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# Close binary progenitors of gamma-ray bursts

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#### **ABSTRACT**

The strong dependence of the neutrino annihilation mechanism on the mass accretion rate makes it difficult to explain the long-duration gamma-ray bursts (LGRBs) with duration in excess of 100 s as well as the precursors separated from the main gamma-ray pulse by few hundreds of seconds. Even more difficult is to explain the Swift observations of the shallow decay phase and X-ray flares, if they indeed indicate activity of the central engine for as long as 10<sup>4</sup> s. These data suggest that some other, most likely magnetic mechanisms have to be considered. Since the efficiency of magnetic mechanisms does not depend that much on the mass accretion rate, the magnetic models do not require the development of accretion disc within the first few seconds of the stellar collapse and hence do not require very rapidly rotating stellar cores at the pre-supernova (SN) state. This widens the range of potential LGRB progenitors. In this paper, we re-examine the close binary scenario allowing for the possibility of late development of accretion discs in the collapsar model and investigate the available range of mass accretion rates, black hole (BH) masses and spins. We find that the BH mass can be much higher than 2-3 M<sub>\(\Omega\)</sub>, usually assumed in the collapsar model, and normally exceeds half of the pre-SN mass. The BH spin is rather moderate, a = 0.4–0.8, but still high enough for the Blandford-Znajek mechanism to remain efficient provided the magnetic field is sufficiently strong. Our numerical simulations confirm the possibility of magnetically driven stellar explosions, in agreement with previous studies, but point towards the required magnetic flux on the BH horizon in excess of  $10^{28}$  G cm<sup>2</sup>. At present, we cannot answer with certainty whether such a strong magnetic field can be generated in the stellar interior. Perhaps, the SN explosions associated with LGRBs are still neutrino-driven and their gamma-ray signature is the precursors. The SN blast clears up escape channels for the magnetically driven gamma-ray burst (GRB) jets, which may produce the main pulse. In this scenario, the requirements on the magnetic field strength can be lowered. A particularly interesting version of the binary progenitor involves merger of a Wolf-Rayet star with an ultracompact companion, neutron star or BH. In this case, we expect the formation of very long-lived accretion discs, that may explain the phase of shallow decay and X-ray flares observed by Swift. Similarly long-lived magnetic central engines are expected in the current single star models of LGRB progenitors due to their assumed exceptionally fast rotation.

**Key words:** accretion discs – black hole physics – MHD – relativity – binaries: close – supernovae: general – gamma-rays: bursts.

# 1 INTRODUCTION

The nature of gamma-ray bursts (GRBs) remains one of the most intriguing problems of modern astrophysics. It is now widely accepted that the gamma-ray emission is generated in ultrarelativis-

tic jets but many basic questions related both to the physics of these jets and to the mechanisms of their production remain open. Although many promising theories have been developed over the years since the discovery of GRBs, we are still some way out from solid understanding of this phenomenon. For example, the observed connection between the long-duration gamma-ray bursts (LGRBs) and supernovae (SNe) indicates that these bursts are connected to deaths of massive stars but the details are not clear. In one model of LGRBs, the stellar collapse results in a normal successful SN

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explosion but the newly born neutron star (NS) is very unusual. It has both exceptionally high magnetic field, and for this reason it is called a magnetar, and extremely rapid rotation (e.g. Usov 1992; Thompson, Chang & Quataert 2004; Metzger, Thompson & Quataert 2007; Uzdensky & MacFadyen 2007). The powerful magnetohydrodynamic (MHD) wind produced by such remnant is capable of both accelerating the SN shell above the expansion speed of normal SNe, to the level of hypernovae, and production of collimated ultrarelativistic polar jets (Komissarov & Barkov 2007; Bucciantini et al. 2009).

In another model, the normal SN explosion fails and the proto-NS promptly collapses into a black hole (BH). However, the rapid rotation of the stellar progenitor prevents the rest of the star from falling directly into the BH and a massive neutrino-cooled accretion disc is formed instead. This allows to turn the failed SN into a successful stellar explosion, as this disc can release enormous amounts of energy (Woosley 1993; MacFadyen & Woosley 1999). One way of 'utilizing' this energy is via the neutrino- or magnetically driven wind from the disc. Such wind is not expected to be relativistic due to the high mass loading at its base. However, the polar region just above the BH is less likely to become mass-loaded by the disc matter and can become relativistically hot via annihilation of neutrinos and antineutrinos emitted by the disc. This opens a possibility of driving ultrarelativistic LGRB jets in the collapsar scenario (e.g. MacFadyen & Woosley 1999; Aloy et al. 2000). However, the efficiency of this type of neutrino heating is a very strong function of both the mass accretion rate and the rotation rate of the central BH (Popham, Woosley & Fryer 1999; Chen & Beloborodov 2007; Zalamea & Beloborodov 2009). According to the calculations by Popham et al. (1999) for the BH with the spin parameter a = 0.5, the energy deposition rate via the neutrino annihilation process drops from  $L_{\nu\bar{\nu}}=4\times10^{48}\,\mathrm{erg}\,\mathrm{s}^{-1}$  for  $\dot{M}=0.1\,\mathrm{M}_{\odot}$  to  $L_{\nu\bar{\nu}} = 6 \times 10^{44} \,\mathrm{erg \, s^{-1}}$  for  $\dot{M} = 0.01 \,\mathrm{M_{\odot}}$  and for the accretion rate of  $\dot{M}=0.1\,\mathrm{M}_{\odot}$  from  $L_{\nu\bar{\nu}}=2\times10^{51}\,\mathrm{erg\,s^{-1}}$  for a=0.95to  $L_{\nu\bar{\nu}} = 3 \times 10^{48} \,\mathrm{erg}\,\mathrm{s}^{-1}$  for a = 0. Therefore, this version of the collapsar model, similar to the magnetar model, requires very rapid rotation of the stellar core prior to the collapse so that the accretion disc is formed early on, when the accretion rate is still high enough, and the BH is born rapidly rotating. The results by Birkl et al. (2007) suggest that Popham et al. (1999) may have overestimated the efficiency of neutrino mechanism for high a. This is because the energy released by the disc powers not only the outflow but also the flow into the BH. As a increases, the inner boundary of the disc moves closer to the BH and a larger fraction of the total neutrino-antineutrino annihilation occurs in the region where the vector of deposited momentum points towards the BH. In fact, Birkl et al. (2007) find that the efficiency of the neutrino annihilation mechanism peaks at  $a \simeq 0.6$ .

It turns out that such a fast rotation cannot be a general result of stellar evolution. Although young massive star often rotate sufficiently rapidly at birth, their cores are expected to experience strong spin-down during the red giant phase and during the intensive mass loss period characteristic for massive stars at the Wolf–Rayet (WR) phase (Heger, Woosley & Spruit 2005). In fact, this theoretical result agrees very well with the observed rotation rates of newly born pulsars. Thus, in order to retain the rotation rate required in the collapsar model, the evolution of LGRB progenitors must proceed along a rather exotic route. Recently, it was proposed that a combination of low metalicity and extremely fast initial rotation, at around 50 per cent of the break-up speed, could lead to such a route (Yoon & Langer 2005; Yoon, Langer & Norman 2006; Woosley & Heger 2006). On one hand, the mass-loss rate decreases significantly with

metalicity, leading to a significant reduction in the total loss of angular momentum. On the other hand, the rotationally induced circulation becomes very effective at such a high rotation rate and may result in chemically homogeneous stars that avoid the development of extended envelops and hence the spin-down of stellar cores via interaction with these envelopes. Moreover, the star remains compact by the time of its collapse so the LGRB jet can break out from the star on the time-scale compatible with the observed durations of LGRBs.

Another exotic scenario involves close high-mass binary systems, where the fast rotation of stellar cores is sustained via the tidal interaction between companions (Tutukov & Cherepashchuk 2003, 2004; Izzard, Ramirez-Ruiz & Tout 2004; Podsiadlowski et al. 2004; Levan, Davies & King 2008; van den Heuvel & Yoon 2007). In this case, the pre-SN is a compact helium star, essentially a WR star, because the extended envelope is dispersed into the surrounding space during the common envelope phase. The stellar rotation in such systems is synchronized with the orbital motion on a very short time-scale (e.g. van den Heuvel & Yoon 2007). The contraction of CO cores during stellar evolution leads to their additional spinup but due to the core-envelope coupling only a fraction of their angular momentum is retained (Yoon et al. 2006). As the result, the core rotation rate is insufficient in the cases where the companion of the helium star is a main-sequence star. According to van den Heuvel & Yoon (2007), the core rotation can be high enough to fit the collapsar model with the neutrino-driven LGRB jet only if the component is also a compact star, namely NS or BH. Three examples of such systems are known to date: Cyg X-3, IC 10 X-1 and NGC 300 X-1. Cyg X-3 has a very short orbital period, only 4.8 h (van Kerkwijk at al. 1992), and the radius of the WR star in this system is less than 3–6 R<sub>☉</sub> (Cherepashchuk & Mofat 1994). The recently discovered IC 10 X-1 and NGC 300 X-1 have the orbital periods of 35 and 33 h, respectively (Carpano et al. 2007; Prestwich et el 2007; Silverman & Filippenko 2008). The masses of WR stars are estimated at 18-40  $M_{\odot}$  for NGC 300 X-1 and  $\simeq$ 35  $M_{\odot}$  for IC 10 X-1 (Clark & Crowther 2004). Given the observed production rate of such systems van den Heuvel & Yoon (2007) predicted one hypernova/LGRB every 2000 yr in a galaxy similar to our own. Levan et al. (2008) examined the separations of known compact object binaries, NS-NS or NS-white dwarf, and concluded that up to 50 per cent of the systems could, at the time of core collapse of one of the components, have been sufficiently rapidly rotating to create an accretion disc around the collapsed core.

