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Alpha clustering in nuclei: another form of shape coexistence?

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Abstract

Shape coexistence, where different deformed minima compete within a small range of excitation energy, appears to be ubiquitous across the chart of nuclides. In many light alpha-conjugate nuclei, experimental data points to the coexistence of highly deformed nuclear configurations. It has long been suggested, with strong theoretical justification, that these deformed states are attributable to nuclear clustering based on building blocks of alpha particles. This short review will consider how well alpha clustering fits within the shape coexistence canon and point to future opportunities for experiments that can place the topic on a firmer footing.

Keywords: shape coexistence, alpha clustering, nuclear models

(Some figures may appear in colour only in the online journal)

1. Introduction

Shape coexistence, where different deformed minima compete within a small range of excitation energy, appears to be ubiquitous across the chart of nuclides [1]. Considering nuclear structure in this way is an insightful approach to the complex behavior of the atomic nucleus. It emphasis the complexity in terms of coexistence of different configurations associated with widely differing nuclear deformations. Even in classic closed-shell 'spherical' nuclei like ^{40}Ca, deformed states lie very close to the ground state and this system is widely recognized as a classic example of nuclear superdeformation [2, 53], a phenomenon more commonly discussed in heavy nuclei [4]. Alpha-conjugate systems such as ^{40}Ca but more typically, lighter alpha-conjugate nuclei like ^{12}C have long been discussed as possible examples of nuclear clustering, where certain intrinsic states may reflect building blocks of alpha particles (or composites of alpha particles such as ^{12}C), not themselves fundamental particles, rather than building blocks of individual nucleons [5–8]. Such a picture finds justification through the strong binding of systems such as ^{12}C and the particularly special character of the alpha particle as the most bound nuclear system, whose first excited state lies above 20 MeV [6].

Starting with a simple two-dimensional cluster model which treats alpha-conjugate nuclei as based on subcomponents of rigid alpha particles leads to the prediction of a rich profusion of exotic nuclear shapes. Figure 1 gives an example of the complex, geometrical configurations predicted by such a model [9] including triangular, tetrahedral and even linear chains of alpha particles. The latter configuration of a six-alpha linear chain state was assigned to a highly excited resonance observed in ^{24}Mg [10] and this topic continues to attract considerable theoretical attention [11]. In practice, these simple alpha-cluster models must be an over-simplification, ignoring interaction with surrounding nuclear levels in regions of high-level density and treating alpha particles as simple rigid objects with no exchange of particles between them. Such alpha cluster models have become more sophisticated in recent years through models such as anti-symmetrized molecular dynamics [12, 13].

From the theoretical side, clustering is associated with a cluster model subspace of the full state space. A nuclear state may be said to exhibit clustering if its wave function is at least partially contained in the cluster subspace. The amount of clustering can then be defined as the expectation value of the projection operator that projects a state onto this cluster subspace [14]. A separate question is to what extent the clusters are spatially separated. A naive picture would be for the clusters to be separated but this concept of separation may be somewhat ambiguous. For example, Zhou *et al* have recently shown in the case of clustering in ^{20}Ne that the wave functions of well-separated two-cluster systems are almost exactly equal to wave functions apparently describing non-localized clusters [15]. Interrogating the details of the wavefunction and obtaining unambiguous evidence for clustering is equally challenging and often ambiguous from an experimental perspective. For the experimentalist, clustering is traditionally viewed as the predilection for a system to behave like a composite of clusters, which is manifested primarily, although not exclusively in its fragmentation properties.

The challenge in studying cluster states is that they are generally unbound and lie at high excitation energy. Ikeda *et al* introduced the concept of a hierachy of configurations based on alpha clustering appearing at the particle-breakup thresholds (see figure 2). The consequence of this from the experimental side is that most studies of clustering focus on nuclear reaction data and the identification of unbound resonances. This allows details relating to the spectrum of states to be obtained but makes it difficult to obtain more detailed structural information such as electromagnetic transitions in and between cluster bands.

