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Article:

On the weight of entanglement

David Edward Bruschi

Racah Institute of Physics and Quantum Information Science Centre, The Hebrew University of Jerusalem, 91904 Givat Ram, Jerusalem, Israel

ABSTRACT

We investigate a scenario where quantum correlations affect the gravitational field. We show that quantum correlations between particles occupying different positions have an effect on the gravitational field. We find that the small perturbations induced by the entanglement depend on the amount of entanglement and vanish for vanishing quantum correlations. Our results suggest that there is a form of entanglement that has a weight, since it affects the gravitational field. This conclusion may lead towards a new understanding of the role of quantum correlations within the overlap of relativistic and quantum theories.

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1. Introduction

Does entanglement have a weight? A positive answer to this question would have far reaching consequences, since entanglement is the core resource of some of the most exciting applications of the field of quantum information. For example, entanglement can be used for teleportation [1], quantum key distribution [2] and quantum computing [3] to name a few. More importantly, a positive answer would also help us deepen our understanding of the overlap of relativistic and quantum theories.

Quantum entanglement is a type of correlation that, to date, is not known to interact with gravity. The role of quantum correlations in gravitational scenarios has been so far ignored, most likely due to the fact that overwhelming experimental evidence shows that entanglement can be well established between different systems in the presence of a gravitational field apparently without noticeable consequences [4]. However, experiments are reaching regimes where small modifications introduced by the mutual effects of entanglement and gravity might be measured [5,6]. Therefore, in the last decade attention has been given to understand the effects of gravity on entanglement [7]. Most approaches indicate that effects of gravity on entanglement should exist, although we lack the theory of quantum gravity that can naturally predict this. Unfortunately, the effects predicted by this body of work do not arise because of a direct coupling between gravity and quantum correlations. In particular, it cannot be shown that entanglement will affect gravity, the necessary step to conclude that gravity and entanglement interact with each other.

In this work, we establish that quantum correlations affect the gravitational field and that small perturbations in the metric are induced by the presence of quantum coherence. We employ Einstein's equations and semiclassical theory to show that, for low energy (few particle) states, a small control parameter naturally arises and is uniquely determined by the energy scales of the problem. We then find that small changes in the metric depend on the amount of entanglement present in the state, as measured by the logarithmic negativity (and, additionally, by the concurrence), and vanish for vanishing quantum correlations. These effects are "radiated away" for times larger than the decoherence time, which we show is proportional to the "size" of the particle. Furthermore, the relative phase of the coherences has a direct influence on the magnitude of the effects. Our results are complementary to previous work which investigated the stability of coherent superpositions of different energy states in the presence of gravity [8]. They are also related, for example, to previous work that investigated spontaneous collapse of the wave function due to gravity [9], to stochastic gravity [10] and the role of coherent superpositions [11]. However, contrary to most of this body of work, we are not interested here in the effects of gravity on quantum states (i.e., collapse of the wave function) but rather on the back-reaction of quantum coherence on gravity. Finally, we argue that the regimes considered in this work are well within the limits of validity of semiclassical gravity [12,13].

We believe that our results have important implications for both quantum and relativistic theories, in particular they aid theoretical and experimental research to look for phenomena which might challenge our current understanding of nature.

E-mail address: david.edward.bruschi@gmail.com.

2. Background

2.1. Quantum field theory in curved spacetime

In this work particles are excitations of quantum fields that propagate on a classical spacetime. We consider for simplicity a massive scalar quantum field $\phi(x^\mu)$ with mass m in $(3 + 1)$-dimensional spacetime [14] with metric $g_{\mu\nu}$ (see [15]). The equation of motion of the field is $(\Box + m^2)\phi = 0$, where the d'Alambertian is $\Box \equiv (\sqrt{-g})^{-1}\partial_\mu[\sqrt{-g}\,g^{\mu\nu}\partial_\nu]$ (a standard reference is [14]).

