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A MULTIAXIAL STRESS-BASED CRITICAL DISTANCE METHODOLOGY TO ESTIMATE FRETTING FATIGUE LIFE

J. A. Araújo [a,*], L. Susmel [b], M. S. T. Pires [a], F. C. Castro [a]

[a] *Department of Mechanical Engineering, University of Brasilia, 70910-900 Brasilia, DF, Brazil*
[b] *Department of Civil and Structural Engineering, The University of Sheffield, Mappin Street, Sheffield S1 3JD, UK*

ABSTRACT. This work presents a methodology for fretting fatigue life estimation based on the evaluation of a multiaxial fatigue parameter at a critical distance below the contact surface. The fatigue parameter is defined using the Modified Wöhler Curve Method together with a measure of shear stress amplitude based on the Maximum Rectangular Hull concept. To apply the approach in the medium-cycle fatigue regime, the critical distance is assumed to depend on the fatigue life. Available fretting fatigue experiments conducted on a cylinder-on-flat contact configuration made of Al-4%Cu alloy were used to evaluate the methodology. Most of the fatigue life estimates were within factor-of-two boundaries.

KEYWORDS. fretting fatigue; multiaxial fatigue, fatigue life estimation; theory of critical distances

1. INTRODUCTION

As defined by the American Society for Metals, fretting is "*a special wear process that occurs at the contact area between two materials under load and subjected to minute relative motion by vibration or some other force*" [1]. In metals subjected to cyclic loading, fretting results in a decrease of the material lifetime as compared with that observed in the un-fretted material [2]. This can be ascribed to the presence of stress/strain concentration phenomena occurring at the contact surface and with the local multiaxial stress/strain fields rapidly decaying from the surface inward [2,3]. Reviews both in the USA and in Europe [4,5] indicate that in-service breakage of structural components costs around 4% of GNP in industrialized nations, 50% to 90% of failures being caused by fatigue. In this scenario, fretting fatigue is always a matter of concern to structural engineers since it can remarkably reduce the in-service lifetime of important structural parts such as, for example, threaded pipe connections, bolted/riveted joints, blade-disk attachments in gas/steam turbines, shrink-fitted shaft, and aero-engine splined couplings.

By taking advantage of the "notch analogue" approach [3], this work presents a design methodology suitable for estimating lifetime under fretting conditions. The methodology is based on the combined use of the Modified Wöhler Curve Method (MWCM) [6] and the Theory of Critical Distances (TCD) [7], the TCD being applied in the form of the so-called Point Method [8]. According to Peterson's intuition [8], the Point

* Corresponding author.
E-mail address: alex07@unb.br (J. A. Araújo).

Method postulates that the gradient effect in fatigue can successfully be taken into account by simply using the stress state at a given distance from the assumed crack initiation point. The MWCM is a bi-parametrical critical plane approach taking as its starting point the assumption that fatigue crack nucleation as well as the initial propagation phase take place on those material planes experiencing the maximum shear stress amplitude. The shear stress amplitude, τ_a, is defined in this work by the Maximum Rectangular Hull (MRH) method [9–13]. The MRH was originally developed to be used in a stress-invariant multiaxial fatigue criterion [9]. It was later used to define the amplitude of the shear stresses acting on a material plane [10]. The MWCM employing τ_a calculated with the MRH method, instead of the MCC (Minimum Circumscribed Circle) [14], can provide more accurate endurance limits for some multiaxial loading paths [14]. The accuracy of the proposed methodology is evaluated using fretting fatigue experiments taken from the literature [15].

2. MULTIAXIAL FATIGUE LIFE ESTIMATION

The MWCM [6,16] is a multiaxial stress-based critical plane approach where the driving parameters for crack nucleation are the shear stress amplitude, τ_a, and the maximum normal stress, $\sigma_{n,max}$, acting on the material plane experiencing the maximum shear stress amplitude (i.e., the so-called critical plane). Once the values of τ_a and $\sigma_{n,max}$ are evaluated, the stress ratio, ρ, is defined as

$$\rho = \frac{\sigma_{n,max}}{\tau_a} \quad (1)$$

It is noteworthy that ρ is sensitive not only to the presence of non-zero mean stresses, but also to the degree of multiaxiality and non-proportionality of the local stress path. It is also worth recalling that for an unnotched specimen under fully reversed uniaxial loading the ρ ratio is equal to unity, whereas under fully reversed torsional loading ρ equals zero.

