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Low-order Modelling for the Feedback Control of Bluff Body Fluid Flows

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Abstract

Control of fluid flows can be beneficial in a large number of situations and in particular for the purpose of drag reduction on road vehicles, boats, and aircraft. Modern control design methods typically rely on linear, low-order plant models. However, fluid flows are governed by a set of coupled, non-linear partial differential algebraic equations with infinite state dimension. This results in models of very high state dimension, which yield very computationally expensive control synthesis problems. This paper proposes a computationally efficient approach to the modelling of such systems using frequency domain information, and its efficacy is demonstrated in an example problem.

Keywords Drag reduction; Flow control; Frequency response; PDAEs.

1. Introduction

The ability to control the flow of a fluid can be of great benefit in a number of different situations; from situations where turbulent flow is desirable – such as in combustion chambers, in order to increase mixing of fuel and air – to those where steady, laminar flow is desired – such as that around the rear end of heavy goods transport vehicles. Modi et al. [1] claimed that in 1995 two thirds of all goods in North America were transported by truck, and that 50-70% of the trucks' power is consumed in overcoming drag caused by aerodynamic forces. Successful drag reduction could result in fuel savings worth billions of dollars annually [2] as well as having profound effects on CO_2 production.

The shedding of vortices from the trailing edges of a bluff body results in a low pressure region behind the body, and is responsible for an overall pressure differential across the body, yielding a drag force. This is depicted in Fig. 1. Successful suppression of vortex shedding by active feedback control could lead to a substantial decrease in pressure drag [3].

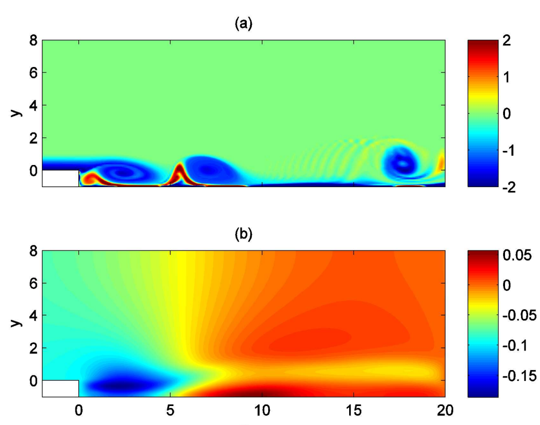

Figure 1. 2D backward facing step flow simulation (flow moves towards the right); (a) Instantaneous vorticity field depicting vortex shedding; (b) Time averaged pressure field showing pressure reduction immediately aft of step rear face; note that all values have been non-dimensionalised.

The main difficulty in controlling flows around complex geometries (such as bluff body flows), is that whilst modern control design methods typically rely on linear, low-dimensional plant models, fluid flows are governed by the Navier-Stokes equations – a set of coupled, non-linear partial differential algebraic equations (PDAEs), from which finite-dimensional approximations typically have very high (~10^5) state dimension. Because of this, existing modelling approaches such as system identification or data driven Galerkin projection methods are very computationally expensive and time consuming. Constructing state-space models by directly spatially discretising the equations results in very large system matrices which are expensive to store/invert and are typically ill-conditioned.

This work presents a computationally feasible modelling approach for systems governed by PDAEs.

2. Methodology

The proposed modelling approach exploits the advantages of working with frequency domain information, namely, avoiding the necessity to construct and store large matrices, the small cost of performing certain operations on frequency response data and the ease with which one can compute the frequency response of descriptor state-space systems (which arise in fluid flow systems due to an algebraic constraint present in the equations). The method is as such: firstly, linearise (if necessary) the governing PD(A)E around the desired operating point such as (1) (an example in one spatial dimension);

$$\frac{\partial \boldsymbol{\varphi}(x,t)}{\partial t} = \mathcal{F}\left(x, t, \boldsymbol{\varphi}(x,t), \frac{\partial \boldsymbol{\varphi}(x,t)}{\partial x}, \frac{\partial^2 \boldsymbol{\varphi}(x,t)}{\partial x^2}\right) \qquad (1)$$

where $\boldsymbol{\varphi}(x,t) \in \mathbb{R}^{\eta}$ is the solution, η is the number of states at each node, and $x \in \Omega \subset \mathbb{R}$ and $t \in \mathbb{R}^{+}$ are the spatial and temporal coordinates, respectively. Next, discretise the PD(A)E in space;

$$\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{\varphi}_i(t) = \hat{\mathcal{F}}\left(\Delta x, t, \boldsymbol{\varphi}_i(t), \boldsymbol{\varphi}_{i+1}(t), \boldsymbol{\varphi}_{i-1}(t)\right) \qquad (2)$$

where Δx is the grid step size. For each node in the spatial domain, Ω, cast (2) into descriptor state-space form with inputs and outputs corresponding to the flow of state information to and from neighbouring nodes;

$$E_i \frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{\varphi}_i(t) = A_i \boldsymbol{\varphi}_i(t) + B_i \begin{bmatrix} \boldsymbol{\varphi}_{i-1}(t) \\ \boldsymbol{\varphi}_{i+1}(t) \end{bmatrix} + B_u \boldsymbol{u}(t) \qquad (3a)$$

$$\begin{bmatrix} \boldsymbol{\varphi}_i(t) \\ \boldsymbol{\varphi}_i(t) \\ \boldsymbol{y}(t) \end{bmatrix} = \begin{bmatrix} I \\ I \\ C_y \end{bmatrix} \boldsymbol{\varphi}_i(t) \tag{3b}$$