The field can be decomposed in any orthonormal basis of solutions $u_{\mathbf{k}}(x^\mu)$ to the Klein–Gordon equation as $\phi = \int d^3k[a_{\mathbf{k}}u_{\mathbf{k}} + a_{\mathbf{k}}^\dagger u_{\mathbf{k}}^*]$, with annihilation and creation operators $a_{\mathbf{k}}$, $a_{\mathbf{k}}^\dagger$ that satisfy the canonical commutation relations $[a_{\mathbf{k}}, a_{\mathbf{k}'}^\dagger] = \delta^3(\mathbf{k} - \mathbf{k}')$ and all other vanish. The annihilation operators $a_{\mathbf{k}}$ define the vacuum state $|0\rangle$ through $a_{\mathbf{k}}|0\rangle = 0 \; \forall \mathbf{k}$. In general, it is convenient to choose the set of modes $\{u_{\mathbf{k}}\}$ if it satisfies (at least asymptotically) an eigenvalue equation of the form $i\,\partial_\tau u_{\mathbf{k}} = \omega\,u_{\mathbf{k}}$, where ∂_τ is some (possibly global) time-like Killing vector and $\omega := \omega_{\mathbf{k}} = \sqrt{\mathbf{k}\cdot\mathbf{k} + m^2}$ is a real eigenvalue [14].

We assume that the spacetime is essentially flat Minkowski with metric $g_{\mu\nu} = \eta_{\mu\nu} = \mathrm{diag}(-1, 1, 1, 1)$ and perturb the flat spacetime metric in the following way

$$g_{\mu\nu} = \eta_{\mu\nu} + \xi\,h_{\mu\nu}, \tag{1}$$

where we have introduced the small control parameter ξ i.e., $\xi \ll 1$. In this work we will consider only effects that are proportional to ξ i.e., we ignore $\mathcal{O}(\xi^2)$ contributions. Here $h_{\mu\nu}$ depends on the spacetime coordinates x^σ and evolves dynamically via Einstein equations. The expansion (1) is known as linearised (or linear) gravity, which has been successfully employed, for example, to predict the existence of gravitational waves [16]. The role of the parameter ξ is pivotal and we will show in the following that it is uniquely determined by the relevant physical energy scales.

2.2. Semiclassical gravity

We wish to take into account the back reaction of the quantum field on the metric, in other words, we wish to take into account the fact that a single excitation of the field is responsible for the perturbation $h_{\mu\nu}$. This can be done within the framework of semiclassical gravity, which has been successfully applied [17–19] but has its own domain of validity [12,13]. Since our work involves only considering mean energy, which is a quantity that can be experimentally measured, the scenario considered in this work falls within this domain of validity and we will comment on this later [11].

In this framework, back reaction is implemented through Einstein's semiclassical equations

$$G_{\mu\nu} = -8\pi\,G_N\,\langle T_{\mu\nu}\rangle_{\mathrm{Ren}}, \tag{2}$$

where $G_{\mu\nu}$ is Einstein's tensor, G_N is Newton's constant, $T_{\mu\nu}$ is the stress energy tensor of the quantum field and "Ren" stands for some choice of renormalisation of the stress energy tensor [14]. The average $\langle\cdot\rangle$ is intended over some chosen initial state ρ of the field. Einstein's tensor contains second derivatives of the metric, which account for its dynamics. However, one needs to be careful with correctly identifying the source of the gravitational field, a process called renormalisation. That care needs to be taken into account in curved spacetime is a well known issue [14]. Many methods have been proposed and employed with different degrees of success [20,21]. However, in this work we will analyse the back reaction of single particle excitations on flat Minkowski spacetime. We believe it is natural to assume that in this case it is sufficient

to subtract the (infinite) zero point energy of the Minkowski vacuum, a procedure known as normal ordering [14]. We therefore have $\langle T_{\mu\nu}\rangle_{\mathrm{Ren}} \equiv \langle: T_{\mu\nu}:\rangle$, where the symbol $:\cdot:$ stands for normal ordering [14].