The neutrino heating is not the only possible mechanism behind the explosions of collapsing stars. Perhaps somewhat less popular, but the magnetic mechanisms are also regarded as potentially important (e.g. Bisnovatyi-Kogan 1970; LeBlanc & Wilson 1970; MacFadyen, Woosley & Heger 2001; Moiseenko, Bisnovatyi-Kogan & Ardeljan 2006; Burrows et al. 2007). Likewise, the LGRB jets can also be powered via a magnetic mechanism, in particular the Blandford–Znajek (BZ) mechanism, which utilizes the rotational energy of the BH (e.g. Blandford & Znajek 1977; Mészáros & Rees 1997; Lee, Brown & Wijers 2000, 2002; Proga et al. 2003; McKinney 2006; Barkov & Komissarov 2008a; Komissarov & Barkov 2009). The total rotational energy of the BH is

$$E_{\rm b} \simeq 1.8 \times 10^{54} f_{\rm l}(a) \left(\frac{M_{\rm b}}{{\rm M}_{\odot}}\right) {\rm erg},$$
 (1)

where  $f_1(a) = 2 - \{a^2 + [1 + (1 - a^2)^{1/2}]^2\}^{1/2}$ . Thus, even for a relatively slow rotation there is plenty of energy to power LGRBs. Since the energy extraction rate of BZ mechanism is proportional to  $a^2$  the BH is still required to rotate quite rapidly. However, the

efficiency of this mechanism is not that sensitive to the mass accretion rate and such rapid rotation does not have to be achieved right after the collapse of the Fe core. Instead, it can be built up gradually during the rest of the stellar collapse. This difference in the sensitivity to mass accretion rate favours the BZ mechanism over the neutrino mechanism in the case of very long-duration LGRBs, more than 100 s long (MacFadyen et al. 2001). The discovery by Swift of the shallow decay phase and late flares in the X-ray light curves of LGRBs (Chincarini et al. 2007; Zhang 2007) also suggests that the central engine may remain active for as long as 10<sup>4</sup> s (e.g. Lipunov & Gorbovskoy 2007, 2008). Since the neutrino mechanism requires the mass accretion rate to stay above few  $\times 10^{-2} \,\mathrm{M}_{\odot} \,\mathrm{s}^{-1}$ , such a prolonged activity implies the progenitor mass in excess of few  $\times 10^2 \,\mathrm{M}_{\odot}$ , which is highly unlikely. Since the BH's magnetic field is actually supported by the disc, the disc is an essential element of the BZ central engine. However, its operation time is not confined to the initial phase of very high mass accretion rates, as in the neutrino model, but extends to the total lifetime of the disc. This means that the nature of the SN explosions accompanying LGRBs is very important. If they are is more or less spherically symmetric and expel most of the stellar envelope, as assumed in Lee et al. (2002), then the central engine will be short-lived, in contradiction with the observations. If they are highly asymmetric and a significant fraction of the stellar envelope, mainly in the equatorial zone, remains bound, then a much longer duration can be expected.

Another problem for the model of neutrino-driven GRBs are the strong precursors sometimes observed before the arrival of the main gamma-ray pulse (Burlon et al. 2008). According to the analysis of Wang & Meszaros (2007) such precursor and the main pulse can be attributed to a single eruptive event only when the precursor and the main pulse are separated by few seconds. However, in some GRBs, the delay can be as long as few hundreds of seconds and in such cases it is much more likely that the precursor and the main pulse correspond to two different events in the life of the central engine. They proposed that the precursor is produced during the SN explosion, in the jet powered by a rotating magnetized NS, and that the main pulse is produced during the fallback phase when the NS collapses into a BH.2 The typical mass accretion rates in the fallback scenario,  $10^{-2}$ – $10^{-3}$  M $_{\odot}$  s<sup>-1</sup>, are too low for the neutrino annihilation mechanism and thus this explanation implies magnetic origin for the main pulse as well (MacFadyen et al. 2001).

Thus, the observations require to include the magnetic mechanism, either in the BH or, in fact, in the disc version, or both, in the collapsar scenario. This widens the range of potential progenitors of LGRBs. Indeed, we no longer need to constrain ourself to the stars with extremely rapidly rotating cores, but can also include the cases with slower rotation where the accretion disc forms much later during the course of stellar collapse.

In this paper, we re-examine the scenario of binary progenitor of LGRBs allowing for the late formation of accretion discs and lower mass accretion rates compared to those required in the collapsar model with the neutrino mechanism. In Section 2, we determine

the parameters of binary systems which allow formation of accretion discs during the collapse of WR companion. We also estimate masses and spins of the BHs by the time of accretion disc formation using simplified analytical model for the structure of pre-SNe due to Bethe (1990). In Section 3, we investigate the degree to which the BH spin can increase later on, during the disc accretion phase, using the same approach as in the recent study by Janiuk, Moderski & Proga (2008). Here, we consider not only the Bethe's model but also the polytropic model and the models of pre-SNe based on detailed calculations of stellar evolution. In Section 4, we describe the numerical simulations of LGRB jet formation with setup based on the results obtained in the previous Sections. In Section 5, we analvse the potential of the binary scenario in the extreme case, which involves merger of the WR star with its ultracompact companion, BH or NS. In Section 6, we summarize our main results and discuss their astrophysical implications.

# 2 FORMATION OF ACCRETION DISC

In a synchronized binary, the tidal torques force the components to spin with the same rate as the orbital rotation:

$$\Omega_c^2 = GM_s(1+q)/L^3,\tag{2}$$

where L is the orbital separation,  $M_s$  is the mass of the star under consideration and  $q = M_{\rm com}/M_s$ , where  $M_{\rm com}$  is the mass of the companion star. Since the orbital frequency decreases with L, the maximum possible spin is reached when the separation is minimum. This corresponds to the case where the star radius is about the size of its Roche lobe. The relation between the minimum separation  $L_{\rm min}$ , the stellar radius  $R_s$  and q can be approximated with sufficient accuracy for 1/100 < q < 100 as (Plavec & Krotochvile 1964)

$$L_{\min} = 2.64q^{0.2084}R_{\rm s}.\tag{3}$$

During the stellar collapse the centrifugal force will halt the freefall of the outer layers and promote the development of accretion disc provided the specific angular momentum on the stellar equator exceeds that of the marginally bound circular orbit for the BH with the same mass and angular momentum as the star. The angular momentum of Kerr BHs is

$$J_{\rm h} = a \frac{GM_{\rm h}^2}{c},$$

where -1 < a < 1 is the dimensionless spin parameter and  $M_h$  is the hole mass. The specific angular momentum of test massive particles on circular orbits in the equatorial plane is

$$l = \frac{(r^2 - 2ar^{1/2} + a^2)}{r^{3/4}(r^{3/2} - 3r^{1/2} + 2a)^{1/2}} \frac{GM_h}{c},$$
(4)

where  $r = R/R_g$  and  $R_g = GM_h/c^2$ , and the radius of the marginally bound orbit is (Bardeen, Press & Teukolsky 1972):

$$r_{\rm mb} = [2 - a + 2(1 - a)^{1/2}].$$
 (5)

The disc formation condition is

$$\Omega_{\rm s}R_{\rm s}^2 > l_{\rm mb},\tag{6}$$

where  $l_{\rm mb}=l(r_{\rm mb})$ . As we shell see later, at the time of the disc formation a is quite small. Using the Taylor expansion, we find that

$$l_{\rm mb} = (4 - a) \frac{GM_{\rm h}}{c} + O(a^2) \simeq \frac{4GM_{\rm h}}{c}.$$
 (7)

Using this result and equation (2), we can now write the disc formation condition as

$$\left(\frac{L}{R_{\rm s}}\right)^3 < \frac{1+q}{16}r_{\rm s},\tag{8}$$

 $<sup>^1</sup>$  Typically, the mass of WR star is 9–2  $M_{\odot}$ , though some observations suggested that it can be as high as  $83\,M_{\odot}$  (Schweickhardt et al. 1999; Crowther 2007).