The MWCM is based on a modified Wöhler diagram where τ_a is plotted against the number of cycles to failure, N_f (Fig. 1). This diagram contains different fatigue curves characterized by different values of ρ ratio. Each modified Wöhler curve is defined by its negative inverse slope, κ, and by a reference shear stress amplitude, $\tau_{A,Ref}$, extrapolated at an appropriate number of cycles to failure, N_A. For a given material, the corresponding modified Wöhler diagram can directly be built provided that the κ vs. ρ and $\tau_{A,Ref}$ vs. ρ relationships are calibrated by running appropriate experiments. The following linear relationships have been demonstrated by Lazzarin and Susmel [16] to correlate a wide range of experimental data:

$$\kappa(\rho) = a\rho + b \quad (2)$$

$$\tau_{\mathrm{A,Ref}}(\rho) = c\rho + d \tag{3}$$

where a, b, c and d are material constants. If these constants are calculated from uniaxial and torsional plain fatigue data, Eqs. (2) and (3) can be rewritten as

$$\kappa(\rho) = [\kappa(\rho = 1) - \kappa(\rho = 0)]\rho + \kappa(\rho = 0) \tag{4}$$

$$\tau_{\mathrm{A,Ref}}(\rho) = \left[\frac{\sigma_0}{2} - \tau_0\right]\rho + \tau_0 \tag{5}$$

where $\kappa(\rho=1)$ and σ_0 are the inverse slope of the modified Wöhler curve and the fatigue limit under uniaxial loading condition, respectively, and $\kappa(\rho=0)$ and τ_0 are the corresponding quantities under fully reversed torsional loading. It should be noted that for materials that do not exhibit a fatigue limit, σ_0 and τ_0 must be defined as fatigue strengths corresponding to an appropriate number of cycles to failure. After determining the material constants in Eqs. (2) and (3), any modified Wöhler curve can be obtained. From the specific modified Wöhler curve for the ρ ratio that characterizes the local stress history being assessed, the number of cycles to failure can be estimated [16]:

$$N_{\mathrm{f,e}} = N_{\mathrm{A}}\left[\frac{\tau_{\mathrm{A,Ref}}(\rho)}{\tau_{\mathrm{a}}}\right]^{\kappa_\tau(\rho)} \tag{6}$$

Although the value of τ_a in the MWCM is usually determined by the Minimum Circumscribed Circle (MCC) method [14], it was found in previous work [10] that fatigue endurance estimates can be improved when the Maximum Rectangular Hull (MRH) method is used. The MRH method involves the combination of shear stress amplitudes associated to mutually orthogonal directions on a material plane, and therefore it is capable to distinguish between the damage caused by proportional and non-proportional shear stress paths. Because of that, the MRH method yields better results than the MCC method for some non-proportional loading paths. The MRH method is schematically illustrated in Fig. 2. The halves of the sides of the rectangular hull with orientation φ are calculated as

$$a_i(\varphi) = \frac{1}{2}\left[\max_t \tau_i(\varphi,t) - \min_t \tau_i(\varphi,t)\right], \qquad i = 1,2 \tag{7}$$

where $\tau_i(\varphi,t)$ (i=1,2) are the components of the shear stress vector $\boldsymbol{\tau}$(t) with respect to the φ-oriented frame. The amplitude of each φ-oriented rectangular hull can be evaluated as

$$\tau_{\mathrm{a}}(\varphi) = \sqrt{a_1^2(\varphi) + a_2^2(\varphi)} \tag{8}$$

An equivalent shear stress amplitude is then defined as the maximum value of Eq. (8) among all φ-oriented rectangular hulls:

$$\tau_{\mathrm{a\,MRH}} = \max_{0 \le \varphi < 90^\circ} \sqrt{a_1^2(\varphi) + a_2^2(\varphi)} \tag{9}$$

3. CRITICAL DISTANCE APPROACH

The stress gradient effect is a crucial aspect in the design of stress raisers such as notches and mechanical contacts. Among the formulations that account for this effect, the TCD [6,7,17] is recognized as one of the most attractive due to its simplicity and good results for a number of notch configurations. The central idea in the TCD is the definition of an effective stress, σ_{eff}, based on an averaging procedure over a volume surrounding the stress raiser. Fatigue failure is expected to occur if σ_{eff} exceeds a reference material strength, σ_{ref}. Simplified methods can also be formulated by considering averages over an area or a line (Area and Line Methods, respectively) or the stress at a point located at a critical distance, L, from the stress raiser (Point Method). The Point Method (Fig. 3) is used in this work.

