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Engineered far-fields of metal-metal terahertz quantum cascade lasers with integrated planar horn structures

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Abstract: The far-field emission profile of terahertz quantum cascade lasers (QCLs) in metal-metal waveguides is controlled in directionality and form through planar horn-type shape structures, whilst conserving a broad spectral response. The structures produce a gradual change in the high modal confinement of the waveguides and permit an improved far-field emission profile resulting in a four-fold increase in the emitted output power. The two-dimensional far-field patterns are measured at 77 K and are in agreement with 3D modal simulations. The influence of parasitic high-order transverse modes is shown to be controlled by engineering the horn structure (ridge and horn widths), allowing only the fundamental mode to be coupled out.

1. Introduction

Terahertz (THz) frequency quantum cascade lasers (QCLs) are semiconductor sources based on intersubband transitions and permit laser emission over a wide portion of the THz spectrum. THz QCLs have undergone considerable improvements in operation since their first demonstration [1], especially in terms of the achievable output power and operating temperature. However, typically these two parameters cannot be optimized simultaneously in a given QCL device. The highest temperature operation of 200 K [2] has been demonstrated through use of metal-metal (MM) waveguides, where a high modal gain is achieved through the high modal confinement. However, a consequence of this is a highly diffracted beam and a large facet reflectivity leading to weak directionality and low output powers [3], respectively. The use of single plasmon based waveguides, on the other hand, have been shown to lead to output power of up to 1 W [4] but such QCLs are typically limited to operation at temperatures below 77 K.

Several monolithic methods have been proposed and realized to address this problem based on photonic structures for surface emission [5,6]. Other powerful demonstrations have exploited 3rd order gratings for emission along the ridge length [7,8]. Although these techniques can extract the QCL power and engineer the far-field emission profile efficiently, they are inherently mono-frequency and not suitable for broadband emission that would be of interest to modelocking [9–11] or frequency comb [12] generation applications. Most broadband methods, however, have relied on manual positioning of external elements such as silicon lenses placed on the facet [13], suspended antenna structures [3] or rectangular waveguides [14]. Owing to the manual nature, the techniques have proved to be difficult or non-reproducible when thermally cycled.

A promising technique has been recently demonstrated in which planar horn-like structures (PHSs) are etched into the QCL surface metal layer, so that the confined mode is better matched to a radiative mode [15]. This simple approach has led to a considerable improvement in directionality and output power without significantly affecting the QCL temperature performance, and has proven to be stable. Indeed injection seeding [16] of bound-to-continuum QCLs with PHS have been recently demonstrated [15]. However, no control of the far-field profile was demonstrated with only one-dimensional profiles of the far-field measured. Further the emitted spectrum from the PHS was not shown to be broadband owing to the use of low spectral bandwidth active regions. In this work, we demonstrate how the far-field emission profile can be controlled by engineering the QCL's transverse modes and the geometry of the PHS, as well as showing the broadband nature of the emission. We further show two-dimensional far-field patterns that are in good agreement with electromagnetic simulations.

2. Experiments

QCLs with a central frequency at 3.1 THz and a spectral bandwidth of 0.6 THz were used, employing an active region design based on longitudinal optical phonon depopulation [17]. These designs have the highest temperature operation when placed in a MM waveguide, and allowed the QCL to be operated at liquid nitrogen

temperatures. The designs also possess an inherently large gain bandwidth, ideal in illustrating the broadband nature of the PHS. The growth was performed using molecular beam epitaxy followed by wafer bonding onto a host GaAs wafer. The original substrate was removed by a combination of mechanical and chemical techniques. The entire doped layer, as well as the etch-stop layer, was removed, reducing the waveguide losses. The MM wafers were then processed with laser ridge widths of 40 μm , 60 μm and 80 μm , and cleaved into cavity lengths of 3 mm, 2 mm and 3 mm, respectively.

