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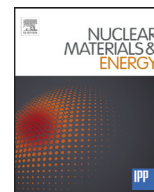
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## Simulation of the interaction between plasma turbulence and neutrals in linear devices

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## ABSTRACT

The interaction between plasma and neutrals within a tokamak dominates the behaviour of the edge plasma, especially in the divertor region. This area is not quiescent, but has significant perturbations in the density and temperature due to turbulent fluctuations. Investigating the interaction between the neutrals and plasma is important for accurately simulating and understanding processes such as detachment in tokamaks. For simplicity, yet motivated by tokamak edge plasma, we simulate a linear plasma device and compare the sources and sinks due to ionisation, recombination, and charge exchange for cases with and without turbulence. Interestingly, the turbulence systematically strengthens the interaction, creating stronger sources and sinks for the plasma and neutrals. Not only does the strength of the interactions increase, but the location of these processes also changes. The recombination and charge exchange have relatively short mean free paths, so these processes occur on the scale of the eddy fluctuations, while the ionisation is mostly unaffected by the turbulence.

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## 1. Introduction

In tokamaks heat is exhausted from the core into the edge plasma where it is then conducted along the magnetic field onto a very thin region on the divertor plates. Future fusion devices such as ITER will deposit so much power onto the divertor plates that, without intervention, severe melting could occur [1]. In addition to heat flux, particle flux through elastic collisions of molecules on the walls and divertor plays an important role in divertor conditions [2]; however, only particle flux is included here, and molecular flux is considered as potential future work. To decrease the heat flux, the plasma can be forced into a detached regime where much of the thermal and kinetic energy is radiated from the edge plasma and spread volumetrically. This occurs when the relatively low temperature plasma in the edge of a tokamak interacts strongly with neutrals near the divertor through charge exchange, ionisation, and recombination [3]. These atomic processes have cross-sections that are functions of the plasma temperature, density, and neutral density meaning that turbulence, which provides fluctuations in these quantities, will have a local effect on the plasma-neutral interactions. To explore these interactions qualitatively and quantitatively, simulations are performed of a linear plasma device.

This provides a simple geometry in which to explore the effect of plasma turbulence on the plasma-neutral interaction. Previous effort has been made in this area with statistically generated turbulence [4] and with two-dimensional SOL turbulence in TOKAM2D [5]; however, we seek to use self-consistent, 3D fluid turbulent simulations.

## 2. Physics models

In the tokamak divertor of existing devices, the plasma is usually collisional and can therefore be treated with a fluid approximation. The model used is based on one described by Simakov and Catto [6], but has been modified and put into finite volume form to conserve particle number and energy. This model, described in the next section, was then implemented in BOUT++ [7], which is a numerical framework for solving systems of differential equations in a range of geometries.

## 2.1. Plasma model

The model is electromagnetic, and evolves electron density  $n_e$ , electron pressure  $p_e$ , parallel ion momentum  $nv_{||i}$ , plasma vorticity  $\omega$ , and poloidal flux  $\psi$ . There is no separation between background and fluctuations in this model, since we wish to study regions such as the tokamak edge where these are of similar magnitude. In the linear device to be simulated the magnetic field

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is constant in space and time, so magnetic drifts and curvature effects are omitted from this model.

$$\frac{\partial n_e}{\partial t} = -\nabla \cdot (n_e \mathbf{V}_{E \times B} + n_e \mathbf{b} v_{||e}) + \nabla \cdot (D_{\perp} \nabla_{\perp} n_e) + S_n - S \quad (1)$$

