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The Theory of Critical Distances to estimate static and dynamic strength of notched plain concrete

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Abstract

The Theory of Critical Distances (TCD) is a well-known design method allowing the strength of notched/cracked components to be estimated accurately by directly post-processing the entire linear-elastic stress fields damaging the material in the vicinity of the stress concentrators being designed. By taking full advantage of the TCD's unique features, in the present study this powerful theory was reformulated to make it suitable for designing notched plain concrete against static and dynamic loading. The accuracy and reliability of the proposed reformulation of the TCD was checked against a set of experimental results generated by testing, under different displacement rates, square section beams of plain concrete containing notches of different sharpness. This validation exercise has demonstrated that the proposed reformulation of the TCD is capable of accurately assessing the static and dynamic strength of notched unreinforced concrete, with the estimates falling within an error interval of ±20%. The level of accuracy that was obtained is certainly satisfactory, especially in light of the fact that static and dynamic strength was estimated without explicitly modelling the stress vs. strain dynamic behaviour of the concrete being tested.

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Keywords: Concrete; notch; static strength; dynamic strength; Theory of Critical Distances

1. Introduction

In situations of practical interest concrete structures have to be designed to withstand high rate of loading. In light of the importance of this complex structural engineering problem, since about the mid-1950s the scientific community has made a tremendous effort to understand and model the mechanical behaviour of concrete materials

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Nomenclature

$f_0(\dot{Z})$	calibration function for σ_0
$f_K(\dot{Z})$	calibration function for K_{Id}
F_f	failure force
$\dot{K}_I$	stress intensity factor rate
K_{Ic}	plane strain fracture toughness
K_{Id}	fracture toughness under dynamic loading
K_t	net stress concentration factor
L	critical distance
Oxyz	system of coordinates
$\dot{F}$	loading rate
r_n	notch root radius
θ, r	polar coordinates
$\dot{\Delta}$	displacement rate
$\dot{\varepsilon}$	strain rate
σ_0	inherent strength
σ_{eff}	effective stress
σ_f	failure stress
σ_{nom}	nominal stress
σ_{UTS}	ultimate tensile strength

subjected to high rate of loading. This issue was addressed extensively by tackling it both from an experimental and a theoretical angle (Malvar & Ross, 1998). However, in spite of the large body of knowledge which is available to structural engineers engaged in designing real structures against dynamic loading, examination of the state of the art shows that a commonly accepted design strategy has not yet been agreed by the international scientific community. In this context, another relevant aspect is that the sensitivity of concrete to the presence of finite radius notches has never been investigated systematically, with this holding true not only under dynamic, but also under static loading.

In this challenging scenario, the present paper reports on an attempt of using the so-called Theory of Critical Distances (TCD) to design notched plain concrete against both static and dynamic loading.

2. Fundamentals of the TCD

Under Mode I static loading, the TCD postulates that the notched component being designed does not fail as long as the following condition is assured (Taylor, 2007; Askes & Susmel, 2015):

$$\sigma_{eff} \leq \sigma_0, \tag{1}$$

In inequality (1), σ_{eff} is the effective stress determined according to the TCD, whilst σ_0 is the so-called inherent material strength. If the TCD is used to perform the static assessment of brittle notched materials, σ_0 can be taken equal to the material ultimate tensile strength, σ_{UTS} (Susmel and Taylor, 2008a). In contrast, as far as ductile notched materials are concerned, σ_0 is seen to be larger than σ_{UTS} (Susmel and Taylor, 2008b; Susmel and Taylor, 2010b), with the determination of σ_0 requiring complex, time-consuming, and expensive experiments (Susmel and Taylor, 2010a).

The second material property which is needed to apply the TCD is critical distance L. Under quasi-static loading, this length scale parameter can be estimated directly from the plane strain fracture toughness, K_{Ic}, and the inherent material strength as follows: (Taylor, 2007):

