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Estimating the orientation of crack initiation planes under constant and variable amplitude multiaxial fatigue loading

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Abstract

This paper investigates the accuracy of the shear strain Maximum Variance Method (γ–MVM) and Maximum Damage Method (MDM) in predicting the orientation of crack initiation planes under both constant and variable amplitude multiaxial fatigue loading. The γ–MVM defines the critical plane as the plane on which the variance of the resolved shear strain reaches its maximum value. In contrast, a specific multiaxial fatigue criterion is needed to be used along with the MDM to predict the orientation of the critical plane under multiaxial fatigue loading. As far as variable amplitude multiaxial loading is concerned, the MDM have to be used by applying with a certain fatigue criterion, a cycle counting method and a cumulative damage rule. In this paper, the MDM is applied with Fatemi & Socie's criterion, Bannantine & Socie's cycle counting method and Palmgren-Miner's linear rule. The MDM assumes that the critical plane is the plane experiencing the maximum accumulated damage. Experimental data for several metals tested under constant and variable amplitude multiaxial fatigue loading taken from literature are used to assess the accuracy of these two methodologies. The results show that the predictions made by both the γ–MVM and MDM have good accuracy for the investigated materials and investigated load histories: 90% of the predictions made by the γ–MVM and 80% of the predictions made by the MDM fall within a scatter band of 20%.

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Keywords: multiaxial fatigue; variable amplitude; critical plane; crack initiation plane; nonproportional loading

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1. Introduction

Critical plane approaches take as a starting point the idea that, at the critical locations, fatigue cracks initiate and propagate on certain planes. As to definition of the critical plane itself, there are different ways. Findley (1956) determined the critical plane by maximizing a linear combination of the shear stress amplitude and the maximum value of the normal stress. Brown and Miller (1973) defined the critical plane as the plane experiencing the maximum shear strain amplitude. Fatemi and Socie (1988) assumed that the critical plane is the plane of maximum shear strain amplitude when the crack initiation process is Mode II governed. As far as variable amplitude multiaxial load histories are concerned, defining the orientation of the critical plane itself becomes very complicate and time consuming. To address this intractable problem, several approaches have been proposed which include: the MDM first proposed by Bannantine and Socie (1991), the MVM proposed by Macha (1989), Susmel (2010), and Wang and Susmel (2015, 2016), and the weight function method.

There always exists an argument on that the predictions of the critical plane approach to the cracking orientation is not coincide with the physical concept which it is based on. And, a limited work has been done in the evaluation of the capability of the critical plane approach to predict the crack directions (Jiang, 2007).

In this scenario, this paper aims to investigate the accuracy in predicting the orientation of Stage I fatigue cracks of the γ–MVM proposed by Wang and Susmel (2015, 2016) and the MDM applied with the shear strain based critical plane approach proposed by Fatemi and Socie (1988) (FS).

2. The shear strain Maximum Variance Method (γ-MVM)

Based on the research by Macha (1989), Susmel (2010) formulated the shear stress-Maximum Variance Method (τ–MVM) to predict, in the high-cycle fatigue regime, the orientation of the critical plane under constant and variable amplitude multiaxial fatigue loading. Then, by extending this idea to in terms of strain, Wang and Susmel (2015, 2016) proposed the shear strain-Maximum Variance Method (γ–MVM) which is suitable to be used in the low/medium-cycle fatigue regime.

The body as shown in Fig. 1a is subjected to a complex system of forces, which results in a tri-axial time-variable stress and strain states at the critical location, point O. The stress and strain states at point O can be expressed as follows:

