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The Capacitated Lot Sizing Model: a Powerful Tool for Logistics Decision Making

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Abstract

Starting from the seminal intuitions that led to the developments of the Economic Order Quantity model and of the formulation of the Dynamic Lot Sizing Problem, inventory models have been widely employed in the academic literature and in corporate practice to solve a wide range of theoretical and real-world problems, as, through simple modifications to the original models, it is possible to accommodate and describe a broad set of situations taking place in complex supply chains and logistics systems.
The aim of this paper is to highlight, once more, the powerfulness of these seminal contributions by showing how the mathematical formulation of the Capacitated Lot Sizing Problem can be easily adapted to solve some further practical logistics applications (mainly arising in the field of coordination of transportation services) not strictly related to manufacturing and production environment. Mathematical formulations and computational experiences will be provided to support these statements.

1. Introduction

The history of inventory problems can be rooted back to the Economic Order Quantity (EOQ) model presented by Harris (1913), also known as the Wilson Lot Size formula, since it was firstly used in practice by Wilson (1934). The EOQ model assumes the presence of a single item whose demand is continuous (with a constant known rate) and an infinite planning horizon. The solution of the model is easy and provides the optimal quantity to be ordered, balancing the setup and inventory holding costs. However, with the same assumptions, in presence of multiple items and capacity restrictions the model becomes NP-hard (Hsu, 1983).
The Dynamic Lot Sizing Problem (in the following, generically referred to as *DLSP* or *Lot Sizing*), first proposed by Wagner and Whitin (1958), can be considered as an extension of the EOQ model. In this new version, on a discrete time scale, deterministic dynamic demand and finite time horizon are considered while the objective function is the same basic trade-off between setup and inventory costs.
Starting from these seminal papers, further variants of the problem have been introduced. These are mainly concerned with the extension to the multi-item case (Barany et al., 1984), the introduction of several conditions about the costs, limitations on production capacities (leading to the *Capacitated Lot Sizing Problem*, in the following CLSP) (Bitran et al., 1982) and possible additional features regarding, for instance, demand uncertainty (Brandimarte, 2006), setup costs and/or times (Trigeiro et al., 1989), linked lot sizes (Suerie and Stadtler, 2003), alternative suppliers (Basnet and Leung, 2005). Combinations of these aspects can provide models with very different complexities.
Interesting reviews about models and methods to tackle Lot Sizing problems have been published by Kuik et al. (1994), Drexl and Kimms (1997), Karimi et al. (2003), Jans and Degraeve (2008), while a rich textbook on the topic has been provided by Pochet and Wolsey (2006).
Jans and Degraeve (2008) compiled a very interesting and complete survey devoted to describe actual and potential variants of the problem. The authors highlight how most of them are inspired by specific real life applications and, in particular, they focus on a variety of industrial production planning problems. Still nowadays, applications of the Lot Sizing model, and its variants, to real-world problems constitute a very active research strand (see, for instance: Rezaei and Davoodi, 2011; Ferreira et al., 2012; Liao et al., 2012).

In this paper we want to show how, through an appropriate interpretation of the elements of the model, lot sizing formulations can also be effectively used to face further practical logistic problems, outside of the classical field of production and manufacturing planning. Therefore, rather than providing original models, the aim of the paper is to show how standard formulations can be used to support decisions in other contexts of applications; in this sense, more established these models are, more powerful and insightful will be their adaptation, as existing results in terms of formulations and solution approaches can be easily exploited.
The remainder of this paper is arranged as follows. In the next section we introduce the mathematical model of the CLSP, considering the single item and the multi-item versions. Then we illustrate a general framework indicating how these models can be used to describe different kinds of logistic problems. In particular three specific examples are introduced and discussed: the optimisation of the departure schedule for a bus terminal; the management of a logistic cross-dock platform; the optimisation of an airport check-in gates configuration. For the above problems, we explain how they can be formulated, through few adaptations, starting from the CLSP model. Furthermore, some case studies (related to real-world situations) are presented, showing how these models can be solved in limited computational times and be used as decision support tools. Finally, some concluding remarks and directions for future research are drawn.

2. Mathematical Models for Lot Sizing Problems: Generalities

By denoting with $t \in \{1..N\}$ one of the N time buckets introduced to divide the planning horizon, the following parameters can be considered, referred to the specific time period *t* and to a single product scenario:

- d_t the demand forecast;
- p_t the unit production or purchasing cost;
- h_t the unit inventory cost;
- f_t the fixed setup or ordering cost;
- C_t the maximum feasible lot size (capacity).

Introducing the variables:

- s_t stock at the end of period *t*;
- x_t quantity to be produced or ordered during period *t*;
- y_t binary variable equal to 1 if units of the product are manufactured (or ordered) in period *t* (0 otherwise).

the DLSP can be formulated as follows:

$$\min \ z = \sum_{t=1}^{N} (p_t x_t + h_t s_t + f_t y_t) \qquad (1)$$

s.t.

$$s_t = s_{t-1} + x_t - d_t \qquad t = 1,..,N \qquad (2)$$

$$s_t = 0 \qquad t = 0 \text{ and } t = N \qquad (3)$$

$$x_t \leq C_t y_t \qquad t = 1,..,N \qquad (4)$$

$$s_t \geq 0; x_t \geq 0; y_t = 0/1 \qquad t = 1,..,N \qquad (5)$$

The objective function (1) represents the total management costs, including the production (and/or purchasing), inventory and setup or ordering costs. Constraints (2) reproduce the demand satisfaction and inventory balance constraint for each period. Conditions (3) impose that inventory levels at the beginning and the end of the planning horizon are equal to zero.

Constraints (4) allow a positive production (constrained between 0 and a value C_t) in period t if and only if the setup variable is equal to 1; in particular, the problem turns out to be uncapacitated for large values of C_t ($C_t \geq \sum_{k=t,N} d_{k,}$ for every specific time period t). Constraints (5) express the non-negativity and binary restrictions on the variables. As known, model (1)÷(5) has O(N) constraints in O(N) variables.

Wagner and Whitin (1958), in order to avoid trivial solutions to the problem, introduced a condition on the production and inventory costs, i.e. $h_t + p_t - p_{t+1} \geq 0$ (Wagner-Whitin cost condition). This condition assures that if setups occur in both periods t and t+1, it is more convenient to produce directly in period t+1, as there is no speculative reason for early production (Pochet and Wolsey, 1995). Zangwill (1966) further clarified that inventory costs h_t and setup/ordering costs f_t should be intended as non-negative.

Zangwill (1969) provided an interesting and fruitful interpretation of the problem as a fixed charge network problem. In Figure 1, a network representation for a generic instance with N periods is provided. The flow on the generic arc $(0, t)$ represents the production in period t (x_t) while flow on arc $(t, t+1)$ reproduces the stock at the end of period t (s_t). This way, constraints (2) can be interpreted as flow conservations conditions at each node t while constraints (4) indicate that, in presence of flow on arc $(0, t)$, this value cannot exceed the capacity of this arc. In practice the problem consists in defining the production inflows (x_t) able to satisfy the outflows (d_t) with the minimum cost, also using possible holdover flows accumulated in the previous periods (s_{t-1}).