$$\begin{aligned} \frac{3}{2} \frac{\partial p_e}{\partial t} = & -\nabla \cdot \left( \frac{3}{2} p_e \mathbf{V}_{E \times B} + \frac{5}{2} p_e \mathbf{b} v_{||e} \right) - p_e \nabla \cdot \mathbf{V}_{E \times B} + v_{||e} \partial_{||} p_e \\ & + \nabla_{||} (\kappa_{e||} \partial_{||} T_e) + 0.71 \nabla_{||} (T_e j_{||}) - 0.71 j_{||} \partial_{||} T_e + \frac{\nu}{n} j_{||}^2 \\ & + \nabla \cdot \left( \frac{3}{2} D_{\perp} T_e \nabla_{\perp} n_e \right) + \nabla \cdot (\chi_{\perp} n_e \nabla_{\perp} T_e) + S_p - \frac{2}{3} Q \end{aligned} \quad (2)$$

$$\frac{\partial \omega}{\partial t} = -\nabla \cdot (\omega \mathbf{V}_{E \times B}) + \nabla_{||} j_{||} + \nabla \cdot (\mu_{\perp} \nabla_{\perp} \omega) \quad (3)$$

$$\begin{aligned} \frac{\partial}{\partial t} (n_e v_{||i}) = & -\nabla \cdot [n_e v_{||i} (\mathbf{V}_{E \times B} + \mathbf{b} v_{||i})] \\ & - \partial_{||} p_e + \nabla \cdot (D_{\perp} v_{||i} \nabla_{\perp} n) - F \end{aligned} \quad (4)$$

$$\begin{aligned} \frac{\partial}{\partial t} \left[ \frac{1}{2} \beta_e \psi - \frac{m_e j_{||}}{m_i n_e} \right] = & \nu \frac{j_{||}}{n_e} + \partial_{||} \phi - \frac{1}{n_e} \partial_{||} p_e - 0.71 \partial_{||} T_e \\ & + \frac{m_e}{m_i} (\mathbf{V}_{E \times B} + \mathbf{b} v_{||i}) \cdot \nabla \frac{j_{||}}{n_e} \end{aligned} \quad (5)$$

with  $E \times B$  drift given by:

$$\mathbf{V}_{E \times B} = \frac{\mathbf{b} \times \nabla \phi}{B} \quad (6)$$

The vorticity is given by

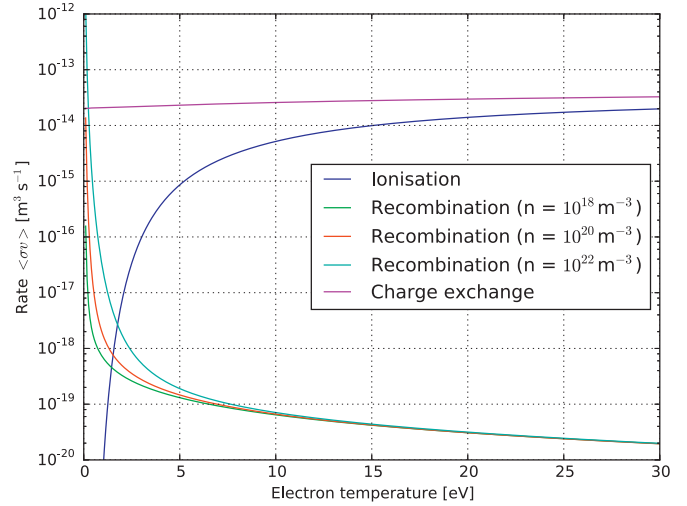
$$\omega = \nabla \cdot \left( \frac{n_0}{B^2} \nabla_{\perp} \phi \right) \quad (7)$$

where the Boussinesq approximation is used, replacing  $n_e$  with a constant  $n_0$  in the vorticity equation. This expression is inverted to find the plasma potential  $\phi$ . A cold-ion approximation is used such that  $p_i = 0$ . The plasma density source is given by  $S_n$  and pressure (energy) source by  $S_p$ . The neutral interactions detailed in the next section provide sources for density  $S$ , pressure  $Q$ , and flux (friction)  $F$ . The density and thermal perpendicular diffusion coefficients are taken to be classical [8]. The parallel heat conductivity is defined as the Spitzer value,  $\kappa_{e||} = n v_{th}^2 \tau_e$  [9], where  $\tau_e$  is the electron collision time. The resistivity,  $\nu$ , is taken to be the Spitzer resistivity [10]. Here we use the notation  $\partial_{||} f = \mathbf{b} \cdot \nabla f$  and  $\nabla_{||} f = \nabla \cdot (\mathbf{b} f)$ .

