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Asgharian, Hossein, Christiansen, Charlotte, Hou, Aijun et al. (1 more author) (2021) Long- and Short-Run Components of Factor Betas: Implications for Equity Pricing. *Journal of International Financial Markets, Institutions and Money*. 101412. ISSN 1042-4431

<https://doi.org/10.1016/j.intfin.2021.101412>

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Journal of International Financial Markets, Institutions & Money

journal homepage: www.elsevier.com/locate/intfin

Long- and short-run components of factor betas: Implications for stock pricing[☆]

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ARTICLE INFO

JEL Classifications:

G12

C58

C51

Keywords:

Long-run betas

Short-run betas

Risk premia

Component GARCH model

MIDAS

ABSTRACT

We propose a new model that estimates the long- and short-run components of the variances and covariances. The advantage of our model to the existing DCC-based models is that it uses the same form for both the variances and covariances and estimates these moments simultaneously. We apply this model to obtain long- and short-run factor betas for industry test portfolios. We find that the risk premium related to the short-run market beta is significantly positive, irrespective of the choice of test portfolio. Further, the risk premia for the short-run betas of all the risk factors are significant outside recessions.

1. Introduction

In this paper, we propose a new model that decomposes return variances and covariances into a long-run (persistent) and a short-run (transitory) component and we obtain long- and short-run factor betas. The short-run component can be interpreted as a correction of or revision to the long-run component, due to arrival of new information. Separating these two components gives us a better understanding of the cross-sectional relationship between risk and expected stock returns.

We apply our model to test the popular [Fama and French \(1993\)](#) three-factor asset pricing model. The long- and short-run components of portfolio betas are based on the market portfolio, the small-minus-big portfolio (SMB), and the high-minus-low portfolio (HML) risk factors. The risk premia are estimated both at weekly and monthly frequency. For comparison with earlier studies we use

[☆] We are thankful for suggestions and comments from participants at the following conferences: World Finance Conference 2017 in Sardinia, Italy; New Methods for the Empirical Analysis of Financial Markets 2017 in Comillas, Spain; China Econometric Society meeting 2018; IFABS 2018 in Porto, Portugal; European Summer Meeting of the Econometric Society 2018 in Cologne, Germany; Annual SoFiE meeting 2018 in Lugano, Switzerland; FMA meeting 2018 in San Diego, USA. We are grateful for helpful comments from Federico Bandi and Oliver Linton. Asgharian and Hou thank the *Jan Wallander* and *Tom Hedelius Foundations* for funding this research. Christiansen acknowledges financial support from the Danish Council of Independent Research and CREATES (Center for Research in Econometric Analysis of Time Series) funded by the Danish National Research Foundation. Wang thanks the German Research Foundation for financial support.

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<https://doi.org/10.1016/j.intfin.2021.101412>

Received 28 March 2021; Accepted 17 August 2021

Available online 21 August 2021

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several alternative test portfolios from French's online data library. Our main analysis is based on the 30 industry portfolios, but for robustness we also consider 49 industry portfolios as well as 25 portfolios sorted on size and book-to-market value.

We find that data frequency matters for risk premia: none of the risk premia estimated at weekly frequency are significant, which is in contrast to the risk premia obtained at the monthly frequency. This may imply that risk premia estimated more frequently than monthly are noisy. Our empirical analysis, at the monthly frequency, shows that decomposing risk across horizons may help explain the anomaly that the traditional market risk premium is not significant, as we find that the risk premium related to the short-run market beta is significantly positive.¹ This result is robust to the choice of the test portfolios. We also find that the short-run risk premia are larger in expansions than in recessions. In fact, the risk premia for short-run betas of all the risk factors are significant if we exclude recessions from our sample. Our results show that for risk management and asset pricing, it is important to differentiation between the short and long horizon. Our results also help obtaining a more accurate measure of risk and a more relevant risk and return relationship for different investment horizons.

This paper is closely related to the literature on conditional asset pricing models with time-varying betas, (e.g., [Bollerslev, Engle, and Wooldridge, 1988](#); [Asgharian and Hansson, 2000](#); [Bali, Engle, and Tang, 2017](#)) and the literature that investigates the risk-return relationship at different frequency components (e.g., [Engle and Lee, 1999](#); [Engle and Rangel, 2008](#); [Lewellen and Nagel, 2006](#); [Cenesizoglu and Reeves, 2018](#)).² We contribute to the literature with a new econometric model, evaluate its theoretical properties, and put it to use in an asset pricing model. To the best of our knowledge, this is the first conditional model that uses a bivariate mixed data sampling (MIDAS) approach to simultaneously estimate the long- and short-run components of the variances and covariances and thereby decompose the total betas into long- and short-run components. The advantage of this bivariate model compared to the existing dynamic conditional correlation-mixed data sampling (DCC-MIDAS) model is that it uses the same structure to define the variances and covariance and estimate these moments simultaneously and we can calculate the betas directly.

The rest of the paper is structured as follows. [Section 2](#) contains an overview of the related literature. [Section 3](#) introduces the econometric framework. We present the data in [section 4](#). In [section 5](#), we discuss the empirical results. Finally, we conclude in [section 6](#). An Appendix provides further details on the econometric model.

2. Literature review

This section contains an overview of the strands of the previous literature that our paper is related to, followed by a discussion of how our paper fills the gap.

Firstly, we build on conditional asset pricing models. For example, [Bollerslev, Engle, and Wooldridge \(1988\)](#) apply a multivariate GARCH model and define the expected return of an asset as a linear function of the conditional covariance of the asset with the market portfolio. [Asgharian and Hansson \(2000\)](#) and [Bali \(2008\)](#) use bivariate GARCH models to obtain time-varying factor betas and then use the estimated betas in monthly cross-sectional regressions to obtain corresponding risk premia. [Bali and Engle \(2010\)](#) use the DCC model of [Engle \(2002\)](#) to investigate if assets' time-varying conditional covariance with the market portfolio predicts the time-variation in the assets' expected returns. More recently, [Bali, Engle, and Tang \(2017\)](#) use the DCC model on individual assets to assess the predictive ability of the factor betas.

In addition to the studies above, there is a big literature considering time-varying beta estimation using various econometric methodologies. For example, [Ang and Kristensen \(2012\)](#) use a nonparametric kernel estimator to estimate and test for conditional betas. [Adrian, Crump, and Moench \(2015\)](#) study a regression-based estimator in a dynamic pricing model setup that nests [Fama and MacBeth \(1973\)](#) regression. [Chen, Smetanina, and Wu \(forthcoming\)](#) consider time-varying data in a semiparametric panel data model context. Recently, [Cai and Juhl \(2021\)](#) extend the time-varying kernel estimator for beta and provide a uniform inference procedure.

This paper is also related to the strand of literature which shows the choice of data frequency and time horizon is important for obtaining an accurate measure of risk and capturing the risk-return relationship. For example, [Gilbert, Hrdlicka, Kalodimos, and Siegel \(2014\)](#) show that there are large differences between high- (daily) and low-frequency (quarterly) stock betas. According to [Lewellen and Nagel \(2006\)](#), compounding implies that betas vary across different frequencies. [Engle and Lee \(1999\)](#) and [Engle and Rangel \(2008\)](#) show that models with both low- and high-frequency volatility and correlation components capture the dynamics of equity returns better than single-frequency models. [Adrian and Rosenberg \(2008\)](#) explore cross-sectional pricing of risk by decomposing equity-market volatility into short- and long-run components. [Cenesizoglu and Reeves \(2018\)](#) use a nonparametric approach and measure market beta with short-, medium-, and long-run components. The short- and medium-run components are estimated from daily returns over one- and five-year periods, and the long-run component is estimated from monthly returns over a 10-year period. [Boons and Tamoni \(2017\)](#) show that dividing risk into long- and short-run components helps uncover a link between risk premia and the macro economy. [Andersen and Bollerslev \(1997\)](#) and [Calvet and Fisher \(2007\)](#) argue that information in financial markets arrives at different frequencies and has different degrees of persistence. Since investors have different investment horizons, their view of systematic risk is horizon dependent, see e.g. [Bansal, Dittmar, and Kiku \(2009\)](#). [Kamara, Korajczyk, Lou, and Sadka \(2016\)](#) find that

¹ Other studies find a significant risk premium for the market beta without decomposition into long- and short run betas. However, these studies are typically based on different test portfolios than the widely used [Fama and French \(1993, 1997\)](#) portfolios. For example, [Maio and Santa-Clara \(2017\)](#) use a number of portfolios sorted on e.g. book-to-market ratio and earnings-to-price and [Kim \(1995\)](#) and [Bali, Engle, and Tang \(2017\)](#) use individual stocks. An exception is [Bali \(2008\)](#) that applies the bivariate GARCH model to [Fama and French's \(1993, 1997\)](#) test portfolios and show that the portfolios' conditional covariance with the market can predict time-variation in the expected return.

