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An agent model of urban economics: digging into emergence

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Abstract

This paper presents an agent-based ‘monocentric’ model: assuming only a fixed location for firms, outcomes closely parallel those found in classical urban economic models, but emerge through ‘bottom-up’ interaction in an agent-based model. Agents make buying and movement decisions based on a set of simple costs they face from their current location. These spatial costs are reduced to two types: the costs of moving people and goods across geographical distances and the costs (and benefits) of ‘being here’ (the effects of being at a particular location such as land costs, amenities or disamenities). Two approaches to land cost are compared: landlords and a ‘density cost’ proxy. Emergent equilibrium outcomes are found to depend on the interaction of externalities and time. These findings are produced by looking at how agents react to changing four types of cost, two spatial and two non-spatial: commuting, wage, good cost and good delivery. The models explore equilibrium outcomes, the effect of changing costs and the impact of heterogeneous agents, before focusing in on one example to find the source of emergence in the externalities of agent choice. The paper finishes by emphasising the importance of thinking about emergence as a tool, not an end in itself.

1 Introduction

In the 'monocentric' model tradition, a market, firm or central business district (CBD) is set at a central location surrounded by land. This tradition examines the choices economic agents make as they optimise between distance to the centre and the cost of land. As well as being useful for investigating real-world settlements with fixed central areas, it is also effective for thinking about the polarity and magnitude of spatial economic forces[1].

The concept of spatial equilibrium is at the heart of traditional monocentric models (Fujita *et al*. 2001 p.17). While originally a verbal argument (Thunen 1826), modern variants of the model have taken Launhardt's initial mathematical approach (Launhardt 1885) to develop the modern version of spatial equilibrium (Blaug 1997 pp.600). This made its way into urban economics via Alonso (1964), becoming in time the Alonso-Muth-Mills (AMM) monocentric framework, as outlined in Glaeser 2008.

As Glaeser puts it (ibid. p.4), spatial equilibrium is "the bedrock on which everything else in the field stands... essentially, there must be no arbitrage across space". No agent can unilaterally make themselves better off by choosing a different location (Lemoy *et al*. 2010 p.7). The assumption that this arbitrage has already taken place provides a solid mathematical foundation - that all agents' utility has become equal and static across distance; $\delta u/\delta d = 0$ (where u is utility and d is distance) as one moves from $d = 0$ at the centre point to the edge of the settlement. All other deductions are built on top of this assumption. In the simplest (and most powerful) finding, spatial equilibrium is used to show that "rents must decline with distance to exactly offset the increase in transportation costs" (Glaeser 2008 p.20).

In contrast, rather than assume equilibrium, Agent Based Modelling (ABM) asks how system-level properties *emerge* through the interaction of individual economic agents, each with their own behaviour. As Bonabeau argues (2002 p.7280), ABM is "the canonical approach to modelling emergent phenomena". Emergence is as central to ABM as equilibrium is for analytic economics. So a natural ABM approach to the monocentric model is to ask whether spatial equilibrium is an emergent property. Can interacting agents create a stable, equal-utility settlement pattern?

A small number of theorists have applied ABM to monocentric models showing, in different ways, how agent interaction can create emergent settlement patterns (the work of Lemoy *et al.* does a particularly thorough job; see below). ABM, however, often suffers from a 'method-centring' malady (Maslow 1966 p.15) where emergence becomes a goal in itself. This paper digs a little deeper into emergent spatial equilibrium, asking: why does spatial equilibrium emerge from agent interaction? What role is space actually playing in spatial equilibrium?

The answer proposed is this: space provides a medium for agent choice *externalities* (the effects of each agent's choices on others) to interact over time. These externalities will accrete

[1] As Blaug says, this was a point made by von Thunen: "in reality, Thunen observes, differences in fertility of the soil which are not themselves related to location will give rise to ground rent in the same manner as do differences in proximity to the central town." (Blaug 1997 p.598)

into a settlement where all agents share the same utility. Time is equally important: the model provides agents with the ability to make independent, objectively valid decisions at their own time point. The spatial environment itself is a store for their decision, simply in the form of their location choice, so that subsequent agents face an altered landscape.