In their recent review of shape coexistence, Heyde and Wood [1] briefly mention clustering in nuclei but the topic forms a very minor part of the review as a whole. A natural bridge between the two topics is the conception of deformed states arising from multi-particle/multi-hole configurations such as 4p−4 h and 8p−8 h, attributed, for example, to the low-lying superdeformed bands in ^{40}Ca [2]. Such configurations are an obvious analogy to excited configurations of alpha particles. Indeed, descriptions of the superdeformed bands in ^{40}Ca and neighbouring nuclei find description within particle-hole models, beyond mean-field models [18] and alpha clustering models among others [13, 19]. This raises the question as to how appropriate it is to consider the multi-faceted aspects of nuclear clustering as a wider manifestation of shape coexistence. In this brief review, we consider some contemporary topics in the clustering in nuclei and seek parallels with the more general phenomenon of

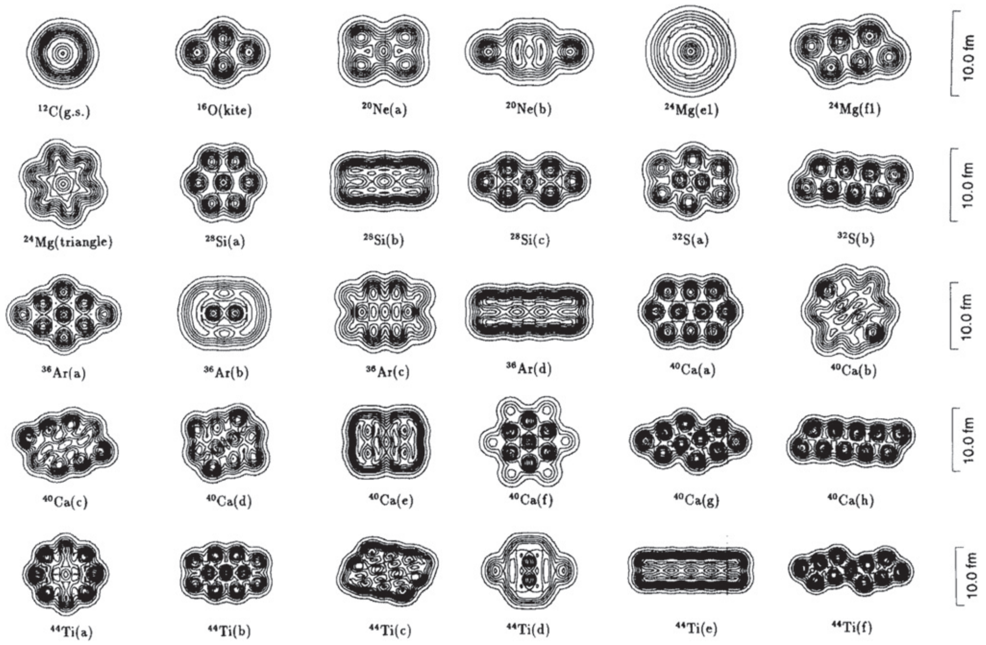

Figure 1. (left) Variety of intrinsic shapes predicted for alpha-conjugate nuclei from two-dimensional alpha cluster model calculations. Reprinted from [9], with permission from Elsevier.