² Please refer to [Section 2](#) for a detailed discussion of the related literature.

cross-sectional risk premiums vary with the return horizon. Bandi, Perron, Tamoni, and Tebaldi (2019) model market excess returns and their predictors are aggregates of uncorrelated components operating over different frequencies and they introduce a notion of scale-specific predictability.

Several studies use the mixed data sampling (MIDAS) approach to estimate systematic risk but without decomposing it into long- and short-run betas. Gonzalez, Nave, and Rubio (2012) use the weighted average of daily returns to estimate monthly betas. Gonzalez, Nave, and Rubio (2018) define the conditional beta with two additive components, a transitory component estimated from daily returns and a long-run component based on macroeconomic state variables. Baele and Londono (2013) use Colacito, Engle, and Ghysels's (2011) DCC-MIDAS model to obtain long-run betas. They find that DCC-MIDAS betas are superior to ordinary betas in limiting the downside risk and ex-post market exposure for the minimum-variance strategy. Ghysels, Santa-Clara, and Valkanov (2005) use MIDAS volatilities to analyze the risk-return trade-off. They investigate the effects of changing the frequency of the returns in the MIDAS risk-return trade-off regressions and find that using high-frequency returns (above monthly) provides excessively noisy estimates. Therefore, they conclude that monthly returns are preferable. Ghysels, Guérin, and Marcellino (2014) continue this analysis by combining regime switching with MIDAS and consider variations across horizons.

Our paper contributes to the literature described above by providing a new econometric model that improves on the existing models for estimation of the short- and long-run components of conditional beta. Our model uses the same structure to define the variance and covariance. In addition, it allows for simultaneous estimation of the variance and covariance in a bivariate framework. Section 3.1 contains a detailed discussion of the advantages of our approach comparing to the existing models.

3. The component asset-pricing model

In this section, we present the new bivariate component GARCH-MIDAS model. The empirical analysis follows a two-step estimation procedure similar to Fama and MacBeth (1973). The first step entails time-series regressions to obtain total, long-, and short-run betas. In the second step, we estimate the corresponding risk premia.

3.1. First step: Bivariate component GARCH model

Within the component GARCH models, there are two general approaches to distinguish short- from long-run movements: the additive approach from Engle and Lee (1999) and the multiplicative approach from Engle, Ghysels, and Sohn (2013). We use the additive approach and extend Asgharian and Hansson's (2000) and Bali's (2008) bivariate GARCH model to a bivariate component GARCH model to decompose the total variance and covariance to a long-run (persistent) component and a short-run (transitory) component.

Our new additive GARCH-MIDAS model differs from the DCC-MIDAS model (see e.g. Colacito, Engle, and Ghysels, 2011; Conrad, Loch, and Ritter, 2014; Asgharian, Christiansen, and Hou, 2016) that applies the multiplicative GARCH-MIDAS volatility (from e.g. Engle, Ghysels, and Sohn, 2013; Conrad and Locch, 2015) as input. The multiplicative approach works well for univariate models but cannot be applied directly to bivariate models because it may give ambiguous estimates of the covariance in case of negative covariance components. In general, the DCC-MIDAS models assume a multiplicative form in the first step of estimating the variances and an additive form in the second step to estimate the correlation (total correlation is modelled as the sum of the long- and short run correlations). There are also several studies that use alternative models to decompose covariances into high and low frequencies. Rangel and Engle (2012) use the two-step DCC model, where the first step is the volatility model from Engle and Rangel (2008) with long- and short-run components. The second step uses the first-step residuals to model the correlations. Similarly, Bauwens, Hafner, and Pierret (2013) use a DCC model to separate long- and short-run correlations of electricity futures returns, where the long-run covariance matrix is deterministic.

Our newly proposed model has several advantages compared to the DCC-MIDAS model. One advantage of our model is that it uses the same additive form for the variances and covariance and estimate these moments simultaneously, in a bivariate framework. Another advantage of the proposed model is that it provides an estimate of the covariance, instead of correlation, which is used directly to calculate betas. Further, in the proposed model, the long-run betas are identical to the conventional rolling-window betas and are thereby directly comparable, and the proposed model still provides direct estimates of the short-run variances (covariances).

We use the subscripts s and t to keep track of time, where s and t denote periods corresponding to the weekly and monthly frequency, respectively. We assume that the mean equations for the weekly excess returns for portfolio i ($r_{i,s,t}$) and the state variable $\times$ ($r_{x,s,t}$) follow a simple form where they are equal to a constant plus an error term ($\epsilon_{i,s,t}$ and $\epsilon_{x,s,t}$, respectively):

$$\begin{aligned} r_{i,s,t} &= \gamma_i + \epsilon_{i,s,t} \\ r_{x,s,t} &= \gamma_x + \epsilon_{x,s,t} \end{aligned} \quad (1)$$

The error terms are assumed to follow normal distributions with mean zero and time dependent variances, $q_{i,s,t}$ and $q_{x,s,t}$ and covariance, $q_{ix,s,t}$.

Engle and Lee's (1999) univariate additive component GARCH model defines the total conditional variance as the sum of a long-run (persistent) component and a short-run (transitory) component where the total variance follows a GARCH(1,1) model. They replace the unconditional variance used in GARCH models by the long-run time-varying variance. The idea of an unconditional time-varying variance is also presented in, for example, Amado and Teräsvirta's (2014) model. We extend the idea in Engle and Lee (1999) to a bivariate GARCH(1,1) model to estimate each portfolio's conditional variances as well as their conditional covariances with the

common factors, one factor at a time.

In the parameterization of the GARCH equation, we use the BEKK specification to reduce the number of parameters, cf. [Engle and Kroner \(1995\)](#). The formulation of the intercept follows [Santis and Gerard \(1997\)](#), while the unconditional moments (the τ 's) are time varying. The total variances and covariance are modeled as:

$$\begin{aligned} q_{i,s,t} &= \tau_{i,t}(1 - a_i^2 - b_i^2) + a_i^2 \varepsilon_{i,s-1,t}^2 + b_i^2 q_{i,s-1,t} \\ q_{x,s,t} &= \tau_{x,t}(1 - a_x^2 - b_x^2) + a_x^2 \varepsilon_{x,s-1,t}^2 + b_x^2 q_{x,s-1,t} \\ q_{ix,s,t} &= \tau_{ix,t}(1 - a_i a_x - b_i b_x) + a_i a_x \varepsilon_{i,s-1,t} \varepsilon_{x,s-1,t} + b_i b_x q_{ix,s-1,t} \end{aligned} \quad (2)$$

Where $\tau_{i,t}$, $\tau_{x,t}$, and $\tau_{ix,t}$ are the long-run variances and covariance. We use the equally weighted moving average of the past five years' observations to estimate the long-run variances and covariances. This is similar to [Colacito, Engle, and Ghysels's \(2011\)](#) approach to estimating long-run correlation in the DCC-MIDAS model.³ The advantage is that the estimated long-run betas are equal to the conventional estimate of the unconditional beta. This facilitates straightforward comparisons with earlier studies:

$$\begin{aligned} \tau_{i,t} &= \frac{1}{K} \sum_{k=1}^K (r_{i,k} - \mu_{i,K})^2 \\ \tau_{x,t} &= \frac{1}{K} \sum_{k=1}^K (r_{x,k} - \mu_{x,K})^2 \\ \tau_{ix,t} &= \frac{1}{K} \sum_{k=1}^K (r_{i,k} - \mu_{i,K})(r_{x,k} - \mu_{x,K}) \end{aligned} \quad (3)$$

where K is the number of months within the past five years, i.e., $K = 60$.⁴ $r_{i,k}$ and $r_{x,k}$ are the monthly returns for portfolio i and state variable x . $\mu_{i,K}$ and $\mu_{x,K}$ are the means of the monthly returns of the past K months for portfolio i and state variable x , respectively. The long-run component is the average of the squared deviations of the monthly returns from their mean within the past five years. In this way, the long-run betas are identical to the conventional rolling-window betas.⁵ Therefore, the short-run variance (covariance) is defined as a function of the deviation of the lagged shock and the lagged total variance (covariance) from the long-run variance (covariance).

