D-particle Recoil Space Times and "Glueball" Masses

Nick E. Mavromatos

Department of Physics, Theoretical Physics, King's College London, The Strand, London WC2R 2LS, United Kingdom,

and

Elizabeth Winstanley

Department of Physics, Theoretical Physics, University of Oxford, 1 Keble Road Oxford OX1 3NP, United Kingdom.

Abstract

We discuss the properties of matter in a D-dimensional anti-de-Sitter-type space time induced dynamically by the recoil of a very heavy D(irichlet)-particle defect embedded in it. The particular form of the recoil geometry, which from a world-sheet view point follows from logarithmic conformal field theory deformations of the pertinent sigma-models, results in the presence of *both* infrared and ultraviolet (spatial) cut-offs. These are crucial in ensuring the presence of mass gaps in scalar matter propagating in the D-particle recoil space time. The analogy of this problem with the Liouville-string approach to QCD, suggested earlier by John Ellis and one of the present authors, prompts us to identify the resulting scalar masses with those obtained in the supergravity approach based on the Maldacena's conjecture, but without the imposition of any supersymmetry in our case. Within reasonable numerical uncertainties, we observe that agreement is obtained between the two approaches for a particular value of the ratio of the two cut-offs of the recoil geometry. Notably, our approach does not suffer from the ambiguities of the supergravity approach as regards the validity of the comparison of the glueball masses computed there with those obtained in the continuum limit of lattice gauge theories.

1 Introduction and Summary

The conjecture of Maldacena [1] that (quantum) correlators on large-N supersymmetric gauge-field theories, living on a space-time manifold $\mathcal{M}$, may be computed by purely classical supergravity methods in the bulk of an Anti-de-Sitter (AdS) space-time with $\mathcal{M}$ as its boundary, found a very interesting application to Quantum Chromodynamics (QCD), after Witten's suggestion [2] that such an approach may lead to an understanding of the confinement-deconfinement transition. This created an enormous interest in the subject [3].

However, most of the approaches so far have used the original formulation, based on critical (super)string theory, which at a certain stage makes explicit use of space-time supersymmetry. The latter is broken explicitly by "temperature" in the approach of [2], in order to provide a regularized version of zero-temperature QCD. In addition, there appear to be some doubts that the calculations of glueball masses based on supergravity calculations in the bulk of AdS geometry [4]-[7], which, from a superstring-theory viewpoint, are lowest order in α', are strictly applicable to the case of glueball masses obtained in the continuum limit of lattice QCD computations [8], which seem to necessitate a resummation of higher order corrections in α'. The latter are still uknown in the context of string theory, although there are claims [6] that such corrections do not affect much the results on the ratios of the glueball masses calculated by means of the supergravity approach.

An alternative to the above-described critical-dimension superstring theory approach, which necessarily makes use of space-time supersymmetry at a certain stage, is the Liouville string approach to QCD [9], where also AdS space times appear but supersymmetry is not a requirement. In this context, but in a different approach from [9], with emphasis on the rôle of magnetic monopoles on the confinement problem, John Ellis and one of the authors have argued [10] that AdS space time appears naturally and *dynamically* within the modern context of D-brane approach to gauge theories. This is a result of quantum fluctuations of the monopole defect, described by "recoil" logarithmic conformal field theories in a world-sheet non-critical (and non-supersymmetric) string approach [11]. More specifically, the D-particle recoil model of [12] has been used in the limit $u_i \to 0$, which, arguably, describes the interaction of a quark-antiquark Wilson loop with a magnetic monopole for the gauge field configuration in QCD [10]. Such configurations have been argued by Polyakov [13] and 't Hooft [14] to play a crucial rôle in the confinement problem.

What we have argued in [10] is that the fluctuating monopole may be represented, in this non-critical string framework, as a quantum fluctuating D(irichlet)-particle. Such fluctuations may be considered as a specific case of "recoil", which in turn, corresponds to logarithmic worldsheet deformations [11] of the pertinent σ-model. The Wilson loop in this picture corresponds to a static macroscopic closed string in interaction with the D-particle defect. Due to the static nature of the loop, the recoil velocity u_i of the defect is taken to be *vanishing*. In the formalism of logarithmic conformal field theory of [11], this limit corresponds to considering only the effects of one of the deformations of the logarithmic pair, namely the C-deformation.

In reference [10], we did not present a complete mathematical analysis of all the properties of

the induced recoil space time, and in particular we did not address the question whether, in this formalism, there is a gap in the associated glueball spectrum, according to the AdS/conformal-field theory correspondence [2]. This is the point of the present article. However, we shall follow a slightly different approach than that in ref. [10]. Here, as in ref. [10] and [12], the Liouville field is identified with the target time which, however, we shall not consider it as constant, in contrast to the situation in [10]. Instead we shall fix a specific combination of time and spatial co-ordinates. As we shall see, this yields a mass gap in the spectrum of scalar matter propagating in the induced space time, and hence is appropriate for a discussion of glueballs in (zero-temperature) QCD. However, the basic philosophy of using D-particle models as regulators of QCD, which underlies both approaches, remains identical between the two scenaria.