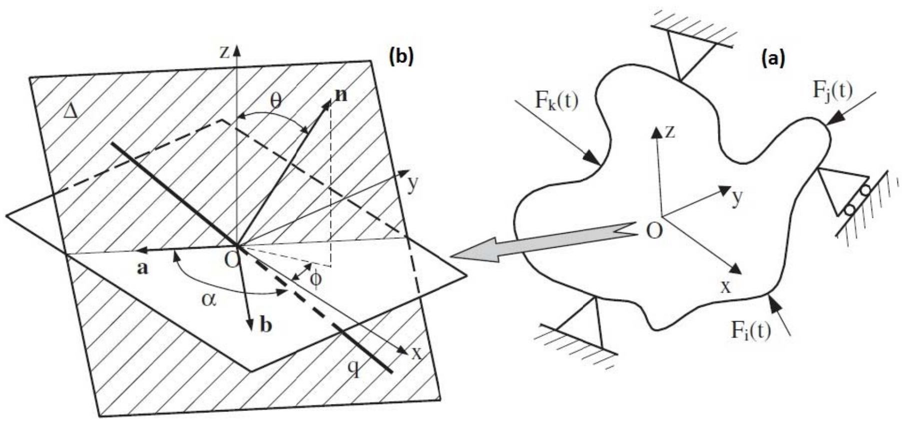

Fig. 1 (a) body subjected to an external system of forces, (b) definition of a generic material plane, Δ, and coordinate system

$$[\sigma(t)] = \begin{bmatrix} \sigma_x(t) & \tau_{xy}(t) & \tau_{xz}(t) \\ \tau_{xy}(t) & \sigma_y(t) & \tau_{yz}(t) \\ \tau_{xz}(t) & \tau_{yz}(t) & \sigma_z(t) \end{bmatrix} \tag{1}$$

$$[\varepsilon(t)] = \begin{bmatrix} \varepsilon_x(t) & \frac{\gamma_{xy}(t)}{2} & \frac{\gamma_{xz}(t)}{2} \\ \frac{\gamma_{xy}(t)}{2} & \varepsilon_y(t) & \frac{\gamma_{yz}(t)}{2} \\ \frac{\gamma_{xz}(t)}{2} & \frac{\gamma_{yz}(t)}{2} & \varepsilon_z(t) \end{bmatrix} \tag{2}$$

Then the shear strain resolved along of a generic direction on a generic plane, $\gamma_q(t)$, can be obtained by tensor rotation of the above strain tensor to this direction. Alternatively, $\gamma_q(t)$ can also be expressed by the following scalar product:

$$\frac{\gamma_q(t)}{2} = \mathbf{e}(t) \bullet \mathbf{d} \tag{3}$$

where, **d** and **e** can be expressed as follows:

$$\mathbf{d} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \\ d_5 \\ d_6 \end{bmatrix} = \begin{bmatrix} \frac{1}{2}\left[\sin(\theta)\sin(2\phi)\cos(\alpha) + \sin(\alpha)\sin(2\theta)\cos(\phi)^2\right] \\ \frac{1}{2}\left[-\sin(\theta)\sin(2\phi)\cos(\alpha) + \sin(\alpha)\sin(2\theta)\sin(\phi)^2\right] \\ -\frac{1}{2}\sin(\alpha)\sin(2\theta) \\ \frac{1}{2}\sin(\alpha)\sin(2\phi)\sin(2\theta) - \cos(\alpha)\cos(2\phi)\sin(\theta) \\ \sin(\alpha)\cos(\phi)\cos(2\theta) + \cos(\alpha)\sin(\phi)\cos(\theta) \\ \sin(\alpha)\sin(\phi)\cos(2\theta) - \cos(\alpha)\cos(\phi)\cos(\theta) \end{bmatrix} \tag{4}$$

$$\mathbf{e}(t) = \begin{bmatrix} \varepsilon_x(t) & \varepsilon_y(t) & \varepsilon_z(t) & \frac{\gamma_{xy}(t)}{2} & \frac{\gamma_{xz}(t)}{2} & \frac{\gamma_{yz}(t)}{2} \end{bmatrix} \tag{5}$$

Therefore, the variance of the shear strain resolved along a generic direction q can be expressed in the following form:

$$Var\left[\frac{\gamma_q(t)}{2}\right] = Var\left[\sum_k d_k e_k(t)\right] = \sum_i \sum_j d_i d_j Cov\left[e_i(t), e_j(t)\right] \tag{6}$$

All the candidate critical planes can be obtained by determining the global maxima of the variance of the resolved shear strain, as discussed by Susmel (2010) and Wang and Susmel (2016).

