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Generation of impulses from single frequency inputs using non-linear propagation in spherical chains

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Abstract

This paper investigates the use of chains of spheres to produce impulses. An ultrasonic horn is used to generate high amplitude sinusoidal signals. These are then input into chains of spheres, held together using a minimal force. The result is a non-linear, dispersive system, within which solitary waves can exist. The authors have discovered that resonances can be created, caused by the multiple reflection of solitary waves within the chain. The multiply-reflecting impulses can have a wide bandwidth, due to the inherent nonlinearity of the contact between spheres. It is found that the effect only occurs for certain numbers of spheres in the chain for a given input frequency, a result of the creation of a nonlinear normal mode of resonance. The resulting impulses have many applications, potentially creating high amplitude impulses with adjustable properties, depending on both the nature and number of spheres in the chain, and the frequency and amplitude of excitation.

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1. Introduction

There have been various studies that have investigated acoustic propagation along chains of spheres, where acoustic propagation is non-linear due to Hertzian contact between each sphere Coste C. et al (1997). This causes interesting features, one of which is the creation of solitary waves, provided certain conditions are met. Nesterenko (1983) has described how solitary waves can be established in long chains of spheres when the contact force holding them together is extremely small. Spadoni and Daraio (2010) demonstrated that solitary waves in chains exist when steel ball-bearings are held within columns and a small pre-compression (F_0) applied, with a later study extending

the analysis using piezoelectric actuation (Donahue et al (2014)). The resultant effects are dependent on the characteristics of the applied transient force (F_m), the diameter of the spheres, and the relative values of F_m and F_0 . If $F_m \approx F_0$, weakly non-linear behavior results, whereas if $F_m \gg F_0$, propagation along the chain is highly non-linear.

Most of the experimental work to date has used relatively low ultrasonic frequencies and large spheres. The cut-off frequency for solitary waves limits the upper frequency that can propagate in chains; hence, higher frequencies will need smaller spheres. In addition, little work seem to have been performed experimentally into establishing whether higher frequencies and non-impulsive waveforms could be used in finite lengths of chain. Of particular interest was the establishment of resonances within such chains. These are likely to be complicated, due to non-linear propagation from one sphere to the next. Various papers have described the concept of Non-linear Normal Modes (NNMs) of vibration, which are expected to exist in such systems (Jayaprakash et al (2011); Starosvetsky and Vakakis (2009); Lydon et al (2013)). NNMs are expected to be very sensitive to the input conditions, in terms of the nature of F_m and F_0 , the types of spheres used, and the boundary conditions.

The present paper describes a series of experiments that have investigated the behavior of spherical chains of variable length, and where the ratio of F_m/F_0 could be varied. To achieve this at ultrasonic frequencies, it was necessary to have a high amplitude ultrasonic input. This was supplied from an ultrasonic horn at a fixed resonant frequency. As will be shown, this leads to the generation of very interesting phenomena, and in some cases, to the creation of a set of impulses (Hutchins et al, 2015).

2. Apparatus and Experiment

The apparatus was designed to allow signals from an ultrasonic horn at 73 kHz to be transmitted into one end of a chain of 1 mm diameter stainless steel spheres, held within a cylindrical holder, as shown in Fig. 1. The last sphere was held in position via an annular end-plate, so that a vibrometer could be used to measure the acoustic velocity waveform at the far end of the chain. A static force F_0 could be applied at known values to this end plate. The signal supplied by the ultrasonic horn was also adjustable in terms of both amplitude (in terms of acoustic velocity amplitude v_m) and the number of cycles in the waveform. Output signals were recorded for later analysis using a digital oscilloscope.

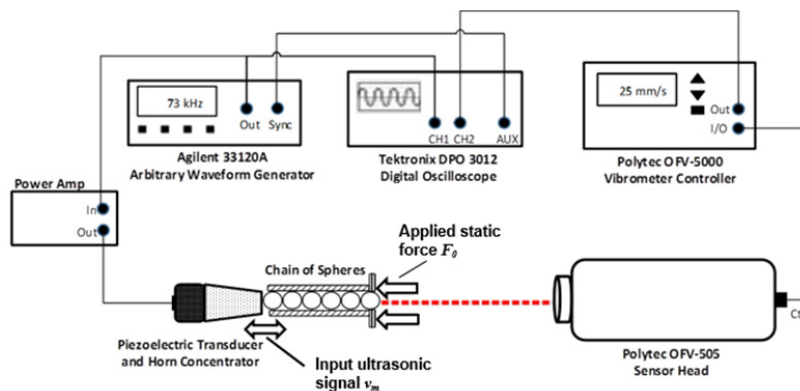


Fig. 1. Schematic diagram of apparatus.

3. Theory

The experimental results could be compared to theory, using solutions to the dynamic equations for spheres in a chain. Details are given elsewhere (Hutchins et al, 2015), but a point to note is that the results are highly dependant on the boundary conditions – the contact of the first sphere to the horn, and of the last sphere to the annular end-plate of the holder, must both be modelled properly. The theory assumes that the centres of each sphere move

relative to each other, but that the spheres themselves do not vibrate internally. The value of F_0 is modelled by assuming an equivalent initial displacement of the centre of each sphere from its equilibrium position.

4. Results and comparison to theory

Signals were recorded for different lengths of spherical chains as v_m was varied for a negligible F_0 . It was found that the output was highly dependant on the input conditions. As an example, Fig. 2 shows the results obtained for a 10-sphere chain when the input amplitude was increased for a small value of static force F_0 . At small input amplitudes, the output signal was of low amplitude; the spectrum shows the main input at 73 kHz, with a small harmonic at 146 kHz. As the input signal v_m was then increased in amplitude, the time waveform started to change so that the waveform started to become periodic, with a period longer than that of v_m . This was associated with the appearance of other distinct frequencies in the spectrum. As v_m increased further, the new periodicity in the time waveform became more obvious, so that at the highest input amplitude a set of distinct regularly-spaced impulses can be seen. This is associated with an equivalent set of regularly-spaced peaks in the frequency spectrum.

