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# Relativity and quantum theory: Under the spell of today's paradigms

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### **Relativity and quantum theory: Under the spell of today's** paradigms

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Abstract. Thomas S. Kuhn described the development of the (natural) sciences as a specific dynamical process. Periods of piecemeal growth of knowledge based on widely accepted paradigms are interrupted by bursts of revolutionary changes which lead to new paradigms incommensurate with the earlier ones. This process is briefly illustrated by recalling the changes to classical physics brought about by Albert Einstein's theory of relativity on the one hand and by quantum theory on the other. Both theories represent fundamental paradigms of contemporary physics. They appear unshakeable to the working physicist but according to Kuhn their paradigmatic status is of a temporary nature only. Does Kuhn's framework help to identify potential future revolutions?

#### 1. Introduction

By its very nature, history is descriptive and comes after the fact. Nevertheless, there is a wide variety of both methods applied and goals declared by historians. The 19th century saw speculative theories which postulated forces driving mankind towards a specific future state of society or they assumed some necessary "dialectic" evolution. However, the idea of "laws" acting throughout history was met with considerable skepticism in the 20th century with philosophers of history cutting back such largescale interpretations. Instead the focus turned towards fairly representing past events, avoiding a judgement of them based on a privileged - later - position and not distorting them in order to support some preconceived idea. These views clearly leave no room for the idea that past events could reliably determine future events in spite of being strongly intertwined.

Thomas S. Kuhn's views ran counter to this trend, albeit in the narrow setting of the history of science where it might be possible to say more [1]. Kuhn suggested that scientific theories – physics in particular - evolve according to a "binary" pattern consisting of longer periods of normal science interrupted by shorter bursts of revolutionary paradigm shifts, induced by anomalies which create *crises.* This conception represented a return to 19th century ideas in the sense that there exists a law – or at least a definite pattern of "scientific revolutions" - which is supposed to govern the way in which sciences develop over time. This approach was at odds with the idea that scientific activities would be purely additive or cumulative, a view shared by many earlier historians of science. However, Kuhn did not suggest that the proposed structured progression would lead to a specific final state. Being "goalless", the evolution rather had Darwinian traits.

For the sake of the argument, let us assume that Kuhn's structure of scientific evolution correctly describes the relation of theories following each other. The presence of law-like development – even if as rudimentary as a sequence of normal and revolutionary periods - raises the question of whether the

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model possesses any "predictive" power. The interesting point here is to ask whether the model may help scientists to judge the status of a currently valid paradigm which defines their world view. In other words, is Kuhn's model limited to retrospective descriptions only – once paradigms have lost their power – or may it be used to identify potential future revolutions from within a given paradigmatic framework?

To answer this question, first Kuhnian terminology will be used to describe the transition from classical Newtonian physics to today's theory shaped by Einstein's theory of relativity and by quantum theory. In the process, fundamental concepts such as simultaneity and particle trajectories were given new meanings incompatible with those of the pre-revolutionary framework. The resulting post-revolutionary framework, based on quantum theory and relativity, defines the currently accepted paradigm of physics. One will then be in a position to ask whether Kuhn's model allows us to transcend the paradigms of today's physics.

In Section 2, this paper briefly describes the key notions of Kuhn's dynamical model of the natural sciences: *paradigms* which underlie normal science, *anomalies* within a given set of paradigms and scientific *revolutions* triggered by the growing weight of inconsistencies. In Section 3, these concepts are illustrated by developments in physics which happened at the beginning of the 20th century when counter-intuitive theories toppled the longstanding paradigm of Newtonian classical mechanics. These scientific revolutions of the past set the scene for potential future revolutions which would have to fundamentally alter relativity and quantum theory or replace them. Section 4 discusses the current status of these theories while Section 5 addresses the question to what extent Kuhn's model may help prepare the next paradigm shift.

#### 2. Paradigms, anomalies and revolutions

A survey of the technologies which reduced the necessity of manual labour, simplified communication or facilitated easier transport, for example, may suggest that they evolve in a cumulative way. Improvements are often based on refining existing methods or devices. It seems natural to assume an equally linear and additive development of the sciences which underpin the technological changes. According to Kuhn, a closer reading of the history of science reveals that this view is inappropriate - it does not do justice to the dynamics of fundamental sciences such as physics.