In the approach developed by Taylor [17], the critical distance L is determined using the fatigue threshold of a cracked (or sharply notched) specimen. In the Point Method, the critical distance takes the following form:

$$L = \frac{1}{2\pi}\left(\frac{\Delta K_{\mathrm{th}}}{\Delta \sigma_0}\right)^2 \tag{10}$$

where ΔK_{th} is the threshold stress intensity factor range and $\Delta\sigma_0$ is the uniaxial plain fatigue limit range. Further work [6,18−21] investigated the extension of the TCD to stress raisers under multiaxial loadings. It was concluded in Refs. [18,19] that the combination of the MWCM with the Point Method requires a critical distance identical to that of Eq. (10).

An extension of the TCD to estimate fatigue life of stress raisers was developed in Refs. [22,23]. To explain the formulation, it is recalled that the appropriate critical distance to estimate static strength of notched members is given as [7,24]:

$$L_{\mathrm{S}} = \frac{1}{2\pi}\left(\frac{K_{\mathrm{Ic}}}{\sigma_{\mathrm{r}}}\right)^2 \tag{11}$$

where K_{Ic} is the plane strain fracture toughness and σ_r is a reference material constant which can be equal to or larger than the ultimate tensile strength, σ_{UTS}. As the values of the critical distances under fatigue and static loading are usually different, it can be assumed that the critical distance at the medium-cycle fatigue regime, L_M, depends on the number of cycles to failure, N_f. A power law relationship is used to relate L_M and N_f, i.e.,

$$L_M(N_f) = AN_f^B \tag{12}$$

where A and B are material constants. Two fitting procedures can be used to obtain these constants: one based on critical distances determined by considering fatigue threshold and static properties (Eqs. (10) and (11), respectively) and another based on fatigue curves generated by testing plain and sharply notched specimens. Although the latter procedure provides more accurate notch fatigue life estimates when compared with experimental data, the former is simpler to work with as the material constants required to determine A and B are usually available or can be obtained from empirical relationships.

4. APPLICATION TO FRETTING FATIGUE

Several attempts to address the fretting fatigue problem using notch methodologies have been developed [3,25–29] due to the similarities between notch and fretting fatigue. In both problems, the stress fields are characterized by stress gradients and multiaxial stresses. An additional feature of the fretting fatigue problem is the modification of the surface due to wear. Notch methodologies generally assume that the wear process can be neglected. This approximation is considered in this work, at least for the partial slip case where the amount of wear debris is usually small [30].

The use of the methodology for a typical fretting fatigue problem is shown in Fig. 4. The contact configuration involves a normal force P, a cyclic tangential loading $Q(t)$, and a cyclic remote stress $\sigma(t)$. The first step is to determine the crack initiation point. This task can be accomplished, for example, by searching the point where a given fatigue parameter achieves its maximum value. The crack initiation point generally occurs at the trailing edge of the contact zone, as illustrated in Fig. 4a. The subsequent stress analysis is carried out along a straight line that emanates from the crack initiation point and is perpendicular to the contact surface. In order to obtain the number of cycles to failure, the critical distance corresponding to a trial number of cycles to failure, N, is determined according to Eq. (12). The stress quantities τ_a, $\sigma_{n,max}$, and ρ are then evaluated at this critical distance, and the number of cycles to failure, $N_{f,e}$, is calculated using Eq. (6). If the calculated $N_{f,e}$ is different from the trial value N, a new iteration is started considering $N = N_{f,e}$. This recursive procedure is repeated until convergence between the distance used to estimate the number of cycles to failure and the distance determined according to Eq. (12) is reached.

