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Capacity management in public service facility networks: a model, computational tests and a case study

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Abstract
In this work, we present a mathematical model to support location decisions oriented to rationalize facility systems in non-competitive contexts. In order to test the model, computational results are shown and an application to a real-world case study, concerning the Higher Education system in an Italian region, is discussed.

1 – Introduction
Facility location decisions represent a critical element in strategic planning in both private and public sectors, as they can have a strong and lasting impact on operational and logistics performance (Drezner and Hamacher, 2002).
A facility location problem is aimed at finding the best position for a set of facilities within a given region in order to optimize a specific objective function. Starting from this general framework, several formulations may be defined in terms of objective function, features of facilities to be located, demand to be served, and location space (ReVelle et al., 2007). In the last decades, there have been many applications concerning the location of both public (i.e. schools or post offices, emergency services, fire stations, hospitals, ambulances) and private (i.e. plants, warehouses, industrial sites) facilities (Aboolian et al., 2007; Başar et al., 2012). Historically, such models have been a viable decision support tool for institutions and firms planning to open new facilities or expand their capacity in a given region (Drezner and Hamacher, 2002).
In recent years, due to the general interest to reduce costs and improve efficiency, companies and institutions have been more interested in the reorganization of the current configuration of facilities either completely closing down some of them or downsizing their capacities (Farahani et al., 2013).
Notably, in the public sector, economic conditions imposing austerity measures and constraints on public expenditure could make the existing supply system in critical areas (such as healthcare, education and public transport) unaffordable and/or unsustainable. Such conditions may require the rationalization of the existing service facilities (ReVelle and Eiselt, 2005). Rationalization actions are intended here as modifications of the current configuration of the existing service facilities (by entirely closing some of them or by downsizing or transferring capacities) performed in order to increase its affordability while still providing a required service level.
These decisions may have a strategic value given their long-term impact, and should be planned by taking into account different perspectives. Indeed, while the planner would be interested in the

improvement of the performance in terms of financial sustainability, users will be affected by the loss of one or more facilities. Apart from possible financial gains (achievable through cost savings for facilities operations), the closure of existing facilities will produce some side effects, such as the increase of costs faced by users (in terms of accessibility) and, potentially, the worsening of the quality of the offered service (in terms of drop of coverage, worsening of user satisfaction and over-utilization of the remaining facilities). Therefore, to effectively solve these kinds of problems, decision support models should be able to find solutions that are capable of maximizing the benefit due to the closure (taking into account the perspective of the planner) by limiting, at the same time, the damage due to the closure itself (taking into account the perspective of the user).

In this work, after a review of the extant literature, we present a mathematical model specifically designed for addressing re-organization decisions about facilities in a public sector context. The model is aimed at identifying, in a given system, the set of facilities to be closed and/or downsized in order to find solutions that represent a good trade-off between the goal of the decision maker and the needs of the users. The model has been tested on a set of randomly generated instances in order to show that a good range of problems can be solved to optimality through the use of a commercially available solver (CPLEX); in addition, it has been adapted to deal with a real-world case from the public sector, concerning the University system of an Italian region.

The paper is organized as follows. In the next section, a literature review about models and methods proposed to deal with the re-organization of facility systems is illustrated. Then, in Section 3, a general mathematical model for the rationalization of a facility system in a public sector context is introduced and described. Computational results based on randomly generated test instances are shown in Section 4, while Section 5 reports the adaptation of the model to a real case study. Finally, conclusions and directions for further research are drawn.

2 – Literature Background

In the literature there are many studies addressing the problem of modifying the configuration of an existing facility system in response to occurred or potential changes in the conditions in which the system operates, through some re-organization actions. According to the adopted approach, we can classify proposed models in the following classes:

- *Ex-ante models*, in which decisions are taken *before* changes occurr, taking into account, in advance, any predictable change (for instance, by means of forecasting tools);
- *Ex-post models,* in which decisions are taken *once* changes have *already* occurred.

Within each class, we may distinguish between single period and multi-period models, thus obtaining the four classes of models described in the following (as reported in Table 1).

Ex-ante single period models can be described as stochastic models that, starting from an initial configuration of the facility system, aim at re-organizing it on the basis of estimated probabilities associated to future scenarios. Berman and Drezner (2008) developed the first formulation for the p-median problem under uncertainty. In this case, the proposed model aimed at locating p facilities, knowing that up to q additional facilities would have been located in the future. However, the proposed approach did not allow the closure of the facilities that have been located during the first round, nor the modification of their capacity. Similarly, Sonmez and Lim (2012) proposed a solution approach that can determine the initial locations and the future relocations of facilities in the case that demand is subject to change and also the number of future facilities is not fixed a-priori. The aim was to minimise the initial and expected future weighted distances without exceeding a given budget for opening and closing facilities. Also in this case, however, there is no possibility of altering the capacity of the facilities.

Multi-period ex-ante models are designed to dynamically adapt the configuration of a facility system to changeable conditions (estimated through forecasting techniques). Van Roy and Erlenkotter (1982) formulated one of the seminal attempts to model the simultaneous closure, opening and relocation of facilities by using a multi-period perspective. Min (1988) solved a real-life relocation problem by employing a fuzzy multi-objective model with constraints on budget and on the maximum number of relocations per period. Shulman (1991) firstly introduced capacity constraints in the model formulation, allowing the expansion of existing facilities to cope with evolving demand conditions. A similar contribution was provided by Chardaire et al. (1996). Canel and Khumawala (1997) provided a model to solve a multi-period problem including estimation of transfer costs for facilities relocations. Within a similar framework, Dias et al. (2006) pointed out that reopening a facility produces in general lower costs than opening it for the first time.
Melo et al. (2006) dealt with multi-period facility reconfiguration decisions in a multi-commodity, dynamic supply chain context, involving reallocation of facilities' capacities assuming that all existing facilities are operating at the beginning of the planning horizon; if an existing facility is closed, it cannot be reopened, while when a new facility is opened, it will remain in operation.
Georgiadis et al. (2011) and Wilhelm et al. (2013) formulated similar problems by allowing the dynamic reconfiguration of the network over time (i.e., by opening or closing facilities, expanding, downsizing or contracting their capacities) to accommodate changing trends in demand or costs.