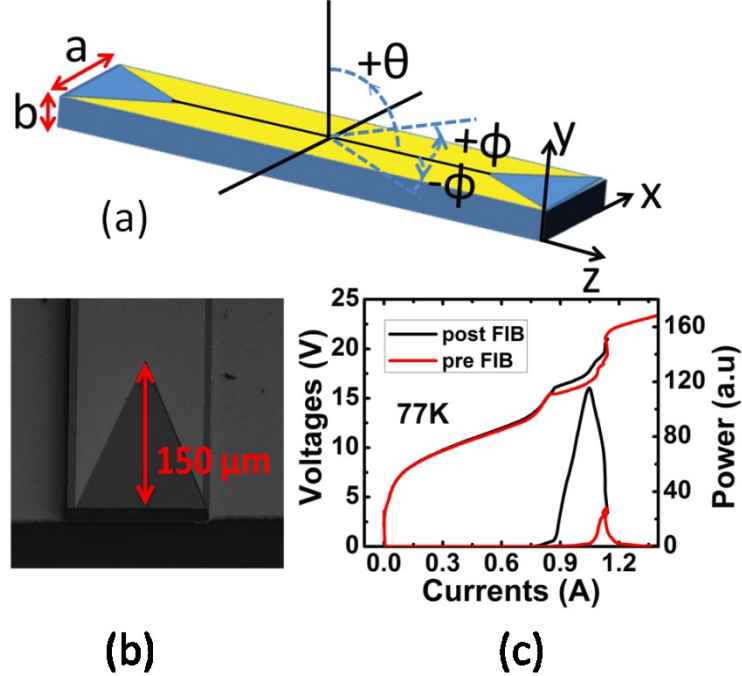


Fig 1. (a) Schematic of metal-metal THz QCLs with $\sim 150\text{-}\mu\text{m}$ -long triangle horn structures at each end of the QCL cavity. (b) SEM image of a horn structure on one end of the QCL. (c) The L-I-V curves at 77 K pre- and post-FIB milling of horn structures.

The PHS were realized by post processing through the use of focused ion beam (FIB) milling to remove the top metal layer as shown in figure 1(a) and 1(b). This allows the strongly confined mode to gradually adapt into a propagating beam, reducing the facet reflectivity, and therefore resulting in a higher output power and a directional far-field. Generally, the longer the horn length, the larger the output power and the more directional the beam. However, a long PHS results in higher effective mirror losses and an increase in laser threshold, as detailed in ref [15]. A horn length of 150 μm was chosen here as a compromise.

Figure 1(c) shows the output light-current-voltage characteristics of an 80- μm -wide ridge pre- and post-FIB milling, with the QCL at 77 K. The QCL was operated in pulsed mode with 500-ns-long electrical pulses at a repetition frequency of 50 kHz. A 20 Hz electrical envelope modulation was used to enable lock-in detection of the output power using a Golay cell detector. A $f/1.67$ parabolic mirror was used to collect the output radiation from the QCL, and a $f/2$ parabolic mirror used to focus it onto the Golay cell. A factor of four increase in measured output power was observed for the QCL incorporating the horn (post FIB). This increase is partly due to a reduced facet reflectivity. The threshold currents of the pre- and post-FIB sample (from the change in differential resistance in the V-I curve) are 0.849 A and 0.875 A, respectively, corresponding to a reduction in reflectivity from approximately 0.8 to 0.65, respectively, with a calculated waveguide loss of 22 cm^{-1} (see below). This reduction in reflectivity would only result in a two-fold increase in output power, and so does not account for the four-fold increase observed. (The L-I was not used to determine the threshold owing to a small overshoot in time of the electrical characteristics, resulting in the threshold in the light emission appearing smaller for the horn antenna sample. The V-I is taken over a large time window that does not include the electrical overshoot and is therefore more robust to determine the laser threshold).

We show below that this difference arises from there being a more directional beam in the devices with horn-like structures. We also demonstrate that the far-field profile can be engineered by controlling the transverse mode that propagates through the QCL waveguide. Indeed, by favouring the fundamental transverse mode, we demonstrate that the far-field profile can be changed from a double to a single lobed emission. The far-fields profile of three samples were measured and simulated; an 80- μm and a 40- μm -wide ridge each with a PHS, and a 60- μm -wide ridge as cleaved.