where $I \in \mathbb{R}^{\eta\times\eta}$ are identity matrices, $A_i, E_i \in \mathbb{R}^{\eta\times\eta}$ and $B_i \in \mathbb{R}^{\eta\times 2\eta}$ are matrices which depend on the spatial discretisation technique employed, and $B_u \in \mathbb{R}^{\eta\times\eta_u}$ and $C_y \in \mathbb{R}^{\eta_y\times\eta}$ are matrices describing how any external control inputs, $\boldsymbol{u}(t) \in \mathbb{R}^{\eta_u}$, and/or measurement outputs, $\boldsymbol{y}(t) \in \mathbb{R}^{\eta_y}$, enter and exit the system, respectively. Evaluate each node's frequency response using (4) [4] at a range of discrete frequencies $\omega_k \in \{\omega_1, \dots, \omega_\ell\}$ (note this is a very cheap operation due to the small state-dimension of a single node model);

$$G_i(\mathrm{i}\omega_k) = \begin{bmatrix} I \\ I \\ C_y \end{bmatrix} (\mathrm{i}\omega_k E_i - A_i)^{-1} \times [B_i \quad B_u] \in \mathbb{C}^{(2\eta+\eta_y)\times(2\eta+\eta_u)} \tag{4}$$

where $\mathrm{i} \coloneqq \sqrt{-1}$. The overall frequency response from control inputs to measurement outputs is constructed using the Redheffer Star Product [5] between adjacent nodes, with which a low-order transfer function model can be fitted and subsequently used for control design.

3. 2D Wave-diffusion System Example

To demonstrate the efficacy of this proposal, the method is applied to a 2D wave-diffusion system, with dynamics governed by (5);

$$\frac{\partial^2\phi(x,t)}{\partial t^2} = c^2\nabla^2\left(1 + \kappa\frac{\partial}{\partial t}\right)\phi(\boldsymbol{x}, t) \tag{5}$$

where $\phi(\cdot,\cdot) : \Omega \times \mathbb{R}^+ \to \mathbb{R}$ is the displacement of the surface, $\Omega \coloneqq [-1,1]^2 \subset \mathbb{R}^2$ is the spatial domain with boundary $\partial\Omega$, $\boldsymbol{x} \coloneqq (x, y)$ is the spatial coordinates, $c, \kappa \in \mathbb{R}$ are constants, and $\nabla \coloneqq \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}\right)$ is the Del operator. It was assumed the boundary conditions were homogeneous Dirichlet, $\phi(\boldsymbol{x}, t) = 0 \;\; \forall \boldsymbol{x} \in \partial\Omega$, control input was direct forcing at the centre of the domain, and measured output was the displacement in the same location, $y(t) = \phi(\boldsymbol{x}_{\text{cent}}, t)$.

Firstly, a high order model was derived by direct spatial discretisation of (5), and the frequency response computed for a finite set of frequencies. Secondly, the modelling approach proposed in this paper was used to obtain the frequency response for the same frequencies. As can be seen from Fig. 2, identical frequency response data were obtained using the proposed method, at much cheaper computational expense, proving the efficacy of the method. Figure 3 demonstrates the reduction in complexity achieved by employing the modelling approach discussed in this paper, as opposed to constructing the full, high order model and subsequently evaluating the frequency response. The former method costs $O(\varrho^4)$ floating point operations (FLOPS), whilst the latter costs $O(\varrho^6)$, where ϱ is the computational mesh density in one spatial dimension measured in nodes per unit length.

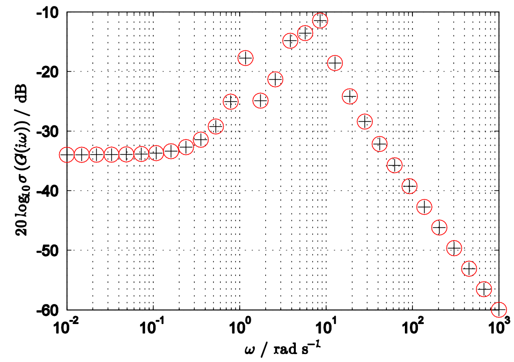

Figure 2. Comparison of singular value for 2D wave-diffusion system obtained through direct (expensive) approach (red circles), and through proposed methodology (black crosses).

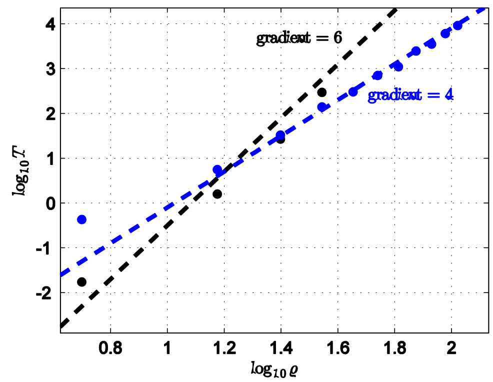

Figure 3. Comparison of computation time, T (seconds), required to obtain frequency response for a single frequency using different computational mesh densities, ϱ, through direct (expensive) approach (black dashed line), and through proposed methodology (blue dashed line), on a log-log scale.

4. Conclusions

A computationally tractable approach to modelling the input-output behaviour of systems governed by PD(A)Es in the frequency domain has been presented and its efficacy demonstrated by application to the 2D wave-diffusion equation.

The assumption of linearity for the case of a controlled backward facing step show has been shown to be reasonable in [3], and so the method discussed in this paper is currently being applied to the linearised Navier-Stokes equations for a 2D backward facing step flow.

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