To do this, an agent-based monocentric model is presented with some additions to the traditional framework. These are kept simple. The *spatial costs* that agents face are broken down into two types. First, **distance costs**: the cost of moving people *and* goods across geographical distances (rather than just people as traditional monocentric models tend to focus on). Second, **proximity costs**: the cost of 'being here' - that is, the effects of being at a particular location such as land rent, amenities or disamenities.

The usual proximity cost in monocentric models is the rent paid to occupy a plot of land. Land costs are the keystone of urban economic models: they provide the reason for distance to exist at all in a model trying to explain spatial morphology "as an endogenous outcome of the economic process" (Storper 2010, p.315). A limited stock of land is an essential component of the traditional monocentric model, and its impact on morphological outcomes well-known in urban economics (land is a normal good; better-off people wish to consume more). In this paper, a land market is implemented in a way that avoids many of the complexities of bidding processes commonly faced in ABM market models.

A second proximity cost is also introduced - a 'density cost' derived solely from the proximity of other agents. This approach is a simple way to avoid the 'black hole' outcome where actors do away with space altogether if they can, collapsing into a single point (Fujita 1999 p.58)[2]. A land market is thus considered a specific case of the more general proximity cost. This point is then used to make the case for the role of externalities working through proximity costs.

The following section looks at ABM and the idea of emergence, as well as agent-based approaches to the monocentric model. The model's structure is then explained in detail, before the results are presented - a series of models illustrating how spatial equilibrium is reached, as well as how agents react to key cost changes, heterogeneous wealth of various forms, and how they react if given heterogeneous preferences.

Both distance and proximity costs are necessary conditions for spatial equilibrium to emerge. Section 4.4 takes a closer look at the decision sets of two- and three-agent scenarios, in order to examine how spatial equilibrium comes about through these two types of spatial cost interacting with externalities.

[2] As Alonso put it, "if the only criteria for residential location are accessibility to the centre and the minimising of the costs of friction, and considerations of the size of the site are excluded, all residence would be clustered around the centre of the city at a very high density." Alonso 1964 p.9

2 Agent-based modelling, emergence and space

In ABM, the 'agents' are distinct code objects, programmed to interact with their environment and each other. ABM's use ranges from the most abstract artificial life to 'autonomous' agents earning their keep controlling real-world infrastructure. ABM developed in tandem with Object-Oriented Programming (OOP); this approach has been the prime determinant of agent modelling theory and practice (Robinson and Sharp 2009 p.211). Wooldridge defines objects as "computational entities that encapsulate some state, are able to perform actions on this state, and communicate by message passing" (Wooldridge 2009 p.28). Objects are created from classes; a real-world metaphor would be that classes are the blueprint and objects the physical form. Thus, a model may have a single 'Firm' class but many 'Firms' created from that blueprint, replicating a structure but each with their own internal state.

ABM's power as a method is rooted in its ability to investigate *emergent* outcomes - system-level properties that result from the interaction of many agents, but that are qualitatively different from those agents (Bonabeau 2002 p.7280). A common physical analogy is that atoms do not have temperature and pressure - these are system-level properties of their interaction that cannot be deduced by examining atoms in isolation (Flake 1998 p.134). Emergence is thus as central to ABM as equilibrium is to classical economic models.

The role of emergence as ABM's default focus has led to some rejecting system-level analyses - adopting honorary proto-ABM theorist (Vriend 2002 p.2; Miller and Page 2007) Friedrich Hayek's earlier insistence that "we must show how a solution is produced by the interactions of people each of whom possesses only partial knowledge" (Hayek 1945 p.530), dismissing system-level descriptions as 'fallacies of misplaced concreteness' (Hoover 2001 p.108). It is perhaps unsurprising, then, that many see ABM as a rejection of equilibrium economics, where an assumed system-level end-state is essential to the whole approach. In this view, ABM offers a "a pioneering break from a moribund Newtonian worldview" (Manson 2001 p.412) that is "simply wrong: stability is not the norm in complex systems" (Colander and Rothschild 2010 p.286).