$$L = \frac{1}{\pi}\left(\frac{K_{Ic}}{\sigma_0}\right)^2 \tag{2}$$

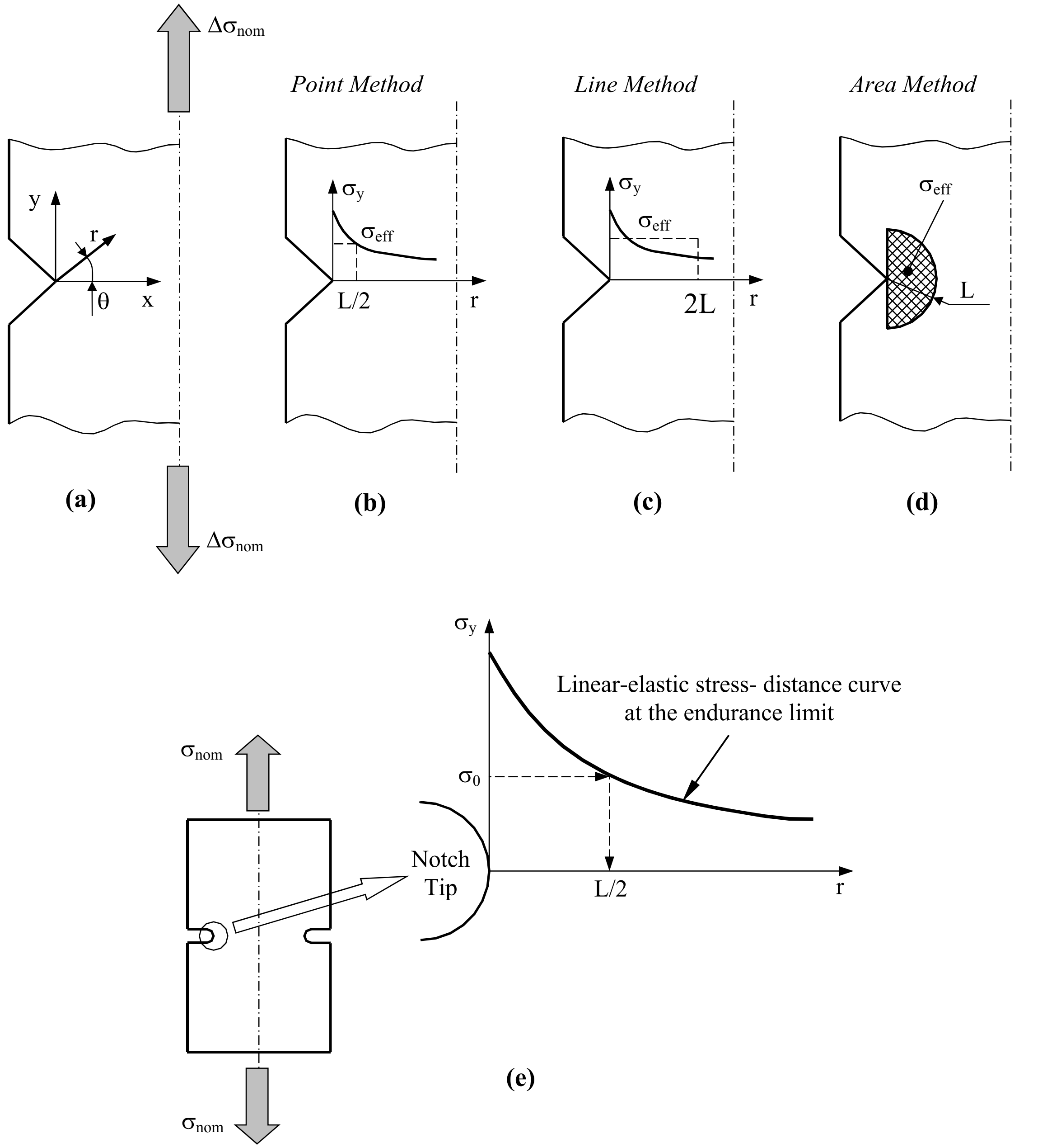

Fig. 1. Definition of the local systems of coordinates (a) and range of the effective stress, $\Delta\sigma_{eff}$, calculated according to the Point Method (b), Line Method (c), and Area Method (d).