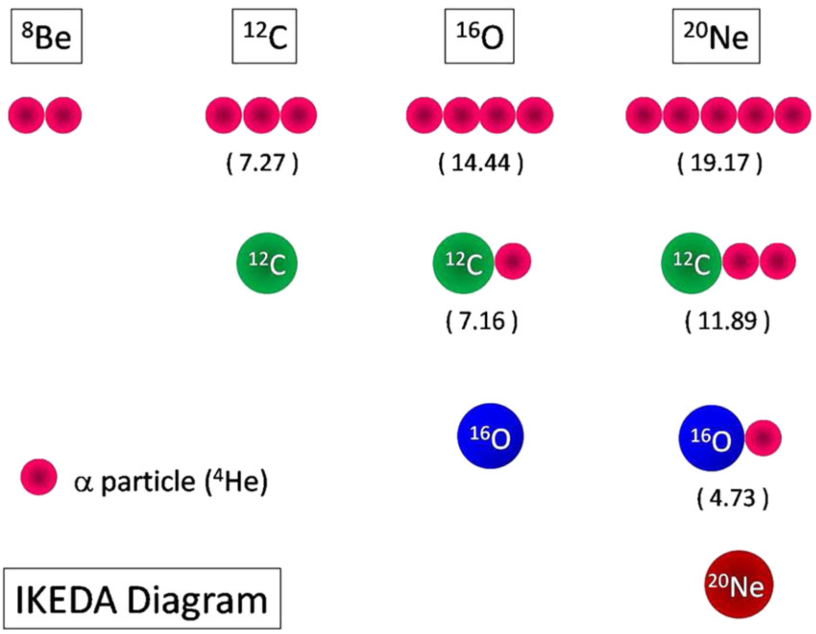

Figure 2. (left) The so-called 'Ikeda' diagram [16] showing how above particle-breakup thresholds, the structure of light alpha-conjugate nuclei can be thought of as comprised of alpha clusters. Reprinted from [17], with permission from IOP Publishing.

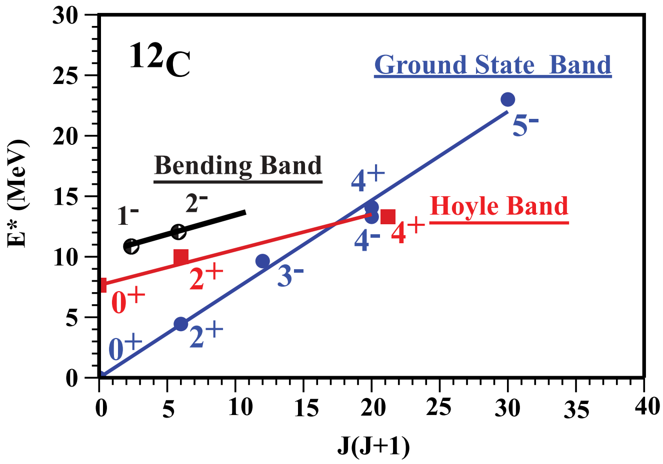

Figure 3. Proposed band structure for ^{12}C based on D_{3h} symmetry and correspondence with a subset of the known excited states in ^{12}C (taken from figure 4 of [28] with permission).

nuclear shape coexistence. We will look to the similarities and differences, and emphasize the strong experimental challenges in obtaining the data needed.

2. Geometric models

Emblematic of the story of clustering in nuclei is the so-called 'Hoyle state' in ^{12}C—an excited 0^+ state lying just above the three-alpha breakup threshold whose existence is essential to the fusion of alpha particles in massive stars without which carbon and life in the form we know it, could not exist. The enigmatic Hoyle state in ^{12}C has long presented a strong challenge to theory since it is as far removed as it could possibly be from being a simple shell model state. Cluster models can account for it at the low excitation energy where it is found but such models usually assume at the outset that the clustering picture is valid. In recent years, there has been an explosion of interest in ^{12}C from the theoretical perspective including the no-core shell model [20] and other models [21, 22]. Significantly, advances in computing power mean that calculations for ^{12}C are now possible within an *ab initio* model [23]. From the experimental side, the recent identification of an excited 2^+ state at around 10 MeV in ^{12}C has elicited particularly strong interest since it has been suggested to correspond to the excitation of the Hoyle state [24, 25]. This starts to provide key information towards answering questions on the character of the Hoyle state, which have been around since the 1950s: is it a deformed state? is it a linear chain of alpha particles? does it rotate [26]? A different window on this long-standing topic is whether simple geometrical models can account for the pattern of excited states in ^{12}C and ^{16}O. A parallel could be drawn with the phase-coexistence, critical-point interpretation of the structure of heavy nuclei [27] which is often in conflict with a shape coexistence picture. The utility of geometric models is clearly a topic of strong interest throughout the nuclear chart.