Ex-post single period models deal with decisions (mainly regarding strategic aspects) motivated by changes that have *already* occurred and can be implemented over a single period. Within this class, the first contribution can be retrieved in Leorch et al. (1996) approach that developed an application of the problem to the military context, dealing with the closure of some sites after the drawdown of an army from a region. Wang et al. (2003) introduced a model addressing the situation in which, due to some occurring changes in the distribution of the demand, relocation of the existing facilities in the location space was required in order to improve the service level, explicitly taking into account user perspective. In this case, the problem was modeled by considering, simultaneously, the possibility of opening new facilities and closing existing ones (however, not including the possibility of altering the capacity of single facilities). With a similar approach, ReVelle et al. (2007) introduced two different models, to deal with both competitive and non-competitive environments. The first one considered firms ceding market share to competitors under situations of pressing financial needs. The second model considered a firm (or a public body) operating in a non-competitive market that has to downsize its services for economic reasons. In this case, authors deal with the situation in which the current organization is the best one in terms of users accessibility (according to parameters such as average and maximum travel distance), but the re-organization is needed due to economic conditions that make the current system economically unaffordable and impose to downsize the service. The goal in this case was to contain as much as possible the degradation of the service level provided to the users; the objective function is thus measured as the number of users that, after the reallocation, had to cover a distance (travel time) longer than a given threshold. It has to be highlighted that the benefit for the planner is modeled in terms of total number of facilities to be closed (as each facility provides the same benefit) and the demand reallocation does not consider constraints on the capacity of the remaining facilities.

Ex-post multi period models deal with re-organization actions, motivated by changes that have *already* occurred, that need to be gradually implemented over a planning horizon. The only contributions related with this class of models can be retrieved in Dell (1998), who developed an approach for dealing with a multi-year programme of army sites closures and downsizing in the US, and Araya et al. (2012), who constructed a model for the rationalization of the Chilean school system.

	Ex-Ante	**Ex-post**
Single-Period Models	Berman and Drezner (2008), Sonmez and Lim (2012).	Leorch et al. (1996); Wang et al. (2003); ReVelle et al. (2007)
Multi-Period Models	Van Roy and Erlenkotter (1982), Min (1988); Shulman (1991), Chardaire et al. (1996); Canel and Khumawala (1997); Melo et al. (2006); Dias et al. (2007), Wilhelm et al. (2013).	Dell (1998) Araya et al. (2012)

Table 1 – Classification of re-organization approaches

It has to be noticed that most of the surveyed models represent facilities that are able to provide only one type of service. In various applications this assumption is not adequate. For example, in the case of public facilities (e.g., hospitals; recycling centers; comprehensive schools), sites often host complex structures capable of providing multiple services to users (e.g., respectively: different wards; different types of waste; different grades). In these cases, it may not be necessary to close the whole facility but just downsize it by reducing the range of offered services. This also applies to production sites within a supply chain, where plants may be both entirely closed down or downsized by dismantling some existing manufacturing or service lines (Melo et al., 2006).
From a practical point of view, just a few explicit applications to public services facilities have been developed, notably in the above-mentioned works from Min (1988) and ReVelle et al. (2007). Furthermore, it has to be noticed that most of the papers concerned with multi-period representations deal with fairly tactical version of the problems, in which facilities can be closed and re-opened at relatively low costs. This may mean that these models could not be suitable to represent situations in which closing or downsizing facilities constitute strategic actions having a long-range effect. Moreover, most of the models reproduce situations in which the reorganization problem arises because of a change in the distribution of the demand that has made obsolete and inefficient the current configuration (Wang et al., 2003), with the only notable difference represented by the cited work of ReVelle et al. (2007), in which the reorganization is needed due to economic conditions that make the current system economically unaffordable, even if currently the best-one from the user perspective.
The process by means of which every user selects his preferred facility represents another crucial aspect to be taken into account in rationalization actions. Indeed, this mechanism will drive the reallocation of users previously assigned to closed (or downsized) facilities to the ones that are still active; it will also play a pivotal role in the assessment of the damage imposed on the users themselves. It has to be highlighted that most of the surveyed papers (regardless of their classification) assume that users choose the closest available facility (Espejo et al., 2012); this assumption implicitly considers all facilities being equally attractive. In real-world applications, it can be reasonably supposed that the distance is not the only factor to be considered in the choice, as facilities are characterized by different attractiveness profiles. In these scenarios, user behavior can

be effectively described by more complex spatial interaction rules, as empirically proved by Bucklin (1971), Hodgson (1981), McLafferty (1988), Lowe and Sen (1996), Bruno and Improta (2008). Indeed, it is possible to define (on the basis of a variety of factors and a given interaction model) the utility of a facility for a given user, that can be also seen as the probability that he will select that facility. Coherently, the distribution of the demand among available facilities will occur according to these probabilities. This is typical, for instance, in gravity models, in which the probability is assumed proportional to the attractiveness of the facility and to a decreasing function of the distance from it (Joseph and Kuby, 2011). It has to be remarked that the integration of such spatial interaction rules into facility re-organization problems is somewhat limited; few examples can be retrieved in the general facility location literature (see, for instance, Aros-Vera et al., 2013).

Compared to the mentioned contributions available in the literature, the model described in this paper will deal with strategic rationalization decisions in a public sector context, in which facilities are offering multiple services. The goal of the model is the closure of some of the existing facilities or the downsizing of the portfolio of services offered by each of them, in order to respond to changes in economic conditions that have affected the affordability of the current system. Assuming that the current configuration is the best one from the user perspective, any rationalization action will impose some damage, because of the potential perturbations on the current optimal allocation. Therefore, in this case, the problem differs from the ones described in the literature. Indeed, apart from possible gains for the planner, any decision will produce some side effects, such as the increase of costs faced by users (in terms of accessibility to the service) and, potentially, the worsening of the quality of the offered service (measurable in terms of drop of coverage, worsening of user satisfaction and over-utilization of the remaining facilities). Therefore, the model will be aimed at maximizing potential benefits, by achieving economic efficiencies, keeping into account the need to limit the damage (or discomfort) imposed on the user (as a consequence of potential reallocation decisions). As another element of novelty, in the model it will be assumed that users are not necessarily assigned to the closest facility, rather being distributed according to their individual preferences; therefore, a coherent user reallocation mechanism will be defined.

3 – A Mathematical Model for the Rationalization of a Public Service Facility Network

Suppose the presence of a given number of facilities in a location space, each providing different types of services. Then, assume that, in order to reduce costs and improve the affordability of the whole system, the planner aims at closing some of the existing facilities or downsizing the portfolio of services offered by each of them. In this process, he could obtain a benefit (for instance, in terms of operating costs reduction); however, the closure of a service involves users that will have to decide which available facility (among the ones offering the same type of service) to patronize. For this reason, the planner could also decide to pay an additional cost to expand the service capacity of some remaining facilities in order to satisfy reallocated users. Under the hypothesis that the planner wishes to achieve a certain level of benefit, a possible objective could be represented by the minimization of the total cost to provide remaining facilities with additional capacity to cope with reallocated demand. The demand reallocation mechanism will be defined taking into account the interaction mechanism between users and facilities underlying the initial demand distribution. Therefore, the following sets can be introduced:

I set of demand nodes, indexed by i ($|I| = n$);

J	set of existing facilities, indexed by j ($\lvert J \rvert = m$);
K	set of different types of services to be provided, indexed by k ($\lvert K \rvert = q$).