The effective stress, σ_{eff}, required to perform the static assessment according to Eq. (1) has to be calculated by directly post-processing the linear-elastic stress field in the vicinity of the stress concentrator being designed. Effective stress σ_{eff} can be determined according to the Point Method (PM), the Line Method (LM), and the Area Method (AM) as follows (Taylor, 2007; Askes et al., 2013; Ameri et al., 2015):

$$\sigma_{eff} = \sigma_y\left(\theta = 0, r = \frac{L}{2}\right) \qquad \text{(PM)} \qquad (3)$$

$$\sigma_{eff} = \frac{1}{2L}\int_0^{2L} \sigma_y(\theta = 0, r)dr \qquad \text{(LM)} \tag{4}$$

$$\sigma_{eff} = \frac{4}{\pi L^2}\int_0^{\pi/2}\int_0^{L} \sigma_1(\theta, r) r dr d\theta \qquad \text{(AM)} \tag{5}$$

The adopted symbols as well as the meaning of the effective stress calculated through definitions (3) to (5) are explained in Figures 1a to 1d, with σ_y being the stress parallel to axis y and σ_1 the maximum principal stress.

According to Eqs (3) to (5), the determination of the effective stress is based on the use of critical distance L, Eq. (2). Given the experimental value of the plane strain fracture toughness, L can be determined directly solely for those brittle materials for which σ_0 is invariably equal to σ_{UTS}. In contrast, when σ_0 is different from σ_{UTS} (as for ductile materials), the required critical distance has to be determined by post-processing the results generated by testing specimens containing notches of different sharpness (Taylor, 2007; Susmel and Taylor, 2010b). This procedure is schematically shown in Figure 1e. In particular, according to the PM's *modus operandi*, the coordinates of the point at which the two linear-elastic stress-distance curves, plotted in the incipient failure condition, intersect each other allow L and σ_0 to be estimated directly. To conclude, it can be recalled here that this experimental procedure based on notches of different sharpness was seen to be very accurate also to estimate K_{Ic} (Susmel, Taylor, 2010c). In fact, as soon as both L and σ_0 determined according to the procedure schematically depicted in Figure 2 are known, the plane strain fracture toughness for the specific material being investigated can directly be estimated through Eq. (2), with K_{Ic} being the unknown variable in the problem.

3. The TCD reformulated to design notched plain concrete against dynamic loading

Much experimental evidence (Malvar & Ross, 1998; Lambert & Ross, 2000) suggests that the mechanical/cracking behaviour of concrete subjected to dynamic loading is different from the corresponding one displayed under quasi-static loading. In particular, both the dynamic failure stress (Malvar & Ross, 1998) and the dynamic fracture toughness (Lambert & Ross, 2000; Reji & Shah 1990) are seen to increase as the applied load/strain/displacement rate increases. If these experimental findings are re-interpreted according to the TCD's *modus operandi*, the hypothesis can be formed that both inherent strength σ_0 and critical distance L vary with the rate of the applied loading. In particular, using $\dot{Z}$ to denote either the loading rate, $\dot{F}$, the strain rate, $\dot{\varepsilon}$, the displacement rate, $\dot{\Delta}$, or the Stress Intensity Factor (SIF) rate, $\dot{K}_I$, the effect of the dynamic loading on σ_0 and K_{Id} can be modelled as follows (Yin et al., 2015):

$$\sigma_0(\dot{Z}) = f_0(\dot{Z}) \tag{6}$$

$$K_{Id}(\dot{Z}) = f_K(\dot{Z}) \tag{7}$$

where functions $f_0(\dot{Z})$ and $f_K(\dot{Z})$ are material properties that have to be determined by running appropriate experiments. According to Eqs (6) and (7), both σ_0 and K_{Id} vary as $\dot{Z}$ increases, so that, in the most general case, also critical distance L is expected to change with $\dot{Z}$, i.e. (Yin et al., 2015):

$$L(\dot{Z}) = \frac{1}{\pi}\left[\frac{K_{Id}(\dot{Z})}{\sigma_0(\dot{Z})}\right]^2 \tag{8}$$

Having defined the critical distance value, the effective stress suitable for designing notched plain concrete against dynamic loading can be calculated by re-arranging definitions (3), (4), and (5) as follows:

$$\sigma_{eff}(\dot{Z}) = \sigma_y\left(\theta = 0, r = \frac{L(\dot{Z})}{2}\right) \qquad \text{(PM)} \tag{9}$$

$$\sigma_{eff}(\dot{Z}) = \frac{1}{2L(\dot{Z})} \int_0^{2L(\dot{Z})} \sigma_y(\theta = 0, r)\, dr \qquad \text{(LM)} \tag{10}$$

$$\sigma_{eff}(\dot{Z}) = \frac{2}{\pi L(\dot{Z})^2} \int_{-\pi/2}^{\pi/2} \int_0^{L(\dot{Z})} \sigma_1(\theta, r)\, r\, dr d\theta \qquad \text{(AM)} \tag{11}$$

To conclude, it is worth observing that, in accordance with the core assumption on which the TCD is based (Taylor, 2007), the stress analysis problem has to keep being addressed by using a simple linear-elastic constitutive law to model the mechanical behaviour of concrete also under dynamic loading.