This representation can also be used by reversing flows x_t and d_t as shown in Figure 2. In this case, outflows (x_t) have to be determined in order to absorb the sum of the demand inflows (d_t) and of holdover flows from the previous period (s_{t-1}). Of course, in this case, in the formulation, constraints (2) have to be written reversing the signs of the variables x_t and parameters d_t, providing the following:

$$s_t = s_{t-1} - x_t + d_t \qquad t = 1,..,N \qquad (2')$$

Insert Figure 1 here

Insert Figure 2 here

In the case of a multi-item problem, introducing the index $j \in \{1..M\}$ representing one of the M items whose production has to be planned, and considering each parameter and variable indexed by both j and t, the formulation of the Multi-item DLSP, also known as Capacitated Lot-Sizing Problem (CLSP) becomes:

$$\min \ z = \sum_{t=1}^{N} \sum_{j=1}^{M} \left(p_{tj} x_{tj} + h_{tj} s_{tj} + f_{tj} y_{tj} \right) \qquad (6)$$

s.t.

$$s_{tj} = s_{t-1,j} + x_{tj} - d_{tj} \qquad t = 1,..,N; j = 1,..,M \qquad (7)$$

$$s_{tj} = 0 \qquad t = 0 \text{ and } t = N; j = 1,..,M \qquad (8)$$

$$x_{tj} \leq C_{tj} y_{tj} \qquad t = 1,..,N; j = 1,..,M \qquad (9)$$

$$\sum_{j=1}^{M} a_j \, x_{tj} + \sum_{j=1}^{M} b_j \, y_{tj} \leq R_t \qquad t = 1,..,N \qquad (10)$$

$$s_{tj} \geq 0; x_{tj} \geq 0; y_{tj} = 0/1 \qquad t = 1,..,N; j = 1,..,M \qquad (11)$$

The objective function and all the constraints, except (10), simply represent the extension of the expressions (1)÷(5) to the multi-item case. Assuming that a_j is the capacity consumed for the production of one unit of item j, b_j is the capacity consumed for the setup of item j and R_t the total available capacity in period t, conditions (10) represent resource capacity constraints. As pointed out by Karimi et al. (2003), although the setup costs may vary for each product and each period, in general they are sequence independent. However, it is also possible to define some variants of the CLSP, where setups are sequence dependent (also known as *complex setup structure*) (see, for instance, Haase and Kimms, 2000; Kovács et al., 2009).

Further constraints can be added to the model in order to describe different production mode options. Examples of these constraints can be the following:

$$\sum_{j=1}^{M} y_{tj} \leq K_t \qquad t = 1,..,N \qquad (12)$$

$$\sum_{j=1}^{M} s_{tj} \leq S_t \qquad t = 1,..,N \qquad (13)$$

$$s_{tj} \leq \sum_{k=1}^{\delta} d_{t+k,j} \qquad t = 1,..,N-\delta; j = 1,..,M \qquad (14)$$

$$y_{t+\lambda,j} \leq (1 - y_{tj}) \qquad t = 1,..,N-\lambda; j = 1,..,M \qquad (15)$$

$$y_{t+1,j} \geq y_{tj} \qquad t = 1,..,N-1; j = 1,..,M \qquad (16)$$

Constraints (12) assume that at most K_t setups per period are allowed, while (13) express a limitation to the total inventory level in each period.
Imposing an upper bound to the inventory level for item j at the end of period t, based on the sum of the demand of δ periods successive to t, constraints (14), in environments that operate according to a strict First-in-First-out (FIFO) logic, assure a maximum duration (δ) for the inventory level.
Conditions (15) impose a minimum interval λ between two consecutive setups for the production of item j; while conditions (16) impose that once the production (or the purchase) of an item j has been started, it will continue until the end of the planning horizon; these constraints are useful to represent semi-continuous production processes.
The reverse representation in terms of network flow problem implies the association of a commodity with each item; also in this case, in the formulation, constraints (7) have to be written reversing the signs of variables x_{tj} and parameters d_{tj}, providing the following equation:

$$s_{tj} = s_{t-1,j} - x_{tj} + d_{tj} \qquad t = 1,..,N; j = 1,..,M \qquad (7')$$

Coherently, constraints (14) should be reformulated as follows, by replacing d_{tj} with x_{tj}:

$$s_{tj} \leq \sum_{k=1}^{\delta} x_{t+k,j} \qquad t = 1,..,N-\delta; j = 1,..,M \qquad (14')$$

In this case, the upper bound for the inventory level at the end of each period t, is defined as the sum of the units to be processed during the δ periods successive to t. Constraints (14') can be then interpreted as constraints (14).

3. Adaptation of CLSP models to logistics applications

As mentioned in the literature review, the above-cited models have been applied, in the last decades, to solve a variety of problems in the field of inventory management; accordingly, many variants of the basic mathematical formulations have been developed. It can be noticed, however, that the structure of the models can be easily adapted even to fields not strictly related to inventory management, belonging to a wider logistics context. Indeed, the Capacitated Lot Sizing model can be regarded as a general model of flow control, through which various optimization problems can be described and formulated. In particular, by interpreting the index j as representative of a logistic service, rather than of an item, it is possible to revisit the meaning of the variables and of the parameters, as reported in Table 1. The model allows solving generic dimensioning and synchronization problems related to logistics services, in which the demand for a given service j in a time instant t (d_{tj}) is known a-priori. It can be seen that the adaptation of all the elements of the basic version of the model is straightforward, as parameters and decision variables typical of the CLSP (inflows, outflows, holdover flows) can be appropriately interpreted in order to describe the specific applications. In particular, variables x_{tj} and s_{tj} refers, respectively, to the demand for service j to be satisfied in period t, and to the residual demand for service j at the end of period t, while variables y_{tj} represent the activation of service j during period t.

Insert Table 1 here

In the following we describe three applications derived from different logistic fields. Even if apparently quite different, all of them can be modeled through the general CLSP, by utilising the revised meaning of variables and parameters described in Table 1 and performing some simple adaptations. For all the cases we suppose that the time horizon is partitioned into N buckets of duration τ each. Then, it is possible to describe the problems using the formulation (6)÷(11) and its representation in terms of reverse multi-commodity network flows (Figure 3). We also assume, for all the models, non-negative setup costs $(f_t \geq 0 \; \forall \, t)$ and the Wagner-Whitin cost condition.

3.1 The Bus Terminal Schedule Optimization Problem

Suppose we have a bus transit terminal, i.e. a facility in a transportation system, where lines starting from a set of origins converge and users can access departing lines in order to reach a set of destinations. A crucial aspect in managing the terminal is represented by the schedule of output lines towards the set of most common destinations. Indeed, there is a need to find trade-off solutions considering the minimization of the activation costs of the output lines and the total users' waiting costs.

The problem known as aperiodic schedule synchronization problem has generally been solved through the adaptation of models and methods for the periodic case (see for instance Liebchen and Stiller, 2009; Michaelis and Schöbel, 2009) while only few specific mathematical models have been developed to explicitly solve the problem (Wong et al., 2008; Schmidt and Schöbel, 2010).

