Proceedings Paper:

Koshuriyan, Zamir, Stirrat, Sarah and Sescu, Adrian (2023) Taylor’s Hypothesis in High-order Turbulence Correlations. In: International Conference on Flow Dynamics. International Conference on Flow Dynamics, 06-08 Nov 2023 , JPN

Taylor's Hypothesis in High-order Turbulence Correlations

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ABSTRACT

Taylor's hypothesis states that a turbulence field will remain relatively 'frozen' within a flow, and as such evolve with respect to a convected variable. In this paper we show results from Large Eddy Simulations (LES) of a round jet at 4 operating points covering the parameter range of static temperature ratio ($TR = 0.84, 1.0, 2.7$) and acoustic Mach number ($Ma = 0.5, 0.9$). Our results indicates that, remarkably, Taylor's hypothesis provides an accurate estimation of the turbulence auto-correlation under an appropriate moving co-ordinate transformation.

1. Introduction

Historically, measurements of small amplitude turbulence downstream of a turbulence grid show the remarkable property that streamwise velocity fluctuation remains 'frozen' within the unsteady flow field. This result was formulated in a seminal paper by Taylor [1] and the subsequent approximation was referred to as the *Taylor frozen turbulence hypothesis*. Taylor's basic assumption was that if the turbulence intensity is sufficiently small, then flow disturbances (eddies of a lengthscale smaller than an $O(1)$ body dimension) are convected, or transported, with the local mean flow without change to their spatial structure.

Expressing these ideas mathematically, Taylor suggests that if we denote the streamwise velocity fluctuation by $v_1'(\boldsymbol{x}, t)$ where $\boldsymbol{x} = \{x_1, \boldsymbol{x}_T\} := \{x_1, x_2, x_3\}$ are a triad of orthogonal directions in a Cartesian co-ordinate system with origin, for example, centered at the turbulence grid and t is time. Here, all lengths have been normalized by an upstream body dimension, Λ_0^* (e.g. grid spacing), velocity by U_0^* (free-stream velocity), time by Λ_0^*/U_0^* and pressure by $(\rho^* U_0^{*2})$. Under Taylor's hypothesis, $v_1'(\boldsymbol{x}, t) = v_1'(x_1 - U_c t, t + \tau_0; \boldsymbol{x}_T)$. In other words, at a later time $t = t + \tau_0$, the streamwise velocity field $v_1'(\boldsymbol{x}, t)$ depends on the (x_1, t) through the streamwise moving co-ordinate, $\xi = x_1 - U_c t$ (Taylor, Townsend, [2] p.65) at a fixed transverse location $\boldsymbol{x}_T = (x_2, x_3)$.

The latter approximate dependence on a moving co-ordinate then implies that the streamwise component of the incompressible Navier Stokes equations expands as follows (Hinze 1959 [3], p.41),

$$\frac{Dv_1'}{Dt}(x_1, t; \boldsymbol{x}_T) := \left(\frac{\partial}{\partial t} + U_c \frac{\partial}{\partial x_1}\right) v_1'(x_1, t; \boldsymbol{x}_T) = O(|\boldsymbol{v}'|^2) \tag{1}$$

where $v'(x_1, t; \boldsymbol{x}_T) = |\boldsymbol{v}(x_1, t; \boldsymbol{x}_T)|$ is normalized magnitude of turbulence fluctuations at some fixed space time point $(\boldsymbol{x}, t)$. This remainder in dimensional terms is a measure of the turbulence intensity and is equivalent to $O((v'^*/U_0^*)^2)$. Hence for small turbulence intensities Eq. (1) necessarily implies that,

$$\frac{dv_1'}{d\xi}(\xi; \boldsymbol{x}_T) = o(1), \tag{2}$$

when $|\boldsymbol{v}'|^2 \ll 1$.