<sup>&</sup>lt;sup>2</sup> This model may struggle to explain delays shorter than the typical fallback time, 100–1000 s, found in one-dimensional simulations MacFadyen et al. (2001). However, due to the rotational effects the SN explosions could be highly aspherical, resulting in shorter fallback time-scales in the equatorial region.

where  $r_s = R_s/R_{gs}$  and  $R_{gs} = GM_s/c^2$ . For the typical parameters of WR stars, this amounts to

$$L < 14R_{\rm s}(1+q)^{1/3} \left(\frac{R_{\rm s}}{R_{\odot}}\right)^{1/3} \left(\frac{M_{\rm s}}{10\,{\rm M}_{\odot}}\right)^{-1/3}.$$
 (9)

The comparison of this result with equation (3) shows that collapse of WR stars in close binaries can indeed lead to formation of accretion discs (Bisnovatyi-Kogan & Tutukov 2004; Podsiadlowski et al. 2004). We can rewrite the above condition in terms of the binary period,  $T_{\rm b}$ , as

$$T_{\rm b} < \frac{1}{4} T_{\rm k} r_{\rm s}^{-1/2} \simeq 48 \, {\rm h} \left( \frac{M_{\rm s}}{10 \, {\rm M}_{\odot}} \right)^{-1} \left( \frac{R_{\rm s}}{{\rm R}_{\odot}} \right)^2,$$
 (10)

where  $T_k$  is the Keplerian period at  $R = R_s$ . This upper limit is about five times higher than that obtained in Podsiadlowski et al. (2004) who required the disc to form immediately after the collapse of iron core.

Effectively cooled accretion discs remain geometrically thin and their inner radius is given by the radius of the last stable circular orbit:

$$r_{\rm ms} = \{3 + Z_2 - [(3 - Z_1)(3 + Z_1 + 2Z_2)]^{1/2}\},\tag{11}$$

where  $r_{\rm ms} = R_{\rm ms}/R_{\rm g}$  and (Bardeen et al. 1972)

$$Z_1 \equiv 1 + (1 - a^2)^{1/3} [(1 + a)^{1/3} + (1 - a)^{1/3}],$$

$$Z_2 \equiv (3a^2 + Z_1^2)^{1/2}.$$
(12)

The corresponding specific angular momentum,  $l_{\rm ms}$ , determines the evolution of the BH spin via the disc accretion.<sup>3</sup>

The outer radius of the disc,  $R_{\rm d}$ , is determined by the specific angular momentum on the stellar equator,  $^4l_{\rm s}=\Omega_{\rm s}R_{\rm s}^2$ . Assuming that  $r_{\rm d}=R_{\rm d}/R_{\rm gs}\gg 1$ , the angular momentum at the outer edge of the disc is simply

$$l_{\rm d} = (GM_{\rm s}R_{\rm d})^{-1/2}$$

Matching  $l_d$  and  $l_s$ , and using equation (2), we find that

$$r_{\rm d} = r_{\rm s}(1+q) \left(\frac{L}{R_{\rm s}}\right)^{-3}$$
 (13)

Thus,

$$r_{\rm d} \sim 47 \left(\frac{R_{\rm s}}{{\rm R}_{\odot}}\right) \left(\frac{M_{\rm s}}{10\,{\rm M}_{\odot}}\right)^{-1} \left(\frac{\tilde{L}}{10}\right)^{-3},$$
 (14)

where  $\tilde{L} = (1+q)^{-1/3}(L/R_s)$ . For the widest orbital separation which still allows disc formation (see equation 9), this equation gives  $r_d \leq 17$  whereas for the closest one (see equation 3) we have  $r_d \leq 5 \times 10^3$ . Thus, the model predicts a wide range of accretion disc sizes. Compact discs and the inner regions of large discs will cool via the neutrino emission, whereas the outer regions of large discs will remain adiabatic. The accretion time of neutrino cooled discs (Popham et al. 1999),

$$t_{\rm d} \approx 2.6 \left(\frac{\alpha}{0.1}\right)^{-6/5} \left(\frac{r_{\rm d}}{100}\right)^{4/5} \left(\frac{M_{\rm h}}{10\,{\rm M}_{\odot}}\right)^{6/5} {\rm s},$$
 (15)

is significantly less than the free-fall time-scale

$$t_{\rm ff} \approx 240 \left(\frac{R}{\rm R_{\odot}}\right)^{3/2} \left(\frac{M_{\rm s}}{10\,\rm M_{\odot}}\right)^{-1/2} \,\rm s. \tag{16}$$

The accretion time of large discs can be estimated using the  $\alpha$ -model for slim discs ( $\delta = H_d/R_d \simeq 0.3$ ) (Shakura & Sunyaev 1973):

$$t_{\rm d} \approx 250 \left(\frac{\alpha \delta^2}{0.01}\right)^{-1} \left(\frac{r_{\rm d}}{10^3}\right)^{3/2} \left(\frac{M_{\rm h}}{10 \,\mathrm{M}_{\odot}}\right) \,\mathrm{s}.$$
 (17)

Thus, with the exception of largest discs, the time-scale of disc accretion is shorter compared to the free-fall time-scale, and hence the growth rate of the BH mass is given directly by the rate of the collapse.

In order to estimate the mass and rotation rate of the BH at the time of the disc formation, one needs to know the mass distribution of progenitor at the onset of collapse. Here, we adopt the power-law model used by Bethe (1990) in his analytical models of corecollapse SNe,

$$\rho(R) = \rho_{\rm c} \left(\frac{R}{R_{\rm c}}\right)^{-3}, \qquad R > R_{\rm c}, \tag{18}$$

where  $R_c$  is the radius of iron core. Simple integration allows us to find the following equations for the mass

$$M(R) = 4\pi \rho_{\rm c} R_{\rm c}^3 \ln(R/R_{\rm c}),\tag{19}$$

and the moment of inertia

$$I(R) = \frac{1}{3} \frac{M(R)R^2}{\ln(R/R_c)}$$
 (20)

of the shell between the iron core and the radius R.

By analogy with the BH theory, it is convenient to describe the rotation rate of collapsing star using the spin parameter

$$a_{\rm s} = \frac{J_{\rm s}c}{GM_{\rm s}^2}.\tag{21}$$

In Bethe's model, it relates to  $\Omega_s$  via

$$\Omega_{\rm s} = a_{\rm s} \frac{3GM_{\rm s}(1+\eta)^2}{R_{\rm s}^2 c} \ln y_{\rm s},\tag{22}$$

where  $y_s = R_s/R_c$ ,  $\eta = M_c/M_s$ , and we ignore the small contribution of compact iron core to the total spin of the star. The condition (3) with q = 1 implies that

$$a_{\rm s} \le \frac{1}{9 \ln y_{\rm s}} r_{\rm s}^{1/2} \simeq 5.2 \left(\frac{R}{\rm R_{\odot}}\right)^{1/2} \left(\frac{M_{\rm s}}{10 \, \rm M_{\odot}}\right)^{-1/2},$$
 (23)

where we used  $y_s = 100$ . This seems to suggest that the stellar collapse may lead to formation of rapidly rotating BHs.

Suppose that the disc is first formed at time  $t^*$  and that by this time the BH has swallowed the star up to the initial radius  $R = R_*$ . Assuming that the BH spin at this point is low,  $a_* \ll 1$ , we have

$$\Omega_{\rm s} R_*^2 = (4 - a_*) \frac{G(M_* + \eta M_{\rm s})}{c},\tag{24}$$

where  $M_* = M(R_*)$ . Using equations (19) and (22), this condition can be written as the following algebraic equation for  $y_* = R_*/R_c$ :

$$y_*^2 = \frac{(4 - a_*)y_s^2}{3(1 + \eta)^2 a_s \ln y_s} \left[ \frac{\ln y_*}{\ln y_s} + \eta \right], \tag{25}$$

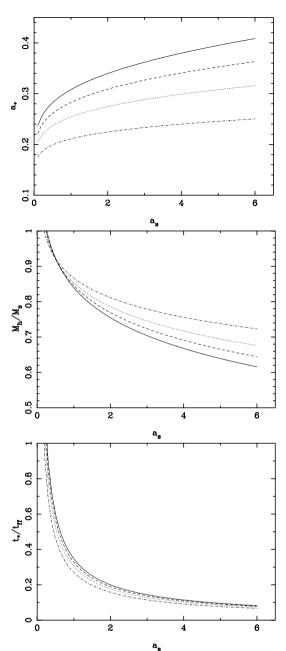
where

$$a_* = \frac{4}{1 + 3(\ln v_* + n \ln v_*)}. (26)$$

This equation is solved numerically and the results are presented in Fig. 1. One can see that the disc is formed relatively late, with the typical time  $t_* > 0.1t_{\rm ff}$ , when more than a half of the star has already collapsed into the BH. However, the BH spin at this moment is relatively low,  $0.2 < a_* < 0.4$ .

<sup>&</sup>lt;sup>3</sup> For simplicity, we ignore the effects of magnetic torques on the evolution of BH spin.

<sup>&</sup>lt;sup>4</sup> In fact, various torques operating in the accretion disc change the angular momentum and hence the location of the outer edge, but this effect is relatively minor (Shakura & Sunyaev 1973; Chen & Beloborodov 2007).



