The Dynamic Excitation of a Chain of Pre-Stressed Spheres for Biomedical Ultrasound Applications: Contact Mechanics Finite Element Analysis and Validation

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Abstract—**There has been recent interest in the transmission of acoustic signals along a chain of spheres to produce waveforms of relevance to biomedical ultrasound applications. Effects which arise as a result of Hertzian contact between adjacent spheres can potentially change the nature of the signal as it propagates down the chain. The possibility thus exists of generating signals with a different harmonic content to the signal input into one end of the chain. This transduction mechanism has the potential to be of use in both diagnostic and therapeutic ultrasound applications, and is the object of the study presented here. The nonlinear dynamics of granular chains can be treated using discrete mechanics models. However, in cases where the underlying assumptions of these models no longer hold, and where geometries are more complex, a more comprehensive numerical solution must be sought. Contact mechanics problems can efficiently be treated using the finite element method. The latter was used to investigate the dynamics of a pre-stressed chain of six, 1 mm diameter stainless steel spheres excited at one end using a tone burst displacement signal with a fundamental frequency of 73 kHz. The final sphere of the chain was assumed to be in contact with a cylindrical matching layer radiating into a half-space of fluid with the properties of water. After addition of the fluid loading, radiated acoustic pressures in the medium were predicted. Comparison with experimental results suggests that finite element analysis is a suitable tool for investigating the design and performance of contact mechanics based transducers. Nevertheless, a better handle on the model input parameters as well as an improved experimental protocol are required to fully validate the model.**

Keywords—**Finite element analysis; Granular chain; Nonlinear systems; Ultrasonic transducers.**

I. INTRODUCTION

Granular chains are generally defined as one-dimensional alignments of usually spherical particles. The interactions between such particles are governed by the laws of contact mechanics. The study of granular chains is of interest to a range of disciplines in science and engineering, and has recently been extended to biomedical applications [1], [2]. Solitary waves are known to be generated in a one-dimensional granular chain of spherical particles, where dispersive and nonlinear effects, due to the discreteness of the system and the Hertzian contact among spheres, balance out [3]. Effects which arise as a result of Hertzian contact between adjacent spheres have the potential to change the nature of the signal as it propagates down the chain. The possibility thus exists of generating signals with a different harmonic content to the signal input into one end of the chain. In [1], the generation of high-amplitude focused acoustic pulses using a one dimensional array of granular chains was investigated, where the amplitude, size, and location of the focus could be controlled by varying the static pre-compression of the chains. Such an array could have important applications to both therapeutic high-intensity focused ultrasound and medical imaging. In this context, it is of interest to assess the potential of a transduction mechanism based on the nonlinear dynamics of granular chains. Alongside experimental measurement, the development of validated theoretical models capable of simulating the nonlinear transduction process is vital. This will generate a more thorough understanding of the evolution of the solitary waves throughout the granular chain. It will also help determine how best to couple the chain to the acoustic medium, so that signals of appropriate amplitude, transient and harmonic content may be produced. Discrete mechanics formulations have been proposed to predict the dynamics of granular chains [3], [4]. These essentially consist of a one-dimensional system of point masses linked by nonlinear springs. When investigating biomedical applications of such devices, it may be required to move beyond such formulations. Indeed, it is expected that the granular chain will couple into an acoustic medium, such as water or soft tissue, via a matching layer. These loading conditions will affect the dynamic behavior of the chain in a way which will be difficult to predict using a

discrete mechanics formulation. Hence, a numerical solution to the design of such a transducer is likely to provide more flexibility in designing an application-specific device. The use of the finite element method to analyze the dynamic behavior of a granular chain was instigated in [5], providing good agreement with the discrete mechanics solution proposed by [3] and [4] and with experimental results. Finite element analysis (FEA) was subsequently employed in [6] to model the dynamics of granular chains with signals relevant to biomedical ultrasound. This yielded good agreement with the discrete mechanics solution and demonstrated that the multiple collisions which occur between the beads of the chain could be accurately modeled using FEA.

In this paper, we describe an extension of the axisymmetric FEA proposed in [6]. The configuration involved a granular chain of six perfectly aligned beads, coupled into an acoustic medium via a Sigradur® K vitreous carbon cylinder. The beads were assumed to be of spherical shape, with a 1 mm diameter, and were made of chrome steel. The first sphere of the chain was excited via a steel cylindrical piston, the axial displacement of which was obtained from a laser vibrometer measurement at the tip of a purpose-built horn ultrasonic transducer, described in [2]. The fundamental frequency of the signal was 73 kHz and the peak normal tip displacement magnitude 3.78 μm. The FEA was carried out using a transient analysis in ANSYS Mechanical version 16.1 [7]. The final sphere of the chain was in contact with a vitreous carbon cylinder of 0.5 mm thickness. This layer was then coupled into a half-space of water. The acoustic pressure was then predicted along the axis of symmetry at 1 cm from the vitreous carbon front face and compared with experimental results using a calibrated Precision Acoustics™ membrane hydrophone.