The common rejection of equilibrium approaches seems to go hand in hand with an overly 'method-centred' (Maslow 1966 p.15) outlook: identifying emergent outcomes has dominated over using it to dig deeper into an understanding of the system being modelled. Epstein's claim that "if you didn't grow it, you didn't explain it" (Epstein 1996 p.xii) is a highly cited case of making emergence the goal rather than the tool. In this view, equilibrium outcomes, if they exist at all, can only be explained through emergence. Explanation is a heavy burden for emergence to bear - if 'growing' it explains it, there is nothing else to ask. The result, as Di Paolo *et al.* say, is often that any deeper understanding of why a system acts as it does is "brush[ed] under the carpet of emergence" (Di Paolo *et al*. 2000 p.8).

Users of the equilibrium assumption seem to suffer less from this method-centring; there is a general self-awareness of its limitations. As Glaeser says of spatial equilibrium, "no-one thinks that this assumption is a literal depiction of reality. Still, models based on the concept ... do a generally good job of actually explaining the real world." (Glaeser 2008 p.4). Most importantly, the equilibrium approach, while acknowledged as limited, is put to use as a tool to answer questions.

This is perhaps why equilibrium approaches have continued to dominate in spatial economics, despite ABM's enormous potential. In ABM, the fundamental spatial questions - "who produces what, where and why?" (Ohlin 1933 quoted in Brakman *et al.*, p.81; cf. Stanilov 2012) - have been overshadowed by the technology itself.

Applying ABM to spatial economics is even more challenging since, historically, spatial economic questions have fallen through the cracks in the research field. Agent-based Computational Economics (ACE) - "the computational study of economies modelled as dynamic systems of interacting agents" (Tesfatsion 2006 p.834) - suffers from the same problem as much of mainstream economics: space is ignored altogether, sticking to a "wonderland of no dimension" (Isard 1956 p.26). Attempts to address this spacelessness in the neoclassical school[3] have (with notable exceptions; see below) not been matched in ACE. In line with ABM more generally, it has also kept its focus on emergence, asking "who does the job of the so-called market adjustment?"[4] (Posada *et al.* 2007 p.102).

While ACE mostly avoids space, there are plenty of spatial ABM models - but again, few tackling topics traditional to spatial economics, despite ABM tackling spatial issues since its earliest precursor models predating OOP. Thomas Schelling's segregation model, first presented in 1969 (Schelling 1969, 1978) was originally a pencil and paper exercise (Schelling 2006). It still finds application in modern agent models (Crooks 2010; Feitosa *et al.* 2011; Jordan *et al.* 2014) and as an exemplar of how good economic explanation links micro to macro (Krugman 1996, p.15). In other areas, ABM land use models with a cellular automata lineage (Batty 2000) loom large (see e.g. Torrens' survey of spatial ABM; 2010), many with highly complex layers of properties (e.g. Zellner *et al.* 2008) or increasingly high-resolution movement detail (Benenson *et al.* 2008; Malleson *et al.* 2010; Torrens *et al.* 2012).

So while space and geography have been part of the ABM story from the start, other fields like geographical economics (Brakman *et al.* 2009) have had traditional spatial economic questions mostly to themselves. None of this is to question the power of emergence. The aim of this section is to suggest it needs to be downgraded from its common role as an end-goal in itself and re-focused as the ABM equivalent of the equilibrium assumption: the method's central tool for addressing research questions that dictates both its strengths and weaknesses. ABM may then be in a better position to tackle spatial economic questions in novel ways.

[3]Krugman developed his core-periphery model (1991) specifically because (echoing Isard almost exactly) "in the late nineteen-eighties mainsteam economists were almost literally oblivious to the fact that economies aren't dimensionless points in space" (Krugman 2010, p.1).

[4]This is despite a hope that ABM would usher in a renewed examination of the oldest Marshallian questions about economics (Leijonhufvud 2006).

2.1 Agent-based monocentric models

There have been applications of ABM to monocentric models in recent years; this section gives a brief overview. In an early example, Sasaki and Box seek to 'verify' the classic von Thunen model with ABM (Sasaki and Box 2003) by showing how spatial equilibria can come about through individual-level action but, aside from referring to von Thunen himself, it sticks to the ABM literature and rejects utility in favour of a 'goal-based' approach (p.2).

