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Modelling Acceleration Decisions in Traffic Streams with Weak Lane Discipline: A Latent Leader Approach

Abstract

Acceleration is an important driving maneuver that has been modelled for decades as a critical element of the microscopic traffic simulation tools. The state-of-the art acceleration models have however primarily focused on lane based traffic. In lane based traffic, every driver has a single distinct lead vehicle in the front and the acceleration of the driver is typically modelled as a function of the relative speed, position and/or type of the corresponding leader. On the contrary, in a traffic stream with *weak lane discipline*, the subject driver may have multiple vehicles in the front. The subject driver is therefore subjected to multiple sources of stimulus for acceleration and reacts to the stimulus from the *governing leader*. Hence, only the applied accelerations are observed in the trajectory data, and the governing leader is unobserved or *latent*. The state-of-the-art models therefore cannot be directly applied to traffic streams with weak lane discipline.

This prompts the current research where we present a *latent leader* acceleration model. The model has two components: a random utility based dynamic class membership model (latent leader component) and a class-specific acceleration model (acceleration component). The parameters of the model have been calibrated using detailed trajectory data collected from Dhaka, Bangladesh. Results indicate that the probability of a given front vehicle of being the governing leader can depend on the type of the lead vehicle and the extent of lateral overlap with the subject driver. The estimation results are compared against a simpler acceleration model (where the leader is determined deterministically) and a significant improvement in the goodness-of-fit is observed. The proposed models, when implemented in microscopic traffic simulation tools, are expected to result more realistic representation of traffic streams with weak lane discipline.

Keywords

Acceleration, weak lane discipline, mixed traffic

1. Background

Microscopic traffic simulation tools, which model individual driver manoeuvres (e.g. longitudinal and lateral movements, route choice, etc.) and deduce the network condition from those, can be used as laboratories for testing the effectiveness of candidate traffic improvement initiatives before their actual field implementation. These tools are increasingly being popular worldwide for selecting the optimum transport scheme. In particular, they can potentially play a very significant role in the context of developing countries where the transport landscape is changing very rapidly but the resources are often constrained.

Acceleration is an important driving manoeuvre and has been significantly modelled for several decades. However, majority of this research is conducted for homogeneous traffic conditions which prevail in developed countries. The developed models range from simple models with minimum parameters, to comparatively complex and comprehensive models with much detailed considerations and can be grouped as car-following models (e.g.*1-10*), psychophysical models (e.g. *11-13*), fuzzy-logic models (e.g. *14-16*), cellular automata models (e.g. *17-19*), and general acceleration models (e.g. *20-23*) . The state-of-the-art acceleration models however have several limitations. For example, as highlighted by Kim et al. (*24*) and Punzo and Simonelli (*25*), some acceleration models (e.g. *1-3, 5, 20*) are based on the assumption that drivers always follow the same driving decision rules. Whereas, in reality, these rules may differ among different drivers, for the same driver in different conditions, or even for same driver in similar or nearly identical situations (*25*). Other limitations include inadequate emphasis on the errors and uncertainty in the data used for calibrating the models (*25*), exclusion of factors beyond vehicle kinematics and surrounding conditions in the model framework, limited stochasticity, etc. Moreover, the most important setback of these models is that these are developed for homogeneous lane-based traffic (Figure 1a) and cannot be directly applied to heterogeneous traffic stream where the traffic characteristics are significantly different (Figure 1b).

<Figure 1>

The heterogeneous traffic scenarios have a mix of motorized and non-motorized vehicles which have wide differences in size, speed and acceleration-deceleration capabilities. The required driving skills for operating motorized and non-motorized vehicles are very different as well. Moreover, in most cases, there is an absence of strict lane discipline in such mixed traffic. In presence of weak lane discipline, a single lane can be occupied by multiple narrow vehicles. Also, even if there are lane-markings, in congested conditions, drivers very often position themselves in between other vehicles in an attempt to make use of the entire available space and thus occupy multiple lanes.