The total betas are calculated from the estimated total covariance and variance, and the long-run betas from the estimated long-run covariance and variance:

$$\hat{\beta}_{x,i,s,t}^{\text{total}} = \frac{\hat{q}_{ix,s,t}}{\hat{q}_{x,s,t}} \text{ and } \hat{\beta}_{x,i,s,t}^{\text{long}} = \frac{\hat{\tau}_{ix,s,t}}{\hat{\tau}_{x,s,t}}. \quad (4)$$

The short-run betas are the differences between the total and long-run betas. We also calculate the short-run betas from the short-run covariance and variance defined in equation (4), results not tabulated. This approach is noisy as it results in extreme values when the short-run factor variance is very small.

In addition to the mixed-frequency model, we also work with a single-frequency model. In the single-frequency model, s and t are identical, i.e. either weekly or monthly frequency.

Several restrictions have been applied to ensure that the conditional variance–covariance matrix is positive definite at each s and t . The details are in the appendix, where we also discuss identification and stationarity of the model. The log-likelihood function for model estimation is also given in the appendix.

3.2. Second step: cross-sectional regressions

In our setting, the expected returns depend on both long- and short-run components of the three risk premia, stemming from the market, SMB, and HML risk factors.

The second step concerns the [Fama and MacBeth \(1973\)](#) cross-sectional regressions. There is one cross-sectional regression for each period s , where $R_{i,s,t}$ is the excess return over that period. When we consider the total betas, it reads as follows.

$$R_{i,s,t} = C_{0,s,t}^{\text{total}} + C_{1,s,t}^{\text{total}} \beta_{M,i,s,t}^{\text{total}} + C_{2,s,t}^{\text{total}} \beta_{\text{SMB},i,s,t}^{\text{total}} + C_{3,s,t}^{\text{total}} \beta_{\text{HML},i,s,t}^{\text{total}} + \varepsilon_{i,s,t}, \text{ for } i = 1, \dots, N \quad (5)$$

We also do cross-sectional regressions with both short- and long-run betas and thereby obtain long- and short-run risk premia. This is new to the literature.

³ We have also used the more general beta-lag polynomial weighting function for describing the long-run moments, where the weighting function is estimated. However, the estimation converges to the equally weighted average in most cases. The model with the beta-lag polynomial weighting function is described in the Appendix.

⁴ The five-year window with monthly returns is conventional for estimating unconditional betas (see e.g. [Fama and French, 1993](#)). We have conducted the estimations for different values of K , where $K = 60, 96, 120$ months (results not shown). The parameter estimates are robust to the choice of K .

⁵ In the GARCH–MIDAS model, the long-run component is calculated as the weighted sum of the realized variances and covariance. We also estimate the model with realized moments based on daily data and exponential weights. The conclusions remain unaltered.

$$R_{i,s,t} = c_{0,s,t} + c_{1,s,t}^{\text{long}} \beta_{M,i,s,t}^{\text{long}} + c_{1,s,t}^{\text{short}} \beta_{M,i,s,t}^{\text{short}} + c_{2,s,t}^{\text{long}} \beta_{SMB,i,s,t}^{\text{long}} + c_{2,s,t}^{\text{short}} \beta_{SMB,i,s,t}^{\text{short}} + c_{3,s,t}^{\text{long}} \beta_{HML,i,s,t}^{\text{long}} + c_{3,s,t}^{\text{short}} \beta_{HML,i,s,t}^{\text{short}} + \varepsilon_{i,s,t}, \text{ for } i = 1, \dots, N \quad (6)$$

The risk premia are the average of the estimated slope coefficients, the c 's. We use the time series of the estimated coefficients to investigate the properties of the factor risk premia such as whether the average coefficients are significant and, if so, whether they are positive or negative. We use [Newey and West \(1987\)](#) corrected standard errors, which is similar to [Bali, Engle and Tang \(2017\)](#). The usage of portfolios rather than individual assets as test assets help reduce the errors-in-variables problem of using estimated explanatory variables (the betas) in the second step regressions.

4. Data

Our analysis is based on the value-weighted excess log-returns for 30 industry portfolios at weekly and monthly frequency. We use market, SMB, and HML risk factors as state variables ([Fama and French, 1993](#)). The sample covers the period from 1945 to 2015 and includes several recessions such as the dotcom bubble and the recent financial crisis. For robustness, we also use 49 value weighted industry portfolios and 25 size and book-to-market double-sorted portfolios.⁶ We use the NBER recession indicator to measure of the state of the macroeconomy.⁷

[Table 1](#) shows descriptive statistics for the monthly excess returns of the 30 industry portfolios. The mean returns are significantly positive and varies from 6.0% per year ("Other") to 11.5% per year ("Smoke"). The standard deviations are relatively large, ranging from 13.3% per year ("Utilities") to 32.4% per year ("Coal"). For all industry portfolios, we observe negative skewness and positive excess kurtosis, revealing extreme negative returns.

5. Empirical results

In this section, we show the empirical results. First, we show the results regarding estimations of betas and following for the estimated risk premia. At the end, we discuss how the risk premia are related to the state of the economy.

5.1. Estimation of the bivariate component GARCH model

We use two different frequency pairs (s, t) to decompose the long- and short-run components. The long-run component, t , is at the monthly frequency and the short-run component, s , varies between weekly and monthly frequency.⁸ Our benchmark model is based on the monthly-monthly (M–M) frequency. That is, the returns in equation (1), the total variance and covariance in equation (2), and the long-run variances and covariance in equation (3) are all based on monthly returns. This is a GARCH specification with time-varying unconditional moments. We use the monthly-monthly approach as the base case to be able to compare our results with earlier studies since it is conventional to use a five-year moving window with monthly returns to estimate betas and the monthly frequency to estimate the cross-sectional regression. We also use an alternative specification of the component GARCH model in which we keep the long-run moments in equation (3) at the monthly frequency while changing the frequency of the bivariate variance and covariance in equation (2) and the returns in equation (1) to weekly (denoted monthly-weekly or M–W). This is a MIDAS specification.

[Table 2](#) shows the means and standard deviations of the parameter estimates of the bivariate component GARCH model in equations (1) to (3) estimated for each of the 30 industry portfolios together with each of the three factors, one at a time, both for the monthly-weekly and monthly-monthly specifications. The parameter estimates show that the volatilities are persistent, because all the b coefficients are much greater than the corresponding a coefficient. The related standard deviations are very small, indicating that the volatility persistence holds for most of the industries. As expected, the estimated mean returns (the γ 's) are larger in the monthly-monthly specification than in the monthly-weekly specification.

To illustrate the estimated betas over time, we use the financial industry ("Fin") as an example. [Fig. 1](#) shows the time series of the total and long-run betas for the monthly-monthly and monthly-weekly frequency for this industry. The long-run betas are smoother than the total betas, especially when we use the monthly-weekly frequency instead of the monthly-monthly frequency. The market betas are less variable than the SMB and HML betas at both frequencies. As expected, the estimated betas are, in general, very large during the recent financial crises, which supports the large contribution of the financial industry to the systematic risk during this period.

A number of studies examine the link between the cross-sectional dispersion of industry betas and the state of the economy. [Gomes, Yaron, and Zhang \(2003\)](#) find that the heterogeneity of betas across firms increases during recessions leading to increasing beta dispersion. This effect is reinforced by the countercyclical behavior of dispersion of the firms' characteristics, which is in line with the findings of [Chan and Chen \(1988\)](#). Similarly, [Baele and Londono \(2013\)](#) find that the empirical cross-sectional dispersion in industry betas increases during recessions.