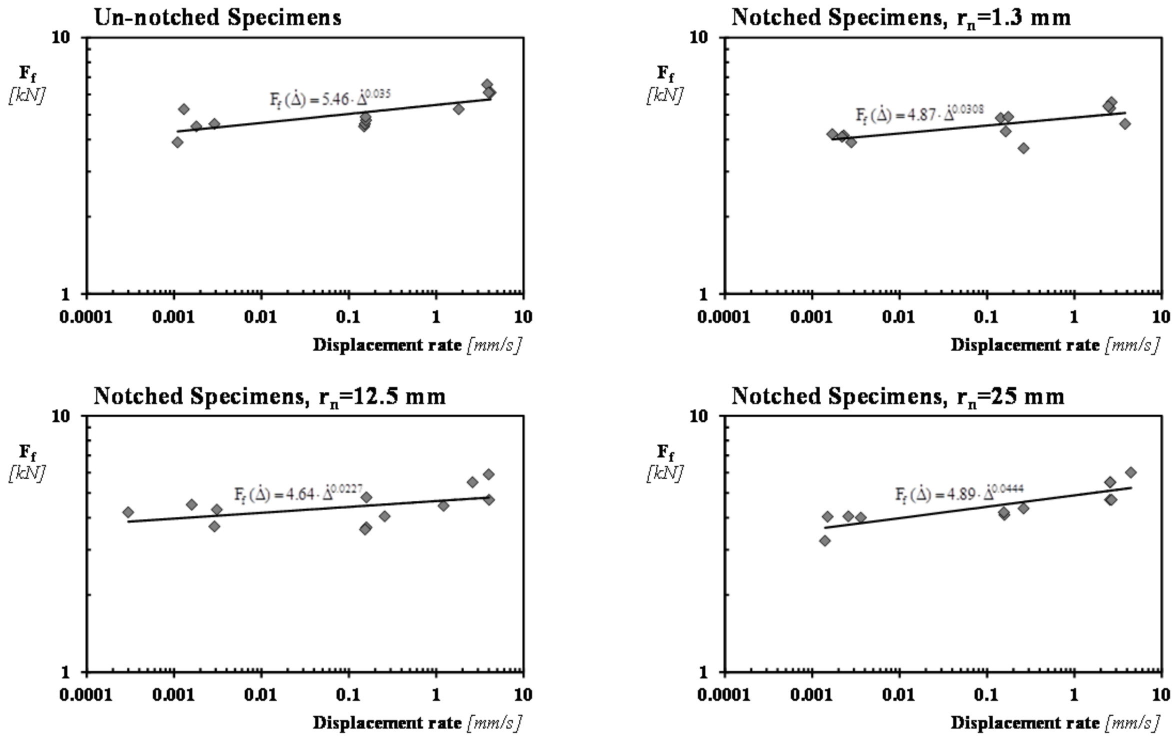

Fig. 2. Summary of the generated experimental results.

4. Experimental details

To assess the accuracy of the proposed reformulation of the TCD in estimating static and dynamic strength of notched plain concrete, 100 mm x 100 mm square section beams weakened by notches of different sharpness were tested under four-point bending. The strength of the un-notched material was determined by testing under three-point bending square section specimens having width equal to 100 mm and thickness to 50 mm. The length of the notched samples was equal to 500 mm and the nominal notch depth to 50 mm, with these beams containing U-notches having root radius, r_n, equal to 25 mm (K_t=1.47), 12.5 mm (K_t=1.84), and 1.3 mm (K_t=4.99).

The concrete mix used to cast the specimens was as follows (Franklin et al., 1997): Portland cement (strength class equal to 30 N/mm^2), natural round gravel (10 mm grading), and grade M concrete sand. The water-to-cement ratio was set equal to 0.45. The specimens were removed from the moulds 24 hours after casting and subsequently cured in a moist room for 28 days at 23°C.