It is important to note, that in our approach and that of ref. [10] there is no need for a large-N assumption as in the supergravity approach [1, 2]; the large AdS radius, and the associated small Regge slope α', needed for the validity of the low-energy supergravity Lagrangian in the bulk of AdS space, where one can calculate reliably, arise naturally [10]. In particular, the radius of the induced AdS space time is found to be proportional to $|\epsilon|^{-2}$, where ϵ is the standard regulating parameter of the logarithmic recoil operators [11], assumed small for the validity of the world-sheet analysis. Closure of the logarithmic world-sheet algebra [11] requires that $|\epsilon|^{-2} \sim \ln A$, where A is the area of the Wilson loop, which in the infrared regime, in which we are interested, is assumed large [10]. Thus in the infrared regime of QCD, the AdS radius is large, independently of the number of colours N of the gauge group, in contrast to the supergravity approach where the AdS squared radius is proportional to $\sqrt{g_s N}$, with the string coupling being connected to the gauge coupling as $g_s = g_{YM}^2$.

More importantly, it seems to us that the calculated glueball masses in our approach are calculated in the same limit [10] as the glueball masses in the continuum limit of the lattice QCD calcuation [8], and hence a direct comparison is possible. This is due to the fact that the glueball masses M are given in terms of a fixed ultraviolet cut-off of the effective theory, Λ_{UV}, as

$$M^2 = f(g_{YM}^2 N)\Lambda_{UV}^2 \tag{1}$$

where f is some function. In the supergravity approach one should consider the (fixed) limit $g_{YM}^2 N \gg 1$, which necessitates a large N approach in order to have a small string coupling $g_s = g_{YM}^2$. On the other hand, in the continuum limit of lattice gauge theories [8], $\Lambda_{UV}^2 \to 0$, which implies $g_{YM}^2 N \to 0$. In our approach [10], since the radius of the AdS is proportional to $|\epsilon|^{-2}$, which is small in the infrared limit of QCD, independently of the number of colours, one can naturally achieve weak string couplings independently of a large N limit, and thus one can compute glueball masses in the same limit as the continuum limit of lattice QCD.

As we shall see, however, under certain conditions, there can be agreement with the results of [4]-[7], which may be taken to be an independent confirmation of the claims [6] that higher-order corrections in α', within the supergravity approach, do not change qualitatively the results, at least as far as the ratios of glueball masses are concerned.

In the present article we first describe in detail how an AdS space time arises from D-particle recoil, as in [12], and subsequently we demonstrate the crucial rôle of the recoil picture in inducing a gap in the scalar glueball spectrum at zero temperatures. This arises due to the presence of *both* ultraviolet and infrared (spatial) cut-offs in the recoil geometry, which appear in order to avoid curvature singularities in the space time. Namely, we shall show that the time variable, which in the present article, as in [10, 12], is identified with the Liouville field, needs to be *strictly positive* in order for the gap to exist. This positivity reflects the fact that we are considering the induced space time as a result of (and hence consequently after) the recoil [10, 12]. However, for the existence of the gap in the spectrum, we need the simultaneous presence of a second cut-off. It is the presence of this cut-off which implies that the space time (for low temperatures) is inside the boundary of AdS, set [10] by the radius $b \propto |\epsilon|^{-2}$.

We emphasize that two cut-offs are necessary for the appearance of a gap in the non-black hole AdS, in contrast to the case of [2], where the space time with a regular AdS (non-black hole) does not lead to a gap in the glueball spectrum. This is the reason why Witten [2] considered AdS black holes, to study the deconfinement transition in QCD starting from the high-temperature regime of the model, where the black hole space times are known to be thermodynamically stable [15].

We stress that in the Liouville string case studied in [10] and here, one is working at zero temperature. In this work we demonstrate the existence of a glueball mass gap for the regular non-black hole AdS, which is the stable space time at low temperatures [15]. This property demonstrates the possibility of having glueballs in our picture at zero temperature, which is the situation met in realistic QCD models.

This picture also complements nicely the approach of [16] where a gas of AdS black holes in various (finite) temperature regimes has been studied. In that work, in the high temperature phase, where the black holes are stable, and a world-sheet perturbation theory breaks down, it was found that the radius of AdS was small, scaling like the inverse of the temperature. On the other hand, at low temperatures, where there are no black holes, the situation may be thought of as the limiting case of a black hole AdS space time with vanishing black hole mass. This is the case we study here, but we approach the problem in a straightforward way, deriving the metric directly in the zero-temperature recoil picture of [10]. Notably, in this regime the world-sheet perturbative approach is reliable [10, 11].

The nature of the phase transition from low to high temperatures, i.e. from confinement to deconfinement, is not completely understood as yet, and the subject needs further study. However, the analogy with the Van der Waals case, pointed out in [16], is suggestive of a first order transition, which seems to be supported by the black-hole thermodynamical approach of refs. [2, 15].

Before commencing our analysis, we note, for the benefit of the reader, that in the present article our convention for the signature of a Lorentzian space time metric is $(-+ + \ldots +)$, and we use units in which $G = \hbar = c = 1$. This will be understood in what follows.