5. COMPARISON WITH EXPERIMENTAL DATA

The experimental results reported in Ref. [15] were used to evaluate the proposed methodology. The tests were carried out with a pair of cylindrical pads pressed against a flat dog-done specimen, both made of Al-4%Cu alloy. Four different series were investigated. In each series, tests were run for an average of eight different pad radii varying from 12.5 to 150 mm. Given the peak contact pressure, p_0, the ratio between the tangential and the normal forces, Q/P, and the remote stress, σ_B, were kept constant so that the magnitude of the stress field was the same for each test, but different stress gradients were induced by varying the pad radius (contact size). Table 1 reports the relevant load conditions for all the data series. The coefficient of friction, f, within the slip zones is 0.75. The available material properties for the investigated aluminum alloy are listed in Table 2. Other fatigue constants, as A and B in the L_M versus N_f relationship, Eq. (12), were determined using L values calculated at the fatigue threshold and at the limit static condition (Eqs. (10) and (11), respectively). The values for κ and $\tau_{A,Ref}$ were obtained from fully reversed uniaxial and torsional fatigue curves. These curves were estimated by fitting fatigue data corresponding to 10^3 and 10^7 cycles. The fatigue strengths at 10^3 cycles were obtained from the ultimate tensile strength by means of empirical relationships.

Due to the geometry of the experimental setup and the applied loadings, analytical techniques [2] can be used to solve the elastic contact problem. The surface tractions are characterized by a Hertzian contact pressure distribution, and by shear tractions that are similar to the Mindlin-Cattaneo distribution except that the stick zone is not symmetrical with respect to the center of the contact zone, but shifted due to the presence of an alternating remote stress. After determining the surface tractions, subsurface stresses are obtained by using a Muskhelishvili potential. Finally, the time varying elastic stress field in any material point in the specimen is calculated by superposing the effects of the contact pressure, shear traction and remote stress.

Estimated and observed number of cycles to failure are shown in Fig. 5. The solid diagonal line represents a perfect correlation, and the dashed lines define factor-of-two boundaries. Most of the estimates are within the factor-of-two boundaries. Considering the well-known scatter of fatigue test data, the obtained level of accuracy is certainly satisfactory.

6. DISCUSSION

Under in-service conditions, components subjected to fretting fatigue often experience variable amplitude loadings. Due to the use of combined amplitudes to evaluate the equivalent shear stress amplitude, as defined in Eq. (9), the methodology is only applicable to constant amplitude loading. Generalizations of the critical plane multiaxial fatigue criterion used in this work to general loading can, in principle, be developed by considering proper directions on the critical plane and then counting fatigue damage on such directions. However, for situations where Mode II and III shear components act simultaneously on the critical plane, it is not clear if the fatigue damages associated with these components should be combined or not. This is due to the fact that the majority of the experimental data which have been used so far to assess the accuracy of critical plane models were produced under combined axial (or bending) and torsion loading. For such loading paths, crack nucleation and early crack growth generally occur on planes perpendicular to the surface, where only the Mode III shear component acts [31,32]. It is also noted that there is a lack of fretting fatigue experiments conducted under variable amplitude loadings. Such kind of experiments are crucial to a critical evaluation of more general approaches for fretting fatigue life estimation.

The methodology described in this work is designed for the prediction of crack initiation life. It is possible to extend the methodology to total fatigue life prediction. This can be carried out by regarding the size of the fatigue process zone and the corresponding number of loading cycles to failure (see Eq. (2)) as, respectively, the size of an initiated crack and the number of cycles to crack initiation. These variables, together with a long crack growth law, can then be used to estimate the total fatigue life. It should be noted that for the fretting fatigue experiments under investigation, the crack propagation life is estimated to be less than 15% of the total life (according to the results reported in Ref. [33], where a similar contact configuration and Al alloy were employed). Hence, the accuracy of the fatigue life estimates is not expected to change significantly if the crack propagation life is included in the current analysis.

Depending on the material, loading range and stress state, different fatigue cracking mechanisms can occur. The present work only focused on a critical plane fatigue criterion designed for shear cracking, namely the Modified Wöhler Curve Method. Similar methodologies employing tensile-based fatigue criteria can also be developed. For example, the formulation presented in Ref. [22] makes use of the maximum principal stress applied at a critical distance. However, for the investigated experimental configuration, a significant improvement on the estimated fatigue life by using a tensile-based approach is not expected. This is suggested by the very similar fatigue lives obtained by Araújo and Nowell [26] using averaged values of the Fatemi−Socie and Smith−Watson−Topper parameters.