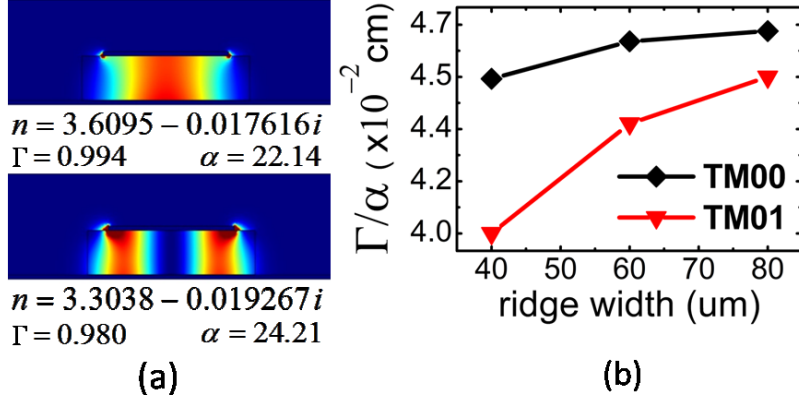


Fig 2. (a) Modal simulations of the fundamental TM_{00} (top) and second order mode TM_{01} (bottom) for a 40- μm -width metal-metal QCL. (b) The figure of merit, Γ/α , as a function of QCL ridge width for the fundamental TM_{00} (black) and second order mode TM_{01} (red).

The effect of the ridge width can be seen by analyzing the modes that are permitted to propagate. For the general case of a waveguide of width a and height b (see figure 1(a)) with no loss, the propagation constant, β , is given by [18]:

$$\beta = \sqrt{k^2 - (k_x^2 + k_y^2)} = \sqrt{\omega^2 n^2 \mu\epsilon - (k_x^2 + k_y^2)} \quad (1)$$

where n is the refractive index, ω is the angular frequency, and $k_x = m\pi/a$ and $k_y = l\pi/b$ are the wavevectors in the x and y directions with propagation in the z -direction. (m and l are the integer mode numbers). For a mode to propagate, $\beta^2 > 0$ and therefore:

$$(k_x^2 + k_y^2) < \omega^2 n^2 \mu\epsilon \rightarrow \left(\frac{\pi m}{a}\right)^2 + \left(\frac{\pi l}{b}\right)^2 < \omega^2 n^2 \mu\epsilon \quad (2)$$

For intersubband transitions, the electric field polarization is perpendicular to the laser surface (transverse magnetic (TM)), and with $b < a$ ($l = 0$):

$$m < \frac{\omega a n}{\pi} \sqrt{\mu\epsilon} = 2 a n f \sqrt{\mu\epsilon} = \frac{2 a n}{\lambda} \quad (3)$$

where λ is the wavelength. From Eq. (3) it can be seen that the number of transverse modes can be limited by reducing the ridge width a . For example, choosing a ridge less than 30 μm will result in only the fundamental transverse mode $TM_{0,0}$ propagating.

To take into account the actual QCL structure with optical losses, simulations of the 2D waveguide modes were performed (COMSOL). Figure 2(a) shows the mode profiles of the fundamental (TM_{00}) and second order mode (TM_{01}) at 3 THz for a 40- μm -wide ridge. The modal overlap with the active region, Γ , and the waveguide losses, α , are found to be 0.994 and 22 cm^{-1} , and 0.98 and 24 cm^{-1} , for the TM_{00} and TM_{01} modes, respectively. The modal overlap is smaller for higher order modes owing to a larger overlap with the surrounding air, and the losses increase owing to a larger overlap with the surrounding metal layers. This results in a reduction in the figure of merit, Γ/α , for higher order modes, which indicates that they are less likely to demonstrate laser action. This is summarized in figure 2(b) that shows the figure of merit as a function of ridge width, highlighting that the difference in the figure of merit between the fundamental and second order mode increases as the ridge width decreases.