In this context, the following parameters can be defined:

α_j^{ik}	fraction of demand d_{ik} initially assigned to facility j $(0 \leq \alpha_j^{ik} \leq 1)$, also representing the probability that users from i, requiring service k, select facility j;
$\bar{b}_{kj}$	benefit deriving from the closure of service k at facility j;
$\bar{\bar{b}}_j$	additional benefit deriving from the closure of the whole facility j;
B	minimum benefit to be obtained.
C_{kj}	maximum demand that can be served for service k at facility j (capacity);
c_{kj}	cost to provide an additional unit of capacity for service k at facility j;
d_{ik}	total demand coming from node i for service k;
l_{kj}	binary indicator equal to 1 if and only if facility j initially provides service k;
L_j	set of services initially provided by facility j $\left(L_j = \{k \in K: l_{kj} = 1\}; \lvert L_j \rvert = N_j\right)$;
U_k	set of facilities that initially provide service k $\left(U_k = \{j \in J: l_{kj} = 1\}\right)$;

Furthermore, the following decision variables can be introduced:

Δ_{kj}	non-negative decision variable denoting the additional capacity needed for service k at facility j to satisfy the reallocated demand in the final configuration.
s_{kj}	binary decision variable equal to 1 if and only if service k, initially provided by the facility j, is closed;
x_j^{ik}	non-negative decision variable representing the fraction of demand d_{ik} allocated to facility j in the final configuration;
y_j	binary decision variable equal to 1 if and only facility j is closed.

Thus, the model can be formulated as follows:

$$\min\ z = \sum_{k \in K} \sum_{j \in J} c_{kj}\, \Delta_{kj} \qquad (1)$$

Subject to:

$$x_j^{ik} + s_{kj} \leq l_{kj} \qquad \forall i \in I, \forall k \in K, \forall j \in J \qquad (2)$$

$$\sum_{j \in J} x_j^{ik} = 1 \qquad \forall i \in I, \forall k \in K \qquad (3)$$

$$x_j^{ik} = f(\alpha_j^{ik}, s_{kj}) \qquad \forall i \in I, \forall k \in K, \forall j \in J \qquad (4)$$

$$\sum_{i \in I} d_{ik} x_j^{ik} - \Delta_{kj} \leq C_{kj} \qquad \forall j \in J, \forall k \in K \qquad (5)$$

$$y_j - \frac{1}{N_j} \sum_{k \in K} s_{kj} \leq 0 \qquad \forall j \in J \qquad (6)$$

$$y_j + \left(N_j - \sum_{k\in K} s_{kj}\right) \geq 1 \qquad \forall j \in J \qquad (7)$$

$$\sum_{j\in J}\sum_{k\in L_j} \bar{b}_{kj} s_{kj} + \sum_{j\in J} \bar{\bar{b}}_j y_j \geq B \qquad (8)$$

$$s_{kj} \in \{0,1\}, y_j \in \{0,1\}, x_j^{ik} \geq 0, \Delta_{kj} \geq 0 \qquad \forall i \in I, \forall j \in J, \forall k \in K \qquad (9)$$

The objective function (1) represents the minimization of the total cost needed to provide remaining facilities with additional service capacities to satisfy demand resulting from the reallocation of users previously assigned to closed services.

Constraints (2) impose that, for each node $i \in I$, the demand for service k can be assigned only to a facility j that offered ($l_{kj} = 1$) and still offers it ($s_{kj} = 0$). Therefore, this constraint also ensure that service k at facility j may be closed ($s_{kj} = 1$) if and only if it was initially offered by that facility ($l_{kj} = 1$).

Conditions (3) guarantee that, for each node i, the demand for each service k is satisfied thanks to the contribution of facilities still providing that service. It can be noticed that this set of constraints assure that, for each service k, there will always exist at least one facility providing it.

Conditions (4) drive the reallocation of demand after the closure of some existing services. The explicit expression of these conditions depends on the assumptions about the interaction model between users and facilities, which are related to the specific application being considered. In the following, an explicit formulation of this group of constraints will be proposed and described.

Constraints (5) indicate that, for each service k, the total demand assigned to j does not exceed the total capacity of facility j, potentially expanded with the additional capacity Δ_{kj}.

Constraints (6) impose that, if the number of closed services at a given facility j $\left(\sum_{k\in K} s_{kj}\right)$ is lower than the total number of provided services (N_j), the facility remains open $(y_j = 0)$.

Constraints (7) assure that, if all services provided by a given facility j have been closed $\left(\sum_{k\in K} s_{kj} = N_j\right)$, facility j has to be closed $(y_j \geq 1)$.

Constraint (8) expresses the need for the planner to obtain a minimum benefit value B. The total benefit is calculated as the sum of the benefits related to the closure of the single services $\left(\sum_{j\in J}\sum_{k\in L_j} \bar{b}_{kj} s_{kj}\right)$ and of an additional benefit achieved closing facilities as a whole $\left(\sum_{j\in J} \bar{\bar{b}}_j y_j\right)$.

Constraints (9) define the nature of decision variables.

3.1 Reallocation rules

A special discussion is required for constraints (4), which drive the reallocation of the demand after the closure of some existing services. As mentioned, the reallocation of the demand has to be performed according to rules that depend on the underlying spatial interaction model, i.e. the process by which users select the facility to patronize. To this aim, the current allocation matrix $\{\alpha_j^{ik}\}$ may be exploited. In the following, we distinguish between two different situations: a first one in which users select the closest facility; a second one in which they distribute themselves among available facilities according to probabilities that depend on a given utility function.

Accordingly, two rules will be defined: a *Closest re-assignment rule* and a *probabilistic re-assignment rule.*

3.1.1 Closest re-assignment rule

Supposing that initially each user i, for the generic service k, patronizes the closest facility j providing k ($\alpha_j^{ik} \in \{0,1\}$, $\forall i,j,k$), it is reasonable to assume that, after the closure of that service at j, he will continue to choose on the basis of the distance and, hence, he will be reallocated to the closest facility j' still providing k.

It is possible to reproduce this mechanism by restricting the assignment variables x_j^{ik} to be binary in constraints (9) ($x_j^{ik} \in \{0,1\}$ instead of $x_j^{ik} \geq 0$) and, replacing constraints (4) with the following ones, obtained by reformulating Berman et al.(2009) proposal:

$$\sum_{t\in J} d_{it}x_t^{ik} + \left(F - d_{ij}\right)(l_{kj} - s_{kj}) \leq F \qquad \forall i \in I, \forall j \in J, \forall k \in K \qquad (10),$$

where F is a very large positive number (for example, $F = max_i\{\sum_{j\in J} d_{ij}\}$).

