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Simulation of Acoustic Wave Propagation in 3-D Sonic Crystals based on Triply Periodic Minimal Surfaces

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Sonic crystals have been investigated in recent years both as a potential form of noise barrier, and as a form of sonic art aimed at enhancing perception of the soundscape. The broader aim of this research is concerned with the auralisation of these structures in the context of the space for which they are intended, which is hoped will enable a useful subjective analysis of a structure prior to its construction. In a previous publication [1], prediction of the acoustic propagation through doubly periodic arrays of solid, cylindrical scatterers embedded in air was performed in 3-D Finite Difference Time Domain simulations. In this study, using the same simulation technique, we investigate the scattering effects of a type of triply periodic structure observed in nature known as a gyroid. It is thought that this type of structure could lend itself well to applications in noise control, first and foremost because they may exhibit more extreme filtering effects than the 2-D sonic crystals we observed previously, but also because - visually and conceptually - we believe them to be far more interesting.

1   Introduction

The broader context of this study is regarding the use of auralisation techniques to assess the potential impact of sonic crystal inspired 'sound sculptures' on the perceived quality of the soundscape - particularly in areas that are adversely affected by environmental noise. In an earlier publication, 3-D simulations of a 2-D sonic crystal were performed in preparation for what is attempted here: the acoustic modelling of a more complex type of periodic structure known as a gyroid. The purpose of the experiment is to synthesise the impulse response (IR) of the gyroid - that is, to characterise how the transmitted sound is affected as it propagates through the structure. Analysis of the IR in the frequency domain ought to reveal the band structure of the gyroid, which may help to 'fine-tune' the gyroid parameters in relation to a particular problem. Auralisation may then be performed by convolution of the IR with recordings of environmental sound.

2   Background

2.1   Sonic crystals

Belonging to the class of acoustic metamaterials, sonic crystals are artificial composite materials consisting of periodically spaced arrays of solid inclusions embedded in air. The concept is founded on the basic principle that sound waves propagating within a crystalline structure may be scattered through interaction with atoms or elements in the material, potentially resulting in positive and negative phase interference. The theory follows that, for certain frequencies and angles of incidence, either or both of the following scenarios may ensue: intense peaks of reflected radiation over a narrow range of frequencies; a pattern of standing waves signifying a band gap in the frequency spectrum of the transmitted signal.

The dimensionality of a sonic crystal refers to the number of dimensions in which the structure has periodicity. 2-D sonic crystals have been broadly investicated in recent years, both as works of 'sonic art' [2][3] and as potential forms of noise control [4], whereas the possibility of producing 3-D sonic crystals for such purposes remains largely unexplored.

2.2 Gyroids

Discovered by Schoen in 1970 [5], the gyroid is an infinitely connected triply periodic minimal surface containing no straight lines. Essentially, it consists of a single surface folded in such a way that the mean curvature at the interface between solid and pore space is zero. The pore space is composed of distinct channels, which interlock to form a labyrinthine structure that is very homogeneous in terms of channel diameter variations. The name 'gyroid' alludes to the spiralling or 'gyrating' movement that a wave must make as it traverses a given channel (figure 1).