If we write Eq. (1) or (2) at another space time field point that is a linear translation of $(x_1, t; \boldsymbol{x}_T)$, say $(\bar{x}_1, \bar{t}; \boldsymbol{x}_T) := (x_1 + \eta_1, t + \tau_0; \boldsymbol{x}_T)$ (where the transverse location $\boldsymbol{x}_T$ remains fixed) we can use the Chain rule to re-write derivatives in terms of (η_1, τ_0). Multiplying this equation by $v_1'(x_1, t; \boldsymbol{x}_T)$ and adding it to the product of Eq. (1) and $v_1'(x_1+\eta_1, t+\tau_0; \boldsymbol{x}_T)$, and performing the time-average, we can easily that Eqs. (1) or (2) implies that:

$$\frac{DR_{11}}{D\tau_0}(\eta_1, \tau_0) := \left(\frac{\partial}{\partial \tau_0} + U_c \frac{\partial}{\partial \eta_1}\right) R_{11}(\eta_1, \tau_0) = 0, \tag{3}$$

where,

$$R_{11}(\eta_1, \tau_0; \boldsymbol{x}_T) =: \overline{v_1'(x_1, t; \boldsymbol{x}_T) v_1'(x_1 + \eta_1, t + \tau_0; \boldsymbol{x}_T)} \equiv \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} v_1'(x_1, t; \boldsymbol{x}_T) v_1'(x_1+\eta_1, t+\tau_0; \boldsymbol{x}_T)\, dt \tag{4}$$

where T is a large time interval and the transverse spatial field point is $\boldsymbol{x}_T = O(1)$. Eq. (3) then implies that $R_{11}(\eta_1, \tau_0; \boldsymbol{x}_T) \equiv R_{11}(\eta_1 - U_c \tau_0; \boldsymbol{x}_T)$ and therefore, following Eqs. (1) & (2) that

$$\frac{dR_{11}}{d\xi}(\xi; \boldsymbol{x}_T) = 0, \tag{5}$$

commensurate with the small turbulence amplitude scale below (2). The convection velocity (Mach number), U_c is defined by a constant velocity at the location (x_1, t); i.e. $U_c := U(\tilde{x}_1, \tilde{t})$ where tilde refers to the fact the (x_1, t) are fixed.

The principal advantage of using Taylor's hypothesis in the form of (3) is the conversion of an Eulerian spatial function to a temporal correlation function inasmuch as [4, 5]:

$$R_{11}(\eta_1, 0; \boldsymbol{x}_T) \equiv R_{11}(0, U_c \tau; \boldsymbol{x}_T). \tag{6}$$

That is, a streamwise spatial correlation function at zero time separation is identical to an Eulerian time

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correlation at zero spatial separation when the time co-ordinate is re-scaled via "convected time unit", $U_c\tau$. (The latter is also the Eulerian auto-correlation in this co-ordinates). Remember that spatial correlations are always more difficult to obtain due to probe displacement effects in real experiments and/or increased streamwise resolution needed in computational simulations for appropriate convergence norm to be acheived.

Indeed, there are two approximations underlying Taylor's hypothesis in the form of Eqs. (1), (2) (and their statistical form in Eqs. 3 & 4). First that the convection velocity (Mach number) is constant. Second, that the flow is incompressible. The interesting extension then arises of whether the hypothesis in the form of (6) is applicable to shear flow turbulence[6, 7] at high Mach number and for high-order correlations. In the next section we compare Eq. (6) to the fourth order correlations $R_{1111}(\eta_1, \tau; \boldsymbol{x}_T)$ and $R_{1212}(\eta_1, \tau; \boldsymbol{x}_T)$ at the transverse location of maximum turbulence (i.e. along the jet shear layer $|\boldsymbol{x}_T| = 0.5$ and azimuthally averaged). These correlations have a particular importance to Aero-acoustics (see Stirrat *et al.* 2023[8]).

2. LES Database

The LES was performed using an in-house implicit LES solver which uses the 2nd order Adams-Bashforth time marching method. The mesh was composed of $13, 244, 832$ cells and the computational time was roughly one week. Four LES simulations were completed providing unsteady data for four jets described in Table. 1.

Table 1 LES database.