3. The Maximum Damage Method (MDM)

The stress and strain states summarized via Eq. (1) and Eq. (2) are also used in the MDM, and the stress and strain components on a generic direction of a generic plane are obtained by tensor rotation.

The MDM always needs to be applied with a specific multiaxial fatigue criterion to predict the orientation of the critical plane. In the present paper, the FS criterion is used along with the MDM. As far as variable amplitude multiaxial loading is concerned, the cycle counting method and the fatigue damage cumulative rule are also required. Bannantine & Socie's cycle counting method (Bannantine and Socie, 1991) and Palmgren-Miner's linear damage rule (Palmgren, 1924; Miner, 1945) are used in this paper.

In more detail, the resolved shear stain and the normal stress can be obtained by projecting the loading history on a generic direction of a generic plane. The resolved shear strain cycles and the maximum normal stresses during the shear strain cycles are identified by using Bannantine & Socie's cycle counting method. The *i*-th loading damage is calculated by using the FS criterion, and the accumulated damage is calculated according to Palmgen-Miner's linear rule. This calculation should be done in every direction on every plane at the critical location.

The shear-strain based multiaxial fatigue criterion proposed by Fatemi and Socie (1988) can be expressed as follows:

$$\frac{\Delta\gamma}{2}\left(1+k\frac{\sigma_{n,\max}}{\sigma_y}\right)=\frac{\tau'_f}{G}\left(2N_f\right)^{b_0}+\gamma'_f\left(2N_f\right)^{c_0} \tag{7}$$

where $\Delta\gamma/2$ is the shear stain amplitude relative to the critical direction on the certain plane, $\sigma_{n,\max}$ is the maximum normal stress occurring during the same cycle of $\Delta\gamma/2$ on this plane, k is a material constant, and σ_y is the material yield strength.

The total damage is calculated according to Palmgren-Miner's linear rule as follows:

$$D_{\text{tot}}=\sum_{i=1}^{j}\frac{n_i}{N_{f,i}} \tag{8}$$

where n_i is the number of loading cycles, $N_{f,i}$ is the total cycles to failure under the *i-th* loading, and D_{tot} is the total value of the damage sum.

4. Experimental valuations

In order to check the accuracy of the γ-MVM and MDM in predicting the orientation of the critical plane under multiaxial loading the experimental research for S45C steel by Kim et al. (1999) under short variable amplitude multiaxial loading, the observation of cracking behavior for S460N by Jiang et al. (2007) under multiaxial constant loading and the observed cracking behavior of 1050QT steel and 304L steel under discriminating strain paths by Shamsaei et al. (2011) were taken into consideration.

According to the research by Forsyth (1961), Socie and Marquis (2000), Marciniak et al. (2014) and Susmel et al. (2014), the propagation process of micro/meso-crack can be described by two stages: Stage I is crack initiation, and Stage II is crack propagation. Usually, for elasto-plastic metallic materials, Stage I cracks initiate on those plane of maximum shear. Stage II cracks tend to propagate perpendicular to the normal stress. Therefore, in the current paper, if the length of the observed crack in the original literature is of the order of millimeters, the observed angle is deemed to be the orientation of the crack initiation plane. If the crack length is in the centimeter scale, with reference to the method used by Susmel et al. (2014) and Marciniak et al. (2014), then an angle of ±45° was added to the observed crack angle in the original literature to obtain the orientation of the crack initiation plane.

The static and fatigue properties of the investigated materials are listed in the Table 1. The investigated loading paths are shown in Fig. 2. Figs 3 and 4 show the comparison of the predicted orientation of crack initiation plane and the experimental observation on the cracking behavior. The ranges of the crack shown in these two figures are from 0° to 260°. Here, the crack plane at angle 0° is the same plane as the one which have orientation angle equal to 180°. The definition of the angle of crack plane is shown down-right in Figs 3 and 4. 80 data taken from literature are used in this evaluation. However, some data are hidden from view because of some other data in front of them. The results show that both the γ-MVM and MDM can predict the orientation of the crack initiation plane with a good level of accuracy. 90% of the predictions made by the γ-MVM fall within the 20% scatter band, and 95% of the predictions fall within the 30% scatter band. The MDM provides good predictions. 80% of the estimates made by the MDM fall within the 20% scatter band, and 90% of the estimates fall within the 30% scatter band.