We use the method in [Baele and Londono \(2013\)](#) to calculate the cross-sectional dispersion of the betas for each month. The cross-sectional dispersion coefficient at a given point in time is the value-weighted sum of squares of each industry's beta minus the average

⁶ We gratefully obtain the data from French's online data library.

⁷ We obtain the NBER recession data from the NBER webpage.

⁸ We also estimate the model with the monthly-daily combination. The results are similar to those with the monthly-weekly combination. For the sake of brevity, those results are not reported.

Table 1
Summary statistics for excess returns of 30 industry portfolios.

	Mean	St. dev	Excess kurtosis	Skewness
<i>Food</i>	8.610***	14.179	2.479***	−0.056
<i>Beer</i>	9.767***	18.303	4.868***	0.412***
<i>Smoke</i>	11.512***	19.633	2.870***	−0.065
<i>Games</i>	9.370***	23.862	2.519***	−0.186**
<i>Books</i>	7.641***	19.509	2.412***	−0.025
<i>Hshld</i>	8.179***	16.119	1.503***	−0.302***
<i>Clths</i>	8.400***	20.611	3.115***	−0.083
<i>Hlth</i>	10.165***	16.960	1.903***	0.066
<i>Chems</i>	8.006***	18.456	2.121***	−0.096
<i>Txtls</i>	8.865***	23.308	9.418***	0.492***
<i>Cnstr</i>	7.811***	19.776	2.431***	−0.210**
<i>Steel</i>	6.138***	23.904	2.335***	−0.240***
<i>FabPr</i>	7.751***	20.155	2.513***	−0.384***
<i>ElcEq</i>	9.906***	20.931	1.443***	−0.160*
<i>Autos</i>	7.768***	22.176	5.893***	0.209**
<i>Carry</i>	9.737***	21.213	1.360***	−0.260***
<i>Mines</i>	6.400***	23.920	2.220***	−0.173**
<i>Coal</i>	9.475***	32.400	2.714***	0.143*
<i>Oil</i>	9.232***	18.199	1.062***	−0.005
<i>Util</i>	7.096***	13.343	1.098***	−0.201**
<i>Telcm</i>	6.486***	14.695	1.837***	−0.174**
<i>Servs</i>	9.935***	21.490	1.514***	−0.151*
<i>BusEq</i>	9.681***	22.163	2.051***	−0.311***
<i>Paper</i>	8.612***	17.269	2.098***	−0.169**
<i>Trans</i>	7.584***	19.354	1.279***	−0.200**
<i>Whlsl</i>	8.294***	18.602	2.283***	−0.298***
<i>Rtail</i>	9.001***	17.528	2.409***	−0.222***
<i>Meals</i>	10.068***	20.450	2.478***	−0.405***
<i>Fin</i>	8.523***	17.779	1.808***	−0.405***
<i>Other</i>	5.959***	19.152	1.752***	−0.388***

The table shows the yearly means, standard deviations, excess kurtosis, and skewness of the excess returns in percentage for the 30 industrial portfolios. The monthly sample covers the period from 1945 to 2015. The data are from Kenneth French's online data library. ***, **, and * indicate significance at the 1%, 5%, and, 10% levels, respectively.

Table 2
Parameter estimates of the component GARCH model.

		γ_t		γ_x		a_i		a_x		b_i		b_x	
		Mean	Std dev	Mean	Std dev	Mean	Std dev	Mean	Std dev	Mean	Std dev	Mean	Std dev
M–W	Market	0.208	0.036	0.190	0.011	0.241	0.015	0.265	0.016	0.963	0.005	0.954	0.007
	SMB	0.201	0.038	0.012	0.005	0.290	0.047	0.262	0.015	0.942	0.021	0.949	0.008
	HML	0.209	0.032	0.062	0.004	0.251	0.023	0.272	0.006	0.959	0.008	0.958	0.002
M–M	Market	0.731	0.146	0.662	0.057	0.282	0.020	0.284	0.026	0.941	0.014	0.943	0.011
	SMB	0.693	0.150	0.008	0.031	0.284	0.030	0.319	0.021	0.939	0.020	0.880	0.037
	HML	0.803	0.192	0.305	0.057	0.289	0.028	0.313	0.016	0.927	0.028	0.939	0.014

The table shows the means and standard deviations of the parameter estimates from the bivariate component GARCH model specified in equations (1)–(3) for the monthly-weekly (M–W) and monthly-monthly (M–M) frequencies. The estimations are based on the 30 industry portfolios and the market, small-minus-big (SMB), and high-minus-low (HML) factors. The sample covers the period from 1945 to 2015.

beta across all industries. Table 3 (top rows) shows the time-series average of the cross-sectional dispersion of the estimated betas based on the monthly-monthly frequency. The dispersions of the short-run betas are on average smaller than those of the long-run betas.

To investigate how the cross-sectional dispersion of industry betas varies across the business cycle, we regress the dispersion coefficients for all the betas on the NBER recession indicator. The regression results are reported in Table 3 (bottom rows). The dispersion of the total betas is larger in recessions than in expansions, which supports the findings of earlier studies (e.g., Gomes, Yaron, and Zhang, 2003; Baele and Londono, 2013). Interestingly, our results show that the larger cross-sectional dispersion of the total betas in recessions depends on the short-run betas, as the all the short-run dispersions are significantly larger in recessions than in expansions, while none of the long-run dispersions are significantly different in recessions and expansions. The long-run betas reflect the slow movements of the factor loadings, while the recession periods are fairly short lived.