The metric is coupled to the field via the semiclassical Einstein equation (2). In order to exploit this relation we need to compute stress energy tensor $T_{\mu\nu}$ which is readily found in literature [14] as $T_{\mu\nu} = \partial_\mu\phi\partial_\nu\phi - \frac{1}{2}g_{\mu\nu}[\partial^\rho\phi\partial_\rho\phi - m^2\phi^2]$. The field ϕ satisfies the equation of motion $(\Box + m^2)\phi = 0$ with the full metric $g_{\mu\nu}$ in (1).

Since we choose to look at effects at lowest order in ξ we can expand the field as $\phi = \phi^{(0)} + \xi\,\phi^{(1)}$, where $\phi^{(0)}$ satisfies $(\partial^\rho\partial_\rho + m^2)\phi^{(0)} = 0$ defined with the flat metric $\eta_{\mu\nu}$. We therefore find that $\phi^{(0)} = \int d^3k[a_{\mathbf{k}}u_{\mathbf{k}} + a_{\mathbf{k}}^\dagger u_{\mathbf{k}}^*]$, where the plane wave modes $u_{\mathbf{k}}$ take the form $u_{\mathbf{k}} = (2\pi)^{-3/2}\omega^{-1/2}\exp[i\,k_\mu x^\mu]$ and $k_\mu x^\mu = -\omega t + \mathbf{k}\cdot\mathbf{x}$. The correction $\phi^{(1)}$ to the field satisfies a more complicated differential equation but turns out to be irrelevant for our purposes. Furthermore, we notice that the state does not depend directly on ξ. It will become evident that the parameter ξ appears in the right hand side of (2) only through the average of the stress energy tensor over the initial state.

By considering a small coupling to gravity it is easy to show that the first order contributions to the semiclassical Einstein equation (2) satisfy the following equation

$$\tilde{G}_{\mu\nu}^{(1)} = -8\pi\,\langle: \tilde{T}_{\mu\nu}^{(0)}:\rangle, \tag{3}$$

where the dimensionless tensors $\tilde{G}_{\mu\nu}^{(1)}$ and $\tilde{T}_{\mu\nu}^{(0)}$ are obtained from the dimensional Einstein tensor $G_{\mu\nu}$ and stress energy tensor $T_{\mu\nu}$ respectively.

2.3. Relevant initial states and physical control parameters

We now wish to determine the parameter ξ in terms of the relevant (energy) scales of the problem. We choose to work in the Heisenberg picture and will analyse the following two-parameter family of initial states

$$\rho(\alpha, \beta) = \alpha\,|01\rangle\langle01| + (1 - \alpha)\,|10\rangle\langle10| + \beta\,|10\rangle\langle01|$$
$$+ \beta\,|01\rangle\langle10|, \tag{4}$$

where $0 \le \alpha \le 1$, the parameter $-1/2 \le \beta \le 1/2$ is real without loss of generality and $(\alpha - 1/2)^2 + \beta^2 \le 1/4$ in order for $\rho(\alpha, \beta)$ to represent a physical state. Notice that for $\alpha = 1/2$ and $\beta = 0$ one has a maximally mixed state while for $\alpha = \beta = 1/2$ one has a maximally entangled state. Furthermore, we underline that the sign of the parameter β might play an important role in the final effects and we will comment on this later.

We note here that the state (4) is the most general state that obeys the "superselection rule" greatly discussed in literature [8], i.e., that it is not possible to superpose states with different masses. In fact, the terms in (4) are the only one particle state terms that have the same (average) energy. Terms such as $|0\rangle\langle0|$ or $|11\rangle\langle11|$ clearly have a different (average) energy.