2 Glueball masses in Supergravity

Witten [2] was the first to show that there is a mass gap, in the three-dimensional sense, for quantum fields propagating on the five-dimensional space time

$$ds^2 = \left(\frac{\rho^2}{b^2} - \frac{b^2}{\rho^2} \right) d\tau^2 + \left(\frac{\rho^2}{b^2} - \frac{b^2}{\rho^2} \right)^{-1} d\rho^2 + \rho^2 \sum_{i=1}^{3} dx_i^2. \tag{2}$$

This metric is constructed from the five-dimensional Euclidean Schwarzschild anti-de Sitter (AdS) geometry

$$ds^2 = \left(\frac{r^2}{b^2} + 1 - \frac{M}{r^2} \right) dt^2 + \left(\frac{r^2}{b^2} + 1 - \frac{M}{r^2} \right)^{-1} dr^2 + r^2 d\Omega_3^2 \tag{3}$$

in the limit of large mass M, as follows (we have set the gravitational coupling constant equal to unity in order to simplify the algebra). Firstly, the co-ordinates r and t are rescaled: $r \rightarrow \left(\frac{M}{b^2} \right)^{\frac{1}{4}} \rho$, $t \rightarrow \left(\frac{M}{b^2} \right)^{-\frac{1}{4}} \tau$ so that, for large M, $\frac{r^2}{b^2} + 1 - \frac{M}{r^2} \rightarrow \left(\frac{M}{b^2} \right)^{\frac{1}{2}} \left[\frac{\rho^2}{b^2} - \frac{b^2}{\rho^2} \right]$. Then the metric becomes

$$ds^2 = \left(\frac{\rho^2}{b^2} - \frac{b^2}{\rho^2} \right) d\tau^2 + \left(\frac{\rho^2}{b^2} - \frac{b^2}{\rho^2} \right)^{-1} d\rho^2 + \left(\frac{M}{b^2} \right)^{\frac{1}{2}} \rho^2 d\Omega_3^2. \tag{4}$$

The factor multiplying the last term in this metric means that the radius of the S^3 is of order $M^{\frac{1}{4}}$ and so diverges as $M \rightarrow \infty$. Therefore one can introduce local flat co-ordinates x_i near a point $P \in S^3$. This gives the metric (2). Witten then considers a massless scalar field on this background, having form

$$\Phi(\tau, \rho, x_i) = f(\rho) \exp \left(i \sum_{i=1}^{3} k_i x_i \right), \tag{5}$$

where f satisfies the equation

$$-\frac{1}{\rho} \frac{d}{d\rho} \left[\rho^3 \left(\frac{\rho^2}{b^2} - \frac{b^2}{\rho^2} \right) \frac{df}{d\rho} \right] + k^2 f = 0. \tag{6}$$

As $\rho \rightarrow \infty$, (6) has two linearly independent solutions, which behave as $f \sim$ constant and $f \sim \rho^{-4}$. For a normalizable solution, the latter behaviour is required. In addition, Witten shows that there are no normalizable solutions when k^2 is positive. At the horizon ($\rho \rightarrow b$), Witten stated that the required boundary condition is $\frac{df}{d\rho} \rightarrow 0$. However, further investigation has shown that the appropriate boundary condition is that f is regular at $\rho = b$. It is shown in [4] that the boundary condition specified by Witten is in fact not realized at the horizon, although specifying this condition just outside the horizon is in fact the best way to proceed numerically [5]. Either way, the boundary condition at the horizon will only be satisfied for discrete, negative values of k^2, leading to a discrete mass spectrum for $m^2 = -k^2$ (which is the mass in the three-dimensional sense, for the Euclidean geometry). These discrete eigenvalues

have been calculated in [4]-[7]. Later, we shall compare the numerical eigenvalues computed in this approach with our predictions from the recoil geometry.

The conclusions of this calculation are applied to glueball states using the AdS/CFT correspondence [1]-[3]. The mass gap for quantum fields in the bulk of the Euclidean, asymptotically AdS, geometry implies that there is also a mass gap for glueball states on the boundary of AdS. In this paper we shall only discuss the massless scalar field equation, from which one can conclude, via the AdS/CFT correspondence, information about the masses of the O^{++} glueball states. Other glueball states may be studied by using other quantum field equations, for example, information about the O^{--} glueball states can be gleaned from the equation for a two-form field in the bulk [6]. Here we consider only the simplest case, in order to compare our approach with the study of glueball masses from supergravity. We hope to come back to a more systematic analysis of the higher-rank glueball states in a future publication.