Table 1: Static and fatigue properties of the investigated materials.

Material	Ref.	E (MPa)	G (MPa)	σ_y (MPa)	k in FS criterion	γ_f'	τ_f' (MPa)	b_0	c_0
S45C	Kim et al. (1999)	186,000	70,600	496	1	0.198	685	-0.12	-0.36
1050 QT Steel	Shamsaei et al. (2011)	203,000	81,000	1009	0.6	3.48	777	-0.062	-0.725
304L steel	Shamsaei et al. (2011)	195,000	77,000	208	0.15	0.211	743	-0.145	-0.394
S460N	Jiang et al. (2007)	208,500	80,200	500	1	0.487	559.8	-0.086	-0.493

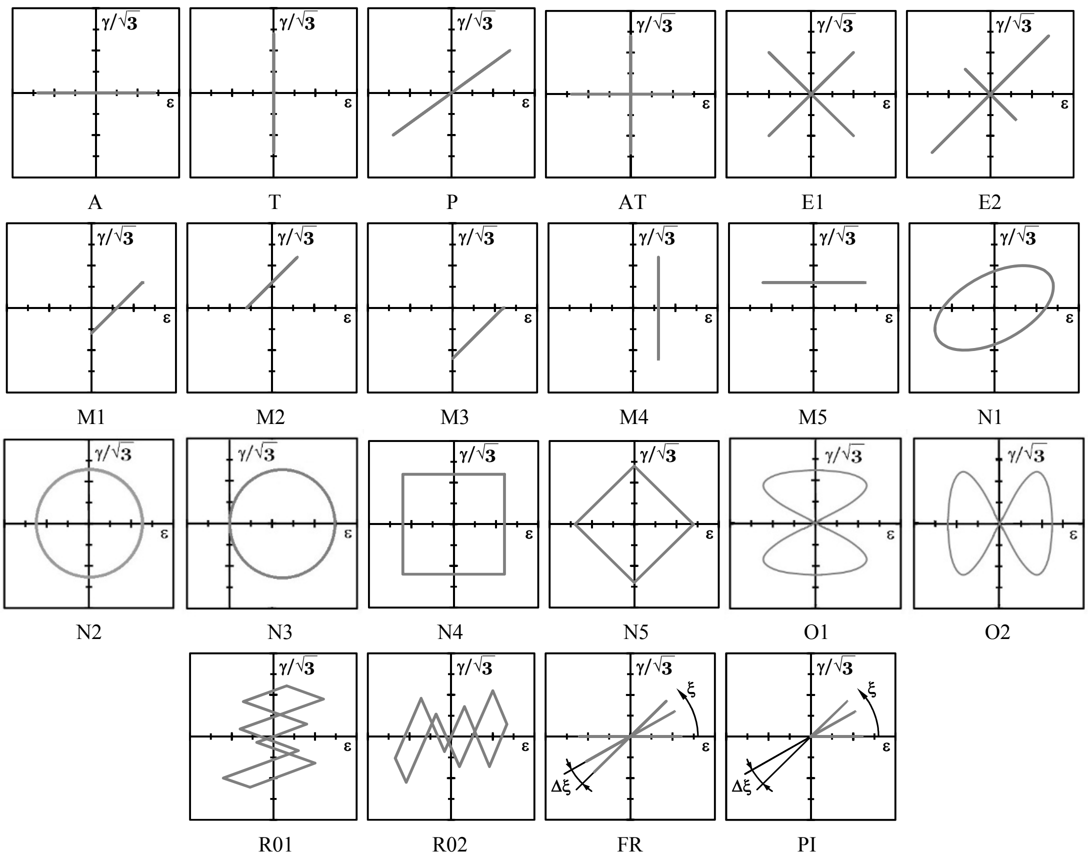

Fig. 2. Investigated strain paths

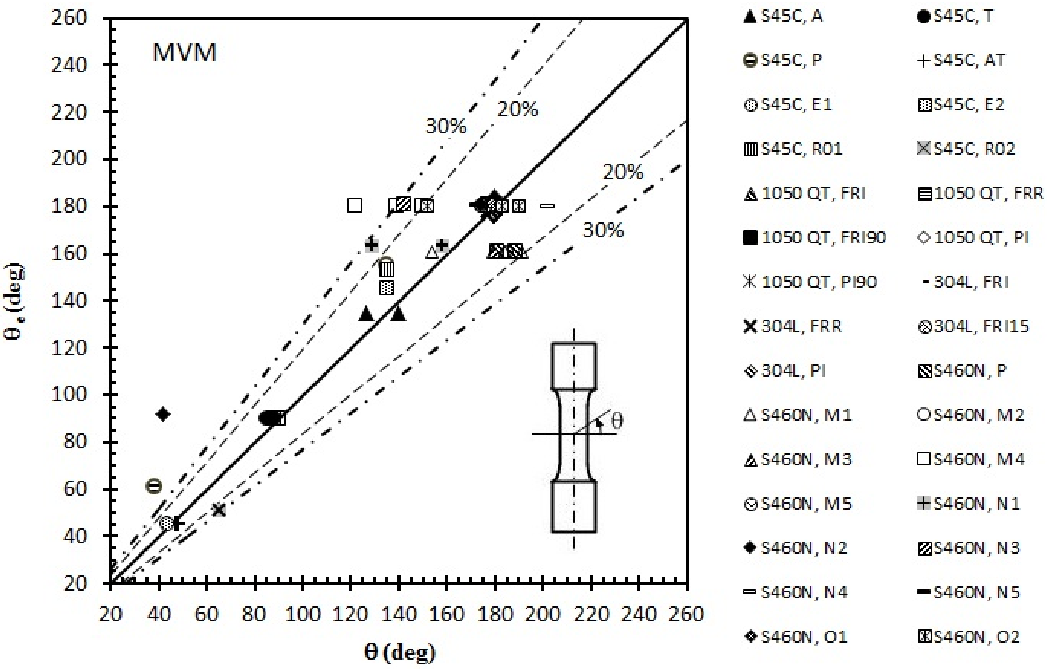

Fig. 3. Comparison of experimental and calculated crack initiaton plane according to the γ-MVM

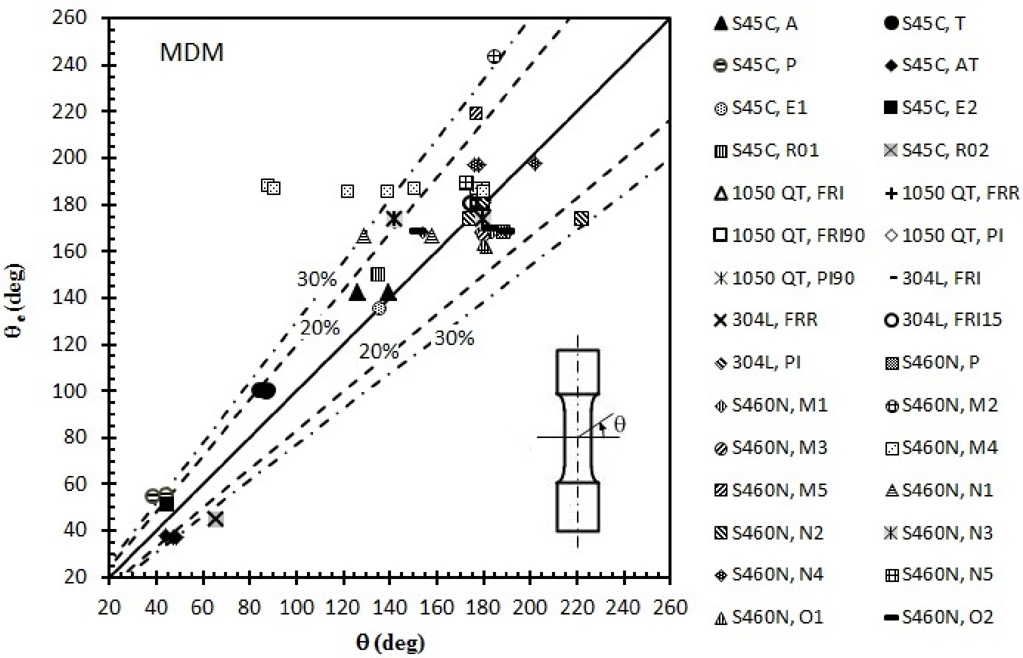

Fig. 4. Comparison of experimental and calculated crack initiaton plane according to the MDM