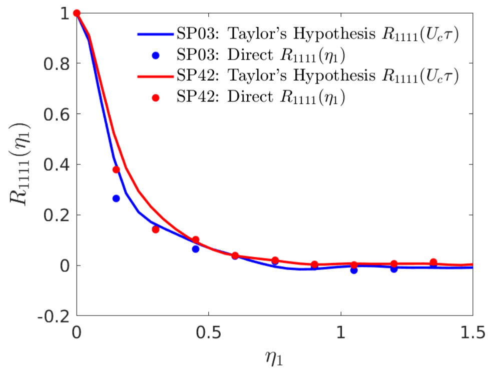
set point	M_a	TR	x_1 start of potential core
$SP03$	0.5	1.0	5.8
$SP07$	0.9	0.87	6.5
$SP42$	0.5	2.7	4.3
$SP46$	0.9	2.7	4.25

3. Taylor's Hypothesis in Jets

Fig. 1 checks Taylor's hypothesis for the four jets, using the fourth order turbulence correlations (R_{1111}, R_{1212}), where we used $U_c = 0.6$. Although Taylor's hypothesis appears to be remarkably accurate at the start of the potential core and along the jet shear layer, there is variation in U_c. Fig. 2 looks at how U_c varies across spatial coordinates (y_1, r), and we can see that it is relatively constant along y_1, particularly at the potential core region which is more or less located $5 < y_1 < 8$. It does vary quite significantly with $r \lessgtr 0.5$, however the LES mesh does have reduced resolution outside of the jet shear layer.

4. Concluding Remarks

In this paper, we have conducted a numerical investigation on the applicability of Taylor's hypoth-



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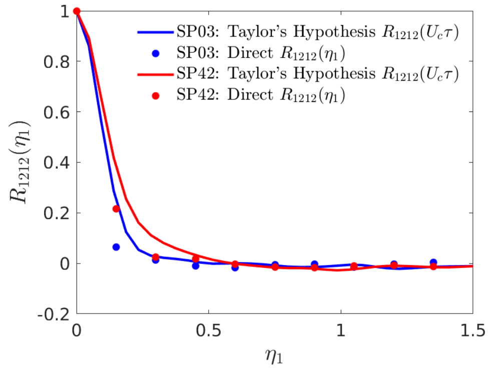

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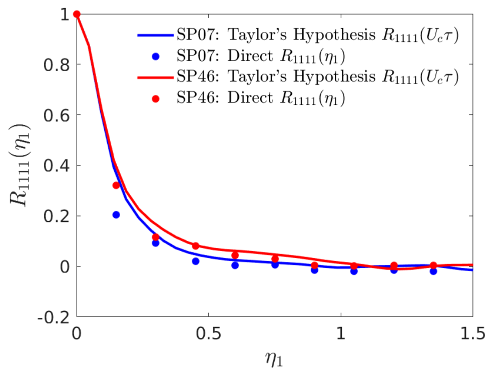

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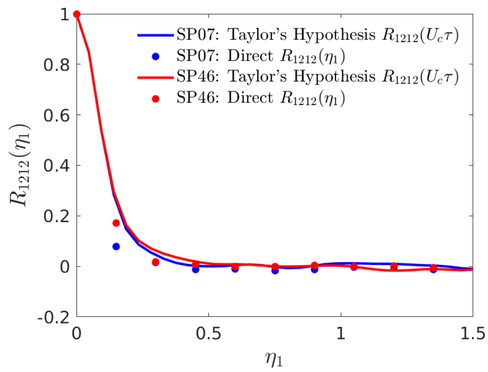


(d)

Fig. 1 Check Taylor's Hypothesis on shear layer ($r = 0.5$) at start of potential core for (a) Ma=0.5 R1111 (b) Ma=0.5 R1212 (c) Ma=0.9 R1111 (d) Ma=0.9 R1212.

esis to turbulent round jets of $O(1)$ subsonic inflow Mach number (where compressibility effects are present). Our results indicate that at least in the location where convection velocity is constant, Taylor's hypothesis remains very accurate. In other words, there is a direct correspondence between the (Eulerian) temporal correlation function and the zero-time delay streamwise spatial correlation.