The gyroid is thought to occur in nature, having recently been discovered on the wings of several species of insect [6]. In such cases it is thought that the influence of the structural geometry on the physical interactions of incident light produces some extreme filtering effects at visible wavelengths - to which the creature's vivid colouration may be attributed. Gyroids have also been investigated on the nano scale in the field of metamaterials [7], that is, artificial materials with unusual magnetic responses such as negative permeability [8]. It is thought here that, on the sonic scale, these structures may exhibit similarly interesting filtering effects when applied to acoustic sources.
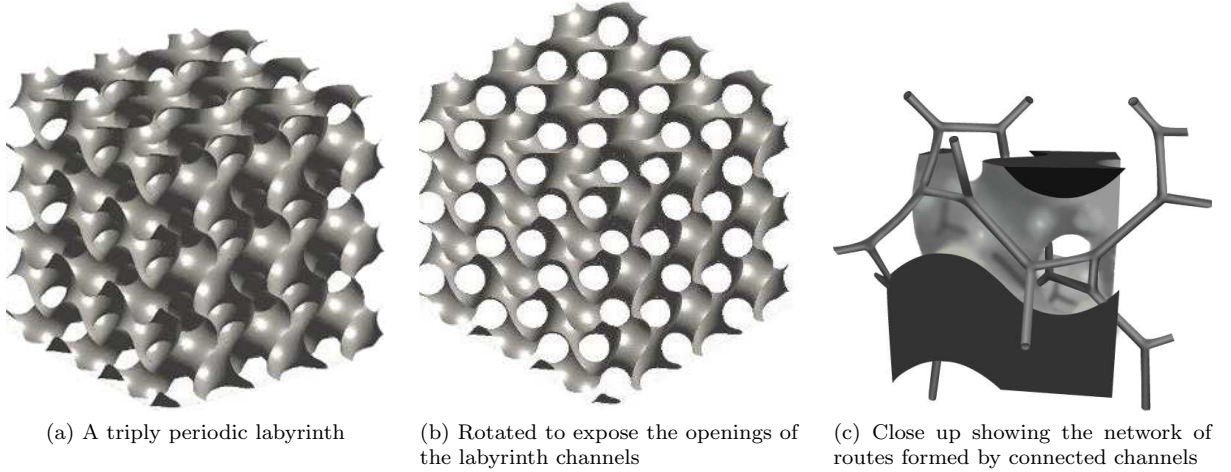

(a) A triply periodic labyrinth   (b) Rotated to expose the openings of the labyrinth channels   (c) Close up showing the network of routes formed by connected channels

Figure 1: The Gyroid

3 Method

3.1 Meshing the gyroid

When mapping triply periodic bicontinuous surfaces, it is standard practice to use the level set method. By this method, three-dimensional shapes are 'built up' from layers of level surfaces represented by functions $F : R^3 \to R$ of points $(x, y, z) \in R^3$, which satisfy the equation $F_{(x,y,z)} = t$, where t is a constant [9]. A value of $t = 0$ defines the boundary of the shape (referred to as the zero level set of F), while the interior of the shape is the set of points on each level surface for which F is positive. Thus, the surface of a gyroid may be defined by Schoen's gyroid level set equation:

$$F_{(x,y,z)} = \sin(x)\cos(y) + \sin(y)\cos(z) + \cos(x)\sin(z) = A \tag{1}$$

Where the parameter A determines the volume fraction of the gyroid labyrinth by the relation $\frac{A}{\varphi}$ in which φ is the golden ratio (approximately 1.61). A theoretically perfect gyroid in which $A = 0$ implies the surface has a mean curvature of zero. In the discretized gyroid of our simulations, exact solutions to (1) do not occur, hence one looks for transitions between positive and negative curvature when mapping the surfaces (figure 2).

Within a fixed volume, we would like to be able to repeat our experiment for gyroids of different compositions. Using a parameter we refer to as the step-size, we can vary the density of the channels (essentially by shifting the frequency of the output in (1). Hence the function used to generate the gyroid dataset becomes

$$F_{(x,y,z)} = \sin(g \cdot x)\cos(g \cdot y) + \sin(g \cdot y)\cos(g \cdot z) + \cos(g \cdot x)\sin(g \cdot z) \tag{2}$$