Here we define the normalised single particle states $|01\rangle$ and $|10\rangle$ as excitations over the Minkowski vacuum of the same particle in different positions in the following way

$$|01\rangle := \int d^3k\,F_{\mathbf{k}_0}(\mathbf{k})\,e^{-i\,\mathbf{L}\cdot\mathbf{k}}\,a_{\mathbf{k}}^\dagger|0\rangle$$

$$|10\rangle := \int d^3k\,F_{\mathbf{k}_0}(\mathbf{k})\,e^{i\,\mathbf{L}\cdot\mathbf{k}}\,a_{\mathbf{k}}^\dagger|0\rangle, \tag{5}$$

where we have introduced the peaked functions $F_{\mathbf{k}_0}(\mathbf{k})$, the constant $\mathbf{k}_0$ defines the location of the peak in momentum space

(aligned along the z-direction without loss of generality, i.e. $\mathbf{k}_0 = (0, 0, k_0)$), the vector $\pm \mathbf{L}$ defines the location of the peak in position space which are located at a distance of $2L := 2\sqrt{\mathbf{L} \cdot \mathbf{L}}$, (again, along the z-direction without loss of generality, $\mathbf{L} = (0, 0, L)$). The creation operators $a_{\mathbf{k}}^{\dagger}$ are the flat spacetime Minkowski operators associated with the zero order field $\phi^{(0)}$. Furthermore, normalisation implies that $\int d^3k \, |F_{\mathbf{k}_0}(\mathbf{k})|^2 = 1$.

We need to make sure that the particle states (4) are orthogonal (at least to good approximation) in order for the concept of entanglement between the two excitations to have a proper meaning. We can choose between two different profile functions. One choice is a Gaussian profile function $F_{\mathbf{k}_0}(\mathbf{k}) = (\sqrt[4]{8\pi^2\sigma^2}\,\sigma)^{-1} \exp[-\frac{(\mathbf{k}-\mathbf{k}_0)^2}{4\sigma^2}]$, where σ is the width of the profile and is assumed to be large, which makes the excitation very localised in position space. We can compute the overlap of the particle states and find $|\langle 10|01 \rangle| = |\int d^3k \, |F_{\mathbf{k}_0}(\mathbf{k})|^2 \exp[-2i\,\mathbf{L}\cdot\mathbf{k}]| \propto \exp[-2\sigma^2 L^2]$ which is negligible for large separations compared to the spread of the wave packet i.e., $\sigma L \gg 1$. This choice might lead to problems when one wishes to look at states with higher numbers of particles. In that case, the overlap of the new states can become larger, which might lead to question the meaning of the following work. We therefore turn to a second choice, the box profile function i.e., $F'_{\mathbf{k}_0}(\mathbf{k}) = (\sqrt{8\sigma}\,\sigma)^{-1} \operatorname{Rect}(\frac{k_x - k_{0,x}}{2\sigma}) \operatorname{Rect}(\frac{k_y - k_{0,y}}{2\sigma}) \operatorname{Rect}(\frac{k_z - k_{0,z}}{2\sigma})$ where $\operatorname{Rect}(x)$ is the rectangle function. In this case, we can choose $\mathbf{L} = (0, 0, n\pi/\sigma)$ with $n \in \mathbb{Z}$ which guarantees orthogonality between the particle states i.e., $\langle 10|01 \rangle \equiv 0$.

We then notice that the parameter σ acts as a natural scale for energies (or equivalently lengths in natural units). In order to understand the interplay of the energy scales of the problem we introduce the dimensionless wave numbers $\tilde{\mathbf{k}} := \mathbf{k}/\sigma$ and the dimensionless coordinates $\tilde{x}^{\mu} := \sigma x^{\mu}$. We then notice that Einstein's tensor $G_{\mu\nu}$ to first order can be written as a combination of second derivatives of the metric. We can therefore introduce $\tilde{G}_{\mu\nu} := G_{\mu\nu}/\sigma^2$, where the dimensionless tensor $\tilde{G}_{\mu\nu} = \xi\, \tilde{G}_{\mu\nu}^{(1)}$ appears in (3). Without loss of generality and to obtain analytical results, we focus on two interesting regimes for the field excitations: that of extremely massive static particles ($m/\sigma \gg 1$ and $k_0 = 0$) and that of massless particles with high momentum ($m = 0$ and $k_0/\sigma \gg 1$). It follows that the average of the stress energy tensor of a particle excitation will be, to good approximation, proportional to $m\sigma^3$ or $k_0\sigma^3$ respectively (this can be found from a straightforward computation of stress energy tensor components i.e., $\langle : T_{00}^{(0)} : \rangle$). We therefore have $\langle : T_{\mu\nu}^{(0)} : \rangle = E_0\sigma^3 \langle : \tilde{T}_{\mu\nu}^{(0)} : \rangle$, where E_0 is proportional to m or k_0 depending on the regime. Putting all together in (2) we have