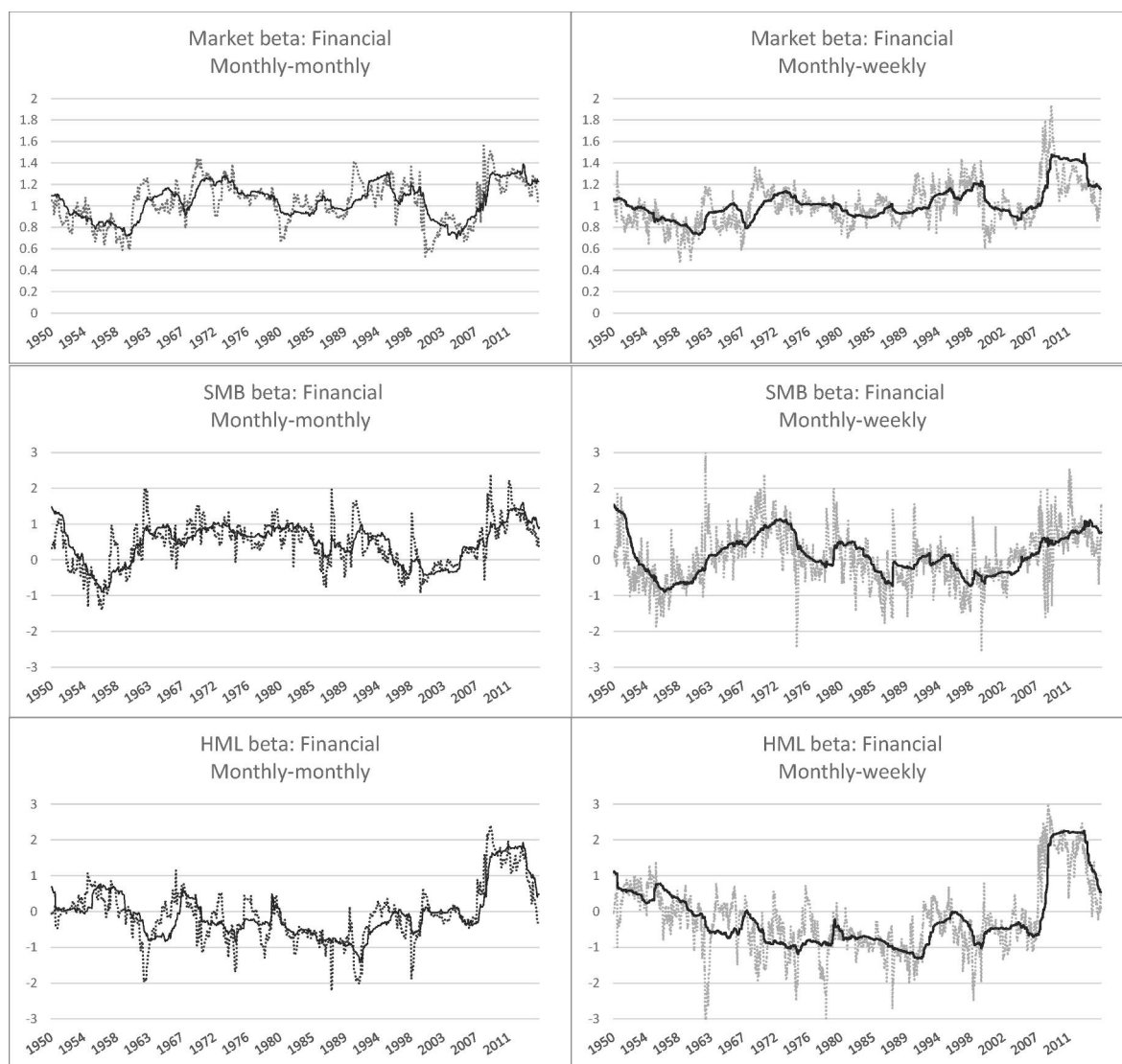


Fig. 1. Factor betas estimated by the component GARCH model. The graphs plot the total (dotted line) and long-run (solid line) market, small-minus-big (SMB), and high-minus-low (HML) betas estimated by the component GARCH model at monthly-monthly and monthly-weekly frequency for the financial industry as the test portfolio. The estimated betas are for the period from 1950 to 2015.

Table 3

Cross-sectional dispersion of betas.

	Total			Long			Short		
	Market	SMB	HML	Market	SMB	HML	Market	SMB	HML
Mean dispersion	0.286***	0.434***	0.523***	0.271***	0.413***	0.478***	0.115***	0.180***	0.233***
Intercept	0.282***	0.431***	0.519***	0.269***	0.414***	0.480***	0.113***	0.174***	0.229***
t-value	74.553	112.830	113.111	79.995	158.618	124.801	72.901	68.935	65.254
Recession	0.031***	0.023**	0.033***	0.009	-0.010	-0.017	0.014***	0.040***	0.024**
t-value	3.084	2.211	2.730	0.981	-1.411	-1.606	3.299	5.878	2.519

The table shows the time-series means of the cross-sectional dispersion of the estimated total, long- and short-run betas, as well as the coefficients and *t*-values from univariate regressions of the dispersion coefficients on the NBER recession indicator. ***, **, and * indicate significance at the 1%, 5%, and, 10% levels, respectively.

Table 4
Risk premia.

Panel A. Risk premia for total betas																
			Intercept		Market		SMB		HML							
			Coef.	t-val	Coef.	t-val	Coef.	t-val	Coef.	t-val	Coef.	t-val				
Unconditional	Weekly		0.175***	4.11	−0.010	−0.12	0.034	0.90	0.026	0.65						
	Monthly		0.858***	4.69	−0.464	−1.55	0.490***	3.01	0.114	0.71						
Bivariate GARCH	Weekly		0.147***	4.21	0.001	0.02	−0.008	−0.27	−0.040	−1.10						
	Monthly		0.643***	3.86	0.039	0.14	0.166	0.93	0.007	0.04						
Component GARCH	M-W		0.156***	4.24	0.019	0.26	0.005	0.16	−0.013	−0.34						
	M-M		0.661***	4.08	−0.020	−0.07	0.463***	2.64	0.217	1.23						
Panel B. Risk premia for component GARCH betas																
			Long						Short							
			Intercept		Market		SMB		HML		Market		SMB		HML	
			Coef.	t-val	Coef.	t-val	Coef.	t-val	Coef.	t-val	Coef.	t-val	Coef.	t-val	Coef.	t-val
M-W			0.150***	3.32	0.018	0.20	0.013	0.30	0.021	0.42	0.050	0.58	0.039	0.90	−0.019	−0.43
M-M			0.844***	4.79	−0.275	−0.89	0.645***	3.01	0.321	1.40	0.818**	2.00	0.669*	1.87	0.402	1.29
Panel C: Risk premia variations across data sets																
			Long						Short							
			Market		SMB		HML		Market		SMB		HML			
			Coef.	t-val	Coef.	t-val	Coef.	t-val	Coef.	t-val	Coef.	t-val	Coef.	t-val		
25 BM-Size			0.876***	4.29	−0.306	−1.05	0.371***	2.55	0.356**	2.01	0.318*	1.76	0.054	0.32	0.102	0.65
30 Industries			0.844***	4.79	−0.275	−0.89	0.645***	3.01	0.321	1.40	0.818**	2.00	0.669**	1.87	0.402	1.29
49 industries			0.467***	3.35	0.202	0.83	0.350**	2.41	0.284*	1.72	0.690***	2.60	0.470**	2.31	0.105	0.61

Panel A shows the risk premia estimated using unconditional betas, betas from a conventional bivariate GARCH model for weekly and monthly frequencies, and total betas from the bivariate component GARCH model for the monthly-weekly (M–W) and monthly-monthly (M–M) frequencies. Panel B shows the risk premia for long- and short-run betas from the bivariate component GARCH model with M–W and M–M frequencies. Panel C shows the risk premia estimated using long- and short-run betas from the bivariate component GARCH model for the M–M frequency using four different data sets as test assets. For each factor, market, small-minus-big (SMB), and high-minus-low (HML), the factor risk premium is estimated as the average of the time-series of the estimated coefficients obtained from the weekly (monthly) repeated multivariate cross-sectional regressions of the weekly (monthly) returns of the 30 industry portfolios on their factor betas. The estimations cover the period from 1950 to 2015. ***, **, and * indicate significance at the 1%, 5%, and, 10% levels, respectively.

Table 5

Recession and risk premia.

Panel A. Risk premia for total betas															
		Intercept		Market		SMB		HML							
		Coef.	t-val	Coef.	t-val	Coef.	t-val	Coef.	t-val						
Unconditional	Entire	0.858***	4.69	−0.464	−1.55	0.490***	3.01	0.114	0.71						
	Expansion	0.834***	4.38	−0.159	−0.52	0.427**	2.49	0.231	1.38						
	Recession	1.004**	2.13	−2.335***	−3.07	0.876**	2.06	−0.604	−1.46						
Bivariate GARCH	Entire	0.643***	3.86	0.039	0.14	0.166	0.93	0.007	0.04						
	Expansion	0.557***	3.23	0.294	1.05	0.132	0.69	0.119	0.63						
	Recession	1.174***	2.75	−1.522**	−2.21	0.375	0.80	−0.681	−1.47						
Component GARCH	Entire	0.661***	4.08	−0.020	−0.07	0.463***	2.64	0.217	1.23						
	Expansion	0.651***	3.88	0.134	0.50	0.433**	2.40	0.316*	1.78						
	Recession	0.720*	1.73	−0.964	−1.45	0.647	1.45	−0.389	−0.89						
Panel B. Risk premia for component GARCH betas															
		Long						Short							
		Intercept		Market		SMB		HML		Market		SMB		HML	
		Coef.	t-val	Coef.	t-val	Coef.	t-val	Coef.	t-val	Coef.	t-val	Coef.	t-val	Coef.	t-val
Entire		0.844***	4.79	−0.275	−0.89	0.645***	3.01	0.321	1.40	0.818**	2.00	0.669*	1.87	0.402	1.29
Expansion		0.777***	4.25	−0.011	−0.03	0.668***	3.19	0.443*	1.94	1.025**	2.37	0.753**	2.07	0.490*	1.64
Recession		1.259***	2.78	−1.897**	−2.40	0.507	0.97	−0.431	−0.76	−0.450	−0.42	0.156	0.17	−0.137	−0.19

The table shows the risk premia for the entire sample and for NBER expansions and recessions. Panel A shows the risk premia using monthly unconditional betas, monthly betas from the conventional bivariate GARCH model, and the total betas from the bivariate component GARCH model with the monthly-monthly frequency. Panel B shows the risk premia associated with the short- and long-run betas from the component GARCH model with the monthly-monthly (M–M) frequency. For each factor, market, small-minus-big (SMB), and high-minus-low (HML), the factor risk premium is estimated as the average of the time-series of the estimated coefficients obtained from the monthly repeated multivariate cross-sectional regressions of the monthly returns of the 30 industry portfolios on their factor betas. The estimations cover the period from 1950 to 2015. ***, **, and * indicate significance at the 1%, 5%, and, 10% levels, respectively.