The research work on modelling such heterogeneous traffic stream is quite limited. In the earlier models for heterogeneous traffic, the acceleration models for homogenous traffic have been recalibrated using simulation runs (e.g. *26-28*). Mallikarjuna and Rao (*29*) have focused on the analysis and modelling of heterogeneous traffic observed on mid-block sections of urban and rural roads in the context of India using a cellular automata approach. Some studies have identified significant effects of lead-vehicle size (e.g. *28, 30*) and type of vehicle pair (e.g. *31*) in vehicle-following behaviour in mixed traffic streams. Lee and Polak (*32*) have developed a desired headway based model for motorcycles which are assumed to have the option to either decelerate or overtake a decelerating front vehicle. But the behaviour of other types of vehicles in presence of motorcycles is beyond the scope of their study. Gunay (*33*) has proposed a staggered car-following model which accounts for lateral discomfort while longitudinal movement is in action. Imran (*34*) has focused on development of car-following model for mixed traffic using fuzzy-logic inference system and in addition to vehicle types and composition, traffic factors (e.g. traffic density) have been identified as a factor affecting acceleration decisions. However, though these models address the issue of lead vehicle type and/or vehicle pair type and traffic factors in detail, there has not been much research on how the models can be extended when there are multiple candidate leaders and the governing leader cannot be determined deterministically. There is a marked research gap regarding identification of governing leader vehicle and the existing models are therefore not robust in the context of weak lane discipline. This has motivated this research where a latent leader model has been formulated and calibrated using detailed trajectory data.

The rest of the paper is organized as follows: the model structure is presented first followed by the descriptions of the data. The estimation results are presented next. The findings are summarized in the concluding section with directions of future research.

2. Model Structure

In the state-of-the-art acceleration models, the observed actions of the lead vehicle in front of the driver have a significant role in predicting the acceleration/deceleration decisions of the subject driver. However, in traffic streams with weak lane discipline, there are often multiple candidate leaders in front of the subject driver (SD), particularly in congested situations (denoted by Front Left (FL), Front Direct (FD) and Front Right (FR) (Figure 2). Therefore, though the longitudinal acceleration/ deceleration of the subject driver are governed by the actions of one of these vehicles either consciously or inadvertently, the leader is often not distinctly observed (hence latent) in the data.

The acceleration decision of the driver is thus modelled using a two level structure. The first level is a dynamic class membership model that predicts the probability of a front vehicle being the governing leader of the subject driver at a given time. The second level denotes the acceleration of the subject driver conditional on the specific leader vehicle (Figure 2).

<Figure 2>

The first level is formulated using a random utility based discrete choice framework where the *choice-set* can consist of up to three vehicles (denoted by Front Left, Direct Front and Front Right). The probability of any front vehicle being the governing lead vehicle can be affected by the relative positions, speeds and types of the vehicles, etc. The second level is formulated using the state-of-the-art, non-linear stimulus-response model, the *GM Non-linear Model* (proposed by researchers at General Motors *(1)* and extended by later researchers *(2, 4-10)*. The parameters of the stimulus and sensitivity functions are allowed to vary depending on the type of the subject and leader vehicle. The modelling challenge lies in the fact that only the acceleration decision of the driver is observed and the choice of the leader and thus the acceleration stimuli is unobserved or latent.

The first level is thus a *Latent Class* membership model (*35, 36*). But the class membership is dynamic (the leader can change over time) and hence similar to *Latent Plan* models which have been used for modelling lane changing decisions for lane-based traffic (*37,38*) . The model components are further described below.

2.1 Latent Leader Component

As shown in Figure 2, a subject driver (SD) may have multiple vehicles in the front (FL, FD and/or FR), particularly in a congested traffic stream. The extent of influence of the leader can vary depending on the position (i.e. lateral overlap between the subject driver and the candidate leader, headway, etc.), type of the vehicles (e.g. if a front vehicle is a heavy vehicle, it may have a more dominating role in the behaviour of the subject driver), speed and accelerations of the candidate leaders (e.g. a decelerating front vehicle may have a more dominating role than an accelerating one, a front vehicle moving much faster may have less impact than a front vehicle with similar relative speed, etc.). To account for these multiple factors, the probability of a front vehicle l being the governing leader of the subject driver *n* at time *t* can be modelled as a random utility based discrete choice model and expressed as follows:

$$P\big(l_n(t)\big) = \frac{exp\big(\beta^l X_n^l(t)\big)}{\sum_{l'} \exp\big(\beta^{l'} X_n^{l'}(t)\big)} \qquad l, l' \in C_n = FL, FD, FR \tag{1}$$

Where:

$X_n^l(t)$: Vector of explanatory variables associated with the front vehicle l of driver *n* at time t

β^l: Vector of estimated parameters associated with front vehicle l

Candidate explanatory variables include proportion of lateral overlap between the subject driver and the front vehicle, headway, type of the front vehicle, type of the subject vehicle, speed and acceleration of the front vehicle, etc.