To simulate the effect of the PHS on the far field, 3D modal simulations were performed using a finite element analysis (RF module). Here, the transverse modes were simulated in MM THz QCLs with a planar horn structure on the top metal layer. The QCLs was modelled in three parts: 1. Top metal gold waveguide layer (2 μm thick) with two 150- μm -long triangle horn shapes on each end of the laser; 2. The active medium layer (10 μm thick GaAs). 3. Bottom gold metal layer (2 μm thick). The model was placed in an air sphere to determine the far-field profile using a $TM_{0,0}$ source to excite the transverse modes within the cavities at one end of the ridge. (See appendix for further details).

The measured and calculated far-field profiles corresponding to the transverse modes are shown in figure 3 (left and right side of figure, respectively). The measurements were performed with a Golay detector rotated around the QCL central axis [19]. In all cases the QCL was operated at a temperature of 77 K. Figures 3(a) and 3(b) show the far-field patterns of a 40- μm - and 80- μm -wide MM THz QCLs with planar horn structures, respectively. Figure 3(c) shows the far-field profile from a QCL with an un-patterned ridge of width 60 μm . The simulations and measurements are in good agreement, and demonstrate the effect of increasing the ridge width. As mentioned above, in the 40- μm -wide cavity, only the fundamental mode $TM_{0,0}$ is present and results in a

single lobed far-field profile. Compared to the 60- μm -wide un-patterned QCL (figure 3(c)), the far-field profile is less diffracted with the power concentrated at $\theta=10^\circ$ and $\phi=15^\circ$ (The slight difference in angle in the ϕ plane when compared to the simulations was a result of the QCL being mounted at a small angle on the cryostat cold finger). In the case of the 80 μm ridge with a horn structure, the QCL demonstrates laser action on a higher order mode ($TM_{0,1}$) that, as a consequence, results in a considerably different far-field profile. Indeed the far-field shows a more diffracted double lobe pattern with $\theta=43^\circ$ and $\phi=\pm 25^\circ$ in agreement with the simulated profile. Figure 3(c) shows the far-field patterns of the 60- μm -wide MM THz QCL without an PHS and is typical of a standard MM QCL with a highly diffractive beam and a concentric ring profile [3] owing to the tight sub-wavelength confinement of the mode.

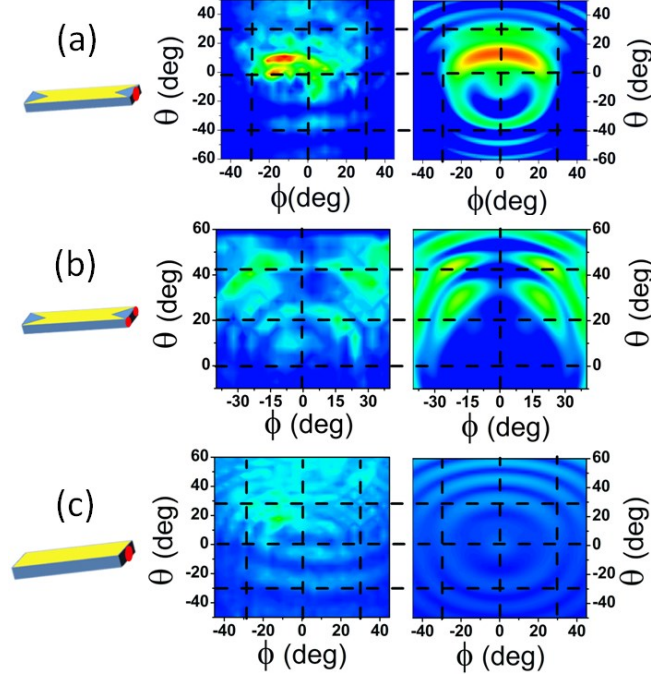


Fig 3. Simulated (right) and measured (left) far-field patterns of metal-metal THz QCLs with different waveguide structures. (a) 3-mm-long 40- μm -wide metal-metal THz QCL with 150- μm -long PHS. (b) 3-mm-long 80- μm -wide metal-metal THz QCL with 150- μm -long PHS. (c) 2-mm-long 60- μm -wide metal-metal THz QCL without any planar horn structure. The measurements were all taken at 77 K.