Recently, Marin-Lambarri *et al* discovered an additional 5^- state in ^{12}C and pointed to the near-degeneracy of excited 4^+ and 4^- states as supporting the expectations of the vibrational/rotational behavior corresponding to a triangular D_{3h} symmetry for ^{12}C [28]. (See figure 3). Similarly, Bijker and Iachello [29] recently performed a calculation of the rotation-vibration

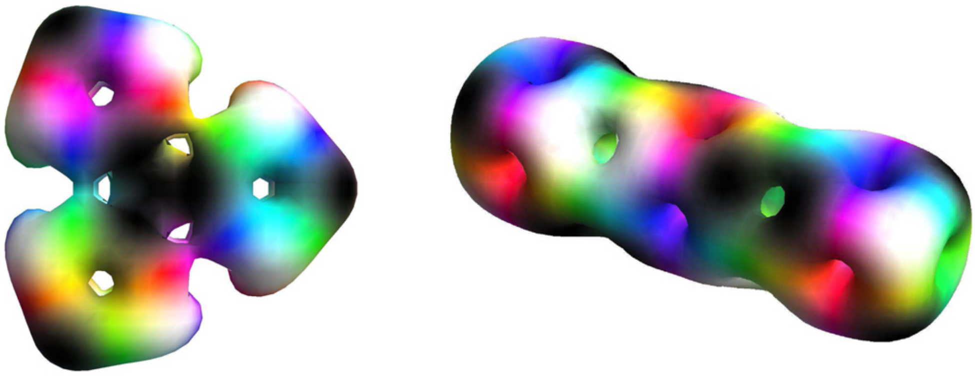

Figure 4. (left) Skyrmion solutions for Baryon number, $B = 12$ with D_{3h} (left) and D_{4h} (right) symmetries (adopted from figures 1 and 2 of [30] with permission).

spectrum of excited states in ^{16}O expected for a tetrahedral arrangement of alpha particles, and suggest that these show good correspondence to the experimental spectrum. In both the case of ^{12}C and ^{16}O, a number of states are omitted from the comparison due to their non-natural parity or non-zero isospin. This demonstrates the complexity of even these light alpha-conjugate nuclei and similar caution needs to be applied in the interpretation of heavy nuclei in terms of shape coexistence where states exist outside simple models and intrude and mix leading to difficulties in interpretation.

An interesting alternative to the more conventional nuclear models are recent calculations of topological solutions coming from the Skyrmion model [30]. These predict a D_{3h} symmetry for the ^{12}C ground state and D_{4h} for the Hoyle state (see figure 4). This is clearly in contradiction to the geometric mode discussed above [28] and would point to a linear alpha chain structure for the Hoyle state. If such different interpretations can give equally reasonable reproductions of the experimental spectra then more discriminating tests of the different models will be necessary. We will return to this topic below under the discussion of electromagnetic transitions.

3. Excited 0^+ states as shape-coexisting bandheads and E0 strengths

One of the characteristic manifestations of nuclear shape coexistence is a multiplicity of excited 0^+ states close to the ground state; the famous example being ^{186}Pb where an alpha decay study of ^{190}Po revealed a triplet of low-lying 0^+ states, including the ground state, attributed to spherical, oblate and prolate minima [31]. Similar examples exist across the nuclear chart such as the first excited state with spin/parity 0^+ in the exotic proton-rich nucleus, ^{72}Kr [32]. Such low-lying 0^+ states are coupled by $E0$ transitions. These transitions have a special status since single-photon decay is ruled out for $E0$ multipolarity and transitions must proceed by internal conversion or, for high energy transitions, by internal pair transitions (or to second order, by two-photon emission). $E0$ transition strengths are of particular significance in the interpretation of shape coexistence since they may be related to difference in the mean-square charge radius of the two different configurations [33].

In light alpha-conjugate nuclei, low-lying 0^+ states are strongly in evidence and in many cases $E0$ transitions strengths and branching ratio to competing $E2$ transitions is known (see table 1). In general, the $\rho^2(E0)$ values of these alpha-conjugate nuclei (excepting ^{32}S) are large

Table 1. Table of $E0$ transitions in light alpha-conjugate nuclei—the data is taken from a recent review [34]. The $E0$ transition energy is presented alongside the transition energy for a competing $E2$ transition where such exists. In the case of ^{16}O and ^{40}Ca the first excited state is the second 0+ state. $\chi(E0/E2)$ is the dimensionless ratio of the absolute transition rate B($E0$) for the 0_i^+ to 0_f^+ transition to B($E2$), the absolute transition rate for the 0_i^+ to 2_1^+ transition. The $E0$ strength is presented as the squared value of the monopole transition strength multiplied by a factor 1000.