A simplified version of Kuhn's main idea splits scientific activity into two distinct phases which inevitably alternate: *normal* and *revolutionary* periods. Widely accepted *paradigms* [1, Section V] define a framework within which mainstream research takes place – normally over an extended period – by providing concepts and theoretical tools which a community of researchers shares. A consistent "world view" determines both relevant problems worthy of investigation and the methods available to study them. Kuhn referred to this activity as "*puzzle-solving*" [1, Section IV]. In other words, the consequences of an unquestioned framework are being unfolded in detail without worrying too much about its foundations.

According to Kuhn, the explanatory power of a framework tends to reach its limits. After a period of normal scientific activity, which sees its successful application and expansion, some experimental data may be found to disagree with the predictions of the framework or internal inconsistencies are discovered. Such *anomalies* [1, Section VI] are often ignored by the majority of researchers as long as they do not directly affect their field.

If more anomalies are found that cannot be resolved within the dominant paradigm, they lead to an increasing number of researchers questioning its validity. A scientific *revolution* [1, Sections IX, X] has occurred once an alternative framework has been put in place which will have similar explanatory power as the earlier one while eliminating the anomalies. Typically the new paradigm is *incommensurate* with the old one in the sense that it does not simply extend or adapt the known concepts; a disruptive conceptual break is necessary to provide new foundations.

Kuhn's binary scenario – periods of normal activity alternate with revolutions triggered by anomalies which necessitate paradigms incommensurate with the earlier ones – has attracted

considerable interest. It has been debated extensively by historians and philosophers of science as well as sociologists (see e.g. [2]).

For the purpose of this paper it will be assumed that Kuhn's view captures essential aspects of how theories such as physics evolve. It begins by illustrating the model and its central notions by briefly describing two developments within physics which happened early on in the 20th century: the emergence of the theory of special relativity and of quantum theory. Only then will it be possible to address the main question.

#### 3. Past revolutions

#### 3.1. The paradigm: Newtonian physics

Isaac Newton's *Principia* [3] established the foundations of classical mechanics in 1689. The theory relies on primitives such as time and a specific reference frame that were not challenged for more than two centuries. The behaviour of material bodies is governed by dynamical laws which require a definition of *time*. In the *scholium*, an early part of his *Principia*, Newton introduced time in an axiomatic way:

*Absolute, true, and mathematical time, of itself, and from its own nature, flows equably without relation to anything external...* [3, p6].

This definition guarantees an unambiguous ordering of events by means of the universal parameter, time. A sunrise observed at the location of Newton's apple tree, say, occurs at a specific moment of time. Any other event will either happen earlier, later or at the same time – the ordering of events is unambiguous when referring to absolute time.

Newton's laws predict the future motion of an object under the influence of known external forces with certainty and arbitrary precision given its exact *position* and its *velocity* at present. Thus, at any given moment of time, a small particle, say, is assumed to occupy a specific position in space and to move in a certain direction with well-defined speed. These data define its *state*. In turn the state can be determined unambiguously by suitable position and velocity measurements. Their original values will not change since the interaction with the particle can, in principle, be made arbitrarily weak.

The motion of celestial – and all other – bodies takes place relative to the *aether*, an unobservable substance permeating the entire Universe. The aether was assumed to exist to provide a medium for light to propagate through a vacuum, by analogy to waves travelling on the surface of water or air carrying sound waves.

#### 3.2. Revolution 1: Theory of relativity

Experiments to determine the motion of the Earth relative to the aether were conducted throughout the second half of the 19th century with ever increasing precision. In 1887 Albert Michelson and Edward Morley devised a method to test whether the speed of light would depend on its direction of travel relative to the aether [4]. Based on interfering light beams, their observations were sufficiently precise to rule out such a dependence. The negative result represented a major inconsistency within the framework of classical mechanics. To account for the result contrived-looking assumptions about the propagation of light had to be made.