For each service k, constraints (10) are trivially satisfied for all those facilities j that didn't offer k ($l_{kj} = 0 \Rightarrow s_{kj} = 0$) or do not offer it anymore ($l_{kj} = 1 \wedge s_{kj} = 1$). In all other cases, i.e., for all those facilities still providing service ($l_{kj} = 1 \wedge s_{kj} = 0$), constraints (10) impose:

$$\sum_{t\in J} d_{it}x_t^{ik} \leq d_{ij} \qquad \forall i \in I, \forall j \in J, \forall k \in K. \qquad (11)$$

Being that the assignment variables are binary, for each demand node i and service k, only one term of the sum at the l.h.s. of (11) will be greater than zero. The inequalities hold for each facility j, if and only if i is assigned, among all those facilities still providing k, to the facility t positioned at the minimum distance $\left(t\colon d_{it} = min_{j\in J:l_{kj}-s_{kj}=1}\{d_{ij}\}, \forall i \in I, k \in K\right)$.

3.1.2 Probabilistic re-assignment rule

If each user initially chooses among the available facilities in a probabilistic fashion, i.e. on the basis of a given measure of the relative perceived utility, it is reasonable to assume that he will continue to choose among the remaining facilities according to the same mechanism.

In particular, it can be supposed that, for each service k, the initial fraction of demand coming from i and being satisfied in j (α_j^{ik}), represents a good estimation of the probability that users from i select facility j to receive service k. After the closure of a given number of services, it would be coherent to assume that users continue to choose among the remaining facilities in a probabilistic fashion; but in this case, the initial probabilities values α_j^{ik} should be updated by taking into account that closed services cannot be patronized anymore.

According to this assumption, a first formulation of constraints (4) may be represented by the non-linear expression:

$$x_j^{ik} = \frac{\alpha_j^{ik}\left(1 - s_{kj}\right)}{\sum_{t\in J} \alpha_t^{ik}(1 - s_{kt})} \qquad \forall i \in I, \forall k \in K, \forall j \in J. \quad (12)$$

First of all, these constraints impose that, for each node i, the demand for service k can be assigned only to facilities j that offered ($l_{kj} = 1$) and still offer it ($s_{kj} = 0$). In fact, if facility j did not offer service k or it has been closed, no fraction of demand will be assigned to it, being, respectively, $\alpha_j^{ik} = 0$ and $s_{kj} = 1$. On the other hand, if facility j still offers service k ($s_{kj} = 0$), the fraction of demand from i assigned to j $\left(x_j^{ik}\right)$ is calculated by normalizing the current fraction α_j^{ik} over the sum of the fractions assigned to the other facilities still providing k. More precisely, if service k remains active at every facility $j \in U_k$, the allocation does not change $\left(x_j^{ik} = \alpha_j^{ik}\right)$, being the denominator equal to 1; on the contrary, if service k has been closed in some facilities, the denominator is lower than 1 $\left(\sum_{t \in J} \alpha_t^{ik}(1 - s_{kt}) < 1\right)$ and then the fraction allocated to each active facility is higher than the current value α_t^{ik}.

It is possible to demonstrate (Appendix A) that conditions (12) are equivalent to the following groups of constraints:

$$x_j^{ik} + s_{kj} \leq l_{kj} \qquad \forall i \in I, \forall k \in K, \forall j \in J \qquad (2)$$

$$\sum_{j \in J} x_j^{ik} = 1 \qquad \forall i \in I, \forall k \in K \qquad (3)$$

$$x_j^{ik} \leq \frac{\alpha_j^{ik}}{\alpha_t^{ik}} x_t^{ik} + s_{kt} \qquad \forall i \in I, \forall k \in K, \forall j, t \in U_k : j \neq t. \qquad (13)$$

This linearization has been obtained by adapting the procedure proposed by Aros-Vera et al. (2013) with reference to a Logit model (see Appendix A).

Then, as constraints (2) and (3) are already included in the proposed formulation, the final form of the model is given by (1-9) replacing (4) with (13).

It must be highlighted that a necessary condition for the consistency of this linearization is that α_j^{ik} values have to be strictly positive $\forall i \in I, \forall k \in K, \forall j \in U_k$. This may be not considered as a restrictive assumption because it is reasonable to assume that, for each service k, the probability that users from i select a facility $j \in U_k$ is larger than zero. If the initial allocation does not satisfy this requirement (i.e., there exists at least one demand node i from which users do not select a facility $j \in U_k$) it will be sufficient to assign an arbitrary low value ε to the corresponding α_j^{ik} and to modify the other ones consequently.

4 - Computational results

The model was tested, in its probabilistic re-assignment version, on randomly generated instances, obtained according to the following procedure.

Step 1: The cardinality of sets I, J, K was fixed; in particular $|I| = 100, 200$; $|J| = 8, 10, 12$; $|K| = 5, 10, 15$. For each triplet $(|I|, |J|, |K|)$, 5 different instances were generated and solved.

Step 2: $|I| + |J|$ points were randomly positioned in a 100x100 square, according to a uniform distribution; then, the Euclidean distance t_{ij} between each demand node i and each facility j was calculated.

Step 3: l_{kj} values were generated according to a Bernoulli probability distribution with parameter equal to 0.3, in such a way to obtain, on an average, the 30% of services K opened at the available facilities J.

Step 4: The demand distribution for each service $k \in K$ across the demand nodes $i \in I$ $\{d_{ik}\}$ and the initial assignment of such demands among the available facilities $\{\alpha_j^{ik}\}$ were generated, according to the two following main assumptions:

- *demand nodes of different magnitudes (for instance, representing the population or the demand for a given service at that node) are present in the location space and, for each service, the demand originated by each node i is related to such magnitude;*
- *the interaction rule between demand nodes and facilities is based on a gravity model.*

According to the first assumption, a parameter p_i, representing the magnitude of node i, was associated with each $i \in I$; demands d_{ik} coming from it, were consequently assumed proportional to such value. This mechanism was introduced as in most of real applications demand for public services can be generally assumed proportional to the population living in each considered area. In this case, p_i values were generated from a probability distribution that reproduces a possible population profile in a regional context; the demand d_{ik} for each node i and each service k was assumed to be proportional to p_i through a factor β_k ($d_{ik} = \beta_k\, p_i$) uniformly distributed in the range $[0.0, 0.2]$.