Nucleus	Transition	$E0$ (keV)	$E2$ (keV)	$\chi(E0/E2)$	$10^3 \times \rho^2(E0)$
^{12}C	$0_2^+ \rightarrow 0_1^+$	7654	3215	2.4(4)	500(81)
^{16}O	$0_2^+ \rightarrow 0_1^+$	6049	—	—	153(22)
^{20}Ne	$0_2^+ \rightarrow 0_1^+$	6725	5091	3.7(13)	364(136)
^{20}Ne	$0_3^+ \rightarrow 0_2^+$	7191	5557	18(6)	298(123)
^{24}Mg	$0_2^+ \rightarrow 0_1^+$	6433	5064	20(3)	294(19)
^{28}Si	$0_2^+ \rightarrow 0_1^+$	4980	3200	0.91 (13)	262(31)
^{32}S	$0_2^+ \rightarrow 0_1^+$	3778	1548	0.047(9)	19(5)
^{40}Ca	$0_2^+ \rightarrow 0_1^+$	3352	—	—	25.6(7)

and far exceed simple shell model estimates of $E0$ strengths in a given shell e.g. sd shell (see [33] for derivation of expected shell model $E0$ strengths). The large $E0$ strengths hence support a shape coexistence picture with deformed configurations based on e.g. 4p−4 h excitations.

The most famous example of a low-lying 0^+ states is the state at 7.654 MeV in ^{12}C—the so-called 'Hoyle state' discussed above. In fact, it is the weak E2 decay of this state (see table 1) that leads to the fusion of ^{12}C in massive stars so, in some sense, the details of the nuclear structure involved are important and the co-existence of the ground state and 'Hoyle state' minima defines the critical parameter from the perspective of nuclear astropysics.

The theoretical literature is rich with predictions of excited 0^+ states as bandheads of strongly deformed cluster configurations, for example in ^{16}O [39, 38] and ^{24}Mg [37]. Many of the relevant states lie at very high excitation energy and experimental data on their properties is limited. A promising technique to address the question of whether excited 0^+ states correspond to the bandheads of cluster configurations is to use the (α, α') reaction at high energies (∼100 MeV) and analyse the states populated with a large spectrometer. Two such systems worldwide are suitable for such studies—the K600 spectrometer at iThemba Laboratory in South Africa and the Grand-Raiden spectrometer at RCNP in Osaka. The break-up of high-lying states can be addressed by adding silicon detectors within the reaction chamber while transition strengths could be obtained by adding germanium detectors to measure de-exciting gamma rays. Experiments of this type have been initiated recently at both iThemba and RCNP. In principle, it would be of high interest to measure $E0$ transitions connecting such excited 0^+ states. Such data exists, for example, for the Hoyle state, where there is high precision on the pair-decay width of the state from electron scattering measurements, which can be used to extract the $E0$ transition strength [35, 36]. Information on pair decay of high-lying 0^+ states in other alpha-conjugate nuclei is likely to be very difficult to obtain given the low probability of this branch against the relatively broad particle decay widths of the states.