**Figure 1.** The BH spin (top panel) and mass (middle panel) at the disc formation time as functions of the progenitor spin,  $a_s$ , for  $M_c/M_s=1/3$  (dash–dotted line), 1/5 (dotted line), 1/9 (dashed line) and 1/31 (solid line). The bottom panel shows the time of disc formation as a function of  $a_s$  for the same models.

#### 3 GROWTH OF BLACK HOLES

In order to explore the evolution of the BH spin for more sophisticated models of LGRB progenitors, as well as its evolution in Bethe's model after the disc formation, one can integrate the following system of dynamic equations:

$$\frac{\mathrm{d}M_{\mathrm{h}}}{\mathrm{d}R} = 4\pi R^2 \rho(R),\tag{27}$$

$$\frac{\mathrm{d}J_{\mathrm{h}}}{\mathrm{d}R} = 4\pi R^2 \rho(R) \int_0^{\pi/2} \tilde{l}(R,\theta) \sin\theta \,\mathrm{d}\theta. \tag{28}$$

Here,  $\rho(R)$  is the stellar mass density prior to the collapse and  $\tilde{l}(R,\theta)$  is the specific angular momentum retained by the fluid element, initially located at the point with the coordinates  $\{R,\theta\}$ , by the time it crosses the event horizon. This quantity is given by

$$\tilde{l} = \begin{cases} l(R, \theta) & \text{if} \quad l < l_{\text{mb}}(M_{\text{h}}, J_{\text{h}}) \\ l_{\text{ms}}(M_{\text{h}}, J_{\text{h}}) & \text{if} \quad l > l_{\text{mb}}(M_{\text{h}}, J_{\text{h}}) \end{cases}$$
(29)

where  $l(R, \theta)$  is the distribution of the progenitor's angular momentum. The initial conditions for equations (27) and (28) correspond to the iron core of the WR star.

$$M_{\rm h}(R_{\rm c}) = M_{\rm c}, \quad J_{\rm h}(R_{\rm c}) = 0,$$
 (30)

where  $M_c$  and  $R_c$  are, respectively, the mass and the radius of the core. When the accretion rate is determined by the free-fall time, R and t can be related via

$$t^2 = \frac{2R^3}{9GM(R)}. (31)$$

As one can see, in these calculations, we assume that the whole of the star collapses into the BH. In fact, a significant fraction of its mass can be expelled during the SN explosion, reducing the final mass of the BH. As to the BH's spin, the outcome is less clear. If the explosion is highly asymmetric and only the slowly rotating polar parts of the envelope are expelled, then the final value of *a* can be higher. On the other hand, the magnetic torques can slow down the BH during the operation of the central engine.

The same approach has been used in Janiuk et al. (2008) in their search for the laws of rotation that would fit the collapsar model of LGRBs. They did not consider the solid body rotation<sup>5</sup> and assumed that initially the BH is rapidly rotating, with a=0.85. They also used the model of geometrically thick and radiatively inefficient disc, with the inner edge located at the radius of the marginally bound orbit, whereas we use the thin disc approximation, which is more suitable for the neutrino-cooled collapsar discs.

#### 3.1 Bethe's model

Fig. 2 shows the typical evolution of the BH mass and spin, as described by equations (27) and (28), for the Bethe's model. One can see that the BH spin increases significantly above the values attained by the time of disc formation. Eventually, it reaches the relatively high values of a = 0.3 - 0.8, the final spin depending mainly on the progenitor spin and less so on the mass fraction of the iron core (see Fig. 3). These higher values of a imply higher potential efficiency of both the neutrino annihilation and the BZ mechanisms of the LGRB jet production (Popham et al. 1999; Barkov & Komissarov 2008a; Zalamea & Beloborodov 2009).

The total mass accretion rate can be easily derived from the mass distribution and the free fall time (see equation 31):

$$\dot{M} = \frac{2}{3} \frac{M_s}{\ln y_s} t^{-1} \simeq 1.45 \left(\frac{M_s}{10 \,\mathrm{M}_{\odot}}\right) \left(\frac{t}{1\mathrm{s}}\right)^{-1} \,\mathrm{M}_{\odot} \,\mathrm{s}^{-1},$$
 (32)

where t is the time since the start of the collapse. As one can see in Fig. 2, soon after the disc formation, the mass accretion rate becomes dominated by disc. Initially, the rate can be rather high, but at around  $t \simeq 100$ s it becomes insufficient for the neutrino annihilation mechanism to operate.

<sup>&</sup>lt;sup>5</sup> The solid body rotation law was studied in Janiuk & Proga (2008), but it was assumed there that the BH was non-rotating.

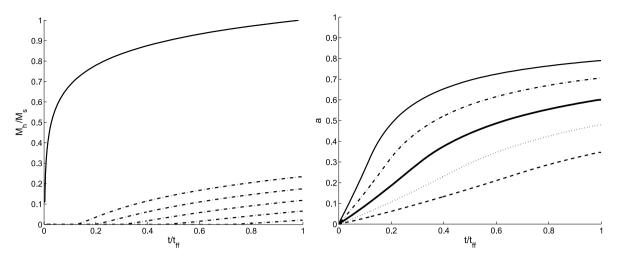
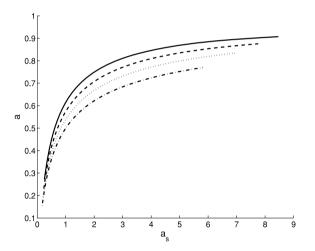


Figure 2. Evolution of the BH mass and spin in the Bethe's model with  $M_c/M_s = 1/9$ . Left-hand panel: the total mass of BH (solid line) and the mass accumulated via the accretion disc (dash-dotted lines) for the progenitor spin  $a_s = 0.33$ , 0.58, 1.0, 1.7 and 3. The higher value of  $a_s$  correspond to the higher fraction of mass processed via the disc. Right-hand panel: the spin parameter a of the BH for the progenitor spin  $a_s = 0.33$  (dashed line), 0.58 (dotted line), 1.0 (thick solid line), 1.7 (dot-dashed line) and 3.0 (thin solid line). The evolution time is given in the units of the free-fall time (see equation 16).



**Figure 3.** The final value of the BH spin in Bethe's model as a function of the progenitor spin for models with  $M_c/M_{\rm star} = 1/3$  (dot–dashed line), 1/5 (dotted line), 1/9 (dashed line) and 1/31 (solid line).

#### 3.2 Stellar evolution models

Although the Bethe's model provides a reasonable zero-order approximation for the structure of pre-SN stars, the more sophisticated models based on numerical integration of the equations of stellar evolution yield somewhat different stellar structure with wealth of finer details. Our next results are based on the pre-collapse structure of massive zero age main-sequence (ZAMS) stars with masses  $M_s = 20 \, \mathrm{M}_{\odot}$  and  $35 \, \mathrm{M}_{\odot}$  described in Heger et al. (2004). Assuming that stars of close binaries lose their extended envelopes, we cut off the mass distributions beyond the C/O core. This results in the progenitors with masses  $M_s = 6.15 \, \mathrm{M}_{\odot}$  (model A) and  $M_s = 12.88 \, \mathrm{M}_{\odot}$  (model B), respectively, and radius  $R_s \simeq 0.3 \, R_{\odot}$ . The

moments of inertia of models A and B are  $I \simeq 0.065 M_{\rm s} R_{\rm s}^2$  and  $0.074 M_{\rm s} R_{\rm s}^2$ , respectively. Given these parameters, equations (2) and (3) imply the spin parameters  $a_{\rm s} < 2.6$  and <1.7 for the models A and B, respectively; somewhat smaller than in the Bethe's model.

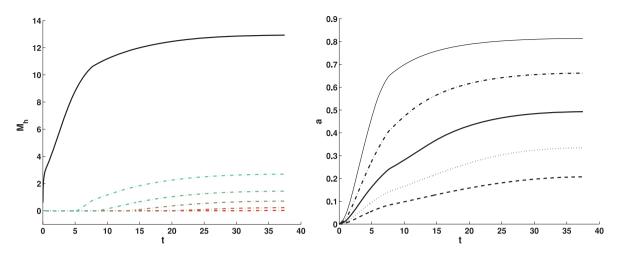
Fig. 4 shows the evolution of the BH's mass and spin in model B for different assumed values of the progenitor's spin. The comparison with the results obtained for Bethe's model shows only relatively minor differences, suggesting that Bethe's model is quite accurate. Fig. 5 shows the accretions rates, both for the disc and in total, for different progenitor spins in models A and B. One can see that initially the disc accretion rate grows rapidly and soon it accounts for most of the total accretion rate. Then, it begins to decay, approximately as  $t^{-1}$  in model A and  $t^{-3}$  in model B. For the cases with faster stellar rotation, the peak disc accretion rate is sufficiently high to ensure effective neutrino cooling of the disc (Chen & Beloborodov 2007).