5. Conclusions

1. For the investigated materials under investigated loading conditions, both the γ-MVM and the MDM can predict the orientation of the crack initiation plane with a good level of accuracy.

2. 90% of the predictions made by the γ-MVM fall within the 20% scatter band, and 95% of the predictions fall within the 30% scatter band. The MDM provides a good prediction. 80% of the data estimated by the MDM fall within the 20% scatter band, and 90% of the estimates fall within the 30% scatter band.

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References

Bannantine, J.A., Socie, D.F., 1991.A variable amplitude multiaxial life prediction method. In: Kussmaul, K., McDiarmid, D. Socie, D.F. (Eds). Fatigue under Biaxial and Multiaxial Loading. ESIS 10. Lodon: Mechanical Engineering Publications, pp. 35.

Brown, M.W., Miller, K.J., 1973. A theory for fatigue under multiaxial stress-strain conditions. In: Proc. Institution of Mechanical Engineering 187, 745-56.

Fatemi, A., Socie, D.F., 1988. A critical plane approach to multiaxial fatigue damage including out-of-phase loading. Fatigue & Fracture of Engineering Material and Structure 11, 149-165.

Findley, W.N., 1956. Modified theory of fatigue failure under combined stress, In: Proc. of the society of experimental stress analysis 14, 35-46.

Forsyth, P.J.E., 1961. A two-stage fatigue fracture mechanisms. In: Proceeding of the crack propagation symposium, Vol. 1, Cranfield, UK, 76-94.

Jiang, Y., Hertel, O., Vormwald, M., 2007. An experimental evaluation of three critical plane multiaxial fatigue criteria. International Journal of Fatigue 29, 1490-1502.

Kim, K.S., Park, J.C., Lee, J.W., 1999. Multiaxial fatigue under variable amplitude loads, Trans ASME Journal of Engineering Materials and Technology 121, 286-93.

Macha, E., 1989. Simulation investigations of the position of fatigue fracture plane in materials with biaxial loads. Materialwiss Werkstofftech 20, 132-6.

Marciniak, Z., Rozumek, D., Macha, E., 2014. Verification of fatigue critical plane position according to variance and damage accumulation methods under multiaxial loading. International Journal of Fatigue 58, 84-93.

Miner, M.A., 1945. Cumulative damage in fatigue. Journal of Applied Mechanics 67, AI59–64.

Palmgren, A., 1924. Die Lebensdauer von Kugellagern. vol. 68. Verfahrenstechnik, Berlin, pp. 339–41.

Shamsaei, N., Fatemi, A., Socie, D.F., 2011. Multiaxial fatigue evaluation using discriminating strain paths. International Journal of Fatigue 33, 597-609.

Socie, D.F., Marquis, G.B., 2000. Multiaxial Fatigue. Warrendale, PA: SAE.

Susmel, L., 2010. A simple and efficient numerical algorithm to determine the orientation of the critical plane in multiaxial fatigue problems. International Journal of Fatigue 32, 1875-1883.

Susmel, L., Tovo, R., Socie, D.F., 2014. Estimating the orientation of stage I crack paths through the direction of maximum variance of the resolved shear stress. International Journal of Fatigue 58, 94-101.

Wang, Y., Susmel, L., 2015. Critical plane approach to multiaxial variable amplitude fatigue loading. Fracture and Structural Integrity 33, 345-356.

Wang, Y., Susmel, L., 2016. The modified Manson-Coffin Curve Method to estimate fatigue lifetime under complex constant and variable amplitude multiaxial fatigue loading. International Journal of Fatigue 83, 135-149.