Before we consider the mass gap produced on the recoil space time, which will be discussed in the next section, we first review why there is no mass gap on the (Euclidean AdS) ball, in order to emphasize the physical differences between the two geometries. Consider D-dimensional Euclidean AdS with metric

$$ds^2 = \frac{|\epsilon|^{-8} \sum_{i=1}^{D} dy_i^2}{|\epsilon|^{-4} - \sum_{i=1}^{D} y_i^2}. \tag{7}$$

Introduce a new co-ordinate r by $r^2 = \sum_{i=1}^{D} y_i^2$ so that the metric takes the form

$$ds^2 = \left(r^2 + |\epsilon|^4\right) dt^2 + \left(r^2 + |\epsilon|^4\right)^{-1} dr^2 + \frac{r^2}{|\epsilon|^4} d\Omega_{D-2}^2 \tag{8}$$

where $d\Omega_{D-2}^2$ is the metric on the $(D-2)$-sphere. Since $|\epsilon| \ll 1$, the sphere has very large radius and is almost flat, so we can introduce co-ordinates x_i near a point on the sphere such that

$$|\epsilon|^{-4} d\Omega_{D-2}^2 = \sum_{i=1}^{D-2} dx_i^2. \tag{9}$$

The metric then becomes

$$ds^2 = \left(r^2 + |\epsilon|^4\right) dt^2 + \left(r^2 + |\epsilon|^4\right)^{-1} dr^2 + r^2 \sum_{i=1}^{D-2} dx_i^2. \tag{10}$$

We now consider a massless scalar field on this background, satisfying the equation:

$$\partial_\mu \left(\sqrt{g} g^{\mu\nu} \partial_\nu \Phi\right) = 0, \tag{11}$$

where we assume that Φ has the form

$$\Phi(t, r, x_i) = f(r) \exp\left(i \sum_{i=1}^{D-2} k_i x_i\right). \tag{12}$$

5

This gives the following equation for f:

$$r^2 \left(r^2 + |\epsilon|^4 \right) \frac{d^2 f}{dr^2} + \left[Dr^3 + (D-2)|\epsilon|^4 r \right] \frac{df}{dr} - k^2 f = 0. \tag{13}$$

Changing variables to $\xi = r^2$ gives the equation

$$4\xi^2 \left(\xi + |\epsilon|^4 \right) \frac{d^2 f}{d\xi^2} + \left[2(D+1)\xi^2 + 2(D-1)\xi|\epsilon|^4 \right] \frac{df}{d\xi} - k^2 f = 0. \tag{14}$$

This can be converted into the standard hypergeometric form

$$v(1-v)\frac{d^2 f}{dv^2} + \frac{1}{2} \left[3 - D - (5-D)v \right] \frac{df}{dv} + \frac{k^2}{4|\epsilon|^4} f = 0, \tag{15}$$

where $v = -|\epsilon|^4/\xi$. The equation (15) has singularities at $v = 0, 1, \infty$, but we are interested only in the region $v \in (-\infty, 0)$. Therefore we shall impose the boundary conditions that f is regular at both $v = 0$ and as $v \to -\infty$. The properties of the hypergeometric equation (15) are well known, and reveal that for generic values of the parameters k^2 and D a solution can be found which is regular at both $v = 0$ and $v \to -\infty$. For some D, there may be certain discrete values of k^2 for which there is no regular solution, resulting in points which are missing from the continuous spectrum. However, almost all values of k^2 are eigenvalues, and in particular, there is no mass gap and no discrete mass spectrum [2].

3 The Recoil Space-Time and Glueball Masses

We are now in a position to study the generation of a mass gap for glueballs on our recoil space time. The reason why this space time is relevant for the QCD problem has been discussed in detail in [10]. Basically we consider the interaction of Wilson loops with magnetic monopoles of the gauge field, which are represented as D-particle defects in a simplified (dual) picture. In this picture, the Wilson loop is viewed as a static "macroscopic closed string", and the D-particle defect is assumed heavy so that only quantum fluctuations in its position are taken into account and not its recoil velocity u_i which is assumed zero. Such fluctuations induce a time-like Liouville field, t, identified with the target time, which dresses up the deformed σ-model, restoring the world-sheet conformal symmetry, which is broken as a result of recoil [10, 11].

Some important remarks are in order at this point. In the approach of [10] we have considered the case of $u_i = 0$ and decoupled the time t by essentially fixing it, and concentrating only in the spatial part of the metric defined over constant time slices. This time was identified, as is the case here, with the first (time-like) Liouville field. There is a second Liouville field in this approach which was space-like, in agreement with the fact that the string theory considered there was assumed to have sub-critical dimension [9]. The result of the time fixing was that the spatial part of the space time was a Euclidean AdS ball, which, as we discussed above, leads to no mass gap in the glueball spectrum.

In the present approach, in agreement with the analysis in [12], and to be more general, we consider (formally) a generic string theory which may even live in its critical space-time dimension [1]. For us the presence of a second (space-like) Liouville field, although probably correct for a QCD description as in [10], however will not play a crucial rôle in inducing a mass gap in the glueball spectrum. In the present approach, we still identify the first Liouville field with the target time, as in [10], however we do not consider it completely fixed. It is only a *particular combination* of space and time co-ordinates that is considered *frozen*. As we shall see, this is the crucial step, sufficient to yield a mass gap in the glueball spectrum.