II. METHOD

A. Finite element analysis

When the surfaces of two separate bodies touch each other so that they become mutually tangential, they are said to be in contact. In the physical sense, the surfaces that are in contact do not interpenetrate and can transmit compressive normal forces and tangential frictional forces. They do not generally transmit tensile normal forces and are thus free to separate and move away from each other. The static frictionless interaction between two adjacent elastic spheres is an exact solution of linear elasticity and is known as Hertz's law [3]. As a result of geometrical effects, there exists a nonlinear relationship between the exerted force on the spheres and the distance of approach of their centers.

It is common to formulate the problem of frictionless contact between two solid bodies as a variational inequality. This presents a special type of minimization problem with inequality constraints, which can be efficiently treated in a standard manner, i.e. with (1) the penalty method, (2) the augmented Lagrangian method or (3) the Lagrange multiplier method [8]. The ANSYS™ Mechanical v16.1 FEA package includes these three options as formulations to establish a relationship between two surfaces to prevent or limit them from passing through each other during the analysis. For an in-depth description of these methods, the reader is referred to

Chapter 4 of [8]. Methods (1) and (2) are both penalty-based and invoke the concept of contact stiffness i.e. the virtual work due to the deformation of imaginary springs at the contact interface term. This inevitably results in some degree of penetration between the two surfaces, depending on the chosen value for the contact stiffness term. Method (3), or the Normal Lagrange Formulation as it is described in ANSYS Mechanical, adds an extra degree of freedom (contact pressure) to satisfy contact compatibility. Consequently, instead of resolving contact force as contact stiffness and penetration, contact pressure is solved for explicitly as an extra degree of freedom. This has the advantage of enforcing near-zero penetration when modelling frictionless contact between two bodies. For this reason, the Normal Lagrange Formulation was opted for over the penalty and augmented Lagrangian formulations.

Propagation in the acoustic medium was assumed to be governed by the linear, inviscid acoustic wave equation, so that the fluid could be defined in terms of its equilibrium density and speed of sound. Coupling at the fluid/structure interface assumed continuity of normal velocity. An absorbing boundary was placed around the acoustic finite element mesh in order to simulate the Sommerfeld radiating condition and propagation of acoustic waves into a half-space. Details of the underlying equations and physical principles may be found in [7].

A mesh of the structural section of the model is displayed in Fig. 1, with a description of the forcing and boundary conditions. This mesh features refinements around the contact regions to improve accuracy of the solution, as well as convergence.
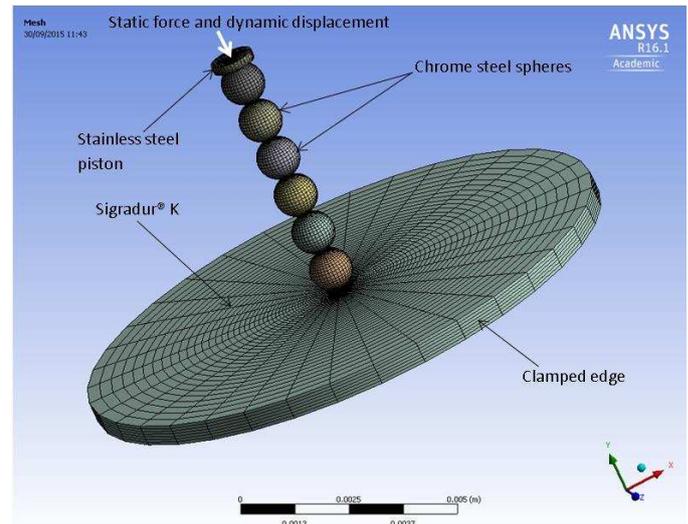

Fig. 1. Structural model mesh: 3D visualization of the axisymmetric model

B. Experimental measurements

The experimental displacement normal to the horn transducer tip was measured using a laser vibrometer in absence of any mechanical loading. The displacement as a function of time is displayed in Fig. 2. The measurement protocol is further described in [2] and [9].