$$\xi\, \tilde{G}_{\mu\nu}^{(1)} = -8\pi\, G_N\, E_0\, \sigma \langle : \tilde{T}_{\mu\nu}^{(0)} : \rangle + \mathcal{O}(G_N E_0 \sigma \xi). \tag{6}$$

We conclude that (6) identifies $\xi = G_N E_0 \sigma \ll 1$ and confirms that (3) holds to lowest order. Higher order terms on the right hand side contain first order correction to the stress energy tensor and do not contribute to the effects of interest here. However, effects to this order would include the direct coupling of quantum correlations with gravity.

3. Interplay of gravity and entanglement

3.1. First order contribution to the curvature

We now proceed to outline our main results. The semiclassical Einstein equation (3) for the initial state $\rho(\alpha, \beta)$ is

$$\tilde{G}_{\mu\nu}^{(1)} = \alpha \langle 01| : \tilde{T}_{\mu\nu}^{(0)} : |01\rangle + (1 - \alpha) \langle 10| : \tilde{T}_{\mu\nu}^{(0)} : |10\rangle$$
$$+ 2\beta \Re (\langle 01| : \tilde{T}_{\mu\nu}^{(0)} : |10\rangle). \tag{7}$$

We conclude from (7) that the Einstein tensor $G_{\mu\nu}^{(1)}$ has a contribution that comes purely from quantum coherence. The term $\beta \Re (\langle 01| : \tilde{T}_{\mu\nu}^{(0)} : |10\rangle)$ is responsible for such difference and its contribution to the metric is therefore proportional to β. We quantify the entanglement present in the state $\rho(\alpha, \beta)$ by employing the logarithmic negativity $E_{\mathcal{N}}$ which is bound by $0 \leq E_{\mathcal{N}} \leq 1$ (see [22]). This is a well known measure of entanglement and is defined as $E_{\mathcal{N}} = \log_2(2\mathcal{N} + 1)$, where the negativity $\mathcal{N}$ is defined as $\mathcal{N} = \sum_n (|\lambda_n| - \lambda_n)/2$ and λ_n are the eigenvalues of the partial transpose of the state $\rho(\alpha, \beta)$. We find that $|\beta| = (2^{E_{\mathcal{N}}} - 1)/2$ which proves that the last term in (7) contributes only when there are some quantum correlations i.e., $E_{\mathcal{N}} \neq 0$. The greatest contribution from this term occurs when $E_{\mathcal{N}} = 1$ i.e., $\alpha = \beta = 1/2$ and the state $\rho(1/2, 1/2)$ is maximally entangled.

Finally, given that (4) is the most general state we can consider, and that the system effectively behaves as a system of two qubits, we are in the position of computing the concurrence C for this system and, if desired, the entanglement of formation E_{oF} [23]. The entanglement of formation, which can be computed in our case as a simple function of the concurrence, captures all of the correlations and enjoys an important information-theoretical and practical interpretation: it quantifies the minimum number of copies of maximally entangled states of qubits necessary to prepare the state with only Local Operations and Classical Communications (LOCC) [23]. For states like ours, the concurrence has been already computed and has the simple expression $C = 2|\beta|$. This corroborates our claim that quantum correlations are responsible for the effects described in this work.