According to the second assumption, we first associated with each pair (k, j), such that $l_{kj} = 1$, an attractiveness value M_{kj} uniformly randomly generated in the range [0,1]. Then, on the basis of these values and of the Euclidean distances t_{ij} between each demand node i and each facility j, we calculated probabilities α_j^{ik}, according to the formula:

$$\alpha_j^{ik} = \frac{u_j^{ik}\left(M_{kj}, t_{ij}\right)}{\sum_{j \in U_k} u_j^{ik}\left(M_{kj}, t_{ij}\right)}$$

where:

$$u_j^{ik}\left(M_{kj}, t_{ij}\right) = \frac{M_{kj}}{t_{ij}^{1.5}}$$

Step 5: For each service k, the total demand D_{kj} assigned to each facility $j \in U_k$ $\left(D_{kj} = \sum_i \alpha_j^{ik} d_{ik}\right)$ was computed; then, capacities were fixed on the basis of the total demand assigned to the most preferred facility $j \in U_k$ ($D_k^* = \max_{j \in U_k}\{D_{kj}\}$). In particular, for each facility j offering k, it was assumed $C_{kj} = \gamma_{kj}\, D_k^*$, being:

$$\gamma_{kj} = \begin{cases} 0.25 & if\ \ 0.00 < \dfrac{D_{kj}}{D_k^*} \leq 0.25 \\ 0.50 & if\ \ 0.25 < \dfrac{D_{kj}}{D_k^*} \leq 0.50 \\ 0.75 & if\ \ 0.50 < \dfrac{D_{kj}}{D_k^*} \leq 0.75 \\ 1.00 & if\ \ 0.75 < \dfrac{D_{kj}}{D_k^*} \leq 1.00 \end{cases}$$

Step 6: c_{kj} and $\bar{b}_{kj}$ values were fixed equal to 1 for each pair (k, j), while $\bar{\bar{b}}_j$ values equal to 0 for each facility j. This way B represents the minimum number of services to be closed and it was fixed as a percentage of 20% of the total number of active services $\left(\sum_{k,j} l_{kj}\right)$.

The test problems were solved using Cplex 12.2 on an Intel Core i7 with 1.86 GHz and 4 GB of RAM. In Table 1 running times (minimum, maximum, average) needed to obtain the optimal solutions are indicated. The corresponding average number of variables and constraints are also specified.

Problem Parameters			Problem Size		CPU Time (Seconds)		
$\|J\|$	$\|I\|$	$\|K\|$	# Variables	# Constraints	Minimum	Average	Maximum
8	100	5	4081	7981	4.67	12.58	28.32
		10	8161	17041	13.00	64.95	157.23
		15	12241	23021	48.38	122.87	237.97
8	200	5	8081	18201	21.71	51.64	93.96
		10	16161	29681	27.39	90.27	179.00
		15	24241	47081	94.33	627.61	1592.13
10	100	5	5101	10841	11.37	23.26	37.40
		10	10201	21241	18.08	597.97	2507.82
		15	15301	31641	210.18	535.48	1052.71
10	200	5	10101	22301	50.96	108.94	183.09
		10	20201	38521	52.61	488.68	1254.12
		15	30301	60101	495.44	1329.25	3251.64
12	100	5	6121	13221	17.64	40.03	67.63
		10	12241	28601	168.01	591.21	1515.92
		15	18361	40611	273.19	2147.29	4124.01
12	200	5	12121	33281	130.06	259.92	766.69
		10	24241	49761	220.55	1604.62	4672.19
		15	36361	69601	1517.38	3960.73	7399.34

Table 1 – Computational Results

Results show that the CPU time generally depends on the combination of the cardinalities of the sets I, J and K; however, the most critical parameter appears to be $|K|$. Problems with lower values of $|K|$ (for example, 5 and 10) can be optimally solved within reasonable times while higher values of $|K|$ (for instance, 15) determine significant increases. Even more critical running times occur for the combination $(|J| = 12,\ |I| = 200,\ |K| = 15)$. Further tests performed on instances with $|J| = 12,\ |I| = 200,\ |K| = 20$ showed that the solver was not always capable to obtain the optimal solution within the time limit of three hours. The size of problems optimally solved appears, however, compatible with those associated with many real-life applications.

5 - Application to a real-world case study

An additional test was performed to model a real problem related to the rationalization of a public university system on a regional scale. In the last decades an attempt to widen general participation in higher education occurred in industrialized countries (Craig 1981; Robinson and Ralph 1984; Garnier and Hage 1991). Coherently to this aim, in Italy there has been a strong increase in the number of academic sites and degree courses. From 1995 to 2009 the total number of institutions rose from 60 to 86; in the same period, while the number of cities hosting main academic sites increased from 45 to 57, the number of cities hosting detached (off-main campus) university sites doubled, increasing from 93 to 185 (see Figure 1). It has to be underlined that the degree of autonomy of Italian Higher Education institutions is still quite low, if compared to similar countries; moreover, Italian universities are still massively publicly subsidized, as 80% of resources are coming from central and regional government transfers. Also, competition among universities appears to be quite low, as one of the peculiarities of the Italian University system is represented by the so-called *legal value of academic qualifications*, providing public recognition of degrees (for instance, for the access to professional careers) regardless of the specific awarding institution (and of its prestige).

In this context, the above-mentioned rise in the number of academic institutions was not driven by market forces, but rather by governmental policies. Coherently to the non-competitive nature of the system, many of these newly created institutions were originating as 'splits' or decentralized campuses from existing universities. For this reason, in many cases, faculty staff of the newly created institutions were just transferred from the 'parent' organization (either permanently or temporarily, on a shared employment basis).

Some recent analyses (performed by the Italian Ministry of University; see, for instance, CNVSU, 2013) revealed that the growth of the supply produced a system characterized by a high percentage of degree programs attracting demand levels much lower than target values fixed by central government. The affordability of such a system has been highly questioned also given the recent application of austerity measures and severe cuts to public expenditure. In this context, rationalization strategies have been considered. Taking into account that the mobility of students across Italian regions is quite low (due to the cited non-competitive nature of the system, and to open access policies), as around 80% of the students choose to study in their home region (Bruno and Genovese, 2012), there is a common agreement on the fact that rationalization strategies have to be defined at a regional level.

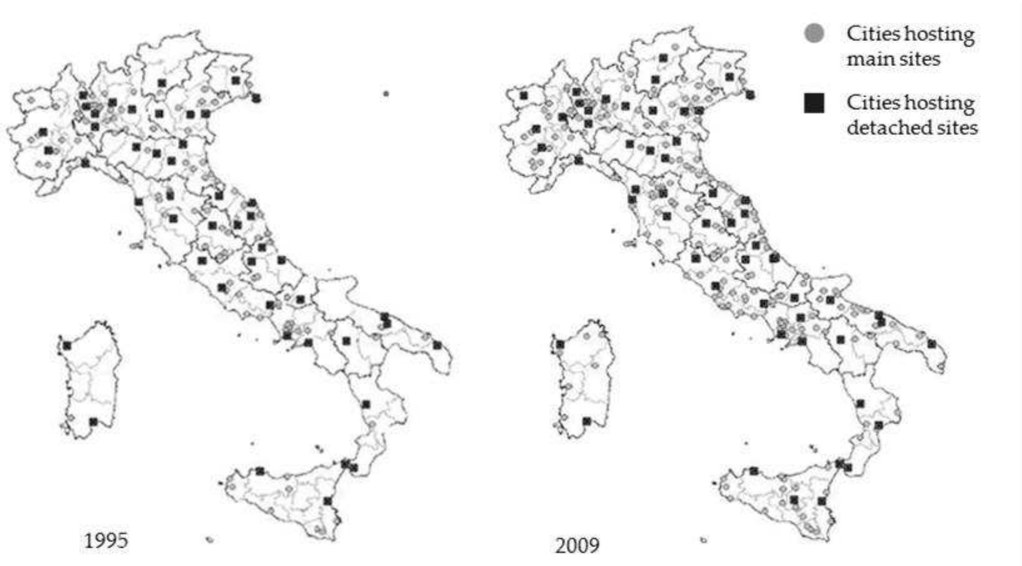


Figure 1 – Cities with main campus and detached sites. A comparison between 1995 and 2009