More recently, ABM has been used in two quite different ways to examine urban economic ideas. First, Huang *et al.* (2013) build an extension to a social-environmental coupled land model with the aim of investigating the impact of agent heterogeneity. They impose this heterogeneity directly on, respectively, preferences for proximity to the centre and for land (or for 'open space amenity', p.191) and on agents' budgets, leading to a mix between "more compact developments in the city centre but more sprawling developments in the suburbs" (ibid. p.199). However, building a preference for proximity or land into the model makes it difficult to inspect the economic impact of those factors.

Lemoy *et al.* (Lemoy *et al*. 2010; Lemoy *et al*. 2012; Lemoy *et al*. 2013) have created a series of ABM models that successfully recreate the equilibrium outcomes of the traditional AMM approach. As with Fowler's approach to producing an ABM version of Krugman's core-periphery model (Fowler 2007; Fowler 2011), their models aim initially to take components from the analytic original and recreate an agent version. They then make an excellent case for the flexibility of ABM in urban economics, for example by analysing polycentric cities.

Lemoy *et al.*'s models are referred to again in the results section below for comparison, where relevant, but it is worth noting some specific differences. This paper's categorisation of spatial costs into two types - distance and proximity costs - is built on adding extra components to the AMM framework (and Lemoy *et al.*'s extension of it). Rather than just commute costs, a delivery cost for goods is used. Rather than just landlords supplying a rented land market, a density cost is introduced. The addition of a delivery cost for goods (rather than distance only impacting via commute costs, as the AMM model and Lemoy *et al.* emplement) is used to make the point that distance costs have a more general impact on urban form: the morphological impact of changes in commute and delivery costs is the same.

In common with most ABM market modelling, Lemoy *et al.* use a form of bidding process for land rent. In contrast, the land market method presented here takes a quite different approach that builds on allowing buyers to have their objective demand met at their point of decision (see section 3.3 for more on this). The introduction of density as a second proximity cost makes it easier to see how agent choices interact through externalities. This then allows for a tight focus on exactly how micro-level choices are responsible for spatial equilibrium outcomes by looking at just two and three agent interaction (section 4.4).

In looking at wealth differences, Lemoy *et al.* impose higher transport costs for richer people, on the basis that their opportunity cost of time is higher. Rather than impose this, it is assumed here that this valuation should come about through agents' endogenous optimisation of their time. They also impose different preferences on rich and poor, giving each separate parameter values in their utility function, rather than using wealth differences alone. Doing so conflates two separate questions: how do agents react to heterogeneous preferences, and what is the impact of differences in wealth? Section 4.3 looks at these.

3 Model structure

This section explains the structure of the paper's model. It begins with an overview that details the model environment, the agents that populate it and how they act, before outlining the cost and utility functions used. The two forms of 'proximity cost' used in the model - landlords versus density cost - are each given a more detailed treatment.

3.1 Environment and agents

The model's spatial environment is a featureless, continuous two-dimensional plane, with each axis having a length of two, so the largest possible circle radius that can fit in the space is 1. Two types of agent are always located in this space: 'People' and 'Firms' (these are capitalised in the text to distinguish when model objects are being discussed). Firms occupy a fixed location at the centre of the space (representing the central market or CBD) from which they provide wages and goods to People, both with exogenously set prices. People are initially given random locations, from which they decide where to move to maximise their utility.

On each iteration, each Person compiles a random sample of potential new locations anywhere in model space (their current spot is also included) and selects the utility-maximising one[5]. People's utility optimisation has two stages. Firstly, they must acquire their wage. They are given time = 1 ('one day per day') to spend on working and commuting. The (exogenously set) commute cost is subtracted from a Person's daily time; the remainder is the time they give to the Firm (or one of four Firms; see below) in exchange for their wage. Secondly, the wage gained is then used to buy goods and land (or goods and density). These two enter into their utility function - the next section explains this function.

As discussed above, the two types of spatial cost People face are **distance costs** and **proximity costs** (the cost of 'being here'). People face distance costs when they commute to a central Firm to acquire their daily