## 2.2. Neutral model

A kinetic approach is often preferred for the description of neutral behaviour in the edge of tokamaks through codes such as EIRENE [11] because the neutral mean-free path can be larger than the machine size. This, however, is computationally expensive, and good qualitative agreement has been seen between simulation and experiment when fluid neutral models have been used with codes such as UEDGE [12]. For our simulations, a fluid-diffusive model is used such that the neutrals are treated as fluid parallel to the field lines so that parallel momentum is conserved in the plasma neutral interactions, with perpendicular motion dictated by diffusion [13,14]. This system evolves neutral gas density  $n_n$ , pressure  $p_n$  and parallel velocity  $v_{||n}$ :

$$\begin{aligned} \frac{\partial n_n}{\partial t} = & -\nabla \cdot (n_n \mathbf{b} v_{||n} + n_n \mathbf{v}_{\perp n}) + S \\ \frac{\partial}{\partial t} (n_n v_{||n}) = & -\nabla \cdot (n_n v_{||n} \mathbf{b} v_{||n} + n_n v_{||n} \mathbf{v}_{\perp n}) - \partial_{||} p_n \\ & + \nabla_{||} (D_{nn} n_n \partial_{||} v_{||n}) + F \end{aligned}$$



**Fig. 1.** The cross-section rates for ionisation, recombination, and charge exchange are pre-calculated for hydrogen species as a function of the plasma temperature [15,16].

$$\begin{aligned} \frac{\partial p_n}{\partial t} = & -\nabla \cdot (p_n \mathbf{b} v_{||n} + p_n \mathbf{v}_{\perp n}) - \frac{2}{3} p_n \nabla \cdot (\mathbf{b} v_{||n}) \\ & + \nabla \cdot (D_{nn} n_n \nabla_{\perp} T_n) + \frac{2}{3} Q \end{aligned}$$

with

$$v_{\perp n} = -D_{nn} \frac{1}{p_n} \nabla_{\perp} p_n$$

The neutral density, momentum, and energy sources ( $S$ ,  $F$ , and  $Q$  respectively) are calculated with expressions for the plasma-neutral interaction through ionisation, radiative and 3-body recombination, radiation, and charge exchange. Although molecular recombination can play an important role in real devices, only radiative and 3-body recombination are considered in this model, which is essentially the same as that used in UEDGE [17]. The rates for these processes are given by

$$R_{rc} = n^2 \langle \sigma v \rangle_{rc} \quad R_{iz} = n n_n \langle \sigma v \rangle_{iz} \quad R_{cx} = n n_n \langle \sigma v \rangle_{cx} \quad (8)$$

where  $n$  is the electron density,  $n_n$  is the neutral density,  $\langle \sigma v \rangle$  is the rate coefficient for the given reaction (in  $m^3 s^{-1}$ ) which is a function of the local electron temperature (and density for recombination). These rate coefficients are calculated through a fit function of a database [15,16], which are shown in Fig. 1. Given the rates in Eq. (8), the density source for the neutrals can be calculated:

$$S = R_{rc} - R_{iz} \quad (9)$$

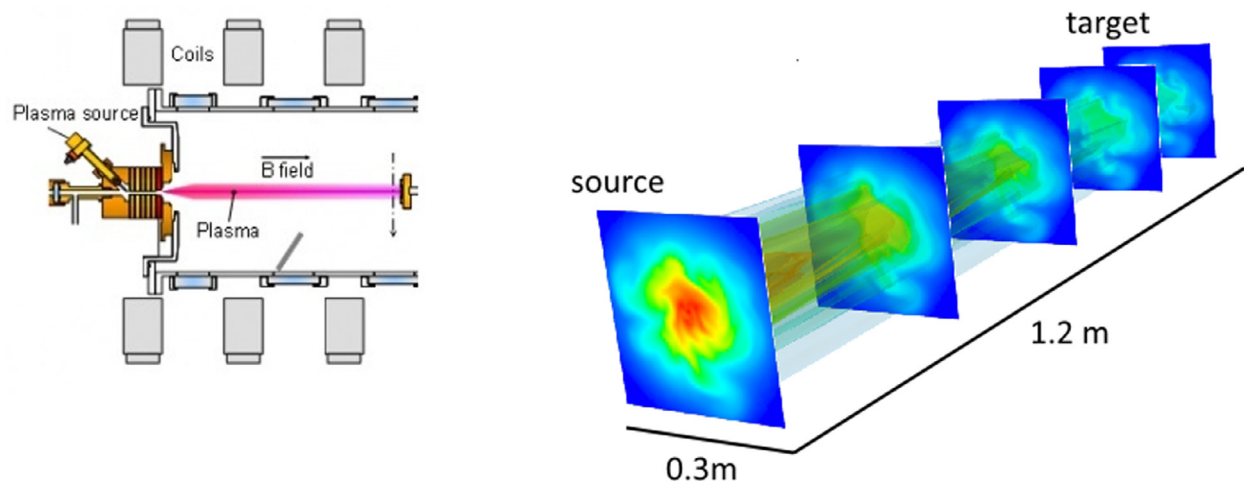
The momentum transfer to the neutrals is given by

$$F = m_i v_{||i} R_{rc} - m_i v_{||n} R_{iz} + m_i (v_{||i} - v_{||n}) R_{cx} \quad (10)$$

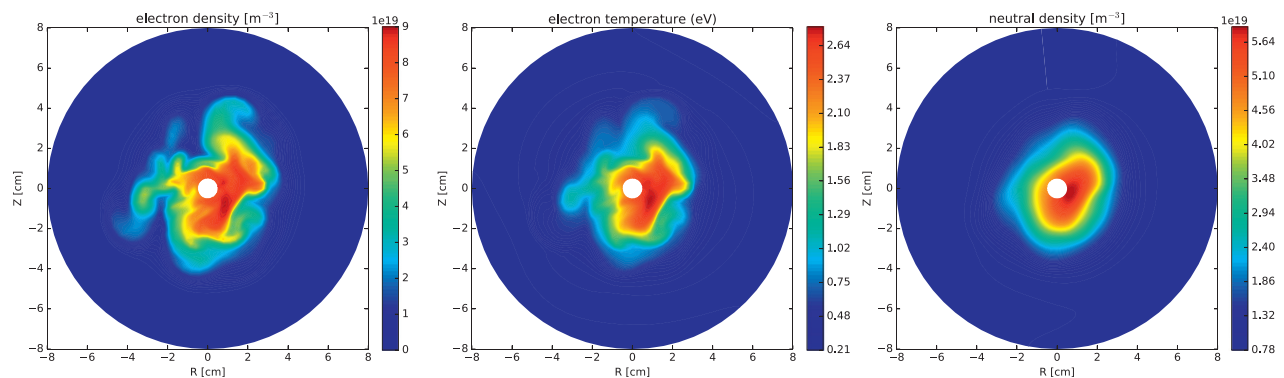
Finally, the energy transfer to the neutrals is

$$Q = \frac{3}{2} T_e R_{rc} - \frac{3}{2} T_n R_{iz} + \frac{3}{2} (T_e - T_n) R_{cx} \quad (11)$$

These sources for the neutrals are subtracted as sinks from the plasma equations as shown in Section 2.1. Due to the cold-ion nature of the plasma model, there are two options in regards to charge exchange - either exclude it from thermal energy exchange or use the electron temperature in the energy transfer term. Since the plasmas of interest here (ie. tokamak edge and similar) has coupled ion and electron temperatures with significant energy loss from charge exchange, the latter method was chosen. A fully consistent model requires a hot ion treatment of the plasma, which will be explored in future work.