The acoustic pressure was measured as a function of time, 1 cm from the Sigradur® K cylinder into the fluid, using a calibrated Precision Acoustics™ D1604 membrane hydrophone. The hydrophone active element was positioned on symmetry of the granular chain (i.e. the Cartesian y-axis). The acoustic pressure measurement is displayed in Fig. 3.
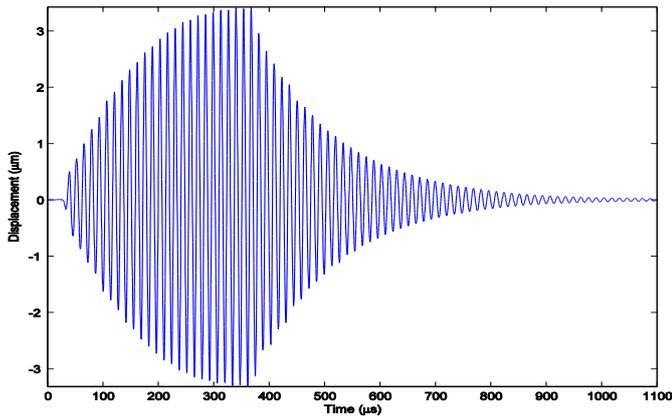

Fig. 2. Normal displacement applied to the outer surface of the stainless steel piston in the FEA model, measured using a laser vibrometer.
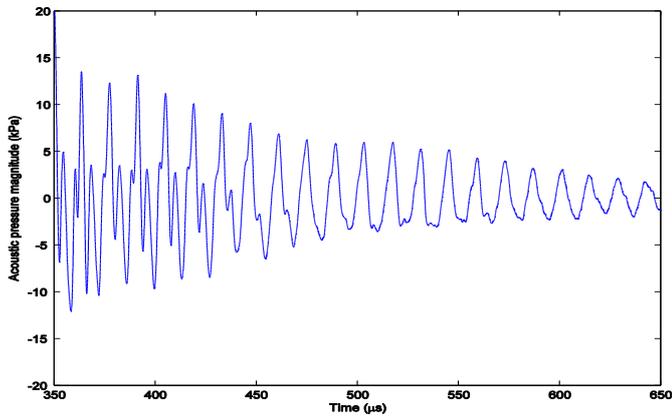

Fig. 3. Acoustic pressure at 1 cm from Sigradur® K front face, measured in water.

While the horn transducer is switched on (i.e. for values of time between 25 µs and 350 µs in Fig. 2), the hydrophone measurement is contaminated by electromagnetic interference [9]. Additionally, as the acoustic pressure decays, the hydrophone measurement suffers from poor signal-to-noise ratio. The acquired acoustic pressure waveform is therefore only displayed and analyzed between 350 µs and 650 µs.

The FFT of the acoustic pressure waveform in Fig. 3 is displayed in Fig. 4. This graph shows a dominant harmonic component at 73 kHz, corresponding to the fundamental frequency at which the horn transducer is driven. In addition to this, the presence of second, third and fourth harmonics are noted, demonstrating the potential of a transduction mechanism based on the nonlinear dynamics of granular chains for generating broadband acoustic signals inside an acoustic medium.
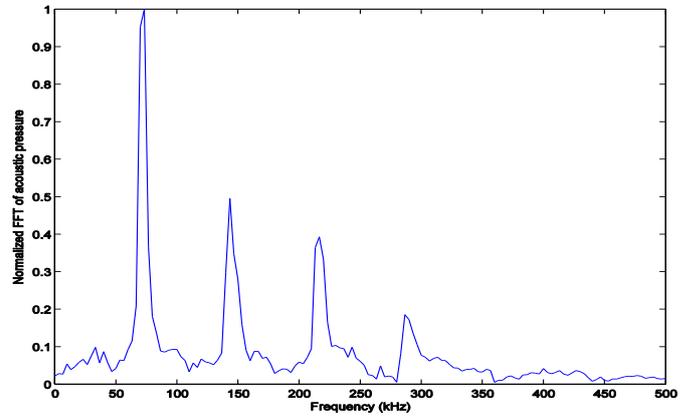

Fig. 4. Normalized FFT of acoustic pressure magnitude at 1 cm from Sigradur® K front face, measured in water.

III. FINITE ELEMENT MODELING RESULTS

The input properties used for each structural material is displayed in table I.

TABLE I.   MATERIAL PROPERTIES

Material	Young's modulus (GPa)	Poisson's ratio	Density (kg m^{-3})
Stainless steel	200	0.35	7800
Chrome steel	201	0.35	7833
Sigradur® K	35	0.15	1540

The fluid region was assigned the properties of water, i.e. a speed of sound of 1500 m s^{-1} and a density of 1000 kg m^{-3}.

The normal displacement excitation described by the time history in Fig. 2 was applied to the disc surface of the stainless steel piston not in contact with the granular chain. In addition to this, a static force of 0.1 N was also applied normal to this surface in the negative y-direction, thus pre-stressing the granular chain. Damping was included in the chrome steel material to in the form of a stiffness matrix damping multiplier, of value 3×10^{-6} [7].