2.2. Acceleration Component

The stimulus-sensitivity framework proposed within the GM Model is adapted for the acceleration model component. In the GM Model framework, the acceleration of the subject driver is a response to the stimulus provided by the leader vehicle in the front. Hence, the acceleration of the subject driver is a direct function of the speed, position and characteristics of the leader vehicle. Following the framework, the acceleration driver *n* applies at time *t* is assumed to be a response to stimuli from the leader *l*:

$$response_n^l(t) = sensitivity_n^l\,(t) \times stimulus_n^l\,(t - \tau_n) \tag{2}$$

Where, τ_n is the reaction time of the driver and *l* denotes the leader.

The stimulus is usually the relative speed of the subject relative to the leader (defined here as the speed of the leader less the speed of the subject vehicle). The sensitivity is a function of explanatory variables like the speed of the subject vehicle, headway, types of leader and subject vehicle, etc. The response to positive and negative stimuli may be different because of the different nature of these situations: the main consideration in the reaction to a negative leader relative speed is safety, whereas the acceleration applied in a positive leader relative speed situation may be affected by speed advantage considerations and by *herd effect* (i.e. human tendency to conform with the actions of others). To capture these differences, the explanatory variables and/or corresponding coefficients may vary depending on if the stimulus is positive (leading to acceleration) or negative (leading to deceleration).

Based on relative speed of leader, the acceleration can thus be car-following acceleration or deceleration:

$$a_n^l(t) = \begin{cases} a_n^{l,acc}(t) & if\ \Delta V_n^l(t - \tau_{n)} \geq 0 \\ a_n^{l,dec}(t) & otherwise \end{cases} \tag{3}$$

Where,

$$\Delta V_n^l(t) = V_n^l(t) - V_n\ (t)$$

$V_n^l(t)$ = *speed of leader at time t*

$V_n\ (t)$ =*speed of subject driver at time t*

The general functional form of acceleration of driver *n* with respect to leader *l* at time *t* can therefore be expressed as follows:

$$a_n^{lj}(t) = s^{l,j}[X_n^l(t)]m^{lj}\left[\Delta V_n^l(t-\tau_{n)})\right] + \varepsilon_n^{lj}(t) \tag{4}$$

Where,

$j \in acceleration, decleration$

$X_n^l(t) = explanatory\ variables\ related\ to\ leader\ l$

$s^{l,j}[.] = sensitivity\ function\ for\ leader\ l$

$m^{lj}[.] = stimulus\ function\ for\ leader\ l$

$\varepsilon_n^{lj}(t) = random\ error\ term$

Assuming that the random error term is normally distributed, $\varepsilon_n^{lj}(t) \sim N(0, \sigma_{lj}^2)$, the probability density function of acceleration can be expressed as follows:

$$f\left(a_n^{lj}(t)|l_{nt}, \tau_n\right) = \frac{1}{\sigma_{lj}} \Phi\left(\frac{a_n^{lj}(t) - s^{l,j}[X_n^l(t)]m^{lj}[\Delta V_n^l(t-\tau_{n)})]}{\sigma_{lj}}\right) \tag{5}$$

In mixed traffic condition, the static and dynamic characteristics of the vehicles are often quite different and the type of the subject vehicle and/or the leader vehicle can affect the sensitivity function (either individual effect or pair-effect). Other candidate variables affecting sensitivity function include speed of the subject vehicle, spacing with the lead vehicle, traffic conditions (e.g. density) and composition, etc.

2.3 Likelihood Function

The trajectory data includes second by second lane changing and acceleration decisions of the driver. The only information about the driver/vehicle characteristics is the length of the vehicle. The following are therefore unobserved in the data:

- The leader(Front Left, Front Direct and Front Right)
- Driver/vehicle characteristics (reflected in reaction time and correlation in the error terms)

The joint probability density of the observed acceleration for driver *n* at time *t*, conditional on the individual specific reaction time is given by:

$$f\left(a_n^l(t)|\tau_n\right) = f\left(a_n^l(t)|l_{nt}, \tau_n\right)P\left(l_n(t)\right) \tag{6}$$

Where, $f\left(a_n^l(t)|\tau_n\right)$ *and* $P\left(l_n(t)\right)$ can be calculated by Equations 1, 3, and 5.