**Fig. 2.** A simple schematic (left) of a linear device (Pilot-PSI in this case [18]) showing the basic geometry of the system. The magnetic field generated by the coils is roughly constant, pointing to the right. The plasma source on the left produces a plasma that streams along the field-lines until impacting the target on the right. The density contours along the plasma device are shown (right) for a turbulent case with the linear device dimensions indicated in meters.



**Fig. 3.** The electron density (left), temperature (middle), and neutral density (right) contours in the drift plane at a snapshot in time.

### 3. Linear device and turbulence

A linear device consists of a series of magnetic coils in a straight line configuration providing a constant magnetic field pointing along the device. A plasma source is then placed at one end, the plasma streams down the device, and hits a target at the far end (see Fig. 2). The linear device simulated here has a length of 1.2 m and magnetic field of 0.15 T. The plasma density and power sources are set as spatial distributions that generate a Gaussian plasma profile with a standard deviation of 3.3 cm, a peak temperature of 3 eV, and a peak density of  $10^{20} \text{ m}^{-3}$  at the source. The boundary conditions at the source end are zero gradient allowing outflow of plasma and neutrals, while sheath boundary conditions [19] are used at the target. Fig. 2 shows a basic schematic of the simulated linear device along with a typical turbulent density contour. The plasma density is at a maximum near the source and decreases along the device towards the target plate. In the simulations presented here, a single species pure Deuterium plasma and neutral gas are assumed.

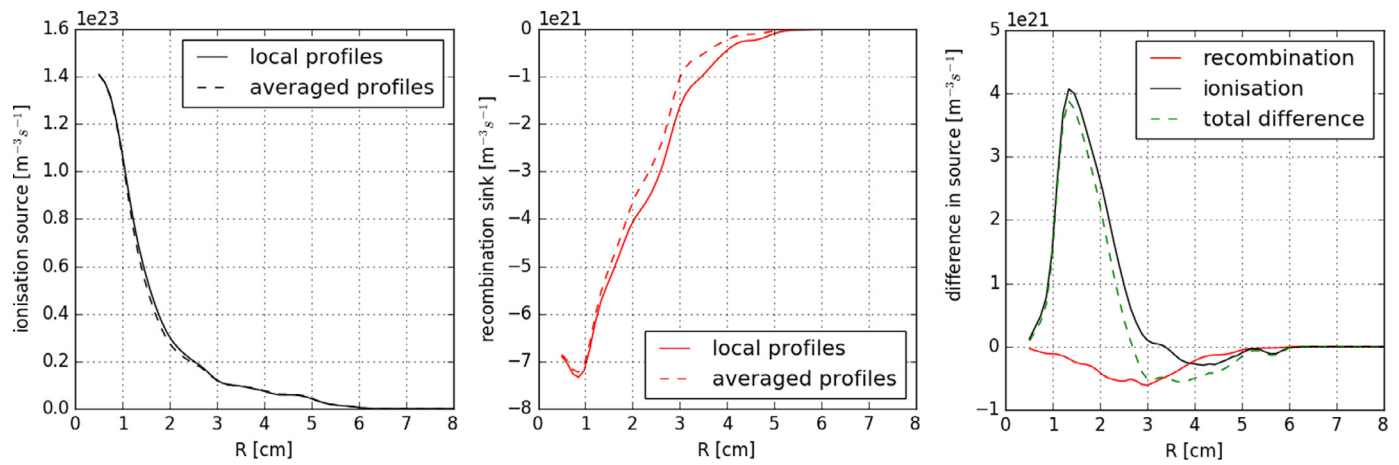
#### 3.1. Turbulence in the linear device