The identification of the material constants of a fatigue life estimation methodology is a crucial part of it, since it can significantly affect the estimated lives. In the current methodology, fatigue curves obtained from smooth specimens subjected to two different loading conditions (e.g., tension-compression and pure torsion) are required to identify the constants of the multiaxial fatigue model. In addition, a fatigue curve obtained from a sharply notched specimen is used to identify the critical distance versus fatigue life relationship. The relative simplicity of such tests is an appealing feature of the methodology when compared with other existing approaches that require data from fretting fatigue tests for their calibration. Unfortunately, since the fatigue curves just mentioned were not available, well-known empirical relationships were used to estimate the material constants. Even so, the methodology was capable of producing satisfactory life estimates. Further work should be accompanied by a careful identification of the material constants by running appropriate fatigue tests. This effort may allow a better assessment of the accuracy of the estimated fretting fatigue lives. Also, different materials and contact configurations should be investigated.

Due to the high stress concentration induced by the contact, some level of plastic deformation is expected around the crack initiation point. However, the stress history required by the current methodology comes from an elastic analysis of the contact problem. This is because in the Theory of Critical Distances the average of the actual elastic-plastic stress field damaging the material is assumed to be related to the average of the stress field that would be produced under purely elastic conditions [34]. In addition, the size of the plastically deformed region is expected to increase as the magnitude of the applied load increases. This is accounted for in the methodology by assuming that the critical distance increases as the number of cycles decreases. Of course, this engineering approximation is expected to fail for significant levels of plastic deformation and, therefore, it is not recommended for life estimation in the low cycle regime. In practice, the methodology should be applied for life estimations in the medium/high-cycle fatigue regime, i.e., for fatigue lives greater than about 10^3 cycles to failure [35].

The proposed methodology combines the Maximum Rectangular Hull concept, a critical plane multiaxial fatigue criterion, and a relationship between the critical distance and the number of cycles to failure. These features were introduced to account for the degree of non-proportionality of the loading path as well as the stress gradient damaging the contact region. The methodology does not take into account a number of features of the fretting fatigue problem as, for example, the surface modification due to wear. A more detailed description of fretting fatigue could be formulated by using a wear law [36] or a crystal plasticity model [37]. However, it is found that the simple approach used in this work can correlate well fretting fatigue data for Al-4%Cu alloy.

7. CONCLUSIONS

An engineering methodology for fatigue life estimation of mechanical couplings subjected to fretting fatigue was presented. The material constants in the methodology can be determined by conducting relatively simple fatigue tests on smooth specimens and on sharply notched (or cracked) specimens. The methodology yielded fatigue live estimates with acceptable accuracy for fretting fatigue experiments conducted on Al-4%Cu alloy. The methodology can be easily incorporated into existing FEM analysis software and be used in the fatigue life prediction of mechanical couplings with complex geometry. Further experimental evaluations of the methodology considering different contact configurations and materials are required.

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TABLE CAPTIONS

Table 1: Experimental parameters used in the fretting fatigue tests [13].

Table 2: Material constants for Al-4%Cu alloy in the proposed methodology.

FIGURE CAPTIONS

Figure 1: Modified Wöhler diagram.

Figure 2: Amplitudes of the φ-oriented rectangular hull bounding the shear stress path.

Figure 3: Schematic representation of the Point Method.

Figure 4: Procedure to estimate fretting fatigue life: (a) definition of the critical distance $L(N)$ and (b) flowchart of the algorithm for fatigue life calculation.

Figure 5: Observed and estimated fatigue lives for the fretting fatigue tests reported in Ref. [13].