5.2. Cross-sectional regressions

To evaluate our component GARCH model, we compare its pricing ability with that of two alternative models for estimating betas: the traditional rolling-window OLS regressions (unconditional betas) and the bivariate GARCH model. For these comparisons, we use both weekly and monthly returns. Table 4 shows all the estimated risk premia. None of the models describe the cross-sectional variation in expected returns perfectly, as all models have significant intercepts (alphas).

Panel A of Table 4 shows the total risk premia obtained from the various models, i.e. the mean of the estimated time-series coefficients from the cross-sectional regressions in equation (5). First, the table shows the estimated risk premia associated with the unconditional betas. The market and HML risk premia are not significant. The SMB risk premium is significantly positive at the monthly frequency, which is in accordance with earlier findings, whereas it is insignificant at the weekly frequency. Table 4 also shows the risk premia obtained from the conventional bivariate GARCH model. None of the risk premia are significant irrespectively of data frequency. Finally, the table shows the total risk premia related to the component GARCH model. Here the total risk premia are qualitatively similar to the unconditional risk premia, namely, that only the SMB risk premium is significant, and only so at the monthly-monthly frequency. So, if we limit our interest to total risk premia, the component GARCH model provides the same information as the traditional model. This also implies that the total risk premia results for our new model confirm previous findings.

Panel B of Table 4 shows the new risk premia of the long- and short-run components of beta. At the monthly-weekly frequency, none of the risk premia in Panels A and B of Table 4 are significant. This indicates that risk premia based on the weekly frequency is too noisy. For the monthly-monthly frequency, several of the risk premia are significantly positive. The risk premia associated with both the long- and short-run SMB betas are significant. Interestingly, the risk premium associated with the short-run market beta is also significantly positive. The results (not tabulated) for the M–M component GARCH model are robust to adding industry control variables (industry average size, the industry average book-to-market, and the industry monthly volatility). In fact, the industry control variables are not significant.

To investigate if our results are robust to the choice of the test assets, we estimate our model for some alternative portfolios. First, we use 25 doubled-sorted Fama and French (1993) book-to-market and size portfolios. Second, we use a finer division into industries (49 industry portfolios). Panel C of Table 4 shows the variations in the long- and short-run risk premia for the three data sets based on monthly-monthly frequency. The risk premium of the long-run beta for HML, as expected, becomes highly significant when we use the portfolios sorted based on book-to-market and size. The risk premia related to the short-run market beta are significantly positive for all three data sets. So, this finding is not specific to the 30 industry portfolios.

5.3. Risk premia across the business cycle

In Table 5, we relate the risk premia to the state of the economy as measured by NBER recessions. More specifically, we calculate the average risk premia from the cross-sectional regressions for the entire period as well as separately for recessions and expansions. Panel A is concerned with the total risk premia and panel B with the short- and long-run risk premia. For the unconditional model, the risk premia during expansions are similar to those for the entire sample period. The values are very different in recessions, where the market risk premium is significantly negative, showing large average ex-post realized return for risky firms, i.e. firms with high market betas. For the bivariate GARCH model, the risk premia of all the factors are insignificant for all the subsamples, except the market risk premium which is significantly negative in recessions. The total betas from the component GARCH model also give significant risk premia for SMB and HML (only at the 10% level for the latter). Overall, none of these estimations give a significantly positive risk premium for the market beta, which is consistent with findings from the previous literature.

For the component GARCH model (Panel B of Table 5), the short- and long-run SMB risk premia are significantly positive and slightly larger in expansions than for the entire sample period. The short-run market risk premium is significantly positive for the entire sample period and during expansions. The short-run market risk premium is larger in expansions than for the entire period, which is caused by the negative (and insignificant) risk premium in recessions. The negative short-run market risk premium in recessions is similar to the negative unconditional risk premium. Risk premia for both short- and long-run HML for expansions are positive and significant at the 10% level. The insignificance of the HML factor for the total period is due to important recession periods that cause a large negative realized mean return and result in an insignificant risk premium. In general, excluding recessions from our sample, i.e. only considering expansions, makes the risk premia for all the betas, except for the long-run market beta, significant and have the expected sign.

6. Conclusion

This paper proposes a new model for decomposing systematic risk into long- and short-run components and provides an important empirical application. The new bivariate component GARCH model enables us to simultaneously decompose total variances and covariances into long- and short-run variances and covariances and thereby to estimate the corresponding components of the factor betas. We model the long-run variances and covariances based on the unconditional variance and covariance of past long-run monthly returns, while the short-run variances and covariances are based on higher or same frequency data (weekly or monthly).

The main analysis is based on the 30 industry portfolios. We investigate the dynamics and determinants of market, SMB, and HML industry betas. We apply our component GARCH model to each factor and an industry portfolio to estimate long- and short-run variances and covariances. From these, we calculate long- and short-run betas and use them in cross-sectional regressions to estimate the long- and short-run risk premia associated with each factor.

We find that the cross-sectional dispersion in short-run betas increases in recessions. Moreover, we find that the data frequency matters for estimation of the risk premium: none of the risk premia estimated at weekly frequency are significant. At the monthly frequency, our analysis of the risk premia highlights the importance of decomposing risk across horizons. Although, the risk premia associated with both the long- and short-run SMB betas are significant, only the risk premium associated with the short-run market beta is significantly positive. The results appear to be robust to the choice of data set, at least for a finer division into industry portfolios and for portfolios based on book-to-market and size. Further, we find that the risk premia of the short-run betas of all the risk factors are significant outside recessions.

Our new model enables us to investigate the cross-sectional variations in the long- and short-run risk premia for industry portfolios. The differences between long- and short-run risk is important for risk management and portfolio choice. In future research it would be interesting to investigate if there are similar differences between long- and short-run risk for other stock markets than the US (e.g. European stock markets) as well as other asset classes such as bonds or currency.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Appendix

This appendix contains technical details about the component GARCH model.

Beta-lag polynomial weighting function

In a more general version of the long-run moments than equation (3), we use the beta-lag polynomial weighting function. Here the long-run moments are:

$$\begin{aligned}\tau_{i,t} &= \sum_{k=1}^K \varphi_k(w_1, w_2) V_{i,t-k} \\ \tau_{x,t} &= \sum_{k=1}^K \varphi_k(w_1, w_2) V_{x,t-k} \\ \tau_{ix,t} &= \sum_{k=1}^K \varphi_k(w_1, w_2) V_{ix,t-k}\end{aligned}\quad (\text{A1.1})$$

where

$$\begin{aligned}V_{i,t} &= (r_{i,t} - \mu_{i,t})^2 \\ V_{x,t} &= (r_{x,t} - \mu_{x,t})^2 \\ V_{ix,t} &= (r_{i,t} - \mu_{i,t})(r_{x,t} - \mu_{x,t})\end{aligned}\quad (\text{A1.2})$$

$\mu_{i,t}$ and $\mu_{x,t}$ are the means of the monthly returns for i and x over five-year historical data before each t , and K is the number of periods within the five years. The long-run component is the average of the squared deviations of the monthly returns from their mean. In this way, the long-run betas are identical to the conventional rolling-window betas.

The weighting scheme is described by a beta-lag polynomial:

$$\varphi_k(w_1, w_2) = \frac{\binom{k/K}{w_1-1} \binom{1-k/K}{w_2-1}}{\sum_{j=1}^K \binom{j/K}{w_1-1} \binom{1-j/K}{w_2-1}}. \quad (\text{A1.3})$$

In the empirical analysis in the paper we use $w_1 = w_2 = 1$.