where g is the step-size. If one considers x, y and z are linear input functions, then step-size can also be thought of as the gradient of these input functions (i.e. their rate of change). A small step-size results in a low frequency output (i.e. fewer channels), whereas a large step-size has the opposite effect. By association, assuming the sampling rate is fixed, then a large step-size implies a lower surface resolution; hence it is necessary to increase the resolution of the mesh for higher step sizes to maintain surface fidelity. Here it was found a step-size > 0.2 necessitated a mesh resolution that was beyond the capabilities of the single CPU used in these experiments. It is also worth acknowledging how increasing the step-size, and effectively lowering the surface resoltion, impacts on the volume fraction - another limiting factor when deciding what step-size to use.
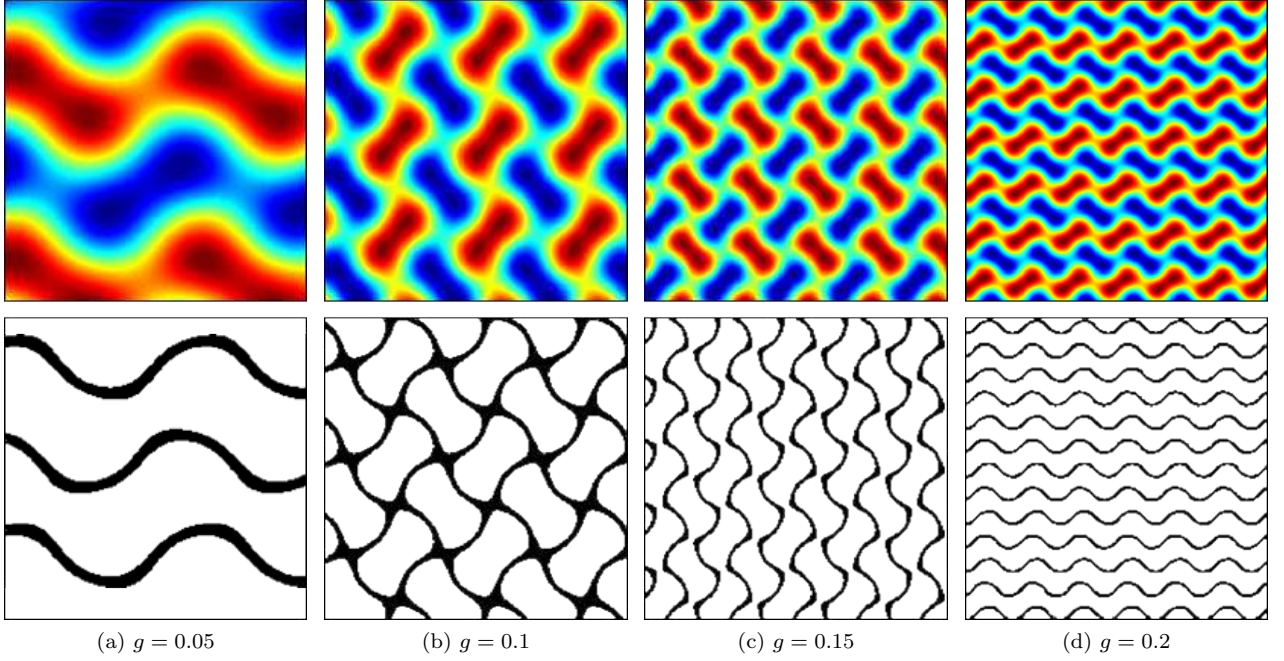
(a) $g = 0.05$	(b) $g = 0.1$	(c) $g = 0.15$	(d) $g = 0.2$

Figure 2: Vertical cross-sections through gyroids of different 'step-size' (g) generated by (2), and the corresponding slices through the simulation domain after meshing the surface. Each slice consists of 200 x 200 mesh elements.

In order to place the gyroid in the simulation domain, we need to determine which elements in the dataset are surface nodes and which are air nodes. We do this by assigning each element in the mesh an absorption coefficient α, the value of which is determined by the gyroid function (2) such that

$$\alpha_{(x,y,z)} = 1 \text{ if } |F|_{(x,y,z)} > A, \text{ and } 0 \text{ if } |F|_{(x,y,z)} \leq A \tag{3}$$

The aim is to keep the volume fraction consistent by fixing the value of A at 0.25. In each case, this yields a volume fraction of approximately 0.16, which we were able to verify by dividing the number of surface elements by the total number of elements making up the gyroid.

3.2   FDTD Simulations

The FDTD method used here is the 3-D compact explicit scheme presented in [10]. The procedure briefly consists of exciting the mesh with a planar Gaussian impulse at one end of the simulation domain and recording the IR from the other end. A control experiment is also performed without the gyroid to test the fidelity of the model, and to enable a comparison between the resulting impulse responses.