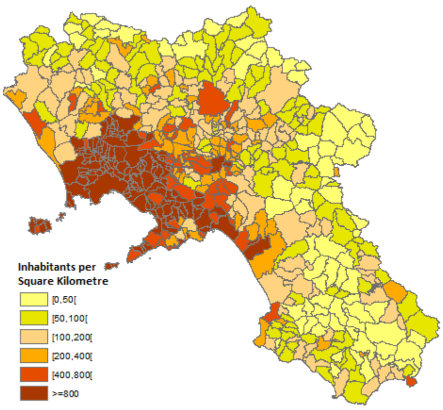
4.1 Model Adaptation and Results

In the described context, we applied the proposed model to analyze the regional university system of the second most populated region in Italy (Campania), hosting 7 public universities, with 5 engineering faculties. These faculties offer a wide number of degree programs which can be classified into 9 different groups.

The case under analysis was modeled as a rationalization problem, in which existing Engineering Faculties may be interpreted as facilities offering a set of services (the different degree programs) to users (the students). A central planner (the regional government) is interested in rationalizing the whole system by shutting down some services (and potentially, entire facilities) in order to find solutions which provide a good balance between affordability purposes and general "public interest" (service accessibility).

Also in the case of the Campania region, as mentioned above for the national average, the outflow of students towards other regions is quite low. Moreover (as shown in Figure 2), the density of population in areas closer to regional borders (that could be the ones in which students are more prone to study in other regions) is quite low compared to remaining areas; therefore, outgoing flows can be neglected and a rationalization strategy can be defined, in a reasonable way, at a regional level. Therefore, this problem can be effectively described by the general model presented in section 3, which was adapted as illustrated below.

In order to aggregate demand data in a manageable way, a discretization of the location space was adopted, as introduced by Bruno and Improta (2008). This way, the Campania region was divided into 58 internal districts and an additional zone (59), representing the rest of the world outside the region. Figure 3 shows the zoning of the study area and the location of the five existing engineering faculties.



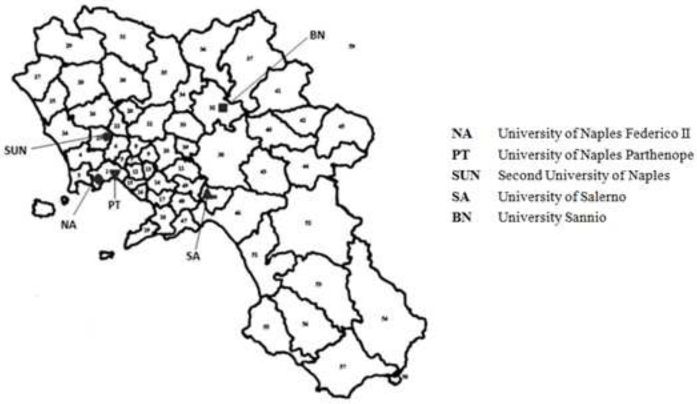


Figure 2 (left) – Population density of the Campania Region
Figure 3 (right) – Zoning System of the Campania Region

According to the introduced notation, the following variables and parameters can be defined:

α_j^{ik} fraction of students coming from zone i initially enrolled at degree program k of faculty j;

B minimum benefit to be obtained.

$\bar{b}_{kj}$ benefit deriving from the closure of degree program k at faculty j;

$\bar{\bar{b}}_j$ additional benefit deriving from the closure of the whole faculty j;

c_{kj} cost to provide an additional unit of capacity to degree program k at faculty j;

C_{kj} maximum number of students that can be served by degree program k at faculty j;

d_{ik} total number of students coming from zone i requiring to enroll at degree program k;

I set of demand nodes coinciding with 59 zones;

J set of existing facilities coinciding with 5 faculties;

K	set of different types of services coinciding with 9 degree programs;
L_j	set of degree programs provided by faculty j;
U_k	set of faculties providing degree program k.

In particular, we assumed $\bar{b}_{kj}$ equal to 1 for any pair (k, j) and $\bar{\bar{b}}_j = 0$ for each faculty j, as a thorough estimation of these benefit parameters would require much deeper analysis and significant efforts. This way, the first term of constraint (9) represents the number of closed degree programs and the second term B the minimum number to be closed. In order to evaluate the solutions provided by the model by varying parameter B from 1 to the maximum feasible value, we considered (9) as an equality constraint. In particular, in order to satisfy constraints (4), which ensure the availability of at least one faculty providing each type of degree program k $(U_k \neq \emptyset, \forall k)$, the maximum number of programs that may be closed is given by the difference between the total number of active programs in the region $\left(\sum_{k,j} l_{kj} = 29\right)$ and the cardinality of set K $(|K| = 9)$. We also assumed c_{kj} equal to 1 for any pair (k, j); this way, the objective function indicates the total additional capacity needed to satisfy the overall demand.

As concerns demand, values for d_{ik} and α_j^{ik} were derived from official historical records and data about enrollments (detailing degree programmes and municipality of origin for each student), obtained by liaising with administrative offices from the five faculties. In particular, the demand for each degree program was assumed equal to the average of enrolments in the last six academic years, i.e. from the 2008/2009 to 2013/2014; more refined forecasts for future demands could include the identification of possible trends in enrolments. In Table 4, for each degree program k and each faculty j, the total number of students enrolled $\left(D_{kj} = \sum_i \alpha_j^{ik} d_{ik}\right)$ and the related capacity C_{kj} are shown. In particular, capacities are fixed according to the requirements defined by the Italian Ministry of Higher Education (CNVSU, 2011), mainly taking into account students/staff ratios and infrastructural issues.