Likelihood function

The bivariate component GARCH model written in matrix form is as follows

$$\begin{pmatrix} r_{i,s,t} \\ r_{x,s,t} \end{pmatrix} = \begin{pmatrix} \gamma_i \\ \gamma_x \end{pmatrix} + \begin{pmatrix} \varepsilon_{i,s,t} \\ \varepsilon_{x,s,t} \end{pmatrix} Q_{s,t}^{1/2} \zeta_t \quad (\text{A2.1})$$

$$\begin{aligned}Q_{s,t} &= \begin{pmatrix} q_{i,s,t} & q_{ix,s,t} \\ q_{ix,s,t} & q_{x,s,t} \end{pmatrix} = \begin{pmatrix} \tau_{i,t}(1 - \alpha_i^2 - b_i^2) & \tau_{ix,t}(1 - \alpha_i\alpha_x - b_ib_x) \\ \tau_{ix,t}(1 - \alpha_i\alpha_x - b_ib_x) & \tau_{x,t}(1 - \alpha_x^2 - b_x^2) \end{pmatrix} + \begin{pmatrix} \alpha_i & 0 \\ 0 & \alpha_x \end{pmatrix} \begin{pmatrix} \varepsilon_{i,s-1,t}^2 & \varepsilon_{i,s-1,t}\varepsilon_{x,s-1,t} \\ \varepsilon_{i,s-1,t}\varepsilon_{x,s-1,t} & \varepsilon_{x,s-1,t}^2 \end{pmatrix} \begin{pmatrix} \alpha_i & 0 \\ 0 & \alpha_x \end{pmatrix} \\ &\quad - \begin{pmatrix} b_i & 0 \\ 0 & b_x \end{pmatrix} \begin{pmatrix} q_{i,s-1,t} & q_{ix,s-1,t} \\ q_{ix,s-1,t} & q_{x,s-1,t} \end{pmatrix} \begin{pmatrix} b_i & 0 \\ 0 & b_x \end{pmatrix}\end{aligned}\quad (\text{A2.2})$$

The error terms in the return equation are assumed to be bivariate normally distributed with Q_t as the conditional variance–covariance matrix and ζ_t is an IID vector process such that $E(\zeta_t \zeta_t') = I$, where I is the identity matrix.

The log likelihood function is

$$L(\Theta) = -\frac{1}{2} \sum_{i=1}^T [\ln(2\pi) + \ln|Q_{s,t}|] + \epsilon_{st}' Q_{s,t}^{-1} \epsilon_{st}. \quad (A2.3)$$

Positive definiteness

Here, we discuss the necessary restrictions on the parameters to ensure the positive definiteness of the conditional variance–covariance matrix, $Q_{s,t}$.

Recall that the residuals from the return equations of the bivariate component GARCH model are assumed joint normal:

$$\epsilon_{s,t} | \mathcal{F}_{s-1,t} \sim \mathbb{N}(0, Q_{s,t}) \quad (A3.1)$$

where, without loss of generality, s and t denote periods corresponding to the higher and lower frequency, respectively, $t = 1, \dots, T$ and $s = 1, \dots, t, \dots, 2t, \dots, S$, and $S = T \times m$, where m is the block size. $[\cdot]$ denotes the floor function of the quotient. Clearly, $t = \lfloor S/m \rfloor + 1$ (note that in practice the block size, m , might be different as we might not have T full blocks of data.).

The returns are an exogenous p covariate and are denoted $R_t = (r_{it}, r_{xt})^T$, where i and x denote portfolios and state variables, respectively. Assume also that R_t is a component-wise stationary, ergodic, strongly mixing process with mixing coefficient $\sum_{n=1}^{\infty} \alpha_n^{1-2/\gamma} < \infty$ ($\gamma > 2$). Q_t is parameterized as being measurable to $\mathcal{F}_{s-1,t}$ and the exogenous variable R_t . $\epsilon_{s,t}$ is a $d \times 1$ matrix and $Q_{s,t}$ is a $d \times d$ matrix. We consider a bivariate model, so $d = 2$. Without loss of generality, $\epsilon_{s,t} = Q_{s,t}^{1/2} \eta_{s,t}$, and $\eta_{s,t}$ is assumed to be IID bivariate Gaussian $\mathbb{N}(0, I_d)$, and is independent of

$$\mathcal{F}_{s-1,t} = \sigma(\epsilon_{s-1,t}, \epsilon_{s-2,t}, \dots, \epsilon_{s-1,t}, \epsilon_{s-2,t}, \dots)$$

The bivariate component GARCH model in matrix form is given as

$$Q_{s,t} = \tau_t - A' \tau_t A - B' \tau_t B + A' \epsilon_{s-1,t} \epsilon_{s-1,t}' A + B' Q_{s-1,t} B \quad (A3.2)$$

where τ_t is a $d \times d$ random variable (long-run exogenous matrix), A and B are $d \times d$ coefficient matrices and that are assumed to be real matrices. In particular, $A = (a_i, 0; 0, a_x)$, $B = (b_i, 0; 0, b_x)$ as in equation (2).

Remark. The term $\tau_t - A' \tau_t A - B' \tau_t B$ resembles the variance-targeting constant term in Pedersen and Rahbek (2014). However, it is worth noting that the constant term is time varying in our case and is driven by low-frequency variables.

τ_t is defined as follows: $\tau_t = V_{tK} \text{diag}(\omega) V_{tK}' = \sum_{k=1}^K \omega_k V_{tK} V_{tK}'$, where V_{tK} is a $d \times K$ matrix of (low-frequency) exogenous shocks, ω is a $K \times 1$ vector, and $\text{diag}(\omega)$ is a $K \times K$ matrix with diagonal elements equal to ω . In our case, $d = 2$ and $\omega = (\omega_1, \omega_2, \dots, \omega_K)^T$ is set to be $(\phi_1(\omega_1, \omega_2), \phi_2(\omega_1, \omega_2), \dots, \phi_K(\omega_1, \omega_2))^T$ defined in equation (A.1.3), and $V_{tK} = (V_{t1}, \dots, V_{tK})$ with $V_{tK} = (r_{i,t-k} - \mu_i, r_{x,t-k} - \mu_x)^T$.

To ensure $Q_{s,t}$ is positive definite at each s and t , we first need to impose a condition to guarantee that $C_{\pi} = \tau_t - A' \tau_t A - B' \tau_t B$ is positive definite. We define the matrix $C = 1 - A' 1 A - B' 1 B$, where 1 is a 2×2 matrix of ones. Then $C = (1 - a_i^2 - b_i^2, 1 - a_i a_x - b_i b_x; 1 - a_i a_x - b_i b_x, 1 - a_x^2 - b_x^2) \stackrel{\text{def}}{=} (c_i, c_{ix}; c_{ix}, c_x)$.

Proposition 1.. ((Positive definiteness of)) C_{π} If $c_i > 0, c_x > 0, c_i c_x - c_{ix}^2 > 0$, then the matrix C is positive definite, and C_{π} is positive definite almost surely.