We commence our analysis by first giving the general expression for the induced space time due to the recoil of a D-particle defect discussed in [12]:

$$ds^2 = -dt^2 + \sum_{i=1}^{D} 2\epsilon \left(\epsilon y_i + u_i t \right) \Theta(t) \, dy_i \, dt + \sum_{i=1}^{D} dy_i^2. \tag{16}$$

For reasons discussed above, to make contact with the QCD problem we take [10] the limit $u_i \to 0$ and consider $t > 0$ only:

$$ds^2 = -dt^2 + \sum_{i=1}^{D} 2\epsilon^2 y_i \, dy_i \, dt + \sum_{i=1}^{D} dy_i^2. \tag{17}$$

Now introduce a new variable r by $r^2 = \sum_{i=1}^{D} y_i^2$, in terms of which the metric (16) becomes

$$ds^2 = -dt^2 + 2\epsilon^2 r \, dr \, dt + dr^2 + r^2 \, d\Omega_{D-1}^2, \tag{18}$$

where $d\Omega_{D-1}^2$ is the metric on the $D-1$ sphere. Before Euclideanizing the metric, we first make a co-ordinate transformation in order to put the metric into diagonal form. To do this, let

$$\tilde{t} = t - \frac{\epsilon^2}{2} r^2 \tag{19}$$

so that the metric is now

$$ds^2 = -d\tilde{t}^2 + \left(1 + Cr^2\right) dr^2 + r^2 \, d\Omega_{D-1}^2, \tag{20}$$

where

$$C = |\epsilon|^4, \tag{21}$$

since ϵ is real. The supergravity analysis of the glueball spectrum takes place on Euclidean space time, so now we perform a Wick rotation $\tilde{t} \to i\tilde{t}$. There is a subtlety associated with this procedure, namely that we must also transform $\epsilon^{-2} \to i\epsilon^{-2}$. This is apparent from either

---

[1]The extra dimensions, which are present in case one starts from a critical string theory and not from a sub-critical one as in [10], play no rôle on the issue of the existence of a glueball mass gap, and eventually may be thought of as being compactified to provide the correct regulator for QCD in this more general approach.

the non-diagonalized form of the metric (18) or the definition of $\tilde{t}$ (19), which shows that the Wick rotation in $\tilde{t}$ results from transforming both t and ϵ^{-2}. The necessity of this manoevre is not surprising since ϵ^{-2} is identified with the "time" in the Liouville string approach [10]. The Euclidean metric then takes the form

$$ds^2 = d\tilde{t}^2 + \left(1 - Cr^2\right) dr^2 + r^2 \, d\Omega_{D-1}^2, \tag{22}$$

where $C = |\epsilon|^4$, as before. Notice that the sign has changed in the g_{rr} component of the metric due to the transformation of ϵ^2. By defining another new variable, $\tilde{r}$ by

$$\tilde{r} = \frac{1}{2}|\epsilon|^2 r, \tag{23}$$

we can also write the metric in the alternative form

$$ds^2 = d\tilde{t}^2 + \frac{1}{C}(1 - \tilde{r}^2) \, d\tilde{r}^2 + \frac{1}{C}\tilde{r}^2 \, d\Omega_{D-1}^2. \tag{24}$$

The $1/C \sim 1/|\epsilon|^4$ term multiplying the metric on the sphere means that the sphere has a very large radius. In this case we may change to Cartesian co-ordinates x_i locally on the sphere, which yields the metric:

$$ds^2 = d\tilde{t}^2 + \frac{1}{C}(1 - \tilde{r}^2) \, d\tilde{r}^2 + \frac{1}{C}\tilde{r}^2 \sum_{i=1}^{D-1} dx_i^2. \tag{25}$$

Before we consider the massless scalar field equation, we need to consider the geometry of our space time. Firstly, the original, Lorentzian geometry (16) has a δ-function singularity at $t = 0$. Therefore, in order to avoid this singularity, it must be the case that, from (19),

$$r^2 > -\frac{2\tilde{t}}{\epsilon^2}. \tag{26}$$

Therefore, for negative $\tilde{t}$, we have a lower bound on the radial co-ordinate r. Secondly, the spatial part of the geometry (24) has the (Euclidean) form:

$$ds_S^2 = \left(1 - Cr^2\right) dr^2 + r^2 \, d\Omega_{D-1}^2. \tag{27}$$

The Riemann tensor of this spatial metric has the form of a constant negative curvature, maximally symmetric, space to leading order in ϵ. This was noted previously in [10] (for the original form of the metric (16), whose curvature tensor components can be found in [12]). Therefore, as observed in a more complex recoil geometry in [17], the geometry is locally isomorphic to AdS. However, this identification is valid only for

$$Cr^2 \ll 1, \tag{28}$$

which implies that

$$r^2 \ll \frac{1}{C} = |\epsilon|^{-4}. \tag{29}$$

In addition, the geometry has a curvature singularity when $Cr^2 = 1$, so this upper bound also means that we are avoiding this second curvature singularity. A comment is in order concerning the magnitude of the cut-offs. The analysis which produced the original recoil geometry is valid only if $|\epsilon| \ll 1$, so that the lower bound (26) has r relatively large. This means that we are in the outer regions of the AdS geometry. However, since $C = |\epsilon|^4$, this is not incompatible with $Cr^2 \ll 1$ being a cut-off for a rather larger value of r. We shall see subsequently that it is only the ratio of the cut-offs which affects the glueball masses. We finally remark that it is only the spatial part of our metric which is AdS. In the conventional analysis of the scalar field equation on AdS [2], the scalar field ansatz is such that the scalar field depends only on the spatial variables (see section 2), not on the (Wick rotated) time. Therefore the fact that the temporal part of our geometry is not AdS does not matter.