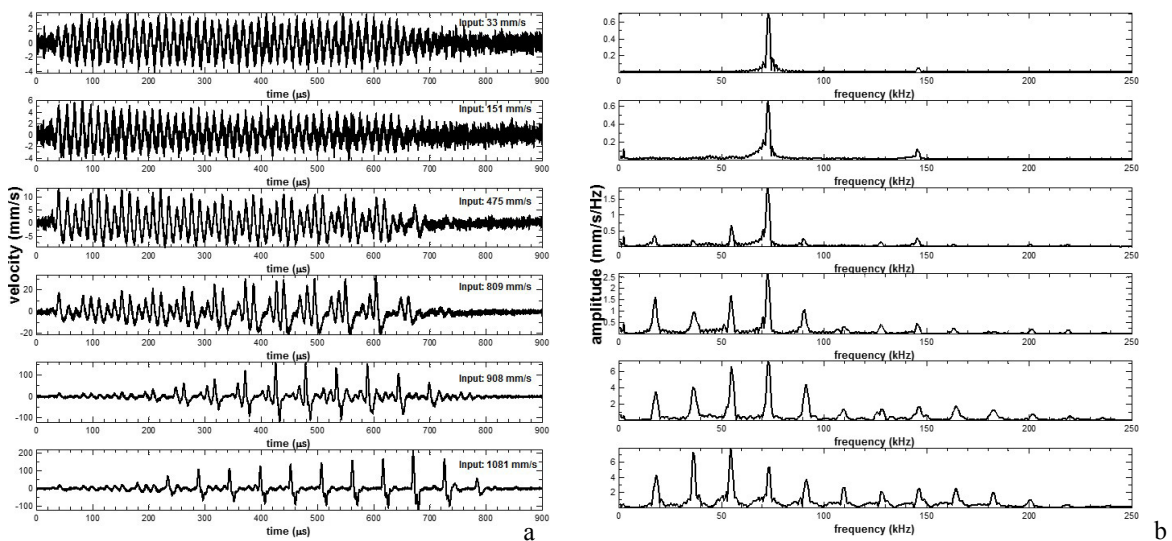


Fig. 2. (a) Waveforms and (b) frequency spectra of signals recorded in a spherical chain containing 10 spheres for a negligible value of F_0 . The excitation waveform v_m was a 45-cycle tone-burst from the ultrasonic horn at 73 kHz, with the pk-pk input amplitude v_m given in each case.

It is interesting to note that the frequency peaks are spaced at a value of $1/4$ of that of the excitation frequency. They thus represent a set of three sub-harmonics, plus additional harmonics, generated as a result of non-linearity. They are not, however, at values expected from simple non-linearity; their exact value is set by the modes of vibration of the spherical chain as a whole, in that it is found that different numbers of spheres lead to a different result in terms of the existence of impulsive behavior and the presence of harmonics. This result depends on whether the conditions are right for the existence of a non-linear normal modes (NNMs) – in effect, a set of resonances of the chain.

It is possible to compare the above results to theory. Fig. 3 presents a comparison of theory and experiment for the maximum value of v_m used in the experiments. The theory assumes a very small pre-compression force, so that the spheres are just held together. It can be seen that the main features are predicted theoretically, in terms of the presence of impulses and the regular pattern of harmonics and sub-harmonics, although there are minor difference in terms of the amplitude of the spectral peaks. Experimentally, the form of the output after ultrasonic propagation along the chain is highly sensitive to the input conditions, as might be expected for a highly non-linear system; modelling of non-linear systems is also known to be highly sensitive to the inputs, but the good agreement

demonstrates that the main features are as seen experimentally. Note that the input amplitude needs to be increased to a certain point before the impulsive behavior is observed – this is the point at which the conditions are met for the creation of the NNM, where the correct balance is achieved between the input and the static pre-compression force.

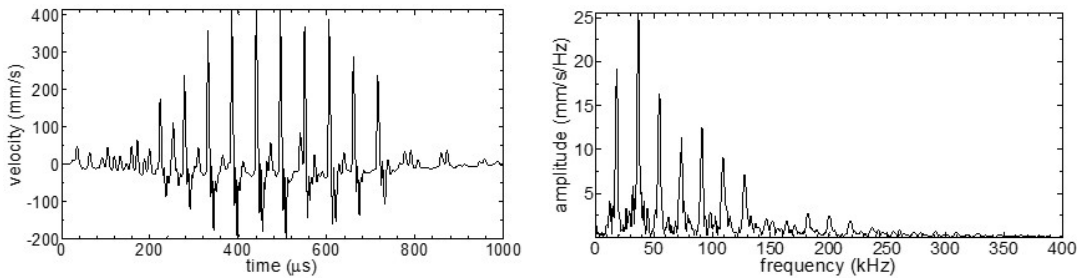


Fig. 3. Theoretical prediction for a 10-sphere chain under negligible precompression, for a 45-cycle tone-burst from the ultrasonic horn at 73 kHz.

5. Conclusions

It has been shown that high amplitude sinusoidal signals can be transformed into a train of impulses of increased bandwidth. This is due to the creation of non-linear normal modes within a chain of spheres. The result is highly dependant on input conditions, but agree with theoretical predictions. Work is now proceeding into making the systems more robust for future applications in biomedical ultrasound.

Acknowledgements

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