Proof.. Because $\tau_t = V_{tK} \text{diag}(\omega) V_{tK}' = \sum_{k=1}^K \omega_k V_{tK} V_{tK}'$, we can define the matrix $\tau_t = (\omega_1^2, \omega_{12}; \omega_{12}, \omega_2^2)$ with $\omega_1^2 = \sum_{k=1}^K \omega_k (r_{i,t-k} - \mu_i)^2$, $\omega_2^2 = \sum_{k=1}^K \omega_k (r_{x,t-k} - \mu_x)^2$ and $\omega_{12} = \sum_{k=1}^K \omega_k (r_{x,t-k} - \mu_x)(r_{i,t-k} - \mu_i)$. Now, we can write that $C_{\pi} = (c_i \omega_1^2, c_{ix} \omega_{12}; c_{ix} \omega_{12}, c_x \omega_2^2)$. We calculate the two eigenvalues of C_{π} . Letting $T_i = c_i \omega_1^2 + c_x \omega_2^2$ and $D_i = c_i \omega_1^2 c_x \omega_2^2 - c_{ix}^2 \omega_{12}^2$,

$$\lambda_1(C_{\pi}) = \left(\frac{T_i}{2}\right) + \left(\frac{T_i^2}{4} - D_i\right)^{\frac{1}{2}}, \lambda_2(C_{\pi}) = \left(\frac{T_i}{2}\right) - \left(\frac{T_i^2}{4} - D_i\right)^{\frac{1}{2}}$$

As $c_i, c_x > 0, T_i > 0$ because $\frac{T_i^2}{4} - D_i = \frac{(c_i \omega_1^2 + c_x \omega_2^2)^2}{4} - c_i \omega_1^2 c_x \omega_2^2 + c_{ix}^2 \omega_{12}^2 = \frac{(c_i \omega_1^2 - c_x \omega_2^2)^2}{4} + c_{ix}^2 \omega_{12}^2 \geq 0$. Further, by the Cauchy-Schwarz

inequality $\omega_{12}^2 \leq \omega_1^2 \omega_2^2$, we have $D_i \geq (c_i c_x - c_{ix}^2) \omega_1^2 \omega_2^2$. Therefore, by $c_i c_x - c_{ix}^2 > 0$, $D_i \geq 0$. This leads to $\lambda_1(C_{\pi}), \lambda_2(C_{\pi}) \geq 0$. Moreover $\lambda_2(C_{\pi}) = 0$ if and only if $\omega_1 = 0$ or $\omega_2 = 0$. As the weights ω are positive and the returns $r_{i,t-k} - \mu_i, r_{x,t-k} - \mu_x$ are continuously distributed, $\omega_1 = 0$ and $\omega_2 = 0$ with probability 0.

Remark. If we would like to extend the model to a multidimensional case, this result can also be proved by considering $C_{\pi} = C^{\circ} r_t$, where $\circ$ denotes the elementwise (Hadamard) product of two matrices. As r_t is a weighted sum of almost surely positive-definite matrices $V_{ik} V_{ik}'$ (symmetric and real), then by Weyl's inequality in matrix theory, the smallest eigenvalue of r_t is almost surely positive as well. Also, C is positive definite according to our conditions. Therefore, we have by the Schur product theory for the Hadamard product, as c_{π} is the Hadamard product of C and τ_t , C_{π} is almost surely positive definite.

Proposition 2.. ((Positive definiteness of)) $Q_{s,t}$ Suppose that diagonal element $b_x \neq 0$ and $a_i > 0$, $b_i > 0$, Q_0 is a positive definite matrix and conditions in proposition 1 hold, then $Q_{s,t}$ is positive definite for all s .

Proof.. If C_{π} is almost surely positive definite, $B' Q_{s-1,t} B$ is positive definite, and $A' \epsilon_{s-1,t} \epsilon_{s-1,t}' A$ is semipositive definite, we have that $Q_{s,t}$ is positive definite. The positive definiteness of C_{π} is addressed by proposition 1. Since Q_0 is positive definite, $B' Q_0 B$ is positive definite, so the positive definiteness of $B' Q_{s,t} B$ follows by iteration. As it can be seen that $\text{rank}(A' \epsilon_{s-1,t} \epsilon_{s-1,t}' A) = 1$, then it follows that $Q_{s,t}$ is positive definite.

Stationarity

Here, we show the identifiability and stationarity results. We rewrite the model in vector form as

$$\text{vec}\{Q_{s,t}\} = (I - A' \otimes A' - B' \otimes B') \text{vec}\{r_t\} + A' \otimes A' \text{vec}\{\epsilon_{s-1,t} \epsilon_{s-1,t}'\} + B' \otimes B' \text{vec}\{Q_{s-1,t}\} \quad (\text{A.4.1})$$

As we do not need all the elements of a symmetric matrix, we write (A.3.2) in terms of the vech operator. The vech form of the bivariate component model specified in equation (A.3.2) can be derived as

$$\text{vech}\{Q_{s,t}\} = C \text{vech}\{r_t\} + A \text{vech}\{\epsilon_{s-1,t} \epsilon_{s-1,t}'\} + B \text{vech}\{Q_{s-1,t}\}, \quad (\text{A.4.2})$$

where the operator vech denotes the vectorized part of the lower diagonal elements of a symmetric matrix. $A = \text{diag}(a_i^2, a_i a_x, a_x^2)$, $B = \text{diag}(b_i^2, b_i b_x, b_x^2)$, and $C = I_{d(d+1)/2} - A - B$.

Proposition 3. (Identifiability) Suppose that $a_i > 0$ and $b_i > 0$. Then the parameters in equation (2) are identifiable.

Proof.. In equation (2), the coefficient attached to $\epsilon_{i,s-1,t}^2$ is a_i^2 , which is identified up to its sign, as is the b_i coefficient. The coefficient associated with $\epsilon_{i,s-1,t} \epsilon_{x,s-1,t}$ is $a_i a_x$. Since a_i is identified, a_x is identified as well. Similarly, b_x is identified.

The stationarity of the BEKK model is studied in Boussama, Fuchs, and Stelzer (2011). Next, we prove that we need to ensure that the spectral radius of $A + B$ is less than one for the stationarity of our model. In particular, this is equivalent to $\max(a_i^2, |a_i a_x|, a_x^2) + \max(b_i^2, |b_i b_x|, b_x^2) < 1$.

Proposition 4.. ((Covariance Stationarity)) If $\max(a_i^2, |a_i a_x|, a_x^2) + \max(b_i^2, |b_i b_x|, b_x^2) < 1$, the model is covariance stationary, and the stationary covariance Σ is of the form $\text{vech}\{\Sigma\} = (I - A - B)^{-1} C \tau_{\infty}$.

The stationary solution of equation (2) is

$$\text{vech}\{Q_{s,t}\} = \sum_{l=1}^{\infty} B^{l-1} A \text{vech}\{\epsilon_{s-l,t} \epsilon_{s-l,t}'\} + \sum_{l=1}^{\infty} B^{l-1} C \text{vech}\{\tau_{\lfloor (s-l)/m \rfloor + 1, t}\}. \quad (\text{A.4.3})$$

Proof.. As in the proof of proposition 2.7 in Engle and Kroner (1995), denote by $\mathbb{E}_t$ the conditional expectation $\mathbb{E}(\cdot | \mathcal{F}_t)$, conditioning on the information set $\mathcal{F}_t$.

$$\mathbb{E}_{s-L} \text{vech}\{\epsilon_{s,t} \epsilon_{s,t}'\} = \sum_{l=2}^L (A + B)^{l-2} C \mathbb{E}_{s-L} \text{vech}\{\tau_{\lfloor (s-l)/m \rfloor + 1, t}\} + (A + B)^{L-1} \text{vech}\{Q_{s-L+1}\} \quad (\text{A.4.4})$$

As $L \rightarrow \infty$, $(A + B)^{L-1} \rightarrow 0$ if $\max(a_i^2, |a_i a_x|, a_x^2) + \max(b_i^2, |b_i b_x|, b_x^2) < 1$.

As we have assumed that $\{R_t\}$ are element-wise strong mixing processes, the elements in τ_t are the weighted sum of functions related to $\{R_t\}$. Mixing series are measure preserving. It can be seen that for the blocks $b = 1, 2, \dots, L/m$,

$\sum_{l=(b-1)m+1}^{bm} (\tilde{A} + \tilde{B})^{L-2} \tilde{C} \mathbb{E}_{s-L} \text{vech}\{\tau_{(s-l+1)/m+1,t}\}$ will be mixing. Note that within block b , as $\text{vech}\{\tau_{[(s-l+1)/m]+1,t}\}$ does not vary with respect to s , therefore the value $\mathbb{E}_{s-L} \text{vech}\{\tau_{[(s-l+1)/m]+1,t}\}$ stays the same within a block. As long as $L/m \rightarrow \infty$, it is not hard to see that

$$\lim_{L \rightarrow \infty} \sum_{l=2}^L (A + B)^{L-2} C \mathbb{E}_{s-L} \text{vech}\tau_{[(s-l+1)/m]+1,t} \rightarrow (I - A - B)^{-1} C_{\tau_{\infty}}, \quad (\text{A.4.5})$$

$$\tau_{\infty} = \mathbb{E} \text{vech}\tau_{[(s-l+1)/m]+1}$$

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