Following these considerations, we shall now fix $\tilde{t}$ to have some (negative) value, and consider the massless scalar field equation on the background

$$ds^2 = (1 - Cr^2) \, dr^2 + r^2 \sum_{i=1}^{D-1} dx_i^2, \tag{30}$$

where we have reverted to the variable r. We shall impose two cut-offs to the geometry (30), at

$$r = a > -\frac{2\tilde{t}}{|\epsilon|^2} \tag{31}$$

and

$$r = b < |\epsilon|^{-4}. \tag{32}$$

We now consider the equation satisfied by a massless scalar field on the background (30):

$$\partial_\mu \left(\sqrt{g} g^{\mu\nu} \partial_\nu \Phi \right) = 0. \tag{33}$$

We assume that the field Φ has the form

$$\Phi(r, x_i) = f(r) \exp \left(i \sum_{i=1}^{D-1} k_i x_i \right). \tag{34}$$

Then the differential equation satisfied by f is:

$$\frac{d^2 f}{dr^2} + \left[\frac{D-1}{r} + \frac{Cr}{1 - Cr^2} \right] \frac{df}{dr} - \frac{k^2}{r^2} \left(1 - Cr^2 \right) f = 0, \tag{35}$$

where

$$k^2 = \sum_{i=1}^{D-1} k_i^2. \tag{36}$$

9

If we now change variables to $\xi = r^2$, we get the equation

$$4\xi^2 (1 - C\xi) \frac{d^2 f}{d\xi^2} + 2\xi \left[D - C(D-1)\xi \right] \frac{df}{d\xi} - k^2 (1 - C\xi)^2 f = 0. \tag{37}$$

This equation can be recast in the form of a standard Sturm-Liouville equation [18]

$$\frac{d}{d\xi} \left[\frac{\xi^{\frac{D}{2}}}{(1 - C\xi)^{\frac{1}{2}}} \frac{df}{d\xi} \right] - \frac{k^2}{4} \xi^{\frac{D}{2} - 2} (1 - C\xi)^{\frac{1}{2}} f = 0. \tag{38}$$

In addition, we require that f satisfies the boundary conditions $f = 0$ when $\xi = a^2$, b^2 (31,32). We can now appeal to standard theorems [18], which imply that there are an infinite number of discrete eigenvalues $k^2 < 0$ for this problem. In other words, we have a mass gap.

Having shown that there is a discrete mass spectrum, we shall now proceed to find the appropriate eigenvalues of (37). Equation (37) does not possess analytic solutions for general D. Therefore we shall use an approximation (which is precisely that under which our geometry can be regarded as AdS), which will enable us to produce exact results. In equation (37), $\xi = r^2$, which ranges from $\xi = a^2$ to $\xi = b^2$, corresponding to our two cut-offs in r. However, although ξ will be finite, it must be the case that $C\xi \ll 1$ in order for our geometry to be approximated by AdS. In this regime equation (37) reduces to

$$4\xi^2 \frac{d^2 f}{d\xi^2} + 2D\xi \frac{df}{d\xi} - k^2 f = 0. \tag{39}$$

Under the variable transformation $u = \log \xi$, this equation becomes

$$4\frac{d^2 f}{du^2} + (2D - 4) \frac{df}{du} - k^2 f = 0, \tag{40}$$

which has solutions of the form

$$f(u) = \mathcal{A} \exp \left(\frac{(2 - D)u}{4} \right) \sin \left(\frac{\lambda(u - \delta)}{4} \right), \tag{41}$$

where $\mathcal{A}$ and δ are arbitrary constants, and λ satisfies

$$-\lambda^2 = 4k^2 + (D-2)^2. \tag{42}$$

The solution f must be sinusoidal if it is to vanish at the two cut-offs in ξ. Therefore it must be the case that λ is real, so that

$$k^2 < -\frac{(D-2)^2}{4}. \tag{43}$$

We require f to vanish when $u = 2 \log a$, $2 \log b$, which implies that

$$\delta = 2 \log a, \qquad \lambda^2 = 4n^2\pi^2 \left[\log \left(\frac{b}{a} \right) \right]^{-2}, \tag{44}$$

10

where n is a positive integer. Substituting in (42) gives

$$k^2 = -\frac{(D-2)}{4} - n^2\pi^2\left[\log\left(\frac{b}{a}\right)\right]^{-2} = -C_0 - C_1 n^2. \tag{45}$$

The glueball masses are determined by this procedure only up to an overall scale factor, and hence only their ratios acquire unambiguous physical meaning. Furthermore, we do not at this stage know what the numerical values of the cut-offs b and a should be. Therefore, the analysis above predicts that the glueball masses should depend linearly on n^2, where n is a positive integer. In the next section we shall determine the value of the ratio b/a by demanding that our results agree with those of the supergravity calculation [4]-[7].