Although the 60- μm -wide ridge can permit propagation of higher order modes, the QCL far field appears to be a result of just the $TM_{0,0}$ mode, suggesting that the ridge is small enough to still favor the fundamental mode only. Although the ridge widths are different, the 2-mm-long 60- μm -wide QCL ridge has the same volume as the 3-mm-long 40- μm -wide ridge and so should have comparable output powers, which allows the effect of the horn to be determined. From the measured far-field profiles, the maximum power output of the horn coupled QCL is ~ 3 times greater than that of the QCL with no horn. The simulations, however, show a five-fold increase. The discrepancy could be a result of imperfections in the horn fabrication where the FIB milling can leave small metallic defects on the surface or on the quality of the cleaved facet.

From the far-fields profiles, the collection efficiency of the parabolic mirror used for the acquisition of the output power of figure 1 was determined. A horn coupled QCL would collect 28% of the total power emitted by the QCL while for the as-cleaved device, this falls to 16%. This adds to the reduction in reflectivity highlighted above for higher output power and explains the four-fold increase in power observed in figure 1.

To illustrate the broadband spectral nature of the PHS, the spectrum of a QCL operating with a similar active region but operating around 2.7 THz with a bandwidth of ~ 600 GHz [20] was investigated. The sample ridge width here was 60 μm with a cavity length of 3mm and the PHS length was 150 μm . The calculated far-fields for 2.4 THz, 2.65 THz and 2.9 THz are shown in figure 4(a). Although there is a slight dependence on the frequency with a larger divergence and more elliptical field for the lower frequencies, all the far fields show a similar single lobed far field. Figure 4(b) shows the resulting spectrum using an injection seeding technique [16] which shows that emission extends from 2.4 THz to 3 THz. Although there appears to be a stronger distribution in intensities of the Fabry-Perot modes when compared to the as cleaved device [20], a similar bandwidth is obtained illustrating the broad spectral response of the PHS.

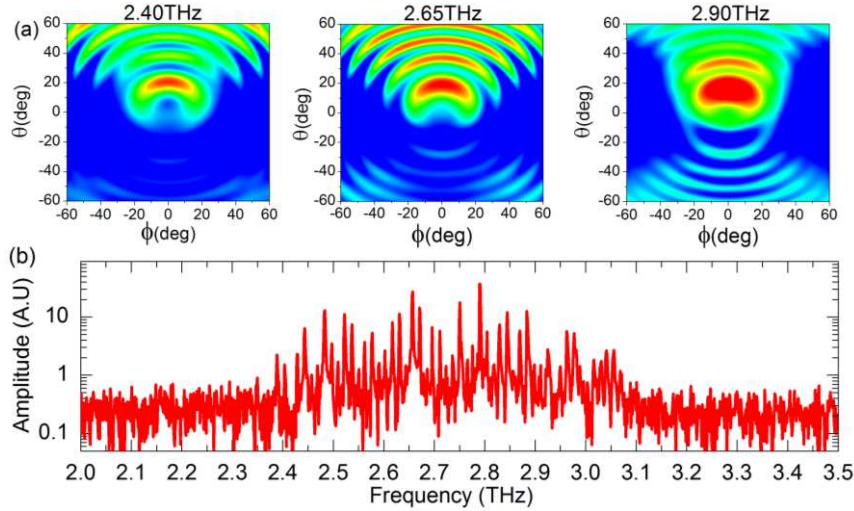


Fig 4. (a) Simulated far field profiles at 2.4 THz, 2.65 THz and 2.9 THz for QCL with central emission frequency of ~ 2.7 THz with a PHS of length $150 \mu\text{m}$. (b) The measured spectrum taken at 10K showing a broadband response.