The marginal probability can be written as follows:

$$f\left(a_n\ (t)|\tau_n\right) = \sum_{l_n} f\left(a_n^l(t)|\tau_n\right), l_n \in FL, FD, FR$$

The behaviour of driver n is observed over a sequence of T_n consecutive time intervals. The joint probability of the sequence of observations is the product of the probabilities of individual observations:

$$f(\mathbf{a}|\tau_n) = \prod_{t=1}^{T_n} f\big(a_n\,(t)|\tau_n\big) \tag{7}$$

Where, **a** is the sequences of observed accelerations of driver *n*.

The unconditional individual likelihood function is deduced by integrating the conditional probability over the distributions of the individual specific variable:

$$L_n = \int_\tau \; f\,(\mathbf{a}|\tau_n) d\tau \tag{8}$$

Where, $f(\tau)$ is assumed to follow double truncated normal distribution.

Assuming that the observations of different drivers are independent, the log-likelihood function for all *N* drivers observed is given by:

$$L = \sum_{n=1}^{N} \ln(\,L_n) \tag{9}$$

For determining the maximum likelihood function, the probability density function (PDF) of the given observations is compared against the PDF of a normal distribution whose mean is specified by the coefficients (estimated) of the influencing variables and the standard deviation (estimated). The set of parameters yielding the best goodness-of-fit is selected. The correlation in the error terms of the observations of the same driver is accounted for using a sandwich estimator (39).

It may be noted that it has been assumed that conditional on the driver characteristics, the actions of a driver is independent over time and state-dependence among the consecutive decisions are not explicitly considered. However, the values of explanatory variables that are derived from the positions and speeds of the subject vehicle and surrounding vehicles depend on earlier decisions made by the driver (e.g. the vehicle speed and position depend on past accelerations) and the inclusion of these variables in the model is expected to indirectly capture some of the effects of previous decisions. The model thus assumes partial independence.

3. Data

3.1 Location

The trajectory data used for estimating the model parameters have been extracted from video recordings from an elevated pedestrian bridge in Mirpur road of Dhaka, Bangladesh (Figure 3a). The video covered the end of the morning peak (9:30-11:00) when the congestion levels are moderate. It may be noted that the site and the schedule have been selected based on a reconnaissance survey and governed by the following conditions:

- Availability of suitable elevated pedestrian bridges
- Presence of mixed traffic and weak lane discipline
- Smaller share of large commercial vehicles (which can obstruct smaller vehicles and cause problems in the image processing)
- Absence of curves
- Minimal side friction and
- Dominance of continuously moving traffic with a reasonable speed (i.e. not free flow or jam conditions)

<Figure 3>

The video was analysed by an image processing software named 'TRAZER' (*40*) and smoothed using Locally Weighted Regression Technique (*41*) using MATLAB. Some data had to be discarded due to high congestion levels (where the image processing software failed or gave unreasonable results) and in total about 45 minutes of usable data has been retained.

3.2. Traffic Characteristics

The cleaned and smoothed trajectory data consists of 895 vehicles. The composition is shown in Figure 4.

As evident in Figure 4a, the traffic stream mostly consists of private cars (48.45%) followed by motorcycle (12.20%). CNG auto-rickshaw also has a fairly good percentage. The percentage of bus, microbus and SUVs are close and considerable. Non-motorized vehicles have a small share (non-motorized vehicles are banned in most of the major roads of the city) and trucks have negligible share (trucks are not allowed in the city between 6am to 10pm).

<Figure 4>

The vehicles are grouped into the following 3 groups depending on size and dynamic characteristics as well as statistical tests of model parameters during the model estimation:

- LMV : Private car, Microbus, Human Hauler, CNG Auto-rickshaw
- HMV : SUV, Bus, Truck[1]
- 2W : Motorcycle, Bicycle, Cycle Rickshaw

This infers that it is theoretically possible to have 9 types of vehicle pairs (e.g. LMV-LMV, LMV-HMV, HMV-LMV, etc.)

The cleaned data has 5507 observations. The average speed is 12.49 km/hr and the average acceleration and deceleration are 1.19m/s^2 and 1.63m/s^2 respectively.

Analysis of lateral overlap of vehicles (Figure 4b) indicated that for majority of the vehicles, there is only a single front vehicle but a substantial portion (937 observations) has more than one front vehicle and multiple candidate leaders.

4. Results

The parameters of the model presented in Section 2, have been estimated using the Broyden–Fletcher–Goldfarb–Shanno (BFGS) algorithm using the software Oxmetrics 6 (*42*). Different model formulations have been tested using the candidate variables listed in Table 1.