4   Comparison with the Supergravity Approach

It is interesting to see how the above analysis compares with previous numerical calculations of the glueball masses in the supergravity approach. As emphasized in the introduction, the important feature of our approach is that the region of the parameters in which perturbation theory applies in the bulk of AdS in our framework is compatible with the continuum limit of the lattice QCD in which the glueball masses have been calculated. It is therefore interesting to examine whether the results obtained above are in agreement with the supergravity results for glueball masses in [4]-[7], which would be an independent confirmation of the claims [6] that higher-order in α' supergravity corrections in the bulk of AdS do not affect qualitatively the results, as far as the ratio of glueball masses are concerned.

Firstly, the paper of Csáki et al [6] predicts, using an analytic WKB approximation, that the squared glueball masses for the O^{++} state in QCD_3 should be proportional to $n(n+1)$. Their numerical calculations confirm this prediction to a high order of accuracy. We note that our prediction of linear dependence on n^2 will be a good approximation to $n(n+1)$ when n is large. The proportionality factor of 6 in [6] was refined by further WKB analysis in [19], although the correction for QCD_3 is small and there is good agreement with the numerical values given in [6]. Minahan [19] extends the WKB analysis to O^{++} glueballs in QCD_4, where the squared masses are proportional to $n(n+2)$.

However, here we consider the numerical values calculated by de Mello Koch et al [4], since they give the glueball masses for much higher values of n than other authors (we note that the authors of [4] are sceptical about the existence of an exact mass formula). We plot in figure 1 the squared masses for the first twelve glueball states given in [4] for the O^{++} glueball in QCD_3 and QCD_4, against n^2. We also show a best fit linear regression line in each case. It can be seen that the linear dependence on n^2 is a very good approximation to these glueball masses. Since there is freedom in the overall scale of the squared masses, only the ratio of the coefficients in (45) is relevant physically, and this determines (or, is determined by) the ratio of the cut-offs, once the dimension of the geometry is fixed.
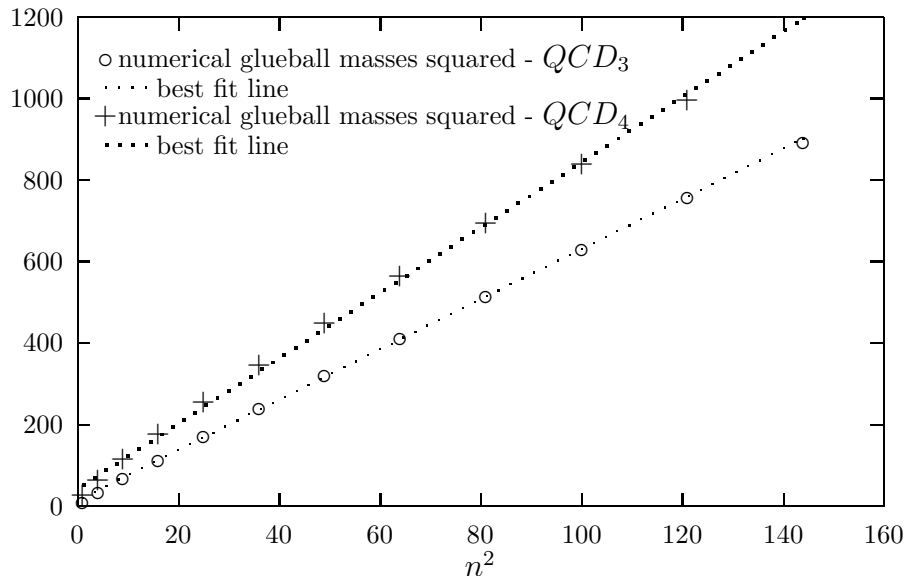
Figure 1: A comparison of the exact glueball masses squared calculated numerically in supergravity [4] and the predicted linear dependence on n^2. Values for the first twelve O^{++} glueball states are used in both QCD_3 and QCD_4.

The (numerical) results of the best fits are given below, and in figure 1. We observe from the best fit lines in figure 1 that there is agreement between our approach and the glueball masses derived from supergravity [4], provided that the ratio of the two cut-offs in our approach (31,32) is fixed to be of order 3. To see this, note that the best fit lines for QCD_3 and QCD_4 are given by, respectively,

$$14.73 + 6.16n^2 \tag{46}$$

and

$$43.12 + 8.02n^2 \tag{47}$$

The ratios of the coefficients in (45) can then be used to predict the ratio of the cut-offs, b/a, once we have specified the number of dimensions, D. For QCD_3, it is appropriate to consider 5-dimensional AdS, so we set $D = 5$, whilst for QCD_4, we need $D = 7$ [6]. The numerical values for QCD_3 lead to $\log(b/a) = 3.2$, whilst those for QCD_4 give $\log(b/a) = 2.9$.

We find it striking that a single number for the two scales in the target space geometry suffices to describe consistently glueball masses in QCD in various dimensions. This agreement may be taken as a confirmation of the claims [6] that the results of the (ratios of) glueball masses obtained using the lowest-order large-N limit in the supergravity approach of [1, 2] to QCD, survive higher-order α' corrections, as more appropriate in that approach for comparison with the continuum limit of the lattice results for the glueball masses.