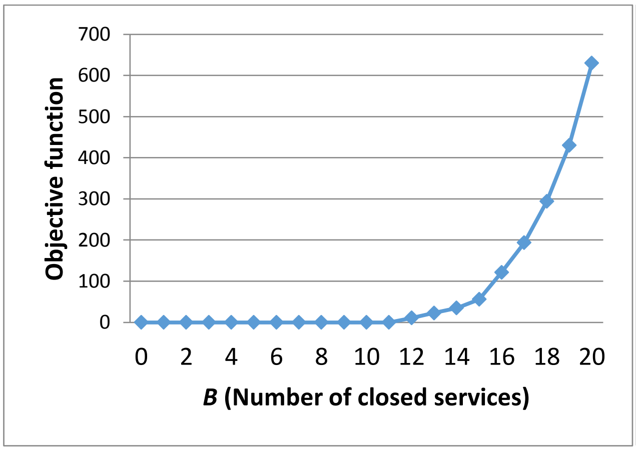
		Faculty j											
		NA		***SUN***		***PT***		***SA***		***BN***		***Total***	
		Enrol.	***Cap.***	***Enrol.***	***Cap.***	***Enrol.***	***Cap.***	***Enrol.***	***Cap.***	***Enrol.***	***Cap.***	***Enrol.***	***Cap.***
Degree Program k	***Civil***	163	300	150	200	47	100	125	300	69	150	**554**	**1050**
	Environmental	87	150	33	100	11	50	63	150	-	-	**194**	**450**
	IT	293	450	88	100	-	-	139	300	81	150	**601**	**1000**
	Electronic	120	300	38	50	-	-	51	150	-	-	**209**	**500**
	Telecommunication	52	150	-	-	34	150	-	-	27	150	**113**	**450**
	Aerospace	229	450	44	50	-	-	-	-	-	-	**273**	**500**
	Mechanical	455	600	97	100	-	-	137	150	-	-	**689**	**850**
	Chemical	171	300	-	-	-	-	88	150	-	-	**259**	**450**
	Management	337	450	-	-	51	150	130	300	-	-	**518**	**900**
	Total	**1907**	**3150**	**450**	**600**	**143**	**450**	**733**	**1500**	**177**	**450**	**3410**	**6150**

NA: University of Naples Federico II; PT: University of Naples Parthenope; SUN: Second University of Naples; SA: University of Salerno; BN: University of Sannio

Table 4 - Enrolments to Engineering Degree Programs offered by Campania Universities

Figure 4a illustrates the pattern of the objective function (representing the required additional capacity) while Figure 4b shows the utilization rate, over the parameter *B*. The utilization rate was calculated as the ratio between the total demand in the region $\left(\sum_{i,k} d_{ik}\right)$ and the total capacity available at the remaining services $\left(\sum_{j,k} C_{kj}\left(l_{kj} - s_{kj}\right) + \Delta_{kj}\right)$; i.e. the capacity of remaining services plus the activated additional capacities. Comparing these figures, it can be noticed that the closure of a significant number of services ($B \leq 11$) can be performed with no investment in terms of additional capacities. This aspect can be interpreted as a signal of the current inefficiency of the overall system, which seems to present a significant level of excess capacity in terms of offered services. This fact is confirmed by the current low value of the overall utilization degree (equal to 3423/6150=0.54, Figure 4b).

The reduction of the number of active services (with no activation of additional capacity) allows the system to increase its degree of overall capacity utilization up to a maximum value of about 0.70. In order to reach better levels of capacity utilization it is necessary to close more services ($B \geq 12$) and invest in additional capacity. However, by closing more services, the remaining ones get more and more utilized; therefore, even large increases in additional capacity produce very limited improvements in the overall utilization rate ($B \geq 16$). Hence, the marginal contribution of the investment in additional capacity on the overall utilization rate decreases significantly.



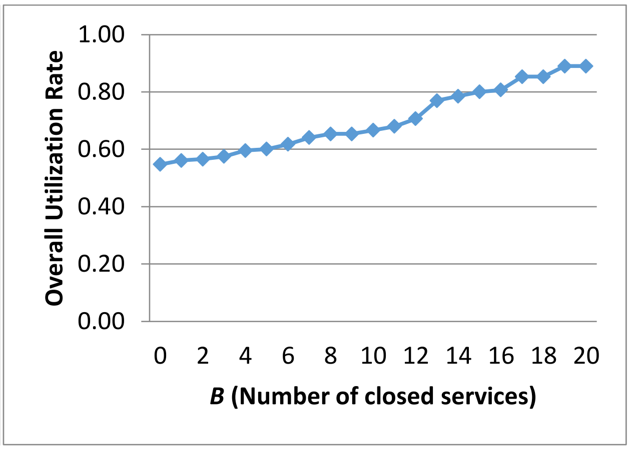


Figure 4a (left) – Objective Function by varying *B* value

Figure 4b (right) - Overall utilization rate by varying *B* value

As an illustrative example, in Table 5 we show the results obtained for B = 11. In particular, the table provides, for each degree program at each faculty, the utilization rate (i.e., the ratio between enrollments and capacity) before and after the closure of 11 services (marked in the table with N/A, meaning that the utilization rate for such services cannot be calculated, as they have been closed). It can be noticed that the model tends to close services with lower enrollment rates, as reallocating these demand portions is the least costly option. In particular, this solution is characterized by the closure of all the degree programs offered by the engineering faculty at Parthenope University.

		Faculty j									
		NA		***SUN***		***PT***		***SA***		***BN***	
		Before	After	Before	After	Before	After	Before	After	Before	After
Degree Program k	***Civil***	0.77	0.88	0.89	0.97	0.61	N/A	0.42	0.45	0.34	0.35
	Environmental	0.64	0.77	0.52	0.57	0.57	N/A	0.35	0.37	-	-
	IT	0.74	0.95	0.99	N/A	-	-	0.29	0.30	0.28	0.31
	Electronic	0.33	0.59	0.83	N/A	-	-	0.23	N/A	-	-
	Telecommunication	0.61	N/A	-	-	0.15	N/A	-	-	0.08	0.84
	Aerospace	0.43	0.53	0.93	N/A	-	-	-	-	-	-
	Mechanical	0.73	0.88	0.91	N/A	-	-	0.60	0.61	-	-
	Chemical	0.66	0.82	-	-	-	-	0.31	N/A	-	-

	Management	0.93	0.99	-	-	0.21	N/A	0.25	0.25	-	-
	Total	0.67	0.77	0.85	0.42	0.32	N/A	0.34	0.30	0.23	0.50
NA: University of Naples Federico II; PT: University of Naples Parthenope; SUN: Second University of Naples; SA: University of Salerno; BN: University of Sannio											

Table 5 – Capacity utilization rate before and after the closure of *B* = 11 services

6. - Conclusions

In this paper we have proposed a mathematical model to support location decisions in order to rationalize facility systems in public sector contexts. The model assumes the presence of a set of facilities offering different types of services that may be closed in order to increase the affordability of the system. Due to the downsizing of existing services, demand is reallocated, assuming that users are attracted by remaining facilities in a probabilistic fashion. The objective function is formulated in terms of the minimization of the total cost needed to provide remaining facilities with additional service capacity to satisfy demand resulting from the reallocation of users previously assigned to services that have then been closed.

