpretation of the whole theory. See 4.9 below.

[13] For present purposes, I again, put aside issues of cardinality and attitudes to various infinities. If taking into account *all* live philosophical positions on such matters one could not so casually rely on ontological commitment to (a plurality of) sets generating further commitment to a set that is their union. I am indebted here to Simon Hewitt.

[14] In writing of one kind of theory (impure) being an instance of the other (pure) I am not expressing serious intent about the metaphysics of theories. Perhaps a more accurate way of articulating the relationship is that the pure theory is a pure set-theoretic representation of the arbitrary impure theory. But I shall forego that more cumbersome way of speaking.

[15] For a range of examples of applications of (what they call) "modal logics" to non-modal subject matters see Blackburn, van Benthem & Wolter (2006).

[16] My present policy on usage is, thus, at odds with reasonably common practice in philosophy, and certainly outside of it, whereby "modal" is used to label any IDO or QUIDO logics and "possible-worlds semantics" to label any interpretation that deploys pure QUIDO structures (or variants on these). See the very title of Blackburn et al (2006). I have already advertised the advantages of my policy.

[17] To underscore the point by means of contrast, it is — of course — a perfectly legitimate project to set out with a notion of "logical" that derives from logic and to *argue* that logically modal space is absolute or maximal in some way or other. See, for example, Hale (1996) following McFetridge (1990).

[18] This line of thought raises a major question about commitment to worlds other than possible worlds and I shall return to that below.

[19] Recall the presumption from the outset that all species of model structure include a *non-empty* set K.

[20] I am putting out of the picture of modal application the pure option of having models in which the domain of the distinguished K-element is empty. But a comprehensive discussion would cover, and may make something of, that case.

[21] Here and throughout this phase, I continue to presume, with Kripke (1963), reflexivity of the accessibility relation, R, as a feature of all model structures.

[22] A notable consequence of this observation is that commitment to a range of necessities and impossibilities, when combined with agnosticism only about contingency, may avoid commitment to a plurality of possible worlds when combined with a Kripkean possible-worlds semantic theory (Divers 2006).

[23] It is noted that a commitment, in the realm of truth, to such contingent non-existence is equivalent to commitment to the invalidity of the converse Barcan Formula (CBF).

[24] Counterpart-theoretic versions of possible-worlds semantics, following Lewis (1968), offer the prospect of *de re* contingency within one-world models (Divers 2004, p.673ff). That is because a single possible world can represent both the possibility of x being F *and* the possibility of x being non-F through one counterpart of x being F and another being non-F. Naturally, the rules governing the operator "at w" require careful handling if that prospect is pursued, lest such a world should also represent the "possibility" that x be both F-and-non-F.

[25] The defence of necessary existence (necessitism) and its consequence in a broadly Kripkean semantic context is undertaken by Linsky & Zalta (1994, 1996) and subsequently by Williamson (1998, 2013). On these views there are no non-actualized possibilia, but the actualized possibilia that are postulated are plenitudinous.

[26] The idea that commitment to impossble worlds is driven by commitment to modal truth is an increasingly popular idea that goes back at least to Salmon (1984). See Nolan (2013) on the extensive literature. To exemplify, it is often suggested that we need to introduce impossible worlds in order to register the falsehood of various modal conditionals that would wrongly be deemed true if their truth-conditions were given by the standard semantic evaluation clauses and had we only *possible* worlds over which to evaluate them. So one driving commitment is to the Falsehood of at least some such conditionals that have (necessary consequents or) impossible antecedents — thus, "If a proof were to appear of the finitude of the set of primes, mathematicians would not be surprised". But those considerations take us beyond the concerns of Kripke (1963) and of this paper.

[27] It might prove illuminating to attempt to relate the relevant requirements of the theory to the intuition about models with which it aligns. But I shall not pursue that option here.

[28] Stalnaker (2012, p.3–4) comes very close to such insistence. But perhaps he is relying on some kind of suppressed inductive considerations about how projects of a certain type, intimated again by possible-worlds semantics, tend to pan out whenever they prove worthwhile.

[29] I would prefer to forgo the qualification of truth as literal, since I do not think that there are any non-literal kinds of truth. But experience suggests that the qualifier may insure against misunderstanding of the intention.

[30] The literal representationalism described here might be thoroughgoing in applying uniformly to the results or theorems of the theory and to the axioms from which they are deduced. However, it is to be noted that for theories in general and for semantic theories in particular, it s not unusual to find *partially* representationalist conceptions involving literalism about theorems and instrumentalism about axioms. See Davidson (1980) for such a view of semantics in general and Stalnaker (2012) for, what appears to be, a view that is at least in this spirit about possible-worlds semantics in particular.

[31] It is granted that support for modal epistemology tends to be off the radar of discussions of applied semantics for QML. But it is an interesting question, and one that bears on my claims about presumptions, why this is so.

[32] For criticism of prefix fictionalism in this regard see Divers (1995).

[33] The point is even more graphically illustrated by a related and familiar explanatory project that is adjacent to our immediate concerns. If the aim is to provide an ontological reduction of — say the properties — by identifying them as sets of possibilia one cannot sensibly claim to have done so while preparing to disavow the prima facie commitment to the existence of possibilia.

[34] Moves of re-interpretation, of a certain kind, within the confines of the pure semantics are,

of course, a different matter altogether. The algebra of structures is a perfectly good discipline and trades in re-interpretation, of a kind, to establish (on the back of embedding or isomorphism) the transferability of results about one kind of structure to equivalent results about related structures. But nothing in that sphere involves either the benefits or the commitments that attach distinctively to impure possible-worlds semantic theories.

[35] This is a matter that, of course, looms large for the champions of second-order versions of modal logics since second-order status itself makes for incompleteness. Thus, for example, see the recent defence by Williamson (2013) of second order quantified **S5** as the right logic for absolute necessity and possibility. I hope to discuss that particular development in some detail elsewhere.

[36] For discussion and sources see Divers (2002, p.215–6).

[37] See Hughes & Cresswell (1996, p.59–60)