#### 3.3 Polytrope model

Finally, we consider the model polytrope with index n=3, which could be used to describe the cores of most massive stars at the pre-SN phase (Tutukov & Fedorova 2007). In this model, the concentration of mass towards the centre is much weaker, resulting in higher moment of inertia and larger angular momentum compared to the Bethe's model with the same mass, radius and rotation frequency. Even if we consider models with the same spin parameter  $a_s$ , the polytrope yields generally higher fraction of mass accreted via the accretion disc and more rapidly rotating BHs (see Fig. 6). Similar to other models, the final value of the BH's spin does not show strong dependence on the iron core mass fraction, at least for  $M_c/M_s \in (1/3, 1/31)$  (see Fig. 7).

The polytrope model was also used to test our calculations against the fully general relativistic simulations by Shibata & Shapiro (2002). For the polytropic star with angular momentum  $a_{\rm s}=1$  our model gives a BH with  $M_{\rm b}=0.90M_{\rm s}$  and a=0.76 by the time of disc formation. This is in excellent agreement with the numerical simulations which give  $M_{\rm b}=0.90$  and a=0.75.

<sup>&</sup>lt;sup>6</sup> This radius is rather small, twice as small compared to the models of WR stars constructed in Schaerer & Maeder (1992) and 10–20 times smaller compared to the observed radii (Cherepashchuk & Mofat 1994; Crowther 2007). We can offer no clear explanation for this discrepancy. Perhaps, the artificial 'removing' of extended H/He envelope is not a particularly accurate procedure.



**Figure 4.** Evolution of BH's mass and spin in numerical model B. Left-hand panel: the total mass of BH (solid line) and the mass accumulated via the accretion disc (dash–dotted lines) for the progenitor spin  $a_s = 0.20, 0.33, 0.57, 0.96$  and 1.6. The higher value of  $a_s$  correspond to the higher fraction of mass processed via the disc. Right-hand panel: the spin parameter a of the BH for the progenitor spin  $a_s = 0.20$  (dashed line), 0.33 (dotted line), 0.57 (thick solid line), 0.96 (dot–dashed line) and 1.6 (thin solid line).

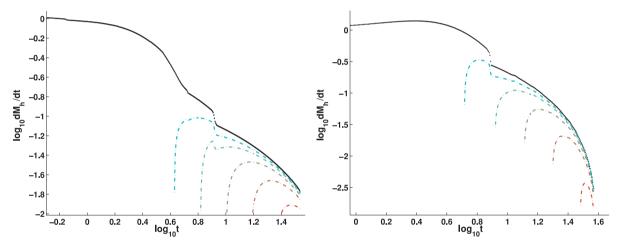


Figure 5. The accretion rate (in the units of  $M_{\odot}$  s<sup>-1</sup>) for model A (left-hand panel) and model B (right-hand panel). The solid lines show the total accretion rate whereas the dash–dotted lines show the disc accretion rates for different spins of the progenitor;  $a_s = 0.32, 0.54, 0.90, 1.52$  and 2.55 for model A and  $a_s = 0.20, 0.33, 0.57, 0.96$  and 1.6 for model B. Higher values of  $a_s$  correspond to earlier formation of accretion disc and higher disc accretion rates.

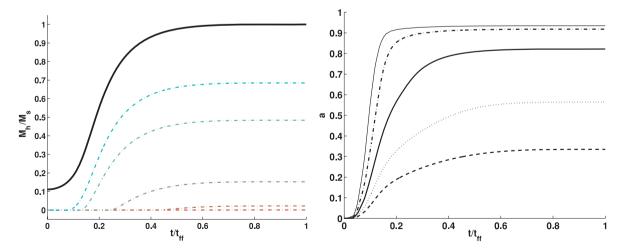
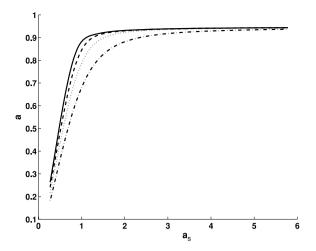


Figure 6. The evolution of BH's mass and spin in the polytrope model of progenitor with the initial BH mass  $M_c = M_s/9$ . The left-hand panel shows the total mass of the BH (thick solid line) as well as the mass accumulated via the accretion disc for different rotation rates of the progenitor,  $a_s = J_s c/G M_s^2 = 0.33, 0.58, 1.0, 1.7, 3.0$  (dash–dotted lines). Faster rising lines correspond to higher rotation rate. The right-hand panel shows the spin parameter of the BH,  $a_s$ , for the same values of  $a_s$ .



**Figure 7.** The evolution of BH spin for polytropic models with  $a_s = 3$  and  $M_c/M_s = 1/3$  (dot–dashed line), 1/5 (dotted line), 1/9 (dashed line) and 1/31 (solid line).

#### 4 JET SIMULATIONS

The analysis carried out in Sections 2 and 3 suggests that during the collapse of a WR star in a very close binary system, the conditions can become favourable to production of LGRB jets either via the neutrino heating or the BZ mechanism. Although the production of jets via the BZ mechanism has already been studied numerically in several previous papers, the conditions suggested by the binary scenario are different from those explored so far. By the time of the accretion disc formation, the BH is much more massive compared to the usually assumed  $M_h \simeq 2\,\mathrm{M}_\odot$ . Its rotation rate is notably lower compared to  $a \simeq 1$ , assumed in the past. Finally, the progenitor's rotation is not differential but uniform. These differences invite additional numerical simulations to explore the new region of parameter space.

#### 4.1 Setup of simulations

The progenitor model describes a compact WR star of radius  $R_{\rm s} = 3 \times 10^{10}$  cm and rotation period  $T_{\rm s} = 1.4$  h; the corresponding specific angular momentum on the stellar equator is  $l_{\rm s} = 1.13 \times 10^{18}$  cm<sup>2</sup> s<sup>-1</sup>. The progenitor's magnetic field is assumed to be purely poloidal and uniform, with the strength  $B_0 = 1.4$ –8.4  $\times$   $10^7$  G.

Simulations of this type are computationally expensive even in two-dimensional. On the other hand, the early stages of the collapse are very simple and can be treated analytically with sufficient accuracy. For these reasons, we start simulations only after the expected time of the disc formation,  $t_s = 17s$ . Based on the analysis given in the previous sections, the BH mass is set to  $M_h = 10 \, \mathrm{M}_{\odot}$  and the mass accretion rate to  $0.14 \, \mathrm{M}_{\odot} \, \mathrm{s}^{-1}$ . The initial radial distributions of mass and velocity are the same as in the Bethe model:

$$\rho \propto R^{-3/2}, \quad v^r = (2GM_h/R)^{1/2}.$$
 (33)

The initial distributions of angular momentum and magnetic field are derived from the progenitor distributions by taking into account the distortions caused by the free-fall collapse over the time  $t_s$ :

$$l(R,\theta) = \Omega_{\rm s}(R\sin\theta)^2 \left[1 + \frac{t_{\rm s}}{t_{\rm ff}(R)}\right]^{4/3},\tag{34}$$

Table 1. Numerical models.

Model	а	$B_0$	$\Psi_{28}$	$L_{\mathrm{BZ}}$	$\dot{M}_{ m h}$	$L_{\rm BZ}/\dot{M}_{\rm h}c^2$
M1	0.6	1.4	0.46	_	_	_
M2	0.6	4.2	1.5	0.44	0.017	0.0144
M3	0.45	8.4	3.1	1.1	0.012	0.049

Note. a is the BH spin;  $B_0$  is the initial magnetic field strength in the units of  $10^7 \, \mathrm{G}$ ;  $\Psi_{28}$  is the magnetic flux accumulated by the BH by the time of explosion in the units of  $10^{28} \, \mathrm{G \, cm^2}$ ;  $L_{\mathrm{BZ}}$  is the total power of the BZ mechanism during the explosion in the units of  $10^{51} \, \mathrm{erg \, s^{-1}}$ ;  $\dot{M}_h$  is the BH mass accretion rate during the explosion in the units of  $M_{\odot} \, \mathrm{s^{-1}}$ .

$$B^{r} = \frac{B_{0} \sin \theta \cos \theta}{\sqrt{\gamma}} R^{2} \left[ 1 + t/t_{\text{ff}}(R) \right]^{4/3}, \tag{35}$$

$$B^{\theta} = \frac{B_0 \sin^2 \theta}{2\sqrt{\gamma}} 2R \left( 1 + t/t_{ff}(R) \right)^{1/3}, \tag{36}$$

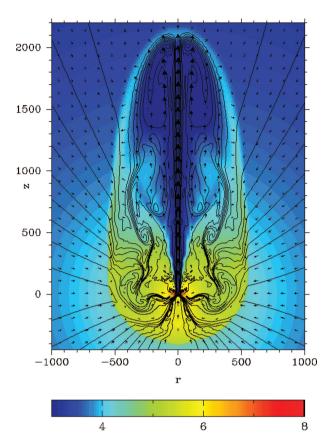
where  $t_{\rm ff}(R) = \sqrt{2R^3/9GM_{\rm h}}$  is the local free fall time-scale and  $\gamma$  is the determinant of the metric tensor of space (see Appendix A). In these simulations, we studied three different cases summarized in Table 1. Since we are able to run simulations only for a relatively short time, we can assume that both the BH's mass and spin, as well as the mass accretion rate, are constant.