Both the un-notched and notched beams were tested by exploring displacement rates, $\dot{\Delta}$, in the range $3 \cdot 10^{-4}$-4.4 mm/s, the generated results being reported in Figure 2 in terms of failure force, F_f. To conclude, Figure 3 shows some examples of the cracking behavior displayed by the notched beams under different displacement rates.

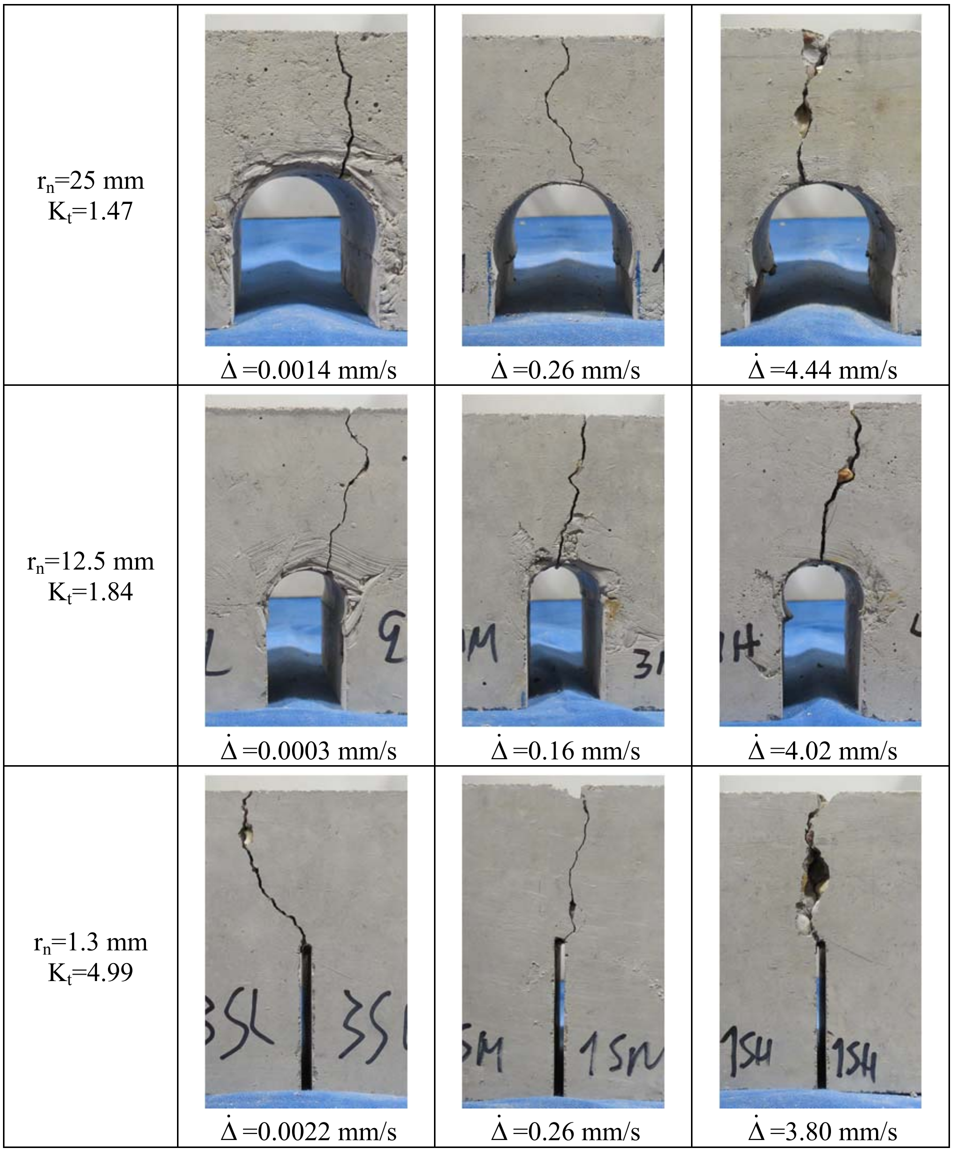

Fig. 3. Examples of the cracking behavior displayed by the tested specimens under different values of the displacement rate.

5. Validation by experimental data

The linear-elastic stress fields in the vicinity of the notches being investigated were determined numerically by using commercial Finite Element (FE) software ANSYS®. The tested concrete was modelled as a homogenous and isotropic material. The FE models were meshed using bi-dimensional elements Plane 183, with the mesh density in the vicinity of the notch tips being increased gradually until convergence occurred.