We could now proceed to compute all (ten independent) terms in (7). This can be done explicitly however, since the main aim of this work is to show that an effect exists in the first place, we find it more convenient to compute the Ricci scalar $R := -G^{\mu}{}_{\mu}$ which gives a more compact result and measures the strength of the curvature locally at each point. To achieve this goal, we note that it is sufficient to compute $\mathcal{D}^{01}{}_{\mu}{}^{\mu} = \langle 01| : \tilde{T}^{(0)}{}_{\mu}{}^{\mu} : |01\rangle$ (or equivalently any other of the terms) since all other terms can be obtained by $\mathcal{D}^{01}{}_{\mu}{}^{\mu}$ with simple modifications. We find

$$\mathcal{D}^{01}{}_{\mu}{}^{\mu} = -\frac{1}{\sigma^2} \int d^3k\, d^3k'\, e^{i\,\mathbf{L}\cdot(\mathbf{k}'-\mathbf{k})}\, F_{\mathbf{k}_0}(\mathbf{k})\, F_{\mathbf{k}_0}(\mathbf{k}')$$
$$\times \left[k'_{\mu} k^{\mu} + 2m^2 \right] u_{\mathbf{k}}^* u_{\mathbf{k}'}. \tag{8}$$

It is straightforward to show that the other diagonal term $\mathcal{D}^{10}{}_{\mu}{}^{\mu} := \langle 10| : \tilde{T}^{(0)}{}_{\mu}{}^{\mu} : |10\rangle$ can be obtained from (8) by replacing $\mathbf{L} \to -\mathbf{L}$ in the integrand and the off diagonal term $\mathcal{D}^{0110}{}_{\mu}{}^{\mu} := \langle 01| : \tilde{T}^{(0)}{}_{\mu}{}^{\mu} : |10\rangle$ can be obtained from (8) by replacing $\tilde{\mathbf{k}} - \tilde{\mathbf{k}}' \to \tilde{\mathbf{k}} + \tilde{\mathbf{k}}'$ in the exponent inside the integrand.

We continue by discussing the contribution of all these terms to the time evolution of the curvature (i.e., Ricci scalar). We start by noticing that all terms contain a factor of the form $\exp[\pm i (\omega - \omega')t]$. When $\sigma^2 \hbar t/m \gg 1$ for extremely massive fields, or $\sigma t \gg 1$ for massless fields, all terms on the right hand side of (7) vanish due to Riemann–Lebesgue lemma. We understand this is a consequence of the spreading of the wave packets $F_{\mathbf{k}_0}(\mathbf{k})$ with time [24].

Focusing on the initial time $t = 0$, one can show that the term $\mathcal{D}^{10}{}_{\mu}{}^{\mu}$ has the expansion $\mathcal{D}^{10}(x, y, z) + \mathcal{O}((\frac{\sigma}{m})^2)$ for massive static particles and $\mathcal{D}^{10}(x, y, z) + \mathcal{O}(\frac{\sigma}{k_0})$ for massless particles with high momentum. It is possible to compute the function $\mathcal{D}^{10}(x, y, z)$ for the box wave-packets and we find $\mathcal{D}^{10}(x, y, z) \sim \operatorname{sinc}^2(\sigma x) \operatorname{sinc}^2(\sigma y) \operatorname{sinc}^2(\sigma (z - L))$. In this case, the term $\mathcal{D}^{01}(x, y, z)$ can be found by the previous one by replacing $z - L$ with $z + L$ and the term $\mathcal{D}^{0110}(x, y, z)$ by replacing $\operatorname{sinc}^2(\sigma (z - L))$ with $\sin^2(\sigma z)/(L^2 - z^2)$. Note that here we have used the fact that $\sin(L\sigma) = \sin(\pi n) = 0$.

The off diagonal terms $\mathcal{D}^{0110}{}_\mu{}^\mu$ do contribute to Einstein's tensor in the fashion described above and to the same order in σ/m or σ/k_0 as the diagonal terms. If one is interested in obtaining the metric itself, one can integrate equation (7) with similar contributions as determined above and obtain the form of the perturbation $h_{\mu\nu}$ for all states, which we have shown will depend on α and β. This could be done numerically however, we are not interested in doing so here, as the aim of this work is to prove that an effect exists in the first place.