A difficulty encountered when attempting to solve an unbounded problem via the FDTD method is that of mesh termination and, consequently, the introduction of unwanted edge reflections into the simulation domain. Whereas in the 2-D case it was possible to eradicate this problem by exploiting the symmetry of the sonic crystal - coincidentally reducing memory requirements and speeding up computation - a unique feature of a gyroid its complete lack of reflective symmetry. Hence in these simulations, we have implemented the 3-D compact explicit boundary model, also described in [10]. Although it is not possible to eliminate edge reflections completely, the standard leap-frog (SLF) frequency-independent implementation of this model results in a significant reduction. To verify this, we performed simulations in an empty mesh, before and after the application of the boundary condition.

The experimental setup is pictured in figure 3. The resolution of the mesh is 5mm, corresponding to a sampling rate of 118 kHz and a total of 4.168×10^7 mesh elements. Memory limitations meant the source needed to be positioned very close to the surface of the gyroid. Likewise, the receiver is placed an equal distance from the rear surface to capture as long an impulse response as possible in a reasonable computation time. Hence our impulse responses are valid in the near field, and some compensatory measures will need to be taken for more distant source and receiver configurations when we reach the auralisation stage.

4   Results and discussion

The results of the simulations suggest the gyroid exhibits a high pass filter effect, where the cut-off frequency and slope of the filter appear to be related to the length of the cross-section of the gyroid channels (apparent from figure 5. In other words, only wavelengths that are small in relation to the channel opening are admitted, whilst longer wavelengths are backscattered. However, the extreme reduction of the low frequencies is somewhat misleading as the model fails to take several important factors into account. For example, edge diffraction and local resonance are both
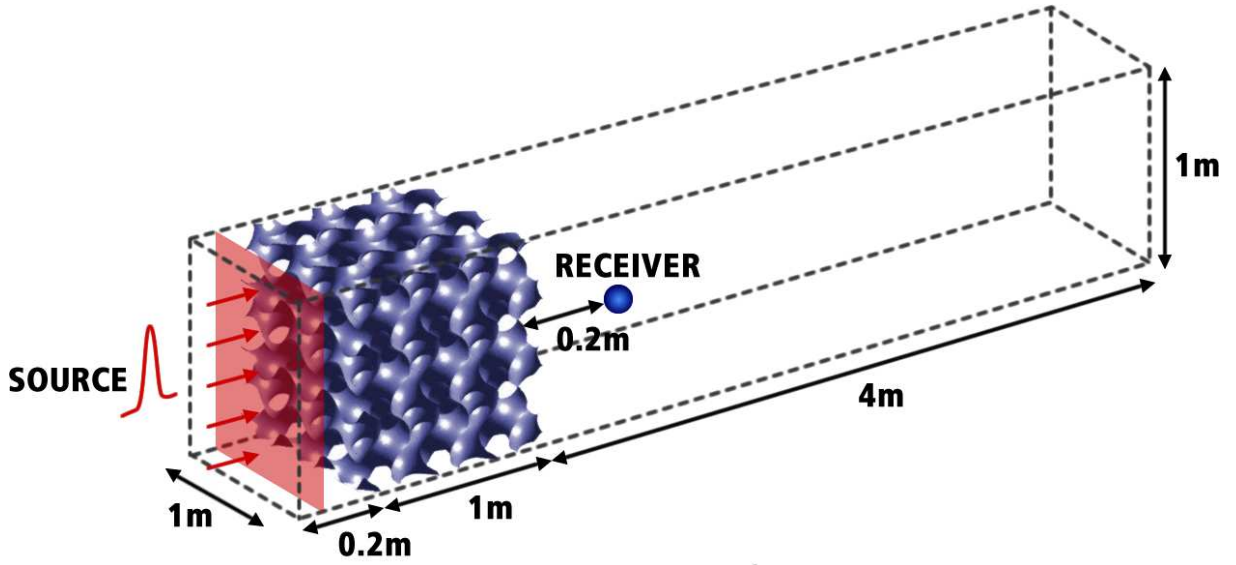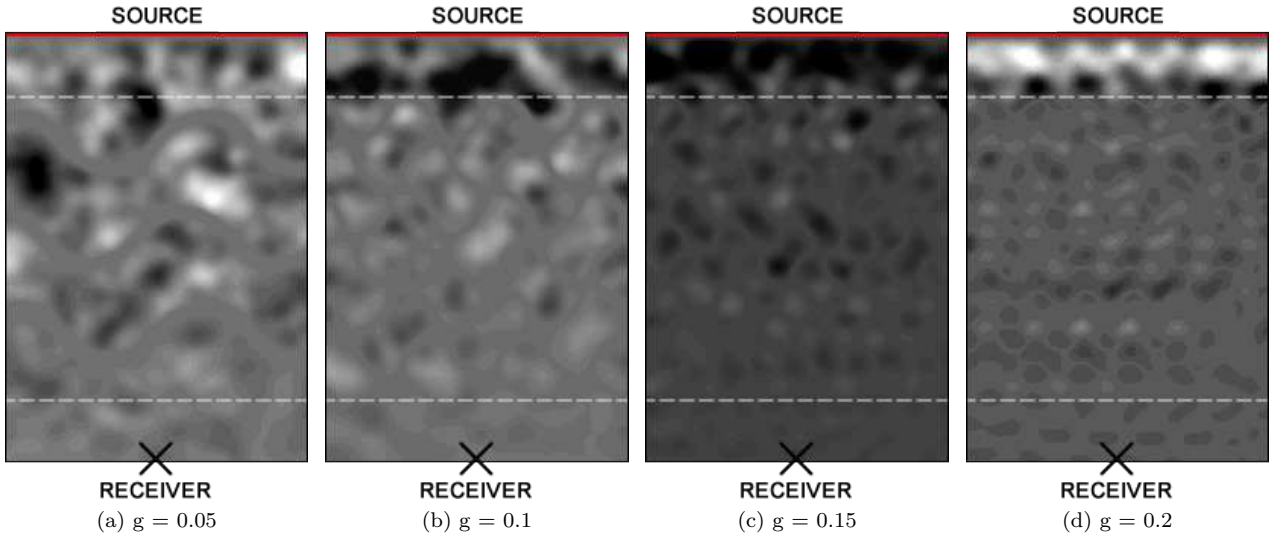
Figure 3: The simulation domain.