Since the tested concrete was characterised by a mechanical behaviour that was predominantly brittle, the hypothesis was formed that inherent strength σ_0 could be taken invariably equal to the un-notched material failure stress (Susmel & Taylor, 2008a). This hypothesis was assumed to hold true independently from the value of the displacement rate being investigated. After making this initial simplifying assumption, owing to the F_f vs. $\dot{\Delta}$ behaviour displayed by the un-notched concrete (see Figure 2), relationship $\sigma_0(\dot{Z}) = \sigma_f(\dot{Z})$ was expressed by adopting a simple power law (Yin et al., 2015), obtaining:

$$\sigma_0(\dot{\Delta}) = 6.71 \cdot \dot{\Delta}^{\,0.0344} \tag{12}$$

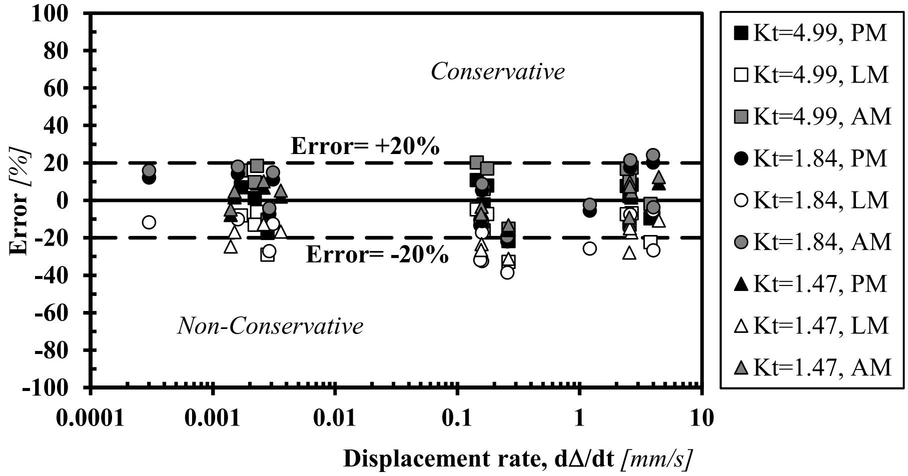

Fig. 4. Accuracy of the TCD applied in the form of the PM, LM, and AM in estimating static and dynamic strength of notched concrete.

The critical distance value, L, needed to calculate $\sigma_{eff}(\dot{Z})$ according to definitions (9) to (11) was estimated by following the simplified procedure schematically shown in Figure 1e. In particular, the results generated by testing both the un-notched and the sharply U-notched specimens were used as calibration information, L being determined by making $\dot{\Delta}$ vary in the range of interest. This procedure returned a value for L that was invariably equal to 4.8 mm. In other words, contrary to what we observed in notched metallic materials subjected to dynamic loading (Yin et al., 2015), for the specific concrete material being investigated the critical distance was seen not to be affected by the rate of the applied loading.

The error diagram of Figure 4 summarises the overall accuracy obtained by applying the TCD in the form of the PM, LM, and AM, with the error being calculated according to the following trivial relationship:

$$Error = \frac{\sigma_{eff}(\dot{\Delta}) - \sigma_0(\dot{\Delta})}{\sigma_0(\dot{\Delta})} \; [\%] \tag{13}$$

As per Figure 4, the use of both the PM and AM resulted in estimates falling within an error interval of ±20%. The LM instead returned predictions that were slightly non-conservative, even if they still fell mainly within the target error band. It is possible to conclude by observing that the level of accuracy that was obtained is certainly satisfactory since, in the presence of stress concentration phenomena, it is not possible to distinguish between an error of ±20% and an error of 0% as a consequence of those problems that are usually encountered when performing the testing as well as the numerical analyses (Taylor, 2007).

6. Conclusions

- The proposed design methodology is suitable for designing notched plain concrete against static and dynamic loading by directly post-processing the linear-elastic stress fields acting on the material in the vicinity of the geometrical features being assessed. Accordingly, reliable static and dynamic assessment can be performed without the need to invoke complex non-linear constitutive laws.

- The TCD used in the form of both the PM and AM was seen to be capable of estimates falling within an error interval of ±20%.
- More work needs to be done to extend the use of this design approach based on the TCD to those situations involving static and dynamic multiaxial loading.

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