The turbulence in the simulations of this linear device is electromagnetic in nature, driven by drift-waves that are a result of the radial pressure gradients from the localised plasma source. After evolving for  $60,000 \omega_{ci}^{-1}$  (roughly 1.25 ms), the turbulence saturates. Fig. 3 shows the neutral and plasma density in the drift-plane (1m from the source for the plasma and at the target plate

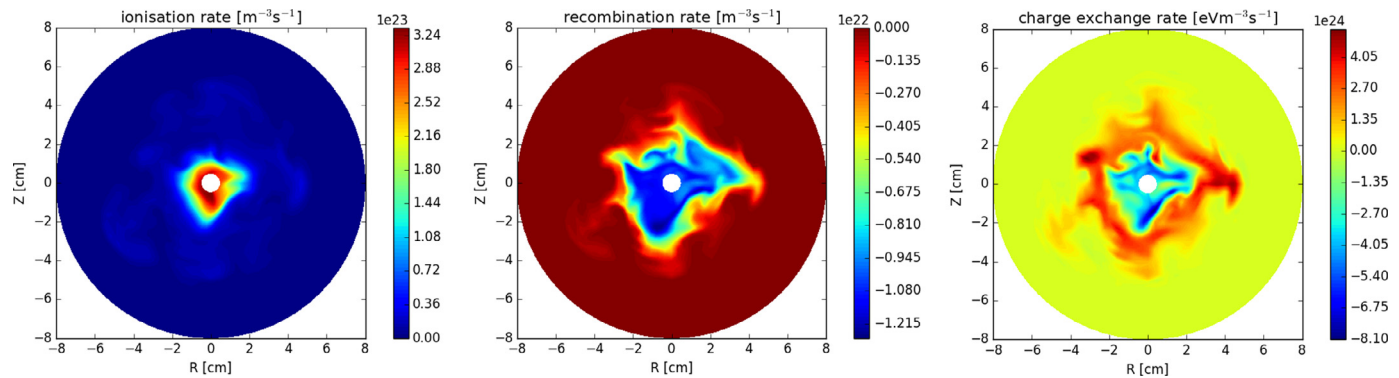
for the neutrals) at a single point in time. The neutral density at the target is highly correlated with the electron density due to the recycling, which is the dominant source for the neutrals. In real machines, gas leakage from the plasma source contributes to the ambient gas density and pressure, but in this simulation recycling and recombination are the only neutral sources. Interestingly, the peak plasma density can lie off-axis despite the density source having a maximum on axis, which is indicative of large perturbations over the background. The neutral density does not show fine scale features seen in the electron density for two reasons: firstly, the fluid model is diffusive, smoothing out features in the drift plane. In addition, the average lifetimes of the plasma and neutrals before recombination or ionisation ( $1/n\langle\sigma v\rangle$  ranging from  $100 \mu\text{s}$  to  $0.1 \text{ s}$ ), are larger than the turbulence correlation time ( $625 \omega_{ci} \approx 13 \mu\text{s}$ ), meaning the neutrals essentially ‘see’ a time-averaged plasma. Using the instantaneous and local temperature to calculate the rate coefficients is justified because the longest ionisation and recombination processes complete in roughly  $1 \mu\text{s}$  [20], which is an order of magnitude below the turbulence correlation time. In this time, an average neutral particle moves roughly 0.1 cm, which is an order of magnitude smaller than the turbulence correlation length. Neglecting the transport of excited states is therefore a reasonable assumption.

### 4. Neutral-plasma interaction

In exploring the interaction between neutrals and plasma turbulence, two questions are answered. Firstly, is there a net effect



**Fig. 4.** A comparison between average sources for turbulent (solid line) and non-turbulent (dashed line) cases. The resulting profiles for ionisation (left) and recombination (middle) are plotted. Finally, the difference between the turbulent and non-turbulent cases is shown (right).