As can be observed from figure 3, although larger output powers can be achieved, a large ridge with a PHS results in a double lobed far-field, which is not ideal for applications. To circumvent this, a tapered shape was realized on the end of the QCL. Here, rather than removing the metal layer from the centre of the ridge, the metal layer was selectively removed from edges of an $80\text{-}\mu\text{m}$ -wide ridge to a $30 \mu\text{m}$ point at the QCL facet over a taper length of $250 \mu\text{m}$ (see figure 5). This scheme increases the losses of the higher order modes and therefore limits laser action to the fundamental mode. Figure 5 shows the measured and simulated far field profiles for an $80\text{-}\mu\text{m}$ -wide ridge. The far field is improved considerably and, in contrast to figure 4(b), is single lobed. (The slight angle in $\phi = 18^\circ$ in the measurement data is a result of a slight asymmetry in the realized PHS fabrication). This shows that the PHS shape can be adapted to the ridge width and not is limited to small ridge widths for lasing on the fundamental mode.

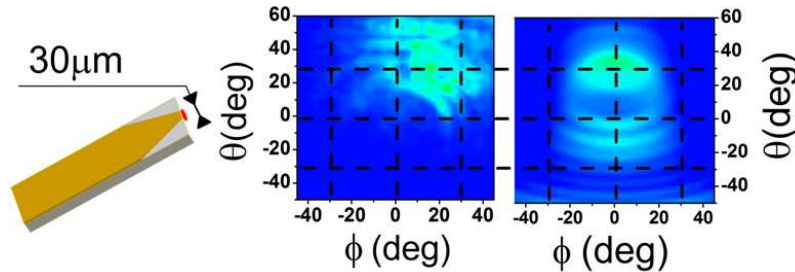


Fig 5. Simulated (right) and measured (left) far-field patterns of a 3-mm-long $80\text{-}\mu\text{m}$ -wide metal-metal THz QCLs with a $250\text{-}\mu\text{m}$ -long inverse planar horn structures. The measurements were all taken at 77 K.

3. Conclusion

In conclusion, the application of a broadband PHS to MM THz QCLs has been demonstrated and the ridge width engineered to favor single lobed emission in the far field. All measurements were performed at temperatures and electrical power dissipation compatible with liquid nitrogen cooling or cryocooler temperatures. The $150\text{-}\mu\text{m}$ -long planar horn structures integrated onto the top metal surface of the QCL reduces the reflectivity and improves the quality of the far-field profiles. This permits a four-fold increase in the detected power whilst conserving a broad spectral response. Further, with longer PHS lengths, the output powers and directionality can be increased considerably further. This stability and the inherent broadband nature of these structures will allow these devices to be employed for applications such as injection seeding of QCLs for coherent detection and the generation of high power frequency combs.

Appendix A: Transverse mode simulations for different ridge width

The far field was simulated using a finite element analysis (RF module). Here, the transverse modes were simulated in MM THz QCLs with a planar horn structure on the top metal layer. The QCLs was modelled with a top metal gold waveguide layer (2 μm thick) with two 150- μm -long triangle horn shapes on each end of the laser, the active medium layer (10 μm thick GaAs) and the bottom gold metal layer (2 μm thick). The model was placed in an air sphere to determine the far-field profile with an appropriate mesh as shown in figure 6. A $TM_{0,0}$ source is employed to excite the transverse modes within the cavities at one end of the ridge (right side of the cavities in figure 7). Figure 7(a) to figure 7(c) shows the resulting simulations of the ridge width on the transverse modes for ridge widths of 40 μm , 60 μm , and 80 μm . For the 40- μm -wide cavity, although the $TM_{0,1}$ mode begins to appear, the fundamental $TM_{0,0}$ mode still dominates. Further increases in the ridge width results in the stronger appearance of other high order transverse modes.