<Table 1>

[1] Out of the 895 vehicles in the dataset, there were 74 buses (8%) and 6 trucks (0.67%). Such proportions are largely representative of the traffic patterns in the Dhaka city (the heavy trucks can only enter the city between 10pm-6am). Due to the small proportions, these have been bunched together with SUVs in a common class Heavy Vehicles during model estimation purposes.

The best model selected based on the coefficient values, robust t-statistics and goodness-of-fit values is presented in Table 2.

<Table 2>

4.1 Latent Leader

This component predicts the probability of a front vehicle for being the governing leader of the subject driver. The choice set includes the front vehicles that have overlap in lateral coordinates and can consist up to three front vehicles: Front Left (FL), Front Direct (FD) and Front Right (FR) (Figure 2).

As presented in Table 2, all else being equal, the probability of a front vehicle being the governing leader is higher if it is a FD vehicle. The constants are however statistically insignificant. The probability also increases with the lateral overlap between the vehicle pair (expressed as a percentage of shared lateral coordinates between the subject vehicle and the front vehicle). The type of the front vehicle also has a significant effect on the probability of it being the governing leader. Estimation results indicate that the subject driver is more likely to be governed by the stimulus from a front vehicle if it is a heavy vehicle. The effect of the type of the subject vehicle has also been tested using a similarity dummy (which is 1 if the front vehicle and the subject vehicle type are the same), but the coefficient is non-intuitive and statistically insignificant. Therefore, the variable has not been included in the final model.

The estimated utilities associated with the candidate front vehicles can be expressed as follows:

$$\begin{aligned} U_n^{FL}(t) &= -1.623 + 2.091\omega_n^{FL}(t) + 0.295\delta_n^{FL}(t) + \varepsilon_n^{FL}(t) \\ U_n^{FD}(t) &= 2.091\omega_n^{FD}(t) + 0.295\delta_n^{FD}(t) + \varepsilon_n^{FD}(t) \\ U_n^{FR}(t) &= -1.451 + 2.091\omega_n^{FR}(t) + 0.295\delta_n^{FR}(t) + \varepsilon_n^{FR}(t) \end{aligned} \quad (10)$$

Where,

$\omega_n^l(t)$= Percentage of lateral overlap of the front vehicle *l* with the subject driver *n* at time *t*

$\delta_n^l(t)$= Heavy front vehicle dummy, 1 if the front vehicle l of subject driver n at time t is a heavy vehicle, 0 otherwise.

The corresponding probabilities can be calculated using Equation 1.

4.2 Acceleration

As presented in Section 3.2, an extended version of the non-linear GM car following model is used in this component. The stimulus function of Equation 4 can be expressed as follows:

$$m^{lj}[\Delta V_n^l(t-\tau_n)] = \left|\Delta V_n^l(t-\tau_n)\right|^{\lambda^{lj}}$$

Where, λ^{lj} = parameter corresponding to relative speed of leader *l*.

A positive correlation between the relative leader speed and the acceleration the driver is expected a-priori. The parameter $\lambda^{lj}$is therefore, expected to be positive for both acceleration and deceleration. Estimation results indicate that λ^{lj} is indeed positive and significantly different for acceleration and deceleration. The stimulus can also have a different effect depending on the type of the front vehicle. This however is not supported in the estimation results and the difference in λ^{lj} among different types of lead vehicles is found to be statistically insignificant. This implies that for a given magnitude of the relative speed, all types of lead vehicles (LMV, HMV and 2W) provide the same magnitude of stimulus to the subject driver. The effects of relative speed on the mean car following acceleration and deceleration are shown in Figures 5a and 6a respectively.

<Figure 5>

The constant terms of the sensitivity functions are positive and negative for car following acceleration and car following deceleration, respectively. The magnitude of sensitivity to a negative relative leader speed is found to be larger than the sensitivity to a positive one. This is expected since a negative relative speed stimulus may have safety implications whereas a positive relative leader speed stimulus only suggests a possible speed advantage to the driver. The sensitivity functions are also expected to vary depending on the type of the subject vehicle/ leader vehicle/ vehicle pair. For, the deceleration function, the constant terms of the sensitivity functions vary significantly depending on the type of the subject vehicle. All else being equal, the magnitude of deceleration is largest for 2W and smallest for HMV (Figure 6a).