Through a simple adaptation process, CLS models can provide a straightforward way of representing the problem. Indeed, here d_{tj} is the number of passengers arrived at transit terminal at time t and directed to one of the M destinations j, while y_{tj} is a binary variable equal to 1 if a bus leaves the terminal at time t towards destination j. Assuming a reverse network flows representation, the problem can be viewed as the determination of passengers leaving the terminal at each time t towards destination j (x_{tj}) through the departure of outbound buses of given capacity C_{tj} (that can be assumed to be constant; therefore $C_{tj} = C$). Within this interpretation, s_{tj} variables represent passengers waiting in the terminal to leave toward destination j at the end of period t.

Assuming $p_{tj} = 0$, the objective function (6) describes a performance measure defined as the sum of the costs associated with users' waiting times and the costs associated with departing lines activation. In particular, in absence of unit production costs ($p_{tj} = 0$), the Wagner-Whitin condition implies $h_{tj} \geq 0 \; \forall \, t, \forall \, j$.
Also in this case the formulation of the model includes flows (passengers) conservation constraints (7'), while conditions (8) simply indicate that no passenger must be in the terminal at the beginning and at the end of the planning horizon. Equations (9) reproduce constraints associated with the capacity of buses leaving at time t with destination j.
In the model, also constraints (12), (13), (14') and (15) can be introduced. The first group imposes that a maximum number of buses (K_t) can leave during the same period t; this may also reproduce physical constraints within the terminal, such as the availability of a limited number of platforms. The second set concerns limitations on the maximum number of passengers waiting in the terminal. Constraints (14') can be introduced to limit waiting times of passengers in the terminal, while conditions (15) to impose a minimum interval λ between two consecutive departures towards the same destination.

A description of basic versions of the model can be found in Bruno et al. (2009) and (2012). Table 2 summarizes the model and the constraints that will be considered in the following implementation; the complete model formulation is reported in Section A.1 in Appendix A.

Insert Table 2 here

3.2 The Cross-Docking Operations Optimization Problem
Within a complex supply chain, a cross-docking platform is a facility that receives goods from suppliers and sorts them into alternative arrangements which have to be delivered to given destinations. According to this logistic practice, storage of goods occurs only for short periods required to assemble and consolidate loads for immediate onward carriage (Vogt and Pienaar, 2007). This way, it is possible to reduce the total distribution costs taking advantage of the benefits of a warehousing strategy in terms of consolidation, but keeping at minimum storage costs. However, in order to properly operate, this kind of systems requires a relevant synchronization between inbound and outbound flows in order to obtain both lower lead times and inventory costs. If the inbound trucks' arrival scheduling along with their composition in terms of loaded lots is supposed to be known, the outbound truck scheduling problem consists in determining the loading and the scheduling sequence of outbound vehicles.
In recent years, several procedures adapted from scheduling models have been proposed, using aggregate representations of the activities within the cross-dock area (see for instance Boysen, 2010; Boysen et al., 2010), or proposing more in-depth and detailed models (see for instance Yu and Egbelu, 2008; Chen and Lee, 2009). Li et al. (2004) consider material handling inside the terminal for a given truck schedule, modelling this task as a machine scheduling problem; a meta-heuristic algorithm is proposed for the solution. Yu and Egbelu (2008) as well as Boysen et al. (2010) consider stylized settings where a terminal consists of

a single inbound and a single outbound door, proposing dynamic programming approaches to solve these problems. Miao et al. (2009) provide scheduling procedures adopted from gate assignment in airports, where trucks are assumed to have given service time windows, which are to be maintained as hard constraints. However, to date, no mathematical programming framework has been proposed to deal with synchronization problems from a more general perspective.

Supposing to know inbound trucks' arrival scheduling along with their composition in terms of lots and their final destination, a crucial aspect in managing the cross-docking terminal is represented by the schedule of outbound trucks towards a set of destinations, after incoming goods have been unloaded and sorted. Indeed, there is a need to find trade-off solutions considering the minimization of the activation costs of the outbound trucks and the total goods storage costs. This problem presents many similarities with that described in the previous application: in practice, the role of the transit terminal is here played by the cross-dock platform and the problem consists in the definition of outbound truck scheduling. Then variables d_{tj}, x_{tj}, and s_{tj} have the same meaning considering freight lots instead of passengers.

In particular, assuming $p_{tj} = 0$ (and, hence, $h_{tj} \geq 0 \ \forall\, t, \forall\, j$), the objective function (6) describes a performance measure defined as sum of the costs associated with storage of goods and the activation of departing trucks.

Constraints (8), (9) and (10) can be easily interpreted on the basis of the above-mentioned analogy (see also Table 3). In this case, it may be worth to reproduce constraints on labour, workforce, and on the structure of the cross-dock expressing that just a limited number of trucks can leave during a given time period; this condition can be effectively formulated through equation (12), assuming that K_t represents this upper bound. Furthermore, conditions (13) and (14') can be used to impose respectively a maximum capacity and a maximum storage time in the cross-dock.

Table 3 summarizes the model and the constraints that will be considered in the following implementation; the complete model formulation is reported in Section A.2 in Appendix A.

Insert Table 3 here

A more complex version of the problem can be introduced, associating with each lot also a deadline, i.e. a time limit for goods to leave the cross-dock. In this case it is possible to define variables d_{tjk} indicating lots arrived at cross-dock at time t that can be transferred to the destination j within the deadline k. Consequently also variables x_{tjk}, and s_{tjk} have to be interpreted considering the deadline. Then the formulation of the problem is the extension to the three index case of the model (6)÷(11) while constraints on the deadline can be easily expressed, imposing that s_{tjk} must be equal to 0 for each period $t > k$.

3.3 The Check-in Service Optimization Problem

In an airport terminal, the check-in service consists in processing and accepting passengers arriving at designated desks. Even if, in recent years, many companies have introduced online procedures aimed at reducing the impact of these operations, the need for an efficient management of such service still arises due to increasing air passengers' traffic and to a concurrent decrease in resources employed in handling operations. The latter phenomenon is related to the necessity of cutting costs for airlines and third party providers, in a context of general economic crisis. These concomitant issues frequently lead to congestions of the terminal infrastructures and long waiting times and queues at check-in desks.

The efficient management of the check-in operations can produce significant benefits in terms of costs and quality of the service provided to the users, especially for long range flights in which passengers need to drop off their baggage. For this reason, some techniques have been proposed in order to support this kind of decisions (TRP, 2010). In particular, as the problem is characterized by different uncertain factors, most of the approaches are based on simulation tools (Kovacs et al., 2010). Even if suitable to represent the actual working of the system, they fail in suggesting near optimal solutions. The most recent trend is to integrate simulation models into optimisation tools (Van Dijk and Van Der Sluis, 2006); therefore, there is the need for designing mathematical programming models capable of handling the problem with limited computational effort.

Suppose to have in the planning time horizon (for instance one day) a given flights departures schedule and, for each time period t, a given check-in capacity (R_t). In particular, $R_t = n_t\, r$, where n_t is the number of accessible desks in period t (related to workforce availability) and r represents the capacity of each single desk (assumed as a static parameter).

In order to optimally allocate this capacity, we have to consider that, in each time period t, only some flights can be processed; indeed, for each flight j, a time window $\left(T_j^-, T_j^+\right)$, within which passengers can be checked in, is defined depending on the departure time of flight j, the characteristics of the flight (i.e. national, international, intercontinental) and/or the company policies.