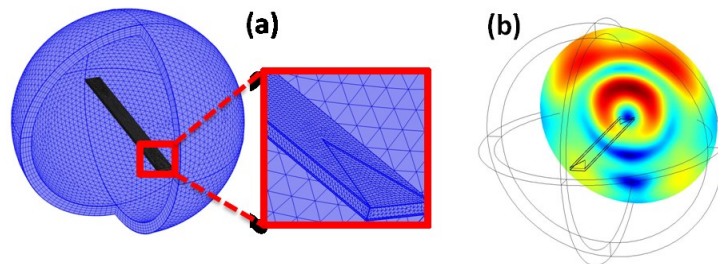


Fig 6. (a) Mesh used in air-field and QCL ridge to determine the far field emitted from the QCL. (b) Example of simulated far field projected on the air-sphere.

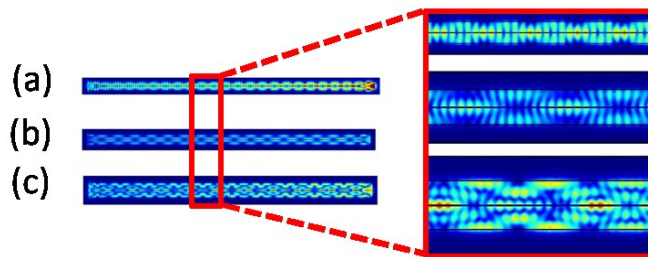


Fig 7. Top view of QCL showing transverse modes excited by a $TM_{0,0}$ excitation on the right side of the ridge for 3-mm-long cavities with 150- μm -long planar triangle structures on the top metal layer on both facets. (a) Ridge width = 40 μm . (b) Ridge width = 60 μm . (c) Ridge width = 80 μm .

