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How Densely Should Networks Be Deployed?

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Abstract

In this work we try to answer the question how densely the wireless networks should be deployed. We modelled the base stations as a Poisson Point Process and derived an analytical form of the coverage probability under the path loss only channel condition. Surprisingly our result shows that the coverage probability is independent of the base station density. This suggests that when only considering distance-dependence path loss channel, the density of the networks has no influence on the network performance.

Keywords Wireless Networks; Stochastic Geometry; Poisson Point Process.

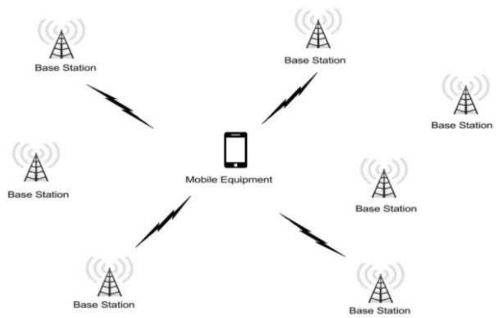

Figure 1. Multi-cell coverage scenario.

1. Introduction

Cellular wireless network deployment is crucial in telecommunication infrastructure. Fig.1 illustrates a typical multi-cell coverage scenario. One of the challenges in the early stages of cellular network deployment is to estimate the wireless base station density that satisfies the communication service requirement. It has long been believed that the majority gain achieved in the wireless networks in the last two decades is attributed to the shrinkage of the cell size. Following this idea of ’smaller cells’, in recent years, dense cell deployment has been advocated to achieve better coverage as a dense cell deployment reduces the coverage range for each base station, which leads to a better signal coverage. However, as the density of the base stations increases, the interference between cells also increases, which can cancel the gain from dense cell deployment. To determine the required base station density in cellular networks has long been a challenge in network planning. The question ’how densely cells should be deployed?’ remains crucial yet unanswered in wireless network planning.

Traditional works model the cellular network as a hexagonal grid. Based on this classic cellular network model, the network performance is determined through network simulators [1]. In such cases, to determine the influence of the base station density requires exhaustive computer simulations to compare the network performance at various base station densities, which costs high computational resources. Under this deterministic perspective, network deployment is solved as an exhaustive searching based computer simulation algorithm. However, such an approach fails to yield an insight to under-standing the impact of the base station has on the network performance.

On the other hand, a statistical approach using the stochastic geometry has been recently introduced to study the network performance. The seminal work [2] established analytical results for the coverage probability and the rate probability in cellular networks using the stochastic geometry tools. Under the Rayleigh fading channel model, the coverage probability and the rate probability are derived as functions of the base station density and path-loss exponent. Such results offer an insight to the complex relationship between the base station density and the network performance. Interestingly, in two special interference-limited cases when the noise is omitted, the network coverage performance is proved to be independent of the base station density. Although the work [2] partially answered the question ’how densely cells should be deployed?’ by concluding that when the noise is ignored the base station density is independent of the coverage probability, this answer is based on a Rayleigh small scale fading channel condition which is normally ignored in wide area network deployment practice. The derivation of the network performance considering only distance while excluding small scale fading remains unsolved.

In wireless network planning practice, the path-loss-only channel model, which only considers propagation distance loss, is widely used especially for wide area deployment. In this work, we study the problem under such a path-loss-only channel model to analyze the relationship between the base station density and the network coverage probability. Similar to the work [2], we use the stochastic geometry framework to model the network. The base stations are modelled as a Poisson point process. However, the techniques used in the work [2] cannot be applied directly to obtain the distribution of interference without small scale fading. By applying the Gil-Pelaez Inversion Theorem, we obtain the distribution of the interference of a path-loss-only channel model. Following this distribution function of the interference, the coverage probability is derived for network with path-loss-only channel model. Surprisingly, our analytical result shows that the coverage probability is independent of the base station density.

2. System Model and Analysis

In this work, we study a typical cellular network deployment scenario illustrated in Fig. 1. We modelled the base stations as a Poisson point process. The coverage of the network is evaluated by the signal- to-interference-plus-noise-ratio (SINR) as:

$$\mathrm{SINR}=p\frac{r^{-\alpha}}{I+N} \quad (1)$$

where p is the transmitter power of the serving base station; r is the distance of the nearest base station to the mobile equipment; α is the path-loss exponent; and I and N denote the interference power and noise power, respectively. The interference comprises the transmitting power from the base stations other than the serving base station. Assuming that the base stations have the same transmitter power, we can thus write the interference in the following form:

$$I = p\sum R_i^{-\alpha} \quad (2)$$

where R_i is the distance of the base stations other than the nearest to the mobile equipment.