Even if the uncertainty in passengers' behaviours could not allow forecasting the exact distribution of the arrivals of passengers, through the analysis of historical data, it is possible to estimate it with a significant reliability (De Neufville and Odoni, 2003).

With reference to a reverse network flows representation, denote by d_{tj} the number of passengers of flight j arriving at check-in desks in the period t. If binary variables y_{tj} indicate whether or not the check-in of passengers of the flight j in the period t is possible, consequently the outflows x_{tj} represent passengers to be checked-in in t, while the holdover flows s_{tj} those queuing at the end of period t. With this interpretation, model (6)÷(11) consists in the optimization of the use of the available check-in capacity. In particular, assuming $p_{tj} = 0$ (and, hence, $h_{tj} \geq 0 \;\; \forall\, t, \forall\, j$), the objective function (6) describes a performance measure defined by the costs associated with maintaining queues and activating check-in operations.

Constraints (7) are passengers' conservation flow constraints. Constraints (8) can be modified in this way:

$$s_{tj} = \sum_{\tau=0}^{t} d_{\tau j} \qquad t = 0,..,T_j^- - 1;\; j = 1,..M \tag{8'}$$

$$s_{tj} = 0 \qquad t = T_j^+,..,N;\; j = 1,..M \tag{8''}$$

Conditions (8') express, for each flight j, the possible presence of queue before its check-in window, represented by all those passengers that arrive at the airport before T_j^- and cannot be processed. It has to be noticed that the arrivals potentially can occur also before the beginning of the planning horizon, especially for those flights, whose check in window starts in period $T_j^- = 1$. In order to take into account this aspect we introduced a non negative demand $d_{0j} \geq 0$, and then the possibility to have a queue at the beginning of the planning horizon ($s_{0j} = d_{0j}$).

Conditions (8'') impose that no passenger of flight j can be in queue from the end of period T_j^+.

Constraints (9) assure that passengers of flight j can be processed in the time interval t if and only if $y_{tj} = 1$. Interpreting a_j as the check-in processing time of a single passenger of flight j, and assuming $b_j = 0$, conditions (10) express the resource capacity constraints for each period. The total capacity R_t, coherently, is the total time available to accept passengers in the period t and, then, it is defined as product between the number n_t of accessible desks in period t and the maximum service time of each desk, equal to the duration τ of the period itself ($R_t = n_t \tau$). It can be noticed that the values of slack variables of constraints (10) indicate the reserve capacity and, therefore, an indication of the number of desks that can be closed with no change in the system performance in each period t.
In this case, constraints (14') on the maximum waiting times of passengers in the airport have been restricted to the check-in windows of the single flights as they are the only periods in which the processing is possible. They are, then, reformulated as follows:

$$s_{tj} \leq \sum_{k=1}^{\delta} x_{t+k,j} \qquad t = T_j^- - 1,..,T_j^+ - \delta; j = 1,..,M \qquad (14'')$$

Similarly, conditions (16) have been introduced here (restricted to the same time window) in order ensure that, once the check-in operations have been activated for a flight ($y_{tj} = 1$), they cannot be stopped until the end of the check-in window.

$$y_{t+1,j} \geq y_{tj} \qquad t = T_j^-,..,T_j^+ - 1; j = 1,..,M \qquad (16')$$

The other constraints derive from the physical meaning of the introduced variables.
Table 4 summarizes the model and the constraints that will be considered in the following implementation. The complete model formulation is reported in Section A.3 in Appendix A.

Insert Table 4 here

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4. Implementation and solution of the described models

The models based on CLSP formulation, introduced to describe three practical logistic applications, have been implemented and used to solve some test problems. The objective is to show how the proposed models are able to find solutions to problems of real dimensions in limited computing times and are capable of providing useful information to support decisions. Models were solved by utilising CPLEX 12.3 (with basic parameters setting) on an Intel Core i7 (1.86 GigaHertz) CPU equipped with 4.00 GB RAM.
It is useful to underline that the experiences that will be shown can also be improved from the computational point of view. In fact, better results in terms of computing times may be obtained using approaches like cutting planes or branch and cuts algorithms, which can exploit alternative formulations of the model (for these aspects see, Pochet and Wolsey, 2009). However this issue is out of the scope of the current paper, that is rather aimed at showing the potential of basic CLS models to solve the described problems.
In the following we show the results obtained by solving the model on test problems appropriately generated to represent practical case studies.

4.1 Application to the Bus Terminal Optimisation Problem

For this application, a test of the model was performed using data related to a public coach and buses company (A.Ir. spa) operating in Southern Italy, in Avellino district.
The aim was to define the optimal number of buses to be used and their departure time during the morning peak period at the transit terminal of Grottaminarda. This facility plays a crucial role in connecting a mainly rural area characterized by a dense presence of towns of limited population to the main towns of the region.

The problem was solved assuming, as time horizon, the morning demand peak period (6.00–11.00 am) divided in buckets of duration $\tau = 5$ min, such that $N = 60$, a value $C_{tj} = C = 50$ as bus capacity in constraints (9) and a value of $K = 1$ as number of buses that can simultaneously leave the terminal in constraints (12). Furthermore, line activation costs (f_{tj}) and waiting costs (h_{tj}) were assumed to be independent of specific destinations and time periods and equal to each other $(f_{tj} = h_{tj} = f = h)$. For the case with $M = 2$ destinations (Avellino and Naples), we used, as d_{tj}, data provided by a passengers' survey performed by the company. Then for $M = 4{,}6$ we produced some randomly generated instances, assigning to each d_{tj} a value according to a uniform distribution in the range $(1, C/M)$. We assumed the value of parameter δ in constraints (14') equal to 4, allowing a maximum passengers' waiting time of 20 minutes.
For each value of M, 5 instances were generated. Average and maximum running times are reported in Table 5.

Insert Table 5 here

In order to further test the behavior of the model, for *M=6*, additional tests were performed. For each considered instance, three different passengers arrival profiles were generated, in order to simulate different congestion levels. In particular, demand values for every destination in each single period (d_{jt}) were randomly generated according to a uniform distribution in the range $[1, \frac{\alpha * C}{M}]$ considering α={0.5; 1.0; 1.5}.

In addition, in order to analyze how the parameters can affect the optimal solutions, a sensitivity analysis on the parameters σ, i.e. the ratio between the single bus activation cost and the waiting costs per period for passengers in the terminal $\left(\sigma = \frac{f_{tj}}{h_{tj}} = \frac{f}{h}\right)$, and δ, i.e. the maximum waiting time, was performed (by not considering, in this case, constraints (12)). In particular, σ has been first fixed equal to 1 and then varied in the range $[10{,}100]$ with step 10, while δ has been fixed equal to $2, 4{,}6$.

It should be highlighted that, when the activation cost of a single bus is assumed to be equal to the waiting cost per period for a single passenger (σ =1), the model activates the maximum number of buses; namely, a bus towards each destination for each time period $(N * M)$. By increasing σ, the number of departing buses decreases and achieves a minimum value, depending on the parameter δ. Figure 3 reports the number of buses, as a percentage of the maximum number of buses ($y_{\%}$=$[\sum_{t=1}^{N} \sum_{j=1}^{M} y_{tj}]/[N * M]$), depending on the parameters σ and δ. Having assumed that each passenger can wait at most δ periods in the terminal, the model has to activate at least a bus every $\delta + 1$ periods towards each destination; therefore, $\frac{N}{\delta+1} * M$. Obviously, it is possible to notice how the arrivals density (ruled by the parameter α) impacts on the slope of the curves and on the convergence to the latter value.