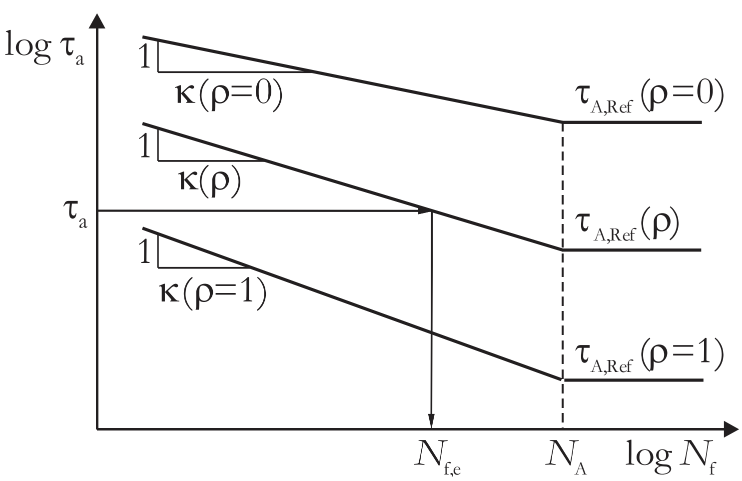
TABLES

Table 1: Experimental parameters used in the fretting fatigue tests [13].

Series	p_0 (MPa)	Q_{max}/P	σ_{Bmax} (MPa)
1	157	0.45	92.7
2	143	0.45	92.7
3	143	0.45	77.2
4	120	0.45	61.8

Table 2: Material constants for Al-4%Cu alloy in the proposed methodology.

σ_{UTS} (MPa)	σ_0 (MPa)	K_{IC} (MPa·m$^{0.5}$)	ΔK_{th} (MPa·m$^{0.5}$)	$\kappa(\rho=0)$	$\tau_{A,Ref}$ ($\rho=0$) (MPa)	$\kappa(\rho=1)$	$\tau_{A,Ref}$ ($\rho=1$) (MPa)
500	124	34	4.4	12.8	161	12.8	115



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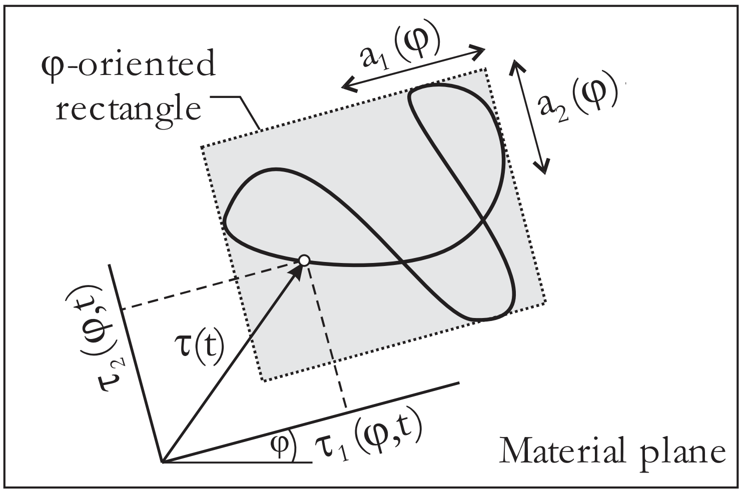

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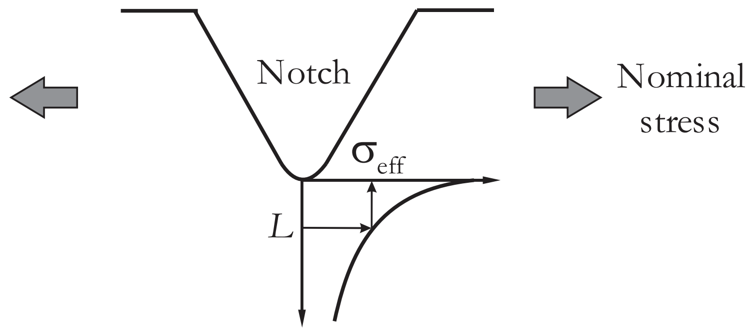