We model the base station locations as a Poisson point process. In this case, the *SINR* is a random variable and we define the coverage probability as the probability where the *SINR* is above a threshold value t. We write the coverage probability as:

$$P(t,\lambda,\alpha) = E_r\left[P(\mathrm{SINR} > \mathrm{t}|r)\right] \quad (3)$$

$$=\int P(\mathrm{SINR} > \mathrm{t}|r) f(r)\,dr \quad (4)$$

$$=\int F_I(\frac{r^{\alpha}}{t} - N) f(r)\,dr \quad (5)$$

where $F_I(\cdot)$ is the distribution function of the interference and $f(r)$ is the distribution of distance to the nearest neighbor in a Poisson point process, given in (6).

$$f(r) = \mathrm{e}^{-\lambda\pi r^2}\, 2\pi\lambda r \quad (6)$$

In order to evaluate the coverage probability in (5), we need the distribution function of the aggregated interference $F_I(rt-N)$. In the work [2], the distribution of the aggregated interference is derived based on the small scale fading channel model. However, in our case, we only consider a path-loss-only channel model. To derive the distribution function of the aggregated interference, we resort to the Gil-Pereaz Inverse Theorem [3], which is given as:

Lemma. (Gil-Pelaez Inversion Theorem) For a univariate random variable X, if x is a continuing point of Fx, then

$$F_x(x) = \frac{1}{2} - \frac{1}{\pi}\int \frac{\mathrm{Im}\left\{e^{-itx}\varphi_x(t)\right\}}{t}dt \quad (7)$$

where $\varphi_x(t)=\mathrm{E}\left(e^{itx}\right)$ is the characteristic function of the random variable X.

To apply Theorem II in deriving the distribution function of the *SINR* in (5), first we use the characteristic function of the interference, which is the Laplace transform of the function given as [2]:

$$L(s)=\exp\left(\lambda\pi r^2\left[1-{}_1F_1\left(-\frac{2}{\alpha};-\frac{2}{\alpha}+1;-sr^{-\alpha}\right)\right]\right) \quad (8)$$

The characteristic function is given by replacing the variable as:

$$\varphi_I(jt)=\exp\left(\lambda\pi r^2\left[1-{}_1F_1\left(-\frac{2}{\alpha};-\frac{2}{\alpha}+1;-sr^{-\alpha}\right)\right]\right) \quad (9)$$

Thus, by inserting (9), (7) and (6) into (5), we have the following result for the coverage probability:

Theorem. The coverage probability in a homogeneous network with a path-loss-only channel model is:

$$P(t,\lambda,\alpha) = \frac{1}{2} - \frac{2}{\pi}\exp\left(\frac{N}{p}\right)\int_0^{\infty}\mathrm{Im}\left\{\exp(-\frac{jx}{t})\frac{1}{{}_1F_1(jx)}\right\}\frac{dx}{x} \quad (10)$$

Where the function ${}_1F_1(\cdot;\cdot;\cdot)$ is the confluent hypergeometric function, and $Im\{\cdot\}$ denotes the imaginary part operator.

Surprisingly the coverage probability result in (10) shows that the coverage probability is independent of the base station density λ. This result echoes with the interference-limited special case results proved in [2]. The explanation for this result is that with high base station density, although the user is closer to the serving base station, the interference from other base stations is also higher. In such case, the gain from a closer distance is cancelled by the higher interference. This conclusion suggests that highly dense cell deployment may not guarantee an improved network coverage performance.

3. Conclusions

In this work, we study the impact of base station density on the coverage probability under a stochastic geometry framework. Adopting the widely used path-loss-only channel model, we derive an analytical form for the coverage probability. Counter-intuitively our analytical result shows that the coverage probability in this case is independent of the base station density, which corroborates special case results in the pioneering work [2]. Such a result also implies that solely increasing the base station density may fail to achieve high network coverage performance.

References

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