(a) g = 0.05   (b) g = 0.1   (c) g = 0.15   (d) g = 0.2

Figure 4: The pressure field at t = 11.8ms recorded on the horizontal plane. The dashed lines indicate the gyroid edges.

likely to undermine the efficiency of the filter in a real-world scenario. The frequency response is also likely to be affected by surface and atmospheric absorption, both of which were omitted in these simulations.

In the range of transmitted frequencies, there are vaguely-periodic regions of attenuation, with one or two narrow regions where it is more distinct (apparent in figure 6). It is difficult to draw any clear conclusions from this as we have not yet anything with which to compare these results, although it may be the case that we learn something about the nature of this filtering effect from the auralisations. A goal of future work will therefore be to attempt some theoretical analysis of the structures in order to predict the results prior to running the simulations. This would be advantageous as we currently have no way of verifying our results other than by performing rigorous checks on the FDTD scheme we are using. Another way would be to compare results with those of another form of numerical modelling technique - the Finite Element Method for example.

It is interesting to observe in figure 4 that the reflections off the gyroid also appear to be undergoing a process of filtration - an effect that was even more pronnounced in the simulations of the 2-D sonic crystals. Although it offers no obvious advantages in this particular case, in other contexts the effect may be more significant. Effectively the barrier surface is behaving as a selective absorber, which could have potential as a form of room acoustic treatment - perhaps even at the architectural level.