The simulations were carried out with 2D axisymmetric GRMHD code described in Komissarov (1999, 2004). The gravity effects are introduced via the Kerr metric with fixed parameters; the Kerr–Schild coordinates are used in order to avoid the coordinate singularity at the horizon. The computational grid is uniform in the polar angle,  $\theta$ , where it has 180 cells and logarithmic in the spherical radius, R, where it has 445 cells. The inner boundary is located just inside the event horizon and adopts the free-flow boundary conditions. The outer boundary is located at  $R = 8.3 \times 10^9$  cm and at this boundary the flow is prescribed according to the Bethe's model.

In the simulations, we used realistic equation of state (EOS) that takes into account the contributions from radiation, lepton gas (including pair plasma) and non-degenerate nuclei (hydrogen, helium and oxygen). This is achieved via incorporation of the EOS code HELM (Timmes & Swesty 2000). The neutrino cooling is computed assuming optically thin regime and takes into account URCAprocesses (Ivanova, Imshennik & Nadezhin 1969), pair annihilation, photo-production and plasma emission (Schinder et al. 1987), as well as synchrotron neutrino emission (Bezchastnov et al. 1997). In fact, URCA-processes strongly dominate over other mechanisms in this problem. Photo-disintegration of nuclei is included via modification of the EOS following the prescription given in Ardeljan et al. (2005). The equation for mass fraction of free nucleons is adopted from Woosley & Baron (1992). We have not included the radiative heating due to annihilation of neutrinos and antineutrinos produced in the accretion disc mainly because this requires elaborate and time consuming calculations of neutrino transport.

# 4.2 Results

In general, the results of these simulations are in agreement with our previous studies (Barkov & Komissarov 2008a,b; Komissarov & Barkov 2009). Because of the modified setup, which corresponds to the later stages of the collapse, the accretion disc if formed straight away. At the same time, the accretion shock separates from the disc surface and quickly expands up to  $R \simeq 100$ –200  $R_{\rm g}$ . In the model M1, the shock then begins to oscillate and no jets emerge by the end



**Figure 8.** Model M2 ( $B_0 = 2.2 \times 10^7$  G, a = 0.6) at the time of t = 1.35 s after the start of simulations (18.3 s after the start of the stellar collapse). The colour image shows the baryonic rest mass density,  $\log_{10} \rho$ , in CGS units, the contours show the magnetic field lines and the arrows show the velocity field.

of the simulations, t = 19.5 s. In contrast, both the models M2 and M3 eventually develop polar jets of relativistic plasma which are powered via the BZ mechanism (see Figs 8 and 9). These results comply with the BZ activation condition,  $\kappa \ge 0.2$ , where

$$\kappa = \frac{\Psi}{4\pi r_{\sigma} \sqrt{\dot{M}c}} \tag{37}$$

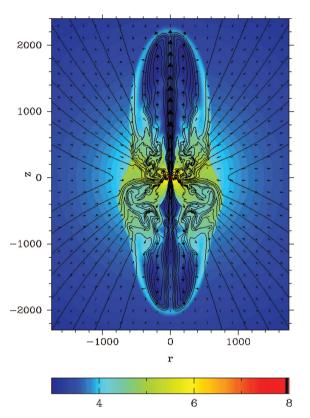
is the activation parameter,  $\Psi$  is the magnetic flux threading the BH and  $\dot{M}$  is the mass accretion rate of the collapsing star (Komissarov & Barkov 2009). For the parameters of the present simulations we have  $\kappa \simeq 0.07 \Psi_{28}$ , where the magnetic flux is given in the units of  $10^{28}\,\mathrm{G\,cm^2}$ , and, thus, one would expect the BZ mechanism to become activated for  $\Psi_{28} > 3.^7$  As one can see from the data presented in Table 1, which was indeed the case.

According to the simple monopole model of BH magnetosphere, the power of the BZ mechanism is

$$\dot{E}_{\rm BZ} = \frac{1}{6c} \left( \frac{\Omega_{\rm h} \Psi}{4\pi} \right)^2, \tag{38}$$

where  $\Omega_h = \{a/[2(1+\sqrt{1-a^2})]\}c^3/GM_b$  is the angular velocity of the BH, which gives us the estimate

$$\dot{E}_{\rm BZ} = 1.4 \times 10^{51} f_2(a) \Psi_{28}^2 \left(\frac{M_{\rm b}}{10 \,{\rm M}_{\odot}}\right)^{-2} \,{\rm erg \, s}^{-1},$$
 (39)



**Figure 9.** Model M3 ( $B_0 = 8.8 \times 10^7$  G, a = 0.45) at t = 1.35 s after the start of simulations (18.3 s after the start of the collapse). The colour image shows the baryonic rest mass density,  $\log_{10} \rho$ , in CGS units, the contours show the magnetic field lines and the arrows show the velocity field.

where  $f_2(a) = a^2(1 + \sqrt{1 - a^2})^{-2}$  (Barkov & Komissarov 2008a). Like in our previous simulations, the direct measurements of energy flux across the BH horizon roughly agree with this result (see Table 1).

One significant difference with the results of previous simulations is the development of a one-sided jet in model M2. Although notable deviations from the equatorial symmetry have been observed before, in particular the asymmetric oscillations of the accretion shock, such a strong deviation is observed for the first time. The initial solution is not exactly symmetric because of the rounding errors, but they are tiny and the observed braking of the equatorial symmetry has to be rooted in the non-linear dynamics of the flow. It appears that the accretion flow, which is deflected towards the equatorial plane at the oblique shock driven by the northern jet, protrudes into the southern hemisphere. There it collides with the accretion flow of the southern hemisphere and together they stream towards the BH's southern pole, thus suppressing the development of a southern jet.

If persistent, such a one-side jet could impart a strong kick on the BH and the binary, significantly altering its motion in the parent galaxy (Fragos et al. 2009). The maximum kick velocity can be estimated as

$$v_{\rm kick} = \frac{E_{\rm jet}}{cM_{\rm h}} \approx 170 \left(\frac{E_{\rm jet}}{10^{52}\,{\rm erg}}\right) \left(\frac{10\,{\rm M}_{\odot}}{M_{\rm h}}\right)\,{\rm km\,s^{-1}}, \tag{40}$$

which is consistent with the observations of the X-ray binary XTEJ 1118+480 (Gualandris et al. 2005). However, at present, we cannot say whether such one-sidedness can persist during the lifetime of LGRB or this is just a transient phenomenon.

<sup>&</sup>lt;sup>7</sup> The magnetic field strength at  $R=2R_{\rm g}$  is related to the magnetic field flux and the BH mass via  $B\simeq 1.8\times 10^{14}\Psi_{28}(M/10\,{\rm M}_{\odot})^{-2}$  Gauss.

#### 5 MERGER SCENARIO

The case of close tidally locked binary considered above involves binaries with orbital separation very close to the size of the Roche lobe of the WR star and this suggests to go one step further and consider the case of even smaller separation which can lead to the common envelope evolution resulting in a merger of the binary (Tutukov & Yungelson 1979) and GRB explosion (Zhang & Fryer 2001). Such a merger can be divided into three phases. During the fist phase, the compact companion spirals inside the extended envelope of the normal star and spins it up via deposition of its orbital angular momentum. The compact star also increases its mass and spin via the Bondi-type accretion. According to the simulations of Zhang & Fryer (2001), during the last 500s of the in-spiral the compact star can accumulate up to  $3.5\,\mathrm{M}_{\odot}$ . Thus, the mean accretion rate is less than  $10^{-2}\,\mathrm{M}_{\odot}\,\mathrm{s}^{-1}$  implying inefficient neutrino heating.

The second stage begins when the compact star approaches the centre of its WR companion and the accretion rate increases. Zhang & Fryer (2001) find that, in the case of  $16\,M_\odot$  companion, the neutrino annihilation mechanism can operate for around 60 s and release about  $10^{52}$ erg. This is more than enough to drive a SN explosion. For the companion mass below  $8\,M_\odot$ , the neutrino heating is too weak and the second phase is absent.