Figures 4 report, for different combinations of values of α and δ, the minimum, maximum and average bus utilization rate depending on parameter σ. In particular, the single bus utilization rate can be defined as $\mu_{tj} = \frac{x_{tj} * y_{tj}}{C}$, being computed just for the couples (t, j) for which $y_{tj} = 1$. It can be noticed that the bus utilization rate increases as the parameters α, δ and σ increase. In particular, these curves show how the model can be employed as a decision support tool to analyze the working conditions of a bus terminal under different demand congestion profiles and service levels to be offered to passengers (expressed both in terms of maximum waiting times and waiting costs).

Insert Figures 3 and 4 here

4.2 Application to the Cross-Docking Operations Optimization Problem

For this specific application, we simulated the typical organization of a cross-dock, considering the real functioning principles of the logistic platform of the Nola dry port in Naples area (Italy). The facility is a huge multi-modal platform designed to handle about 30 million tons of freight per year. Its strategic position on the corridor between the south Mediterranean harbours and North Europe gives it the crucial role as consolidation point utilised for re-sorting shipments. In particular, within the platform, incoming goods are re-arranged and carried forward by truck towards a limited number of other main cross-dock platforms in Northern Italy; then from there, goods can reach the final destination on by means of less-than-truckload shipments. The vast area of the dry port is sub-divided in logistics areas with few gates where freights directed to a limited number of predefined destinations are consolidated.

On the basis of this reference real case, we generated some test problems representing operations to be performed at cross-docking platform. For this reason we assumed a time horizon of 12 hours (from 6am to 6pm) divided in periods of different duration ($\tau = 60, 30, 20$ min) with, consequently, different number of periods ($N = 12, 24, 36$). These time intervals are quite reasonable for a cross-docking platform, as, due to handling and sorting procedures, the time between subsequent shipments is usually not negligible.

As regards the number of destinations, we considered three possible values ($M = 2, 4, 6$). This intends to reproduce the above mentioned case of a single logistic area. The capacity of outbound vehicles $C_{tj} = C$ which appears in (9) was fixed equal to 38 which represents the typical capacity (in EPAL pallet) of a heavy truck.

Incoming goods, constituting the demand d_{tj} for shipments, have been generated at each period assigning a value according to an uniform probability distribution in the range $[0, C/(M * \varepsilon)]$, where ε is a corrective factor depending on the considered number of periods N (in particular we fixed $\varepsilon = N/12$). This way, the number of periods in which the planning horizon has been divided, does not affect the congestion level of the arrivals but only the partition of the total demand across the periods. In addition, the parameter K_t representing the maximum number of trucks that can leave the platform in a given time period, according to constraints (12), was kept equal to 1, to simulate a very constrained situation in terms of workforce. Furthermore, constraints (13) about the maximum cross-docking capacity were implemented assuming $S_t = 2C$, corresponding to the capacity of two trucks, in order to simulate the stringent “just in time” logic that rules the cross-docking practice. As already stated, in this case, constraints (14’) were not considered.

For each pair (N, M), we generated 5 different test instances. In Table 6, we report the average running times (in seconds) obtained to solve problems with different values (N, M). Results show that, even for problems of more significant dimension, the model is capable of providing optimal solutions in reasonable times.

Interesting managerial implications can be derived from a sensitivity analysis of the model considering its parameters (C_t, S_t, K_t). As an example, we present a typical output obtainable by varying the cross-dock capacity (S) and the truck capacity (C). The model is capable of providing the optimal result in terms of objective function for every pair of value (C, S). This way, it is possible to divide the space (C, S) in regions on the basis of the values of optimal objective function. The typical pattern is provided in Figure 5: it shows that the minimum value (z_{min}) is reached when both C and S belong to the extreme North-East region; the objective function gradually increases when the capacity of the terminal and/or the truck capacity decrease; a non-feasibility area can be also identified.

Insert Table 6 here

Insert Figure 5 here

4.3 Application to the Check-in Service Optimization Problem

Test problems were obtained by considering daily timetables of a typical working day of main airports (reporting at least 500,000 passengers per annum) in Southern Italy (including the two main islands of Sardinia and Sicily). Table 7 shows the characteristics of each test problem.

In particular, the beginning (t_0) and the end (t_N) of the time horizon (defined accordingly to the first and the last flight departure time from each airport) is indicated. The number of periods N is calculated assuming a duration $\tau = 10$ minutes for each period; therefore, $N = (t_N - t_0)/\tau$. For a given flight j with departure time t_j, a completion time T_j^+ for check-in operations is determined. In particular, T_j^+ is equal to $t_j - D_j$, where D_j represents the time between the end of check-in operations and the departure time of the flight. We considered D_j equal to 30 and 60 minutes for national and international flights respectively. T_j^- was determined subsequently, by assuming theoretical check-in windows equal to 120 and 240 minutes for national and international flights.

Demand for each flight d_{tj} has been randomly determined using a uniform distribution in the range $[max_j/2, max_j]$, where max_j is the capacity of the aircraft assigned to flight j. This number is then distributed across the time window $(1, T_j^+)$ using passenger arrival profiles at the check-in area derived from a survey performed at Naples International Airport, taking into account the percentage of passengers from each flight requiring a physical check-in service (as they have not checked-in online) and their arrival profiles. The duration of single check-in operations a_j is fixed equal to 90 and 180 seconds for national and international flights respectively. Equation (16) is assumed as the objective function; the parameter δ in constraints (14') is assumed equal to 4, i.e. a maximum passengers' waiting time of 40 minutes is allowed. Table 7 shows that the computational times are extremely low in all the cases; indeed, all real-world cases are solved at optimality within 6 seconds.

Furthermore, the model can be solved by varying its parameters (for instance δ), this way providing useful information for the decision maker.

With simple adaptations, the model can also be used to solve a dimensioning problem for automated check-in services, in which passengers have to access self-service kiosks for printing their boarding cards.

Insert Table 7 here

4.4 Remarks and Potential Further Applications

The previous sub-sections have shown how the proposed models can be adapted to solve some real-world cases with limited computational efforts, thanks to the structural properties derived from the CLSP mathematical models. In addition, it has been shown that a wide set of logistics constraints can be reproduced, without losing the generality of the framework,

just by including equations and conditions that have already been utilized in the extant literature.
With a similar adaptation process, further dimensioning problems arising in logistic scenarios could be modelled, such as, for example:

- dimensioning of airport security gates, in which the number of gates to be opened at specific time instants needs to be decided, keeping into account passengers waiting times;
- dimensioning of motorways toll lanes, in which the number and the type (automated semi-automated, staff-attended) of tolling stations to be opened at specific time instants needs to be decided, keeping into account vehicles waiting times and staff costs;
- optimization of railway transit terminals, in which the number of outbound services towards multiple destinations need to be decided, keeping into account demand profiles based on transiting passenger arrivals and their waiting times within the terminal.