5 Conclusion and future work

In this paper we performed 3-D acoustic simulations of triply periodic bicontinuous surfaces known as gyroids using the FDTD technique. It was found that these structures exhibit a distinct high-pass filter effect where the
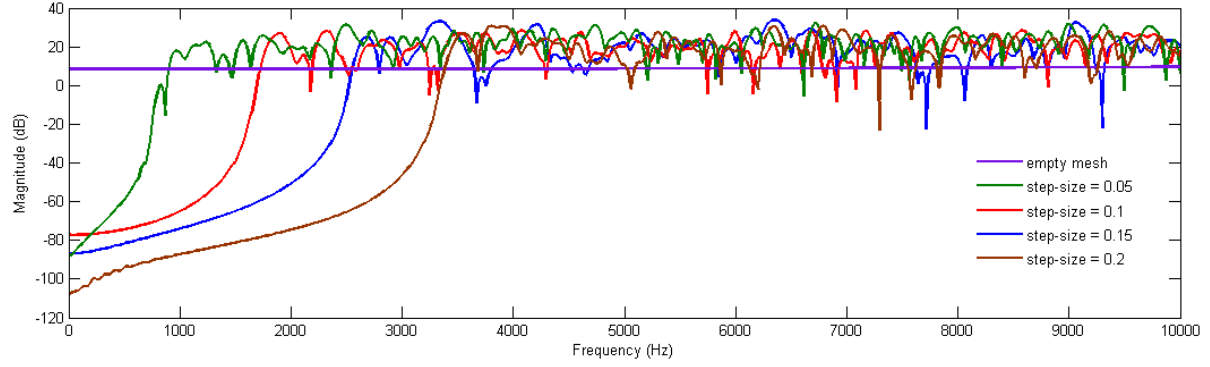
Figure 5: Fourier transforms of the IRs revealing the band structure of each of the 4 gyroids simulated.

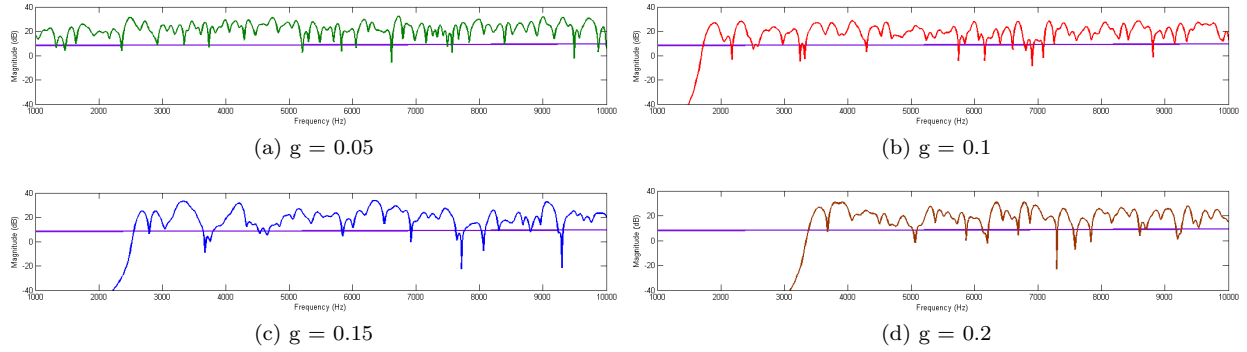(a) g = 0.05

(b) g = 0.1

(c) g = 0.15

(d) g = 0.2

Figure 6: Fourier transforms of the IRs revealing the band structure of each of the 4 gyroids simulated. Close up of 1-10kHz region.

cut-off frequency of the filter is directly related to the diameter of the channel openings in the gyroid. Above the cut-off frequency there appears to be quasi-periodic regions of attenuation in the frequency domain, potentially causing audible 'colouration' of the incident sound. As to whether this colouration is a desirable quality or not will be the focus of future auralisation work.

Beyond the simulation work, our intention is to use the artificial impulse responses to design and auralise a gyroid sound barrier/sculpture in the context of a real world soundscape. The aim will be to present our interpretation of the soundscape before and after the installation of the barrier to groups of listeners for subjective evaluation. In view of this, we believe the results presented here will be useful despite their shortcomings, as the intention has never been to use these impulse responses in isolation when performing the proposed auralisations. Instead we intend to combine them with other elements in order to create a reasonable, albeit impressionistic, representation of the soundscape. Essentially, our approach would be to partition the problem into several parts. For example, by modelling the diffraction around a barrier of finite size in a separate simulation, we may be able to recapture some of the missing low frequency components. Empirical methods could also be used to reintroduce atmospheric absorption into the problem. Given the opposite, low pass filter effect characteristic of edge diffraction and atmospheric absorption, it seems reasonable to suppose that the reintroduction of these elements may result in something more reminiscent of a band-stop filter, yet with the added colouration captured in the gyroid simulations. Exactly how we apply these processes to the raw audio is a matter for consideration in future work.

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