3.2. Physical regimes

We have shown that correlations affect gravity and that, for small perturbations of flat spacetime, the coupling is governed by the dimensionless parameter ξ. Furthermore, this parameter is fully determined by the relevant physical scales of the scenario i.e., energy scales. Let us now restore dimensions in order to understand which is the magnitude of the effects governed by ξ and the time τ it takes for the gravitational field to completely "wash out" all the effects. We start by looking at the control parameter ξ. We have

$$\xi = \frac{G_N E_0 \sigma}{c^4}, \tag{9}$$

where we have noted that $E_0 = mc^2$ for massive static particles and $E_0 = \hbar k_0 c$ for massless particles with high momentum. For a single massive particle whose rest mass $m \sim 10^{-21}$ kg is much larger its "size", of the order of $1/\sigma \sim 10^{-22}$ m (see [25]), we see that $\xi \sim 10^{-26}$. For a single massless particle with high momentum (frequency) $\omega_0 \sim 10^{14}$ Hz compared to its spread $\sigma c \sim 10^9$ Hz we find $\xi \sim 10^{-63}$, which is extremely small. However, for much heavier particles, for ultra-energetic massless particles or for states with a high number of excitations (i.e., N00N states, which have already been employed to greatly enhance estimation of parameters due to their "high" quantum nature [26]), one could hope to increase the above result by several orders of magnitude. This could in principle make the effect measurable.

We notice that, for a very massive and static particle, the parameter ξ can be re-written as $\xi = r_S/r$, where $r_S = \frac{2 G_N m}{c^2}$ is the Schwarzschild radius of a particle of mass m and "size" $r = 2/\sigma$. The predictions of this work become unreliable when the Schwarzschild radius of the particle becomes comparable and exceeds the size of the particle.

Let us turn to the time τ it takes for these effects to become negligible. We have seen that the components of Einstein's tensor vanish after times that depend on the particle being massive (τ_m) or massless (τ_{k_0}). In particular

$$\tau_m := \frac{m}{\sigma^2 \hbar}, \quad \tau_{k_0} := \frac{1}{\sigma c}. \tag{10}$$

Given the numbers considered above we have $\tau_m \sim 10^{-32}$ s and $\tau_{k_0} \sim 10^{-31}$ s respectively. A possible way to increase the lifetime of the contributions would be to consider particles that have very well defined momentum i.e., lower σ.

Surprisingly, it appears that the sign of β affects the results and can make the final effect (slightly) bigger or smaller. This can be generalised to complex β. Furthermore, notice that although the timescale is independent of the parameter β, the vanishing effect occurs equally to all for first order contributions. There is no such behaviour for zero order contributions to the off diagonal terms. Finally, we notice that the magnitude of the effects, or changes, depends on the amount of entanglement (on the absolute value of β), while the direction of the contribution (an increase or decrease) depends on the phase (equivalently, the sign of β).

We now comment on the consistency of the methods and the results. It has been argued that criteria for the validity of semiclassical gravity should depend on the state considered and on the scales probed [12,13]. In particular it has been showed that, for Minkowski space and lower than Planck scales [13] and smeared fields (as the ones considered here) which do not probe scales much smaller than the smearing size [12], the semiclassical treatment is valid and should give correct predictions. As a consistency check on the results, we note that if $E_0 = 0$ or $G_N = 0$ the effects described in this work vanish. This is to be expected since in this case there would be no excitations to produce the perturbation of the metric or no dynamical gravity.

3.3. Considerations on the scope and validity of the results

A few final comments are in place. First, we have analysed states that do not have coherent superpositions or mixtures of single particle states with *different* mass (energy). This property is crucial to our results. On the one hand, our results are not affected by arguments that suggest that gravity should collapse states that are coherent superpositions of states with different energy (in line with [8]). On the other, it guarantees that the states (4) are the most general one particle states that we can consider. This in turns guarantees that entanglement is directly responsible for the effects described in this work. Second, we note that not all entanglement affects gravity. For example, we could look at states of particles entangled in the spin degree of freedom. In the absence of magnetic fields, spin up and spin down are both eigenstates of the same hamiltonian operator (i.e., the energy levels are degenerate in the spin degree of freedom). In this case, entanglement between spins would not interact with gravity. Third, it may be tempting to draw an analogy between the semiclassical equations used here and, for example, semiclassical electromagnetism. One might seek for a direct analogy between equation (2) and, for example,