Nevertheless, stochastic queuing models can be integrated into the proposed optimization models in order to simulate demand profiles.
Furthermore, the possibility of formulating logistic problems using the general framework of a CLSP permits to exploit the vast literature dedicated to this model. In particular it may be possible to consider results about complexity and reformulation aspects and to implement effective and well-established solution methods (exact and/or heuristic).

5. Conclusions

Inventory models have been widely employed in the academic literature and in corporate practice to solve a wide range of theoretical and real-world problems, mainly related to production planning and scheduling.
However, if we look at Capacitated Lot-Sizing model as a general model of flow control, it is possible to use it to describe and formulate a wide variety of optimization problems. The aim of this paper has been highlighting opportunities of using, through simple adaptations of the basic version, this model to solve some practical logistics applications not strictly related to the manufacturing and production environment. In particular, we have illustrated how three different applications can be effectively formulated through this approach. The application of the implemented models to real-world case studies has shown the possibility of obtaining optimal solutions in reasonable computational times by utilizing a commercial solver.
In addition to this aspect, many other advantages can be derived from the use of CLSP models: the possibility of immediately including operational constraints and conditions able to effectively describe real case situations; the availability of a vast developed literature which can be exploited to derive mathematical conditions to describe real-life constraints, to benefit from theoretical results and to implement effective and well-established solution methods (exact and/or heuristic).
Future research could consider a more in-depth analysis of the described applications in order to verify the potential of this model framework to reproduce effectively more complex operational aspects that can occur in real cases; nevertheless, new contexts and fields in which the adaptation of this approach can be effective and useful could also be explored.

Appendix A: Mathematical Models

In this appendix, the different adaptations of the Capacitated Lot-Sizing models utilized to cope with the specific problems introduced are reported. Even though the adaptations are straightforward and detailed in Tables 2, 3 and 4, the models are reported here for the benefit of the reader.

A.1 The Bus Terminal Schedule Optimization Problem Mathematical Model

$$\min \ z = \sum_{t=1}^{N} \sum_{j=1}^{M} \left(h_{tj} s_{tj} + f_{tj} y_{tj} \right) \tag{6}$$

s.t.

$$s_{tj} = s_{t-1,j} - x_{tj} + d_{tj} \qquad t = 1,..,N; j = 1,..,M \tag{7'}$$

$$s_{tj} = 0 \qquad t = 0 \text{ and } t = N; j = 1,..,M \tag{8}$$

$$x_{tj} \leq C y_{tj} \qquad t = 1,..,N; j = 1,..,M \tag{9}$$

$$\sum_{j=1}^{M} y_{tj} \leq K \qquad t = 1,..,N \tag{12}$$

$$s_{tj} \leq \sum_{k=1}^{\delta} x_{t+k,j} \qquad t = 1,..,N-\delta; j = 1,..,M \tag{14'}$$

$$s_{tj} \geq 0; x_{tj} \geq 0; y_{tj} = 0/1 \qquad t = 1,..,N; j = 1,..,M \tag{11}$$

A.2 The Cross-Docking Operations Optimization Problem Mathematical Model

$$\min \ z = \sum_{t=1}^{N} \sum_{j=1}^{M} \left(h_{tj} s_{tj} + f_{tj} y_{tj} \right) \tag{6}$$

s.t.

$$s_{tj} = s_{t-1,j} - x_{tj} + d_{tj} \qquad t = 1,..,N; j = 1,..,M \tag{7'}$$

$$s_{tj} = 0 \qquad t = 0 \text{ and } t = N; j = 1,..,M \tag{8}$$

$$x_{tj} \leq C y_{tj} \qquad t = 1,..,N; j = 1,..,M \tag{9}$$

$$\sum_{j=1}^{M} y_{tj} \leq K \qquad t = 1,..,N \tag{12}$$

$$\sum_{j=1}^{M} s_{tj} \leq S \qquad t = 1,..,N \tag{13}$$

$$s_{tj} \geq 0; x_{tj} \geq 0; y_{tj} = 0/1 \qquad t = 1,..,N; j = 1,..,M \tag{11}$$

A.3 The Check-in Service Optimization Problem Mathematical Model

$$\min \quad z = \sum_{t=1}^{N} \sum_{j=1}^{M} \left(h_{tj} s_{tj} + f_{tj} y_{tj} \right) \tag{6}$$

s.t.

$$s_{tj} = s_{t-1,j} - x_{tj} + d_{tj} \qquad t = 1,..,N; j = 1,..,M \tag{7'}$$

$$s_{tj} = \sum_{\tau=0}^{t} d_{\tau j} \qquad t = 0,..,T_j^- - 1;\ j = 1,..M \tag{8'}$$

$$s_{tj} = 0 \qquad t = T_j^+,..,N;\ j = 1,..M \tag{8''}$$

$$y_{t+1,j} \geq y_{tj} \qquad t = T_j^-,..,T_j^+ - 1; j = 1,..,M \tag{16'}$$

$$x_{tj} \leq C_j y_{tj} \qquad t = 1,..,N; j = 1,..,M \tag{9}$$

$$\sum_{j=1}^{M} a_j\, x_{tj} \leq R_t \qquad t = 1,..,N \tag{10}$$

$$s_{tj} \leq \sum_{k=1}^{\delta} x_{t+k,j} \qquad t = T_j^- - 1,..,T_j^+ - \delta; j = 1,..,M \tag{14''}$$

$$s_{tj} \geq 0; x_{tj} \geq 0; y_{tj} = 0/1 \qquad t = 1,..,N; j = 1,..,M \tag{11}$$

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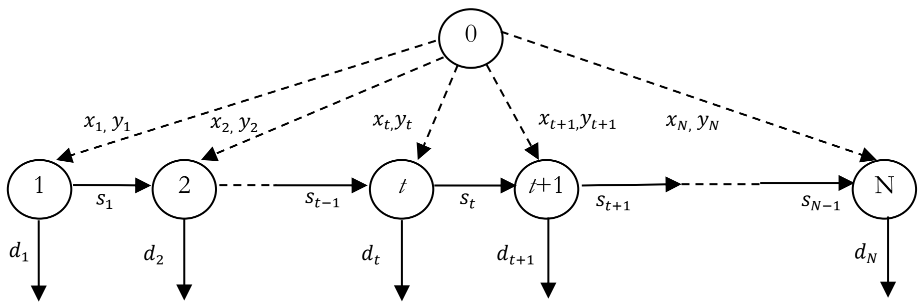