Paradigm change came about in 1905 when Einstein scrutinised the concept of time used in Newtonian mechanics:

## We must take into account that all our judgments in which time plays a part are always judgments of simultaneous events. [5, p893]

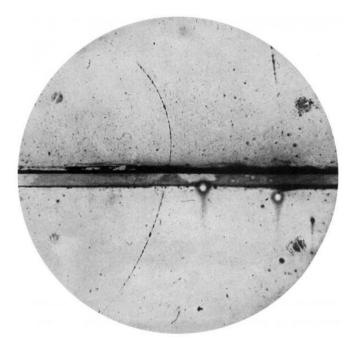
Reference to the absolute time introduced by Newton turns out to be insufficient to define simultaneous events: it is not obvious how to ensure that clocks in different locations actually display the same time. What exactly does it mean to say that events do happen at the same time? Einstein addressed this question by proposing an *operational* approach to synchronise clocks, i.e. by spelling

out a physical procedure for people to synchronise their clocks. The observers are assumed to operate within *inertial reference frames* which move relative to each other along straight lines and at constant speed. Einstein's method to synchronise clocks is based on a new postulate compatible with the findings of Michelson and Morley: the speed of light is constant in all inertial reference frames. This assumption establishes the (satisfying) physical equivalence of all such frames but implies, somewhat counter-intuitively, that two events happening at the same time for one observer are not necessarily simultaneous for another one.

Eliminating simultaneity based on absolute time is a "revolutionary" conceptual step as it does away with one of the fundamental notions on which classical mechanics is built. Newton's conception of time is replaced by a new notion, "incommensurate" with absolute time. The change was triggered by the inexplicable, "anomalous" outcome of an experiment which the new paradigm, the theory of relativity, was able to predict correctly.

#### 3.3. Revolution 2: Quantum theory

The first quarter of the 19th century saw another paradigm shift. Microscopic objects were found to behave in ways which could not be explained by Newtonian mechanics. Atoms turned out to be aggregates of particles with opposite electric charge. Classical models imply that they must attract each other, rendering the atoms unstable, an obvious contradiction with the permanent material structures we live in. Experiments suggested that atoms are emitting radiation of specific frequencies only, a property impossible to derive from classical mechanics. Many other anomalous properties of microscopic objects could not be explained using the age-old paradigm.

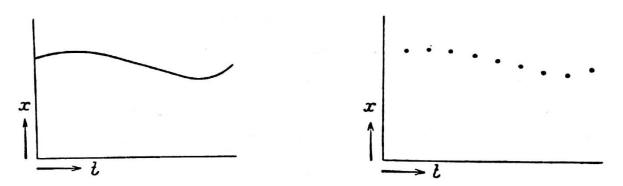


**Figure 1.** The first record of a positron in the presence of a magnetic field; cloud chamber photograph from 1933 [6].

In 1927 Werner Heisenberg took stock of these developments and analysed the observed anomalies from a conceptual point of view [7]. In particular he contemplated experiments with so-called cloud chambers. These devices, dating from the early 1900s, were essential to understand the properties of small particles invisible to the naked eye such as electrons (figure 1). When traversing a cloud chamber, particles leave traces which resemble trajectories of material objects, possibly subject to the laws of classical mechanics. Heisenberg explained that in light of the new theory this interpretation of the tracks cannot be upheld.

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The classical Newtonian equations of motion for a point particle predict continuous trajectories by ascribing to it definite location and velocity at each moment of time (see the sketch on the left in figure 2). Heisenberg pointed out that the tracks seen in cloud chambers are in fact not smooth: careful inspection shows that they rather consist of sequences of condensate droplets which result from the repeated interaction of the particle with the saturated vapour filling the entire chamber (see the sketch on the right in figure 2).



**Figure 2.** The smooth trajectory of a classical point particle (left); a schematic representation of the track of a quantum particle recorded by a cloud chamber (right) [7, p173].

A sequence of rather imprecise particle positions cannot be used to reliably determine the velocity of a massive particle. One might hope that refined measurements would resolve this difficulty, a strategy known to be successful when observing large macroscopic objects such as cars or planets. Quantum theory, however, comes with a built-in finite limit on the possibility of simultaneously attributing exact values of position and velocity to a microscopic article. This property, the content of *Heisenberg's Uncertainty Relation*, means that one has to give up the idea that "*it will be possible to trace the trajectory…, with the tangent to the curve indicating the velocity*" [7]. In other words, an electron simply does not "possess" values for its position and velocity which could be read off using a suitable device.