$$\partial_\mu F^{\mu\nu} = \frac{\mu_0 q}{c} \langle : J^\nu : \rangle, \tag{11}$$

where $F^{\mu\nu} := \partial^\mu A^\nu - \partial^\nu A^\mu$ is the classical Faraday tensor, A^μ is the classical four-vector potential, $J^\nu := -i[\phi\, \partial^\nu \phi^\dagger - \partial^\nu \phi\, \phi^\dagger]$ is the current of the now charged scalar field ϕ, the constant μ_0 is the magnetic permeability of the vacuum and q is the charge of the field excitations. In the same fashion as done in this work, one seeks to expand four potential and current as $A_\mu = A_\mu^{(0)} + \xi\, A_\mu^{(1)}$ and $J_\mu = J_\mu^{(0)} + \xi\, J_\mu^{(1)}$ respectively, where $\xi \ll 1$ is a parameter to be determined. Note that, in order to compare with the gravitational case, we consider a perturbation of the vector potential around the zero order $A_\nu^{(0)}$ which satisfies the homogeneous Maxwell equation $\partial_\mu \partial^\mu A_\nu^{(0)} - \partial_\mu \partial^\nu A_\mu^{(0)} = 0$. This allows us to compare this scenario with the gravitational case, where the zero order component of the metric (i.e., the Minkowski metric $\eta_{\mu\nu}$) satisfies $G_{\mu\nu} = 0$. One then looks for the dimensionless version of $\partial_\mu F^{\mu\nu} = \frac{\mu_0 q}{c} \langle : J^\nu : \rangle$ and wishes to obtain the analogous of equation (3). However, since both vector potential and current are dimensional, after simple algebra one finds $\xi\, [\partial^\mu \partial_\mu \tilde{A}_\nu^{(1)} - \partial^\mu \partial_\nu \tilde{A}_\mu^{(1)}] = \langle : \tilde{J}_\nu^{(0)} : \rangle$. Here quantities with a tilde are dimensionless and the derivates are with respect to a normalised coordinate. The expansion parameter ξ is arbitrary and is not fixed by the physics of the problem. Furthermore $\tilde{A}_\nu^{(1)}$ and $\langle : \tilde{J}_\nu^{(0)} : \rangle$ are independent of ξ. Therefore, the relation $\xi\, [\partial^\mu \partial_\mu \tilde{A}_\nu^{(1)} - \partial^\mu \partial_\nu \tilde{A}_\mu^{(1)}] = \langle : \tilde{J}_\nu^{(0)} : \rangle$ cannot be satisfied and this perturbative expansion is inconsistent. We conclude that, although the main equations of these two semiclassical theories are formally similar, the physics they describe are essentially different and cannot be compared. We understand that this difference is a consequence of the universality of gravity, which couples to all energy, while the electromagnetic field couples only to charge.

Finally, our results suggest that entanglement is responsible for the effects described in this work. The initial state (4) is the most general one-particle state that can be conceived given the constraint that superpositions of different masses (or energies) are not allowed [8]. Entanglement in this state is always present when off diagonal terms are, which corroborates our claims.

4. Discussion

To summarize, we have shown that entanglement can affect the gravitational field. This suggests that entanglement "has a weight". The perturbations in the gravitational field depend on the amount of entanglement and vanish for vanishing quantum correlations. The effects studied in this work decay with a time scale proportional to the characteristic "size" of the particle but that does not depend on the amount of entanglement. Furthermore, relative phase of the coherence term seems to directly affect the strength of the effect. A prospective theory of quantum gravity must be able to account for this phenomenon and explain its origin.

Experiments designed to measure these effects will have to carefully balance the different parameters, in particular the distance at which the entanglement is established and the energy of the particle. We believe that our results can help in better understanding the overlap of relativity and gravity theories and, ultimately, in the quest of a theory of quantum gravity.

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