Figure 1 – Representation of a DLSP as network flow problem

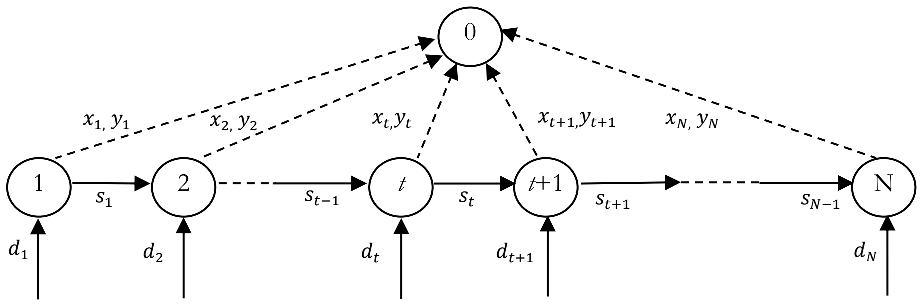

Figure 2 – Reverse representation of a DLSP as network flow problem

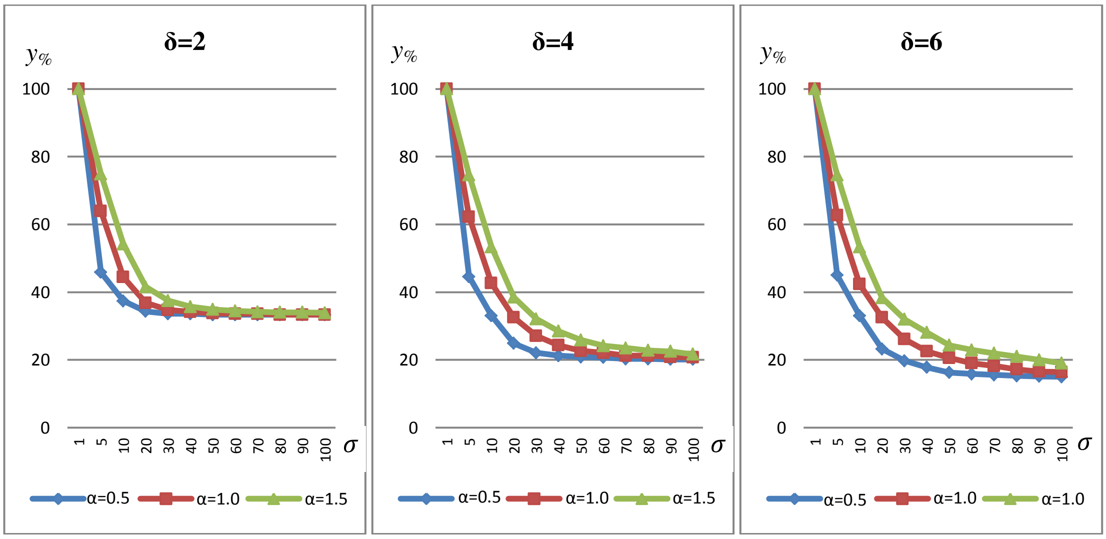

Figure 3 – Bus departures ($y_{\%}$) variation against α, σ and δ parameters

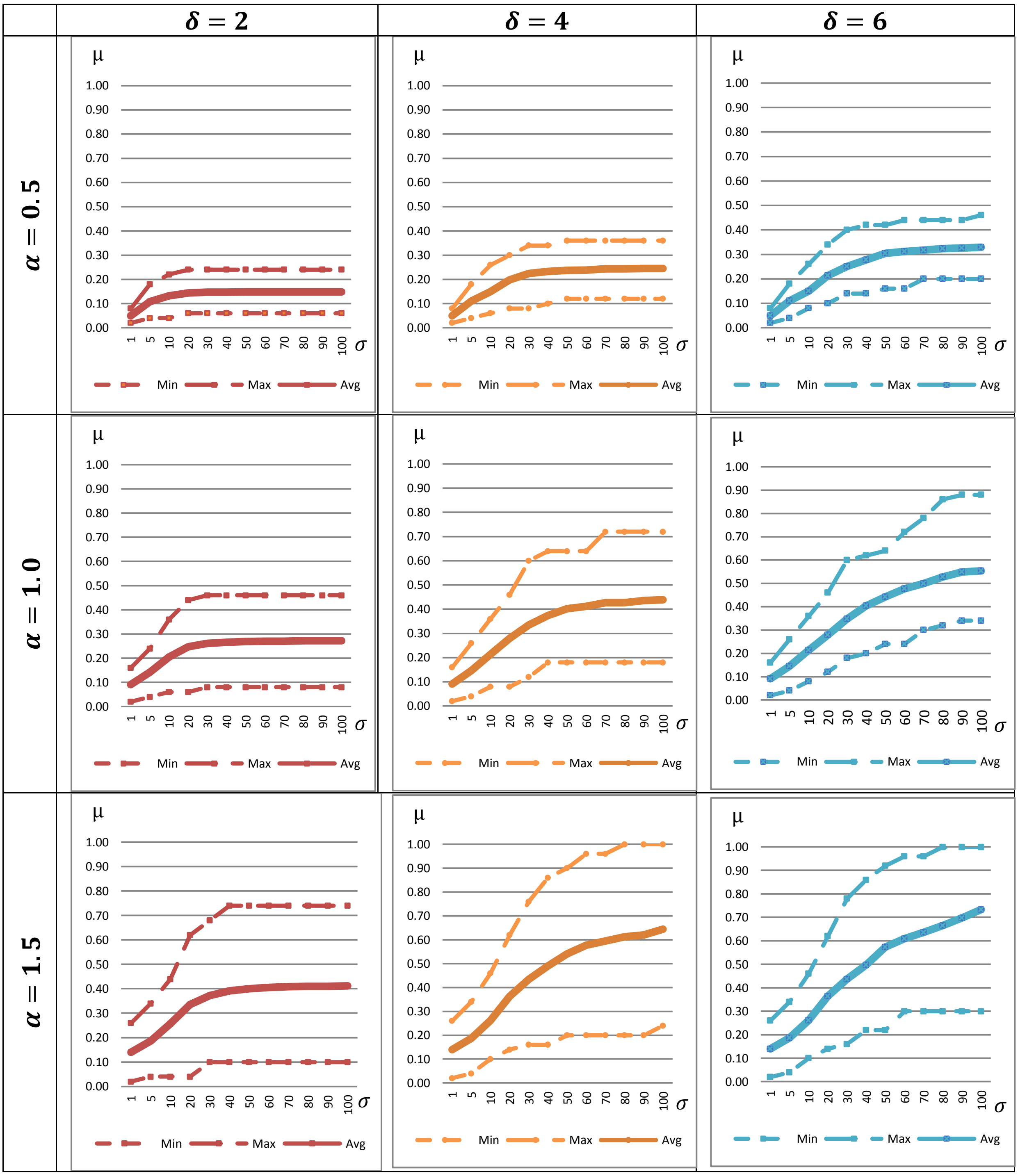

Figure 4 – Bus utilization rate (μ) variation against α, σ and δ parameters

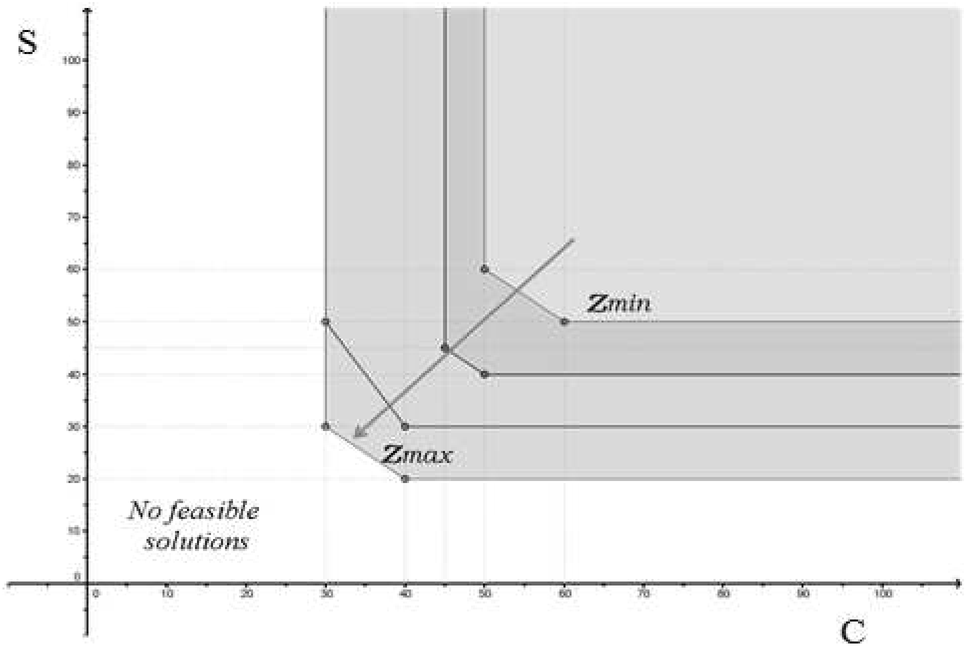

Figure 5 – Sensitivity analysis varying the dock capacity S and the truck capacity C

		Basic Version	*Logistics Adaptations*
Parameters	$\boldsymbol{j}$	Item	Service
	$\boldsymbol{d_{tj}}$	Demand for item *j* in period *t*	Units of demand (for instance, passengers or goods) for service *j* arising in period *t*
	$\boldsymbol{f_{tj}}$	Setup cost incurred for the production (or ordering) of item j in period t	Cost associated with the activation of service j in period t
	$\boldsymbol{p_{tj}}$	Unit production cost for item *j* in period t	Cost for satisfying a unit of demand for service j in period t
	$\boldsymbol{h_{tj}}$	Unit holding cost for item *j* at the end of period t	Cost for maintaining (or storing) a unit of demand for service j in queue (or in a storage facility) at the end of period t
	$\boldsymbol{C_{tj}}$	Maximum feasible lot size	Maximum number of units of demand for service j that can be satisfied in period t
	$\boldsymbol{R_t}$	Total available capacity in period *t*	Total service capacity in period *t*
	$\boldsymbol{a_j}$	Unit capacity consumption for the production of item *j*	Capacity consumption for satisfying a unit of demand for service *j*
	$\boldsymbol{b_j}$	Capacity consumption for the setup of item *j*	Capacity consumption for the activation of service *j*
	$\boldsymbol{K_t}$	Maximum number of setup in period t	Maximum number of services that can be activated in period t
	$\boldsymbol{S_t}$	Maximum inventory level for period t	Maximum demand still to be satisfied (i.e., in queue) at the end of period t
	$\boldsymbol{\delta}$	Maximum duration for the inventory level	Maximum waiting time for service demand
Variables	$\boldsymbol{x_{tj}}$	Quantity to be produced or ordered during period *t*	Units of demand (for instance, passengers or goods) for service *j* being processed in period t
	$\boldsymbol{s_{tj}}$	Stock at the end of period *t*	Residual demand units (for instance, passengers or goods) for service j waiting to be processed at the end of period t
	$\boldsymbol{y_{tj}}$	Binary variable concerning the production activation (or the issuing of an order) (or not) of item *j* in period *t*	Binary variable concerning the activation (or not) of service j in period t, and, therefore, the possibility (or not) of processing demand units for it.

Table 1 – Adaptation of the CLSP model to a general logistics context

Equation	***Meaning***	***Variations from Original CLSP formulation***
(6)	Total cost associated with the activation of departing buses	$p_{tj} = 0 \quad \forall t,j$ $f_{tj} = f > 0 \quad \forall t,j$ $h_{tj} = \sigma f \quad \forall t,j$