References

[1] G.I, Taylor, *Proc. R. Soc. London A*, 164(919), (1938), 476–490.

[2] A.A.Townsend, *Structure of Turbulent Shear Flow*, Cambridge University Press, (1976).

[3] J.O. Hinze, *Turbulence*, McGraw Hill Co., USA, (1959).

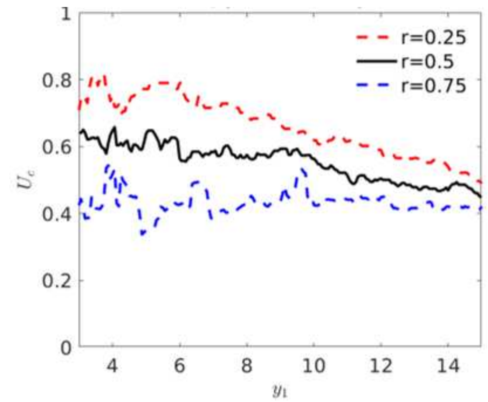
[4] A.S. Monin, A.M. Yaglom, *Statistical fluid mechanics, volume II: mechanics of turbulence*, Dover Publications, (2013), 11.

[5] G. K. Batchelor, *The theory of homogeneous turbulence*, Cambridge University Press, (1953).

[6] P. Moin, *J. Fluid Mech.*, 640, (2009), 1–4.

[7] J.A.B. Wills, *J. Fluid Mech.*, 20(3), (1964), 417–432.

[8] S. Stirrat, M.Z.A. Koshuriyan, A. Sescu, *ICFD* (2023).



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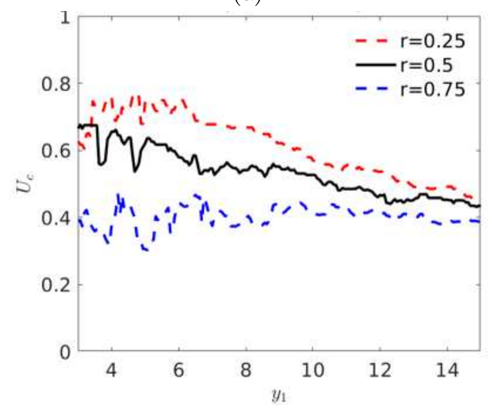

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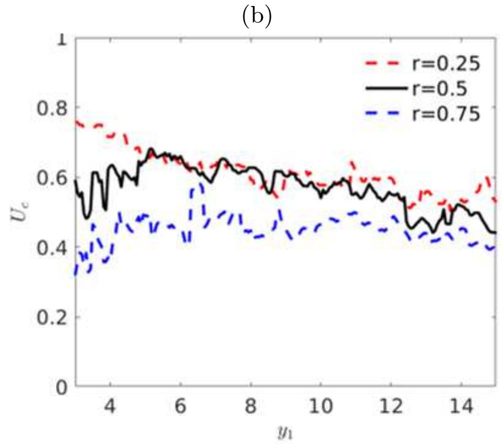

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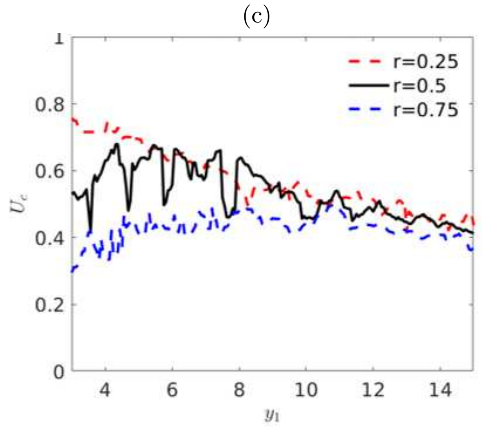

(d)

Fig. 2 Spatial variation of U_c for (a) SP03 R1111 (b) SP03 R1212 (c) SP07 R1111 (d) SP07 R1212.