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References

1. R. Köhler, A. Tredicucci, F. Beltram, H. E. Beere, E. H. Linfield, A. G. Davies, D. A. Ritchie, R. C. Iotti, and F. Rossi, "Terahertz semiconductor-heterostructure laser," *Nature* 417(6885), 156–159 (2002).
2. S. Fathololoumi, E. Dupont, C. W. I. Chan, Z. R. Wasilewski, S. R. Laframboise, D. Ban, A. Mátyás, C. Jirauschek, Q. Hu, and H. C. Liu, "Terahertz quantum cascade lasers operating up to 200 K with optimized oscillator strength and improved injection tunneling," *Opt. Express* 20(4), 3866–3876 (2012).
3. W. Maineult, P. Gellie, A. Andronico, P. Filloux, G. Leo, C. Sirtori, S. Barbieri, E. Peytavit, T. Akalin, J.-F. Lampin, H. E. Beere, and D. A. Ritchie, "Metal-metal terahertz quantum cascade laser with micro-transverse-electromagnetic-horn antenna," *Appl. Phys. Lett.* 93(18), 183508 (2008).
4. L. Li, L. Chen, J. Zhu, J. Freeman, P. Dean, A. Valavanis, A. G. Davies, and E. H. Linfield, "Terahertz quantum cascade lasers with > 1 W output powers," *Electron. Lett.* 50(4), 309–311 (2014).
5. G. Xu, R. Colombelli, S. P. Khanna, A. Belarouci, X. Letartre, L. Li, E. H. Linfield, A. G. Davies, H. E. Beere, and D. A. Ritchie, "Efficient power extraction in surface-emitting semiconductor lasers using graded photonic heterostructures," *Nat. Commun.* 3, 952 (2012).
6. J. A. Fan, M. A. Belkin, F. Capasso, S. Khanna, M. Lachab, A. G. Davies, and E. H. Linfield, "Surface emitting terahertz quantum cascade laser with a double-metal waveguide," *Opt. Express* 14(24), 11672–11680 (2006).
7. M. I. Amanti, M. Fischer, G. Scalari, M. Beck, and J. Faist, "Low-divergence single-mode terahertz quantum cascade laser," *Nat. Photonics* 3(10), 586–590 (2009).
8. M. Wienold, B. Röben, L. Schrottke, R. Sharma, A. Tahraoui, K. Biermann, and H. T. Grahn, "High-temperature, continuous-wave operation of terahertz quantum-cascade lasers with metal-metal waveguides and third-order distributed feedback," *Opt. Express* 22(3), 3334 (2014).
9. J. R. Freeman, J. Maysonnave, H. E. Beere, D. A. Ritchie, J. Tignon, and S. S. Dhillon, "Electric field sampling of modelocked pulses from a quantum cascade laser," *Opt. Express* 21(13), 16162 (2013).
10. J. Maysonnave, K. Maussang, J. R. Freeman, N. Jukam, J. Madéo, P. Cavalié, R. Rungsawang, S. P. Khanna, E. H. Linfield, A. G. Davies, and others, "Mode-locking of a terahertz laser by direct phase synchronization," *Opt. Express* 20(19), 20855–20862 (2012).
11. S. Barbieri, M. Ravaro, P. Gellie, G. Santarelli, C. Manquest, C. Sirtori, S. P. Khanna, E. H. Linfield, and A. G. Davies, "Coherent sampling of active mode-locked terahertz quantum cascade lasers and frequency synthesis," *Nat. Photonics* 5(5), 306–313 (2011).
12. D. Burghoff, T.-Y. Kao, N. Han, C. W. I. Chan, X. Cai, Y. Yang, D. J. Hayton, J.-R. Gao, J. L. Reno, and Q. Hu, "Terahertz laser frequency combs," *Nat. Photonics* 8(6), 462–467 (2014).
13. D. Burghoff, T.-Y. Kao, D. Ban, A. W. M. Lee, Q. Hu, and J. Reno, "A terahertz pulse emitter monolithically integrated with a quantum cascade laser," *Appl. Phys. Lett.* 98(6), 061112 (2011).
14. M. C. Wanke, C. Nordquist, C. L. Arrington, A. M. Rowen, A. D. Grine, E. A. Shaner, M. Lee, "Integration of terahertz quantum cascade lasers with lithographically micromachined waveguides," in *Infrared, Millimeter and Terahertz Waves, 2008. 33rd International Conference on*, vol., no., pp.1-1, 15-19 Sept. 2008.
15. A. Brewer, J. R. Freeman, P. Cavalié, J. Maysonnave, J. Tignon, S. S. Dhillon, H. E. Beere, and D. A. Ritchie, "Coherent detection of metal-metal terahertz quantum cascade lasers with improved emission characteristics," *Appl. Phys. Lett.* 104(8), 081107 (2014).
16. D. Oustinov, N. Jukam, R. Rungsawang, J. Madéo, S. Barbieri, P. Filloux, C. Sirtori, X. Marcadet, J. Tignon, and S. Dhillon, "Phase seeding of a terahertz quantum cascade laser," *Nat. Commun.* 1(6), 1–6 (2010).
17. M. A. Belkin, J. A. Fan, S. Hormoz, F. Capasso, S. P. Khanna, M. Lachab, A. G. Davies, E. H. Linfield, and others, "Terahertz quantum cascade lasers with copper metal-metal waveguides operating up to 178 K," *Opt. Express* 16(5), 3242–3248 (2008).
18. F. Hyo J. Eom, *Electromagnetic Wave Theory for Boundary-Value Problems: An Advanced Course on Analytical Methode* (Springer, 2004), Chap. 3.
19. P. Gellie, W. Maineult, A. Andronico, G. Leo, C. Sirtori, S. Barbieri, Y. Chassagneux, J. R. Coudeville, R. Colombelli, S. P. Khanna, E. H. Linfield, and A. G. Davies, "Effect of transverse mode structure on the far field pattern of metal-metal terahertz quantum cascade lasers," *J. Appl. Phys.* 104(12), 124513 (2008).
20. F. Wang, K. Maussang, S. Moudji, R. Colombelli, J. R. Freeman, I. Kundu, L. Li, E. H. Linfield, A. G. Davies, J. Mangeney, J. Tignon, and S. S. Dhillon, "Generating ultrafast pulses of light from quantum cascade lasers," *Optica* 2(11), 944 (2015)