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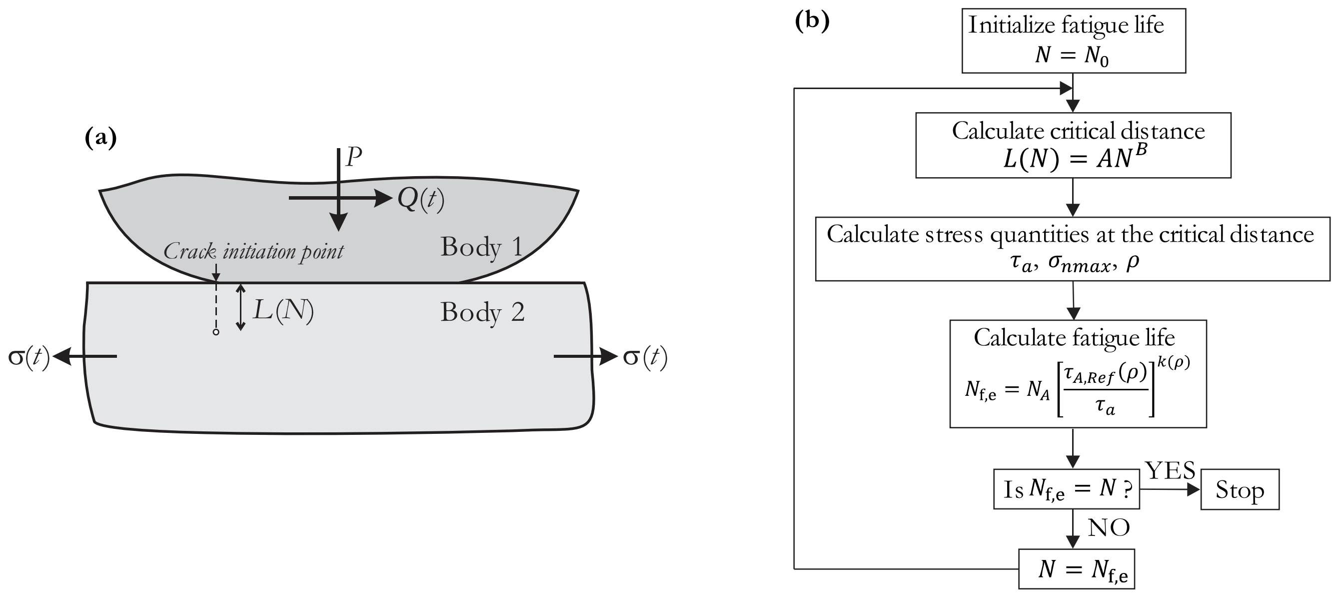

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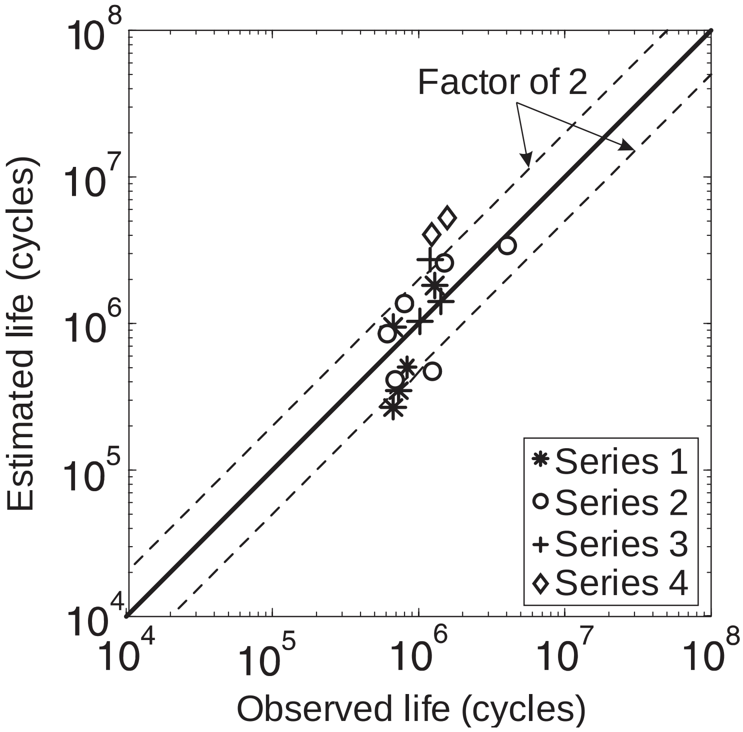

Figure 5: Observed and estimated fatigue lives for the fretting fatigue tests reported in Ref. [13].