4. Electromagnetic transitions

Electromagnetic transitions are a discriminating and well-established test of various models regarding shape coexistence. In the more usual canon of shape-coexisting nuclei of medium and heavy mass, the states of interest are bound and decay by gamma-ray emission. This affords a range of relevant techniques to determine transitions strengths both within rotational bands and between different configurations. Such techniques include lifetime measurements but a particularly powerful technique is Coulomb excitation. Coulomb excitation not only informs on the strength of the collectivity in different rotational configurations but also through the second order reorientation process can give information on the sign of the spectroscopic quadrupole moment which is related to the sign of the nuclear deformation. This latter technique was established many years ago in studies of light stable nuclei like ^{24}Mg and ^{28}Si [40] but in the past decade has been increasingly applied to exotic nuclei with Coulomb excitation of accelerated radioactive beams [41]. The nature of cluster configurations, classically, is that they lie on or just beyond the particle-breakup thresholds. This means that the electromagnetic decay branch is a small fraction of the decay of the state. To give a concrete and typical example, consider the simplest and perhaps most convincing example for alpha-clustering in light nuclei, namely, ^{8}Be. Here, even the ground state is suggested to be clustered within various cluster models [42, 43] and the clustering appears naturally in the intrinsic state from *ab initio* calculations [44]. This means that that the intrinsic structure of ^{8}Be may be pictured as a dumbbell-shaped configuration of two alpha particles. This rotating object has a high moment-of-inertia associated with strong transitions between excited states. The transition strength for the $4^+ \rightarrow 2^+$ transition has been obtained in a brute-force measurement and is consistent with both alpha-cluster and *ab initio* models [45, 46]. An intriguing prediction is that B(E2) for the $2^+ \rightarrow 0^+$ transition should be a factor of four larger than for the $4^+ \rightarrow 2^+$ transition, which would suggest that the configuration is contracting as a function of angular momentum [47]. Confirming this would be a spectacular verification of the alpha-clustering model but the branching ratio of the γ-ray transition from the 2^+ state may be as small as 10^{-9}. A further possibility would be to the measure an internal transition within the 4^+ resonance as this would provide information on the sign and magnitude of the spectroscopic quadrupole moment.

Electromagnetic transitions are also of considerable interest in tracing clustering in heavier systems. Returning to the example of the Hoyle state in ^{12}C, it is clear that discriminating between different models will require more than the observation of patterns of energy levels, and that information on transition strengths would be of high value. Matrix elements from the ground state to the first $2^+, 3^-$ and 4^+ states in ^{12}C are known from electron scattering but not for states of interest to the alpha clustering picture. There are a number of calculations including recent *ab initio* models [23, 48] which make testable predictions on the electromagnetic transition between the 2^+ and 0^+ state in the Hoyle-state band. At present the perspective for observing this transition is poor as even populating the 2^+ state and disentangling it from other states such as the nearby broad 0^+ resonance is extremely challenging. It has been demonstrated though that a clean way to excite the state is with a real photon beam [25] since a real photon beam cannot excite the neighbouring 0^+ state. The ELI-NP faciity under construction in Bucharest will provide very intense, monoenergetic gamma-ray beams. This may provide the scope in the longer term to measure the strength of the 2^+ and 0^+ transition in the Hoyle-state band and compare with theory.

As discussed earlier, there is good evidence for superdeformed bands, associated with cluster configurations, in ^{40}Ca and neighbouring nuclei. Such configurations are also predicted in lighter alpha-conjugate nuclei such as ^{24}Mg, ^{28}Si and ^{32}S. Such structures have largely eluded identification due to their lying at high excitation energy, compared to ^{40}Ca where their bandheads are relatively close (a few MeV) to the ground state. Turning to one contemporary example, the alpha-conjugate nucleus, ^{28}Si provides a well-known example of shape coexistence with predictions of an oblate ground state band in competition with an excited prolate band [49]. At higher excitation energy, cluster models predict a superdeformed (SD) band [12, 49] largely based on ^{24}Mg+α clustering (see figure 5). A review of the