Heisenberg knew that a radical departure from classical concepts was required to formulate an alternative theory. In his 1971 autobiography he described the role of cloud chamber photographs when searching for a new conceptual framework:

We had always said so glibly that the path of the electron through the cloud chamber existed. But perhaps what we really observed was something much less. Perhaps we merely saw a series of discrete and ill-defined spots through which the electron has passed. In fact, all we do see in the cloud chamber are individual water droplets which must certainly be much larger than the electron. [8, p77–78]

To provide a valid description of electrons and other elementary particles, quantum theory rejects basic assumptions about particles made by classical mechanics. Without the concepts of particle position and velocity it becomes impossible to write down Newton's equations of motion. Instead of the lacking dynamical variables, the "wave function" of a system is introduced to describe its state. The paradigm shift is completed by postulating Erwin Schrödinger's equation which governs its evolution in time, replacing Newton's equations. As a major consequence quantum mechanical predictions are typically not deterministic but only probabilistic.

#### 4. Today's paradigms of physics

About a century after their inception, quantum theory and the theory of general relativity (which generalises the early theory considered in Section 3.2 beyond inertial frames) continue to define our view of how nature works at a fundamental level. Over time the theories have been applied

successfully to an ever increasing range of phenomena without, however, the need to modify the basic assumptions on which they rest. It seems fair to say that taken together they define a highly successful paradigm which leaves us with tools to solve puzzles we encounter when studying nature on both microscopic and astronomical scales.

Quantum theory possesses considerable explanatory power regarding the behaviour of molecules, atomic nuclei and their constituents. Recent developments exploit non-classical properties of quantum systems leading to novel applications and technologies such as quantum-cryptographic schemes and efficient ways to process information encoded quantum mechanically. General relativity is essential to model the early Universe and to describe its current state. The accuracy of global positioning systems depends on appropriately taking into account relativistic effects.

According to Kuhn, the discovery of anomalies which do not fit an existing paradigm signals the end of a period of normal science. Having summarised today's paradigm of physics, the paper is now in a position to approach the main question being considered here: can anomalies be identified which might indicate a future revolution upending quantum theory and the theory of relativity? To do so from within a paradigm is qualitatively different from retrospectively applying Kuhn's terminology to a paradigm of the past.

It is common practice in a period of normal science to look for theoretical inconsistencies within the accepted theory or to predict previously unobserved phenomena. An experiment crucial for the acceptance of the theory of (general) relativity was carried out by Eddington in 1919. It confirmed that light grazing the sun was bent away from a straight path twice as much as predicted by a Newtonian approach [9,10]. Later tests of the theory have been carried out with increasing precision and were positive throughout, culminating in the direct observation of gravitational waves [11] and black holes [12]. No widely acknowledged anomalies seem to exist which would suggest the need to alter special or general relativity. Of course there are open questions such as the existence of "dark matter" but they do not seem to create full-fledged conflicts with relativity theory as it stands.

As for quantum theory, an early apparent inconsistency was described in a paper by Einstein, Boris Podolsky and Nathan Rosen in 1935 [13]. Their thought experiment suggests that quantum theory does not provide a complete description of particles such as electrons, hence requiring a fundamental modification. The "EPR paradox" led to long-standing discussions about the foundations of quantum theory. The paradox was finally resolved experimentally in favour of the counter-intuitive predictions made by quantum theory [14], [15].

A long-standing conceptual problem within quantum theory concerns the ambiguous status of *measurements*, i.e. the interactions of an experimenter with a quantum system in order to extract information about its state. If one considers quantum theory as universally valid, then it should be possible to use the theory to describe measurements in a consistent and satisfactory way – in the end they are nothing else than interactions between physical systems. It is, however, not obvious how to set up such a description. From a practical point of view the difficulty is normally eliminated by assuming an artificial split of the laboratory into a quantum part (the observed system) and a classical part (the measuring device). In spite of its conceptual ambiguity, this approach to measurements is effectively being used whenever experiments are performed with quantum systems, be it to test quantum theory or to apply it to achieve some task.