The third phase takes place if the second phase does not result in the SN explosion or if the explosion is highly non-spherical and does not remove the equatorial layers of the WR star. During this phase, the compact object, already a BH, accretes these layers, which have been spun up during the first phase. Assuming that the mass of the compact star is small compared to the mass of its WR companion, its orbital angular momentum can be found via the Keplerian law,

$$J_{c}(R) = M_{c} \sqrt{GM(R)R},$$

where M(R) is the WR mass inside the radius R. As the compact star moves from the radius R to  $R + \mathrm{d}R$ , it transfers the angular momentum  $\mathrm{d}J_{\mathrm{c}}(R) = (\mathrm{d}J_{\mathrm{c}}/\mathrm{d}R)\mathrm{d}R$  to the envelope of the WR star. Assuming that most of this angular momentum is transferred to the mass  $\mathrm{d}M = (\mathrm{d}M/\mathrm{d}R)\mathrm{d}R$  of the envelope located between R and  $R + \mathrm{d}R$ , we obtain the specific angular momentum of the envelope after the merger as

$$l \simeq \frac{\mathrm{d}J_\mathrm{c}}{\mathrm{d}M} = \frac{\mathrm{d}J_\mathrm{c}/\mathrm{d}R}{\mathrm{d}M/\mathrm{d}R}.$$

For the Bethe's model, where M(R) is given by equation (19), this gives

$$l \simeq \frac{M_{\rm c}}{2} \left[ \frac{GR}{M(R)} \right]^{1/2} [1 + \ln(R/R_{\rm c})],$$
 (41)

which is smaller than the local Keplerian angular momentum provided  $M(R) > M_{\rm c}(1 + \ln R/R_{\rm c})/2$ . This suggests that if  $M_{\rm s} \gg M_{\rm c}$ , then only a small fraction of the common envelop is lost during the merger. For  $R = R_{\rm s}$ , this equation gives

$$l \simeq 5.2 \times 10^{18} \left(\frac{M_{\rm c}}{2\,{\rm M}_{\odot}}\right) \left(\frac{R_{\rm s}}{{\rm R}_{\odot}}\right)^{1/2} \left(\frac{M_{\rm s}}{10\,{\rm M}_{\odot}}\right)^{-1/2} {\rm cm}^2\,{\rm s}^{-1}.$$
(42)

In the  $\alpha$ -model, the accretion time-scale of the disc with such angular momentum can be estimated via

$$\begin{split} t_{\rm d} &\simeq \frac{1}{\alpha \delta^2} \frac{l^3}{(GM_{\rm s})^2} \\ &\simeq 8000 \, {\rm s} \left(\frac{\alpha \delta^2}{0.01}\right)^{-1} \left(\frac{R_{\rm s}}{{\rm R}_\odot}\right)^{3/2} \left(\frac{M_{\rm c}}{2\,{\rm M}_\odot}\right)^2 \left(\frac{M_{\rm s}}{10\,{\rm M}_\odot}\right)^{-7/2}. \end{split} \tag{43}$$

This is significantly longer than the duration of the stellar collapse (see equation 16). In fact, such a long time-scale suggests the possibility of explaining the phase of shallow decay and late flares in the X-ray light curves of LGRBs discovered by *Swift* (Chincarini et al. 2007; Zhang 2007).

To find the mass accretion rate as a function of time, we note that

$$\dot{M} = \frac{\mathrm{d}M}{\mathrm{d}t_\mathrm{d}} = \frac{\mathrm{d}M/\mathrm{d}R}{\mathrm{d}t_\mathrm{d}/\mathrm{d}R}.$$

Using equations (19, 43, 41) to evaluate dM/dR and  $dt_d/dR$ , we obtain<sup>8</sup>

$$\dot{M} \simeq \frac{2}{3} \frac{M_{\rm s}}{\ln(R_{\rm s}/R_{\rm c})} \frac{1}{t} \simeq 1.45 \left(\frac{M_{\rm s}}{10\,\mathrm{M}_{\odot}}\right) \left(\frac{t}{1\mathrm{s}}\right)^{-1} \frac{\mathrm{M}_{\odot}}{\mathrm{s}}.\tag{44}$$

Thus, on the time-scale of  $10^3 - 10^4$  s, the mass accretion rate is very low,  $\dot{M} \simeq 10^{-3} \div 10^{-4} \, \rm M_{\odot} \, s^{-1}$ , ruling out the neutrino mechanism and leaving the BZ mechanism clear favourite. Indeed, the maximum possible amount of magnetic flux that can be accumulated by the BH is given by the balance of magnetic pressure and the gas pressure of the accretion disc,

$$\frac{B_{\max}^2}{8\pi} \simeq P_{\rm g} \simeq \rho c_a^2,\tag{45}$$

where  $c_a$  is the sound speed. If we utilize, the model of  $\alpha$ -disc and estimate the magnetic field strength at the gravitational radius, then the corresponding magnetic flux will be

$$\Psi_{\rm max} \simeq 3 \times 10^{29} \left( \frac{\alpha \, \delta}{0.03} \right)^{-1/2} \left( \frac{M_{\rm b}}{10 \, {\rm M}_{\odot}} \right) \dot{M}_{\rm 1}^{1/2} {\rm G \, cm}^2,$$
 (46)

where  $\dot{M}_1$  is the mass accretion rate in the units of  $\rm M_{\odot}~s^{-1}$ . Even for  $\dot{M}_1$  as small as  $10^{-4}$ , this equation gives the substantial value of  $\Psi_{\rm max} \simeq 3 \times 10^{27} \, \rm G\,cm^2$ . The corresponding BZ power,  $\dot{E}_{\rm BZ} \simeq 2.2 \times 10^{49} \, \rm erg\, s^{-1}$ , is more than sufficient to explain the X-ray observations, allowing the magnetic field to be even weaker compared to the value suggested by equation (45).

# 6 DISCUSSION AND CONCLUSIONS

One of the main issues of this study was to investigate the efficiency of the tidal spin up in close massive binaries in the context of the collapsar model of LGRBs. In particular, we wanted to find out the typical masses and spins of the BHs formed during the collapse of WR companion. It turns out that the BH spin in this model is rather modest. For example, in the most optimistic case of a binary with the smallest possible orbital separation, the spin parameter of the WR star is relatively high,  $a_s \simeq 6$ , and one may have expected the BH to be rapidly rotating. However, we find that the spin parameter is only  $a \simeq 0.4$  at the time of the accretion disc formation, and  $a \simeq 0.8$  by the end of the stellar collapse, which is significantly lower than the maximally possible value a = 1. This is mainly due to the significant loss of angular momentum suffered by the mass accreted via the disc; the rejected angular momentum is either stored in the remote part of the accretion disc or removed by a disc wind. Indeed, as soon as the accretion disc is formed, the rate of accretion of angular momentum slows down significantly. Moreover, by the time of the disc formation the BH mass is already rather high, exceeding half of the progenitor mass prior to the collapse. Thus, the BH simply runs out of accreting matter before its rotation can approach the maximal possible rate (cf. Thorne 1974).

<sup>&</sup>lt;sup>8</sup> This equation is the same as equation (32), but *t* spans a different range of time-scales, now dictated by the disc accretion time.

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The mass accretion rate in this scenario is much lower compared to the usual  $\dot{M}=(0.1\text{--}1)\,\mathrm{M}_{\odot}\,\mathrm{s}^{-1}$  invoked in the standard collapsar model (MacFadyen & Woosley 1999). This makes the neutrino mechanism less attractive compared to the magnetic mechanisms, and the BZ-mechanism in particular. In fact, the very rapid decline in the efficiency of neutrino mechanism below  $\dot{M} \leq 0.02\text{--}0.05\,\mathrm{M}_{\odot}\,\mathrm{s}^{-1}$ (Popham et al. 1999; Zalamea & Beloborodov 2009) makes the explanation of the LGRB bursts with duration  $\geq 100\,\mathrm{s}$  rather problematic even within the standard collapsar model due to the low mass accretion rate expected on such time-scale (see equation 44 and Fig. 5).

However, the BZ mechanism could have its own difficulties in this scenario. Indeed, it requires very strong ordered magnetic field. For example, in order to provide the power of  $10^{50}\,\mathrm{erg}\,\mathrm{s}^{-1}$ , the BH of mass  $10\,\mathrm{M}_{\odot}$  and a=0.6 should accumulate the magnetic flux of order  $\Psi=8\times10^{27}\,\mathrm{G}\,\mathrm{cm}^2$ . The magnetic flux necessarily to activate the BZ mechanism soon after the formation of accretion disc is even higher. According to Table I, this is of the order of few  $\times\,10^{28}\,\mathrm{G}\,\mathrm{cm}^2$ . Perhaps the free-fall accretion rate set up in our simulation is a bit too high and could have been reduced by a factor of 10. However, according to equation (37), this would reduce the critical value of magnetic flux only by a factor of 3.

The origin of such strong field is not clear. It could be generated via magnetic dynamo in the accretion disc (e.g. Brandenburg et al. 1995) or in the convective core of the progenitor (e.g. Charbonneau & MacGregor 2001). It may also be inherited by the progenitor from the interstellar medium (ISM) during its formation (e.g. Braithwaite & Spruit 2004). The current status of both the stellar and disc dynamo theories does not really allow to make reliable conclusions. Even the issue of advection of externally generated magnetic field by the accretion disc on to the central BH is still unresolved (e.g. Bisnovatyi-Kogan & Ruzmaikin 1974; van Ballegooijen 1989; Spruit & Uzdensky 2005; Igumenshchev 2008; Rothstein & Lovelace 2008; Guan & Gammie 2009). There seems to be a general agreement that accretion discs produce mainly azimuthal magnetic field and unable to generate poloidal field on scales exceeding the disc height. The magnetic dynamo in convective cores of massive stars could be more promising in this respect. For example, from the results of Charbonneau & MacGregor (2001), it seems possible to generate up to  $\Phi \simeq 10^{28} \, \mathrm{G \, cm^2}$  in the convective cores of B stars.