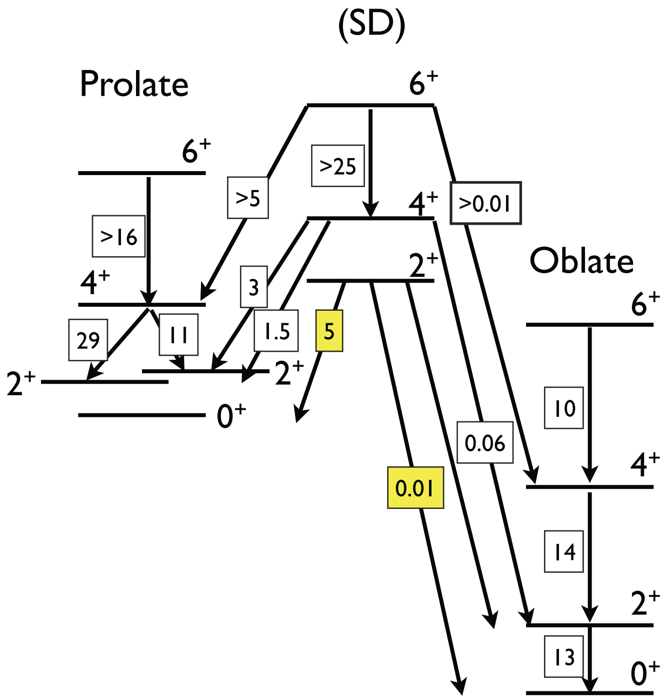

Figure 5. Current state of knowledge of EM transition strengths connecting states in the candidate SD band in ^{28}Si with the known oblate ground state band and excited prolate band (adopted from figure 5 of [50] with permission. Transition strengths are in Wu.).

experimental literature [51] on ^{24}Mg(α, γ) suggests possible candidates for the 2^+, 4^+ and 6^+ states in the SD band and this appears to be in good conformity with the results of recent experimental work using the Gammasphere array [50]. Confirming the hypothesis that these states are indeed part of the SD configuration will require improved knowledge on electromagnetic transition stengths. The 6^+ to 4^+ transition in the candidate SD band is known to have B(E2) $>$ 25 Wu [51] (see figure 5). This lower limit is significantly smaller than cluster model predictions [12, 49]. New data is clearly warranted to discriminate between different nuclear models, including cluster models, in this region. However, it is clear that more standard approaches such as fusion-evaporation reactions struggle to populate the states of interest, simply due to the reaction mechanism which prefers the population of yrast or near-yrast states. Moreover, the states of interest are alpha-unbound and have strong competition with particle-breakup channels. An overlooked opportunity for future progress on this topic would be to use heavy transfer reactions. For example, figure 6 shows the states populated in the ^{32}S(^{12}C, α) reaction with a beam energy of 30 MeV, analysed with a spectrometer [53]. This work precedes the identification of the ^{40}Ca SD band in fusion-evaporation [2] by 30 years. The spectrum shows strong selectivity of the two strongly deformed bands in ^{40}Ca associated with 4p$-$4 h and 8p$-$8 h configurations (or single and multi-alpha clustering depending on the model framework considered). Two of the strongest states produced in this reaction (see figure 6) are the 8^+ states in the strongly deformed bands [2] although this was not appreciated at the time. Moreover, there is a hierachy in the population mechanism in that the 8p$-$8 h states are more strongly populated than the 4p$-$4 h ones. Similar interesting population of ^{28}Si SD states is seen in a similar study of the ^{20}Ne(^{12}C, α) reaction by Kubono

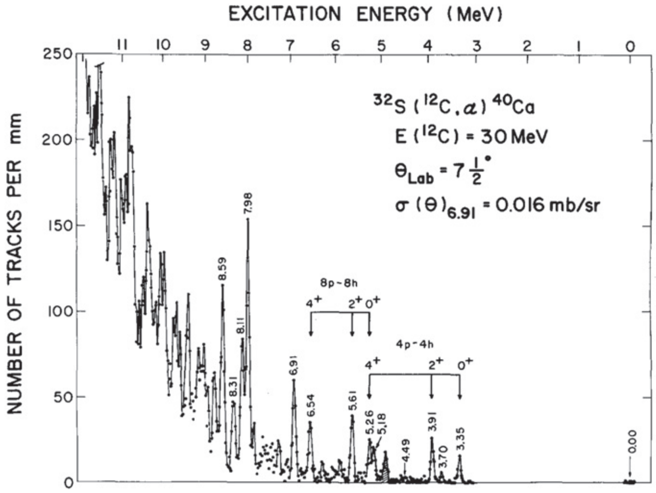

Figure 6. Spectrum of states populated in the ^{32}S(^{12}C, α) reaction at 30 MeV. Reprinted from [53], with permission from Elsevier.