(7')	Flow (Passengers) conservation constraint	
(8)	Passengers waiting in the terminal at the beginning/end of the planning horizon	
(9)	Capacity of available buses	$C_{tj} = C \quad \forall tj$
(12)	Maximum number of buses that can leave the terminal in each time period t	$K_t = K \quad \forall t$
(14')	Limitation on waiting times of passengers in the terminal	
(11)	Physical meaning of variables	

Table 2 – Adaptation of the CLSP to the Bus Terminal Schedule Optimization problem

Equation	***Meaning***	***Variations from Original CLSP formulation***
(6)	Total activation cost of the scheduled outbound trucks	$p_{tj} = 0 \quad \forall t,j$ $f_{tj} = f > 0 \quad \forall t,j$ $h_{tj} = \sigma f \quad \forall t,j$
(7')	Flow (Freights) conservation constraint	
(8)	Freight stored in the cross-dock at the beginning/end of the planning horizon	
(9)	Capacity of outbound vehicles	$C_{tj} = C \quad \forall t,j$
(12)	Maximum number of trucks can leave the cross-dock during time period t	$K_t = K \quad \forall t$
(13)	Maximum storage capacity of the cross-dock	$S_t = S \quad \forall t$
(11)	Physical meaning of variables	

Table 3 – Adaptation of the CLSP to the Cross-Docking Operations Optimization problem

Equation	***Meaning***	***Variations from Original CLSP formulation***
(6)	Number of check-in services activated over the planning horizon	$p_{tj} = 0 \quad \forall t,j$ $f_{tj} = f > 0 \quad \forall t,j$ $h_{tj} = \sigma f \quad \forall t,j$
(7')	Flow (Passenger) conservation constraint	
(8)	Passengers waiting at check-in desks outside the check-in time window	See Equations (8',8'')
(9)	Passengers that can be processed in time period t for flight j	$C_{tj} = C_j \geq \sum_{\tau=1,..,N} d_{\tau j} \quad \forall j$
(10)	Overall available capacity in time period t	$b_j = 0 \quad \forall j$
(14')	Limit waiting times of passengers in queue at check-in desks	See Equation (14'')
(16)	Check-in services continuity	See Equation (16')
(11)	Physical meaning of variables	

Table 4 – Adaptation of the CLSP to the Check-in Service Optimization problem

Destinations (M)	Periods (N)		
	12	24	36
2	2.14	3.23	3.63
4	2.80	4.10	5.18
6	3.09	4.20	8.10

Table 5 – Average computational times (in seconds) for test problems

Destinations (M)	Periods (N)	Running Times (s)	
		Average	Maximum
2	60	3.02	3.91
4	60	4.30	7.21
6	60	6.64	9.49

Table 6 – Computational times (in seconds) for the considered test problems

Airport	Time horizon		Periods (N)	Flights (M)	National	International	Running times (s)
	Start (t_o)	End (t_N)					
Alghero	06.00	22.00	96	22	14	8	4.13
Bari	05.30	22.00	99	45	30	15	5.07
Brindisi	05.30	22.00	99	37	28	9	4.98
Cagliari	05.10	22.20	103	51	35	16	5.34
Catania	04.50	23.00	109	89	50	39	6.36
Lamezia	05.00	22.00	102	39	28	11	5.10
Naples	05.10	22.20	103	76	42	34	6.03
Olbia	05.30	22.00	99	30	19	11	4.78
Palermo	05.00	22.00	103	60	38	22	5.58
Reggio Calabria	06.00	21.00	90	12	11	1	3.98
Trapani	05.30	22.00	99	26	16	10	4.67

Table 7 – Characteristics and computational times for the considered test problems