By design, our numerical simulations cannot address the issue of magnetic field generation in accretion discs and strictly speaking deal only with the fossil model of magnetic field. The numerical results by Braithwaite & Spruit (2004) suggest that strong fossil field can relax to a simple ordered configuration with dipolar poloidal field on a relatively short time-scale, which makes our setup not that unrealistic. However, further studies are required to verify this model. The observations of massive stars do not support magnetic flux of order  $10^{28}\,\mathrm{G\,cm^2}$  and higher. The current record is held by  $\theta^1$  Ori C, whose dipolar magnetic flux  $\Psi \simeq 2 \times 10^{27}\,\mathrm{G\,cm^2}$  (Donati et al. 2002). One may speculate that most of the magnetic flux is hidden in the stellar interior. Indeed, the resistive time-scale across the extended radiative outer layers of massive stars exceeds their lifetime by many orders of magnitude Braithwaite & Spruit (2004).

The fact that the magnetic flux of NSs is less than  $10^{27}$  G cm<sup>2</sup> also seems to be working against the fossil hypothesis. However, NSs are collapsed compact Fe cores of massive stars. The typical cross-section of such a core is several orders of magnitude below that of the whole star and, thus, the core may account only for a small fraction of the total magnetic flux hidden inside the SN progenitor.

The host galaxies of LGRBs show strong evidence of enhanced star formation (Bloom et al. 1998; Sokolov et al. 2001; Fruchter et al. 2006). It is interesting that the recent observations of such starburst galaxies also indicate strong ISM magnetic field, in fact, up to 10 times stronger compared to the Milky Way (Beck & Krause 2005; Beck 2008). This suggests that magnetization of young stars in the host galaxies of LGRBs can be abnormally high as well.

Another interesting proposal stems from the theory of Sun's magnetic activity proposed by Uzdensky (2007). In particularly, he argued that fast reconnection can only operate in collisionless plasma and in the collisional regime the reconnection rate reduces to the much slower rate of Sweet–Parker. Since the collapsar plasma is collisional even in the rarefied funnel of the accretion disc then, according to this theory, the reconnection rate in the BH magnetosphere can be relatively slow. An additional unexplored factor in the LGRB context is the effects of quantum physics on magnetic reconnection. Indeed, the expected magnetic field strength is well above the quantum value of  $B_q = m_e^2 c^3/\hbar e = 4 \times 10^{13} \, \mathrm{G}$ . One may speculate that, under these conditions, the reconnection rate becomes even slower.

In the case of slow reconnection, the BH may be able to build strong magnetic field via collecting the alternating magnetic field generated in the accretion disc. Since the magnetic stresses are invariant with respect to change of magnetic polarity such striped structure of magnetic field has no effect on the efficiency of the BZ-mechanism. Further downstream of the LGRB flow, where its plasma becomes collisionless or the magnetic field becomes sufficiently weak, the reconnection accelerates. However, as long as this occurs beyond the Alfven surface, which for a BH with reasonable spin does not greatly exceed the gravitational radius, this does not disrupt the near magnetosphere of the BH and does not reduce the efficiency of the BZ-mechanism. Moreover, such delayed reconnection could promote bulk acceleration of the LGRB flow (Drenkhahn & Spruit 2002).

Finally, the neutrino heating of the polar region, not included in our analysis and simulations, may also play a very important role, by initiating the LGRB outflow and creating the low-density channel in the polar direction early on, when the mass accretion rate is still sufficiently high for effective neutrino-antineutrino annihilation. This would allow the BZ-mechanism to be activated along the field lines filling the channel even if the BH magnetic flux is much lower compared to the values quoted above. Later on, when the mass accretion rate drops and the neutrino mechanism can no longer provide sufficient power, the BZ-mechanism can take over the role of main driver of the LGRB flow. One may even contemplate the scenario where the GRB precursors are related to the neutrinodriven stellar explosions and the main bursts to the magnetically driven BH jets unleashed in the space cleared up by the blast. The delay between the two phases could be related to the disruption of the accretion flow by the SN blast (Wang & Meszaros 2007). Because of the rotational effects, the disruption may not be as severe in the equatorial direction, compared to the polar direction, as in the one-dimensional simulations by MacFadyen et al. (2001), leading to shorter fallback time-scales. The magnetic jets, though very powerful, could be less disruptive compared to the neutrino-driven jets because the magnetic hoop stress, associated with the azimuthal component of magnetic field, makes the sideways expansion of the jet cocoon less effective.

The most interesting, in view of the recent *Swift* observations of LGRB afterglows, version of the close binary scenario for GRB progenitors is the common envelope case, where the compact star, either a BH from the onset or a NS which eventually collapses into a

BH, spirals inside the normal WR star. The large angular momentum transferred to the external layers of the WR star quite naturally leads to long accretion time-scales,  $\simeq \! 10^4 \, \mathrm{s}$ . Thus, the central engine of LGRB jets arising in this scenario could operate for a sufficiently long time to explain the shallow phase of the X-ray light curves discovered by Swift (Zhang 2007). The X-ray flares, which are often seen during this phase, may result from the gravitational instabilities developing in this disc (Perna, Armitage & Zhang 2006). Although the BZ mechanism is not that sensitive to the mass accretion rate as the neutrino mechanism, some dependence is still expected. For example, equations (39) and (46) suggest that the power of the BZ mechanism may be proportional to the mass accretion rate. This can explain why the gamma-ray emission becomes undetectable on the time-scale of the shallow decay of X-ray afterglows.

The extremely high rotation rates, about 50 per cent of the break-up speed, assumed in the single progenitor model by Yoon & Langer (2005) and Woosley & Heger (2006) imply that in this model the outer layers of the collapsing star can also develop long-lived accretion disc. Indeed, in the 'showcase' model 16TI of Woosley & Heger (2006), the outer  $\simeq\!2\,M_\odot$  have the specific angular momentum increasing outwards from  $10^{18}$  to  $10^{19}\,{\rm cm^2\,s^{-1}}$  at the pre-SN phase. According to equation (42), this corresponds to the disc accretion time-scales of the order of  $10^4\,{\rm s}$ . However, such a long time-scale still rules out the neutrino annihilation as the mechanism for powering the collapsar jets.

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# APPENDIX A: EVOLUTION OF ANGULAR MOMENTUM AND MAGNETIC FIELD IN THE BETHE'S MODEL OF STELLAR COLLAPSE

The free fall model by Bethe (1990) approximates the kinematics of stellar collapse by the model:

$$\frac{dR}{dt} = \begin{cases} 0 & \text{if } t \le 0, \\ -(2GM(R)/R)^{-1/2} & \text{if } t > 0, \end{cases}$$

where *R* is the radius of collapsing shell and M(R) is the mass inside this radius. Since dM/dt = 0, this equation is easily integrated

$$R_0(R, t) = R[1 + t/t_{\text{ff}}(R)]^{2/3} = \text{const},$$

where  $R_0 = R(0)$  and  $t_{\rm ff}(R) = \sqrt{2R^3/9GM(R)}$  is the local free fall time.

Given the initial distribution of angular momentum,  $l_0 = \Omega(R_0 \sin \theta)^2$ , the conservation of angular momentum yields

$$l(R, t) = l_0[R_0(R, T)] = \Omega_s(R \sin \theta)^2 [1 + t/t_{\rm ff}(R)]^{4/3}.$$

Similarly, the conservation of magnetic flux requires

$$\Psi(R, \theta, t) = \Psi_0[R_0(R, t), \theta].$$

For the uniform initial magnetic field,

$$\Psi_0(R_0,\theta) = \pi B_0 \sin^2 \theta R_0^2.$$

Thus.

$$\Psi(R, \theta, t) = \pi B_0 \sin^2 \theta R^2 [1 + t/t_{\rm ff}(R)]^{4/3}$$

The poloidal magnetic field can be found via

$$B_p^i = \frac{1}{2\pi} e^{ij\varphi} \partial_j \Psi,$$

where e ijk is the Levi-Civita tensor of space. This gives us

$$B^{r} = \frac{B_0 \sin \theta \cos \theta}{\sqrt{\gamma}} R^2 \left[ 1 + \frac{t}{t_{\text{ff}}(R)} \right]^{4/3} \tag{A1}$$

an

$$B^{\theta} = \frac{B_0 \sin^2 \theta}{\sqrt{\gamma}} R \left[ 1 + \frac{\mathsf{t}}{t_{\rm ff}(R)} \right]^{1/3},\tag{A2}$$

where  $\gamma$  is the determinant of the metric tensor of space and the vector components are given in the non-normalized coordinate basis,  $\partial/\partial x^i$ . This approach has been used in Bisnovatyi-Kogan & Ruzmaikin (1974).

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