The model is quite general and can be adapted to a wide range of real applications, especially in the public sector context. Computational results show that, using a commercial optimization solver, optimal solutions can be obtained, in reasonable computational times, for instances whose size can be also representative of real problems. Furthermore, the model has been tested on a real-world case study, based on the rationalization strategy of the university system of an Italian region.

Future directions of research may concern the proposal of different versions of the model to take into account various scenarios in terms of objective function and constraints; similarly, a thorough estimation of the financial parameters involved in the model (related to extra-capacity costs and potential benefits deriving from closure and downsizing decisions) could be conducted. Also, wider impact of downsizing and closure of facilities could be investigated, in terms of both modifications to future demand and quantification of further discomfort imposed on users. Moreover, in order to cope with larger instances, heuristics should be developed.

Appendix A

In the following, we prove that the combination of the three sets of constraints (2), (3), (13) guarantees that fractions of demand assume the same values defined by the expressions (12).

For each service k, consider the following subsets of J:

T_k subset of facilities that did not provide service k $\left(T_k = \{j \in J: l_{kj} = 0\}\right)$;

V_k subset of facilities at which service k has been closed $\left(V_k = \{j \in J: l_{kj} = 1, s_{kj} = 1\}\right)$;

W_k subset of facilities that still provide service k $\left(W_k = \{j \in J: l_{kj} = 1, s_{kj} = 0\}\right)$.

Note that the above introduced subsets form a partition of J; in fact, V_k and W_k form a partition of the set of facilities providing k ($V_k \cup W_k = U_k, V_k \cap W_k = \emptyset$) and T_k is the complement set of U_k to J ($T_k = J - U_k$).

For each service k and user i, the equivalence between (2, 3,13) and (12) is trivially proved for any facility j not providing k in the final configuration; i.e., $\forall j \in T_k \cup V_k$.

Indeed, conditions (3) impose:

$$x_j^{ik} = 0 \qquad \forall i \in I, \forall k \in K, \forall j \in T_k \cup V_k$$

similarly to conditions (11), being respectively in T_k and V_k $\alpha_j^{ik} = 0$ and $s_{kj} = 1$.

Then, the equivalence has to be proved only for facilities j still providing k; i.e., $j \in W_k$.

Conditions (12), for each service k, define a proportional relationship between the fractions of demand assigned to each pair of facilities j and r belonging to $U_k = V_k \cup W_k$.

For each pair $(j, r) \in U_k \times U_k$, one of the following conditions can occur:

a) $j \in W_k, r \in V_k$: facility j still provides service k ($s_{kj} = 0$) while r not anymore ($s_{kr} = 1, x_r^{ik} = 0$);

b) $j \in V_k, t \in W_k$: facility r still provides service k ($s_{kr} = 0$) while j not anymore ($s_{kj} = 1$, $x_j^{ik} = 0$);

c) $j, r \in V_k$: service k has been closed at both facilities j and r ($s_{kr} = s_{kj} = 1, x_r^{ik} = x_j^{ik} = 0$);

d) $j, r \in W_k$: : service k is still provided by both facilities j and r ($s_{kr} = s_{kj} = 0$).

We now demonstrate that conditions (13) become active only in the last case. With this aim, consider the paired conditions associated with (j, r):

$$\begin{cases} x_j^{ik} \le \frac{\alpha_j^{ik}}{\alpha_r^{ik}} x_r^{ik} + s_{kr} \\ x_r^{ik} \le \frac{\alpha_r^{ik}}{\alpha_j^{ik}} x_j^{ik} + s_{kj} \end{cases} \qquad \forall i \in I.$$

From Table 6, in which the expressions of the above conditions in the single cases are reported, it is easy to understand that in the first three cases the constraints are trivially satisfied $\forall i \in I$.

a	*b*	*c*	*d*
$\begin{cases} x_j^{ik} \leq 1 \\ 0 \leq \frac{\alpha_r^{ik}}{\alpha_j^{ik}} x_j^{ik} \end{cases}$	$\begin{cases} 0 \leq \frac{\alpha_j^{ik}}{\alpha_r^{ik}} x_r^{ik} \\ x_r^{ik} \leq 1 \end{cases}$	$\begin{cases} 0 \leq 1 \\ 0 \leq 1 \end{cases}$	$\begin{cases} x_j^{ik} \leq \frac{\alpha_j^{ik}}{\alpha_r^{ik}} x_r^{ik} \\ x_t^{ik} \leq \frac{\alpha_r^{ik}}{\alpha_j^{ik}} x_j^{ik} \end{cases}$

Table 6 – Possible expressions of conditions (12) for a generic pair $(j, r) \in U_k \times U_k$

In case *d* the two conditions become equivalent to the following one:

$$x_r^{ik} = \frac{\alpha_r^{ik}}{\alpha_j^{ik}} x_j^{ik} \qquad \forall i \in I.$$

Hence, for a particular user i and service k, it is possible to express all the fractions of the demand assigned to the facilities in $W_k = \{r_1, \dots, r_w\}$ as a function of the same variable x_j^{ik} $(j \in W_k)$. Therefore, replacing:

$$x_r^{ik} = \frac{\alpha_r^{ik}}{\alpha_j^{ik}} x_j^{ik} \qquad \forall i \in I, \forall r \in W_k,$$

in (4), we have

$$\sum_{j \in J} x_j^{ik} = \sum_{j \in T_k} x_j^{ik} + \sum_{j \in V_k} x_j^{ik} + \sum_{j \in W_k} x_j^{ik} = \sum_{j \in W_k} x_j^{ik} = x_{r_1}^{ik} + \dots + x_{r_w}^{ik} = \left(\frac{\alpha_{t_1}^{ik}}{\alpha_j^{ik}} + \dots + \frac{\alpha_{r_w}^{ik}}{\alpha_j^{ik}} \right) x_j^{ik} = 1 .$$

Hence:

$$x_j^{ik} = \frac{\alpha_j^{ik}}{\alpha_{r_1}^{ik} + \dots + \alpha_{r_w}^{ik}} = \frac{\alpha_j^{ik}}{\sum_{r \in W_k} \alpha_r^{ik}} \tag{14}$$

For a given service k, equation (14) holds for all the facilities in the set W_k and each user i; then we can generally write:

$$x_j^{ik} = \frac{\alpha_j^{ik}}{\sum_{r \in W_k} \alpha_r^{ik}} \qquad \forall i \in I, \forall k \in K, \forall j \in W_k \tag{15}$$

which is equivalent to (12) $\forall j \in W_k$.

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