The velocity at the final sphere of the chain was obtained as a function of time, along the y-direction, on the axis of the chain and on the side where the bead and the Sigradur® K cylinder are in contact. This result is shown in Fig. 5. The sharp impulses generated in the velocity signal displayed in Fig. 6 are consistent with the signals measured and predicted in [2]. Oscillations occur beyond 750 µs due to residual motion of the granular chain and vitreous carbon cylinder, possibly indicating that additional damping in the model is required to simulate the experimental configuration. The resulting acoustic pressure radiated by the front face of the Sigradur® K cylinder at 1 cm into the medium is displayed in Fig. 6. For the purpose of comparison with experimental data, the acoustic pressure is only displayed for values of time between 350 µs and 650 µs.
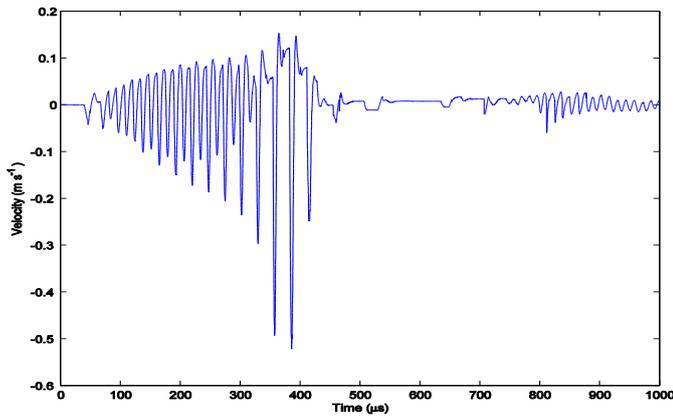
3

Fig. 5. y-component of velocity of final sphere of granular chain as a function of time.
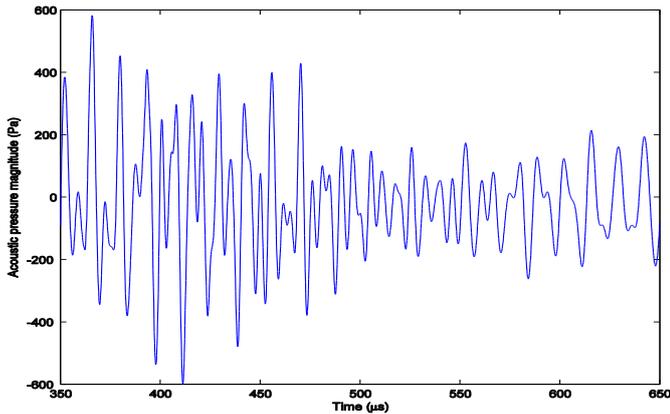

Fig. 6. Acoustic pressure at 1 cm from Sigradur® K front face, predicted in water using FEA.

The normalized FFT magnitude of the acoustic pressure signal in Fig. 6 is shown in Fig. 7, demonstrating the presence of higher order harmonics and spectral content beyond 200 kHz.
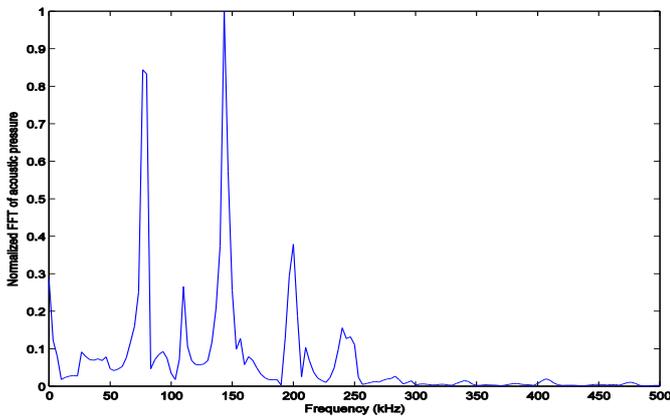

Fig. 7. Normalised FFT of acoustic pressure at 1 cm from Sigradur® K front face, predicted in water using FEA.

Despite similar features between the experimental measurements and the FEA results, it can be seen that discrepancies between these exists. At this stage, it is thought that this is due to a combination of uncertainties in the FEA input parameters, such as damping and pre-stressing force, and the need for refinement in the measurement protocol. Indeed, the hydrophone sensitivity below 300 kHz has not been directly measured, and a sensitivity of 213 nV Pa^{-1} at all frequencies was assumed to obtain the pressure waveform. This figure corresponds to the sensitivity of the device at 300 kHz.

IV. CONCLUSION

A finite element model has been developed for the analysis of acoustic signals resulting from the coupling of a pre-stressed, dynamically excited granular chain, into a fluid. Preliminary comparisons with experimental data show that a better handle on the FEA input parameters and further refinement of the experimental protocol are required for experimental validation. Nevertheless, this preliminary study suggests that FEA will be a useful tool in assisting the design of biomedical transducers which employ a mechanism based on the nonlinear dynamics of granular chains.

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