12

5 Conclusions

In this article we have discussed in detail the geometric properties of a space time induced *dynamically* as a result of the recoil of a heavy D-particle defect during scattering with a macroscopic closed string loop, in the limit of vanishing recoil veolcity $u_i \to 0$. As emphasized in [10], such space times may be relevant for understanding nonperturbative infrared properties of QCD-like gauge theories, within the framework of Liouville strings [9].

In the present article we have derived the properties of the D-particle-recoil induced space time pertinent to the mass gap, and compared with the case of AdS (both with, and without an event horizon). The large mass black hole space time (2) has a co-ordinate singularity at the horizon $\rho = b$, and a genuine singularity at $\rho = 0$, for example:

$$R_{\mu\nu\sigma\lambda}R^{\mu\nu\sigma\lambda} = \frac{8(5\rho^8 + 9b^8)}{b^4\rho^8}. \tag{48}$$

According to Witten [2], the presence of a "cut-off" in the metric (2) is crucial for the mass gap. However, performing a similar calculation in AdS with a cut-off (i.e. the same as in section 2, but with a cut-off at some value of $v = v_0 \in (-\infty, 0)$) does not give a mass gap, since the value of v at the cut-off will be a regular point of the equation (15). Therefore, a cut-off is not, on its own, sufficient to generate a mass gap. This means that the co-ordinate singularity in (2) must be crucial.

The recoil space time has a genuine singularity when $t = 0$, which leads to the imposition of a lower cut-off on r, at $r = a$. Note that it is not possible to set $a = 0$ and get the same result. The point $\xi = 0$ is a regular singular point of the equation (37), and a series solution about this point can be found by the usual Frobenius method. In order to have $f(0) = 0$, it must be the case that $k^2 > 0$, which is not what we want for the mass gap. Therefore, it is necessary to impose a positive cut-off on r, no matter how small. By choosing ϵ sufficiently small, it is possible to take the upper cut-off value b as large as we like. This means that the argument of Witten [2], which led to the interpretation of the mass gap in terms of glueball masses on the boundary Minkowski space, is still valid for our situation since we can cover as large an area of AdS as we like (in particular, we can go as near as we like to infinity).

It is interesting to continue the computation of higher-tensor "matter" fields in our approach and compare with higher-order glueball calculations in the QCD and supergravity approaches, as well as to probe our analysis further towards an understanding of the nature of the confinement-deconfinement phase transition. Moreover, it would be interesting to place our "recoil" approach in a wider context, and to compare it with other approaches to QCD using Liouville-strings, such as the holographic renormalization-group flow [20], especially in connection with the infrared running of the gauge coupling, which in our approach has been discussed briefly in [10]. These are left for future work.

Acknolwedgements

References

[1] J. Maldacena, Adv. Theor. Math. Phys. **2** (1998) 231

[2] E. Witten, Adv. Theor. Math. Phys. **2** 505 (1998)

[3] O. Aharony, S. Gubser, J. Maldacena, H. Ooguri, and Y. Oz, Phys. Rept. **323** 183 (2000), and references therein.

[4] R. de Mello Koch, A. Jevicki, M. Mihailescu and J. P. Nunes, Phys. Rev. **D58** 105009 (1998)

[5] H. Ooguri, H. Robins and J. Tannenhauser, Phys. Lett. **B437** 77 (1998)

[6] C. Csáki, H. Ooguri, Y. Oz and J. Terning, J. H. E. P. **9901** 017 (1999)

[7] M. Zyskin, Phys. Lett. **B439** 373 (1998)

[8] M. Teper, Phys. Rev. **D59** 014512 (1999); hep-th/9812187 and references therein; C. Morningstar and M. Peardon, Phys. Rev. **D56** (1997) 4043

[9] A.M. Polyakov, Nucl. Phys. Proc. Suppl. **68** 1 (1998) (hep-th/9711002); S. Gubser, I. Klebanov and A.M. Polyakov, Phys. Lett. **B428** 105 (1998)

[10] J. Ellis and N. E. Mavromatos, Int. J. Mod. Phys. **A14** 3761 (1999)

[11] I. Kogan, N.E. Mavromatos and J.F. Wheater, Phys. Lett. **B387** 483 (1996)

[12] J. Ellis, P. Kanti, N. E. Mavromatos, D. V. Nanopoulos and E. Winstanley, Mod. Phys. Lett. **A13** 303 (1998)

[13] A.M. Polyakov, Phys. Lett. **B59** 79 (1975); Nucl. Phys. **B72** 461 (1974)

[14] G. 't Hooft, Nucl. Phys. **B190** 455 (1981)

[15] S. W. Hawking and D. N. Page, Comm. Math. Phys. **87** 577 (1983)

[16] J. Ellis, A. Ghosh and N. E. Mavromatos, Phys. Lett. **B454**, 193 (1999)

[17] J. Ellis, N. E. Mavromatos and E. Winstanley, Phys. Lett. **B476** 165 (2000)

[18] G. Birkoff and G.-C. Rota, *Ordinary Differential Equations*, Wiley, New York (1959)

[19] J. A. Minahan, J. H. E. P. **9901** 020 (1999)

[20] E. Alvarez and C. Gómez, hep-th/0003205 and references therein;
I. Kogan, M. Schvellinger and B. Tekin, hep-th/0004185