Candidate variables affecting the sensitivity function include the speed of the subject driver, the space headway, density, types of the subject driver and the leader, etc. However estimation results indicate that only the effect of space headway is statistically significant (both for acceleration and deceleration), the estimated coefficients being negative. This is in agreement with findings of previous researches (e.g. 43-44). For deceleration, this is expected since the underlying safety concern increases when the spacing is reduced. In the case of acceleration, it may be related to a reduced perception of the leader as a stimulus the driver needs to react to. Leader type specific and vehicle pair specific headway coefficients have been tested both for acceleration and declaration functions and found to have a statistically significant difference in case of the deceleration function only where the coefficients of space headway differs significantly depending on the type of the leader vehicle. This indicates that for a particular subject driver, given the headway and relative speed differences are same, the magnitude of deceleration varies significantly depending on the leader type. Except for small headways, the magnitude of deceleration is highest if the leader is an HMV and smallest if it is a 2W (Figure 6a).

<Figure 6>

The estimated acceleration function can be expressed as follows:

$$a_n^{l\,acc}(t) = 0.051[H_n^l(t)]^{-0.0498}\left[\Delta V_n^l(t-\tau_{n)}\right]^{0.0269}$$

$$\varepsilon_n^{l\,acc} \sim N(0, 0.242^2)$$

The estimated deceleration functions are presented in Table 3.

<Table 3>

4.3 Model Comparison

The latent leader specification was statistically compared with a reduced form/ or naïve model where acceleration is assumed to be influenced only by the vehicle with the highest lateral overlap. The goodness-of-fit of both models have been compared and presented in Table 4.

<Table 4>

As seen in the table, the latent leader model, in spite of having more parameters, has a statistically significant improvement compared to the reduced form model.

5. Conclusions

The paper presents a novel model structure for predicting acceleration behaviour in presence of weak lane discipline where the subject driver may have multiple vehicles in its front and there may not be a distinct leader vehicle. The estimated model parameters are intuitive and in agreement with previous studies. For instance, similar to acceleration models developed for homogenous traffic streams, the space headway is found to be a critical variable, especially in the context of declaration.

Though there are some similarities in the model parameters, there are substantial differences as well. For instance, the vehicle pair specific coefficients capture the unique acceleration properties of the mixed traffic streams. Moreover, the flexibility offered by the latent leader framework as opposed to rule based identification of the governing leader makes the models widely applicable in different traffic scenarios of varying congestion levels including, but not limited to traffic streams with weak lane discipline. The improvements due to this additional flexibility are also supported by a significant improvement in the goodness-of-fit. When implemented in microscopic traffic simulation tools, the proposed model is expected to result more realistic representation of traffic streams in presence of weak lane discipline.

However, the research has some limitations as well. For example, in this research, car-following acceleration has been investigated in isolation. Whereas, in reality there can be significant interdependency between the longitudinal movement and lateral movement decisions. The reaction time and desired headways can also be a function of the lead vehicle types and need to be explored in further detail. Further, as mentioned in Seciton 2.3, the model assumes partial independence in calculation of the Maximum Likelihood Functions. Research on state-dependence in the context of lane-changing (*45*) has demonstrated that if there are significant correlations among the unobserved variables, the assumption of partial independence can result over estimation of the coefficients of the serial correlation term. In Choudhury 2007 (*45*) (and later in Toledo and Katz 2009 (*46*)), a Hidden-Markov Framework has been used to explicitly capture the state-dependence among the lane-changing decisions of merging vehicles. Similar formulations in the context of the latent leader acceleration model that allow efficient integration of Markovian processes in the context of leader choice may lead to further improvements in the results.

Moreover, there may be large heterogeneity among the drivers in terms of reaction time, desired headway, comfortable acceleration/deceleration levels, etc. However, as in the other video trajectory based datasets, no driver specific information is available in the collected data. The effect of driver heterogeneity in this research is partially captured in this study by means of statistical distribution of the reaction time but better capturing the driver characteristics and associated human factors holds the promise to further enhance the models. It may be noted that in the context of lane-changing, alternate ways of data collection (e.g. focus group studies (e.g. *47, 48*), "in-vehicle experiments" (e.g. *48-50*), driving simulator studies (e.g. *51*) have been successfully used in capturing the effects of driver characteristics in detail. Deploying similar techniques in the context of acceleration behaviour can be a very interesting direction of future research.

Moreover, the estimated models are based on data from a single site with limited variation in congestion level. Transferability of the model parameters to other sites and time periods can be an interesting direction of future research.

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