**Fig. 5.** Contours of the ionisation (left) and recombination (middle) density rates per cubic metre, and charge exchange energy source (right) are shown 1 m down the linear device for a single time slice. The recombination is negative because it acts as a sink for the plasma density. The charge exchange demonstrates that the plasma heats the neutrals at the centre of the device, while the neutrals heat the plasma towards the edge.

of the turbulence on the sources and sinks for plasma density, momentum, and energy? Secondly, where are the interactions localised in the plasma column? These two questions are addressed in the following sections.

#### 4.1. Net impact of plasma turbulence

By examining the data in two separate ways, the effect of the turbulence is isolated. First, the neutral density source is calculated (Eq. (9)) using the local density and temperature and then averaged axially to produce radial profiles for the sources (turbulent ionisation source, for example:  $\bar{S}_t = \bar{n}n_n\langle\sigma v\rangle$ , where the bar over a variable indicates an axial average). This represents the average sources from turbulent profiles. Contrastingly, radial profiles for the sources can be obtained using axially averaged density and temperature profiles - this represents the average source for a case where there is no turbulence (ionisation source, for example:  $\bar{S} = \bar{n}\bar{n}_n\langle\sigma v\rangle(\bar{T}_e)$ ).

The results from these two cases are shown in Fig. 4. These radial profiles of the density sources are averaged over 10,000  $\omega_{ci}^{-1}$  (roughly 16 fluctuation correlation times) and along the axis of the device. The ionisation density source is increased with turbulence by 10% at maximum. The recombination sink is also stronger with turbulence by a factor of 50% at maximum. This can be seen clearly in the plot on the right in Fig. 4. The interaction of the neutrals with the local turbulent fluctuations consistently gives rise to

stronger sources and sinks than the equivalent averaged profiles would. This consistent strengthening of the interaction appears to be universal due to the nonlinear dependence of the sources on the plasma density and temperature.

#### 4.2. Localisation of interactions

The existence of turbulence strengthens the interaction between neutrals and plasma, and it also changes the location of these interactions. With Gaussian profiles, symmetric axially about the device, the interaction is also symmetrical. However, with turbulence, the location of the interactions can change significantly, as shown in Fig. 5. The mean free path of the neutrals for ionisation is large, on the order of the machine size, so the ionisation source does not show strong features from the turbulence. The maximum is at the areas of highest plasma density and temperature, and the features are blurred. For recombination, however, the mean free path is much smaller, on the order of a centimetre, so the sink is correlated with the intricate features of the plasma turbulence. This process occurs strongest in the areas of high density but low temperature. The energy transfer between neutrals and plasma due to charge exchange is shown in the right plot of Fig. 5. An interesting flow of energy can be seen - the plasma loses energy to the neutrals at the hottest regions (ie. near the centre of the plasma column), and gains energy from the neutrals at the plasma edge in the lowest temperature regions.



## 5. Conclusion

Through simulations of a plasma turbulence in a linear device, the interaction between turbulence and neutrals has been examined. The net impact of the turbulence is to amplify the interactions, both sources and sinks. The local perturbations in density and temperature, combined with a nonlinear relationship between the sources and these fields, gives rise to a systematic increase in the interaction. Not only does the turbulence affect the strength of the interaction, but also the location. The ionisation is mostly localised in the centre of the device, unaffected by the turbulence, but the recombination and charge exchange conform to the features of the turbulence due to their relatively low mean free paths. Understanding the fundamental characteristics of the turbulent plasma-neutral interaction is not only interesting, but useful because many codes for tokamak study currently approximate this in 2D without turbulence. Including the features of the turbulent interaction in these 2D codes is feasible, but this work also motivates the use of full turbulent codes when simulating detachment and other phenomena dependent on neutrals.

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