The unsatisfactory – if not anomalous – status of measurements hangs over the theory ever since it was noticed in the early days of quantum theory [16]. Two strategies have been applied to resolve the problem. There are *interpretations* of the existing theory which claim to remove the difficulty at the expense of other unpalatable assumptions. Alternatively, slight *modifications* of the standard formulation of quantum theory introduce mechanisms which allow one to avoid the problem. Since these approaches do not lead to observable differences, the conceptual difficulty simply persists in one way or the other. Recently the measurement problem has been made more explicit by refining a paradoxical thought experiment which dates back to the 1960s [17], [18]. In any case, there is no candidate theory to replace quantum theory in its current form which would maintain or even extend its predictive power.

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Quantum theory and relativity, each on their own, represent successful, seemingly independent tools to describe the properties of matter on very small and very large scales, respectively. However, if elementary particles move at high speeds, both special relativity and quantum theory are relevant. *Relativistic quantum theory* or *quantum field theory* combine the two into a single consistent framework. Its predictions have been confirmed with remarkable accuracy in particle accelerators.

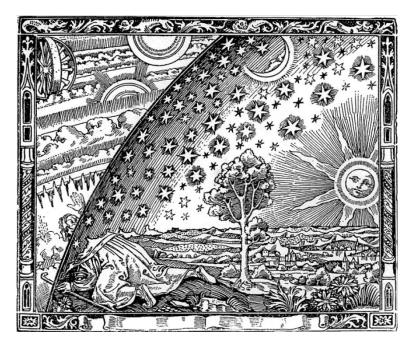
In contrast, it is not known how to combine the basic assumptions of quantum theory with the *general* theory of relativity. Einstein was aware of the rigid, monolithic structure of general relativity, anticipating the difficulty to modify it in a letter to the *London Times* in 1919:

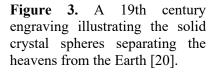
The chief attraction of the theory lies in its logical completeness. If a single one of the conclusions drawn from it proves wrong, it must be given up; to modify it without destroying the whole structure seems to be impossible. [19, p105]

One of today's major challenges remains to develop a theory which unifies quantum theory with general relativity. A theory of *quantum gravity* is needed to describe the early stage of the Universe when extreme densities of matter occurred at very small scales and in the presence of strong gravitational forces. However, to perform controlled experiments in such circumstances is unfeasible. In that sense, physics is – for now – not necessarily facing an anomaly as such but rather an open, admittedly difficult problem within current normal science.

#### 5. Future revolutions?

Let us finally try to go beyond the original scope of Kuhn's historical approach: do his concepts also apply to contemporary science or are they limited to past paradigms only? Do they allow one to construct a "meta-scientific" vantage point to judge the status of a currently accepted paradigm? Can one identify anomalies which hint at developments potentially overthrowing the theories which determine our current view of the world?





The idea of "normal science" clearly applies to today's physics, easily characterised as a paradigmbased undertaking which provides puzzles and the tools to solve them. Quantum theory has reached a state in which it generates new technologies, in spite of the long-standing problem of measurement. Thus, the measurement problem may not count as a full-fledged anomaly in Kuhn's sense. The consequences of general relativity are being unfolded in astrophysics by matching an ever-larger body Journal of Physics: Conference Series 2877 (2024) 012071

of data; solvable puzzles rather than crisis-inducing anomalies are the norm. Thus, taken by themselves, the theories which form today's well-established framework of physics do not seem to head towards an imminent crisis.

Historically, the unification of theories has been a strong driving force within physics. In this respect, the current framework of two coexisting and largely independent theories is unsatisfactory. Camille Flammarion's engraving (see figure 3) depicts the Copernican revolution: an inquiring mind has finally broken through the venerable celestial model dating back to antiquity. Quantum theory and relativity represent today's crystal spheres which shape our views of the material world. We do not know where to look in order to transcend them. Physicists almost routinely search for inconsistencies of theories by testing their limits. The notion of anomaly does not seem to supply an alternative handle with predictive power which would make it easier to recognise the start of a revolutionary period.

To conclude, it has been seen that Kuhn's proposed structure of scientific revolutions is an instructive scheme capable of describing how the physical paradigm valid today came into existence. It is not difficult to point to anomalies which triggered revolutionary crises, ultimately causing the removal of Newtonian mechanics, the earlier paradigm. However, Kuhn's concepts appear to remain purely descriptive: they do not provide us with clues regarding the future development of today's paradigms, the theories of relativity and quantum theory. They continue to cast their spell on us.

#### Acknowledgements

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