et al [52]. This provides a strong steer that if we are to understand the details of the structure of states in the 'Ikeda' region above the particle-thresholds, then these neglected class of reactions are of high value. With the advent of efficient and high-resolution germanium detector arrays, this affords a clear opportunity to populate such states and define their decay branching and its associated electromagnetic transition strengths in detail. In-band transitions in e.g. ^{28}Si are of the highest interest and these branches will be small (at the 10^{-4} or less level) given competition with both out-of-band decay and particle break-up. However, the strong selectivity and simplicity of the resulting gamma-ray spectrum should simplify matters and some efforts should be made to consider this approach in the near future.

A further area where electromagnetic transitions would be of high value in support of the cluster model is in the case of resonances in the ^{12}C–^{12}C reaction. Oscillatory behavior was first seen in this reaction over fifty years ago in the fusion channel [54] and (in)elastic scattering [55]. These resonances in the ^{12}C+^{12}C system have commonly been associated with a non-statistical molecular origin on top of a statistical background [56, 57]. The associated molecule would have a very strong deformation and constitute an excellent of shape coexistence. The ^{12}C+^{12}C resonances are not merely of narrow interest from a nuclear structure perspective, they impact strongly on our understanding of fusion in massive stars [58]. Resonances have been observed to very low energies, approaching the upper limit of the Gamow window ($E_{\mathrm{Gamow}} = 1.5$ (0.3) MeV for a temperature of $T = 5 \times 10^8$ K). Resonances in the fusion cross-section in this astrophysical region would have dramatic consequences on ^{12}C burning at thermonuclear energies in the later phases of massive stars. Indeed, such resonances in the ^{12}C+^{12}C fusion cross section appear to persist as far as it has been practical to make measurements, the current limit being $E_{c.m.} = 2.1$ MeV [59].

Despite the long-standing discussion of the ^{12}C+^{12}C resonances within the cluster model, the discriminating evidence in favour of this model has not been demonstrated namely identifying electromagnetic transitions within the molecular band. Such an identification would not be easy. The situation is significantly more complex than the example of ^{8}Be discussed above, with the strength apparently spread around between multiple resonances of the same J^{π}. Nevertheless, there has been an attempt to directly observe such transitions in the

^{12}C+^{12}C system by Haas *et al* focussing on transitions between 10^+ and 8^+ resonant states [60]. Position sensitive silicon detectors were mounted at the center of the Chateau de Cristal array of barium fluoride detectors. Triple γ-^{12}C–^{12}C coincidences were recorded and it was possible to observe a few events in the expected energy window corresponding to the $10^+ \rightarrow 8^+$ transition [60]. However, the data were not sufficiently clean to rule out these events as due to experimental background [60]. It would be very interesting to revisit this earlier experiment taking advantage of new experimental techniques and developments in detector technology for the detection of γ-rays and/or of fragments. For example, novel scintillator materials like lanthanum bromide offer superior energy resolution for the gamma ray of interest while improved silicon detector performance and solid angle coverage could lead to a significant improvement in sensitivity and statistics.

5. Conclusion

In conclusion, clustering in nuclei is a topic of long-standing which continues to attract substantial interest both from theory and experiment. There are many strong parallels with the topic of shape coexistence since clustering is naturally associated with strong nuclear deformations and competition between different shape minima. Much of our information on clustering rests only on a spectrum of states derived from nuclear reactions such as (in)elastic scattering. These spectra are often complex with an observed splitting of states associated with different deformed minima, along with mixing and interaction with non-clustering states. Analysis of shape coexistence has traditionally relied on key observables such as the presence of excited 0^+ states and their peculiar $E0$ decay, along with strengths of both in-band and out-of-band electromagnetic transitions. As discussed in this article, with a few exceptions, such data are significantly more challenging to obtain even though most of the nuclei of interest are stable isotopes. This is because the states of interest lie at high excitation energy and their electromagnetic decay competes unfavourably with particle-emission. Nevertheless, significant opportunities exist for obtaining new and discriminating data on this topic, with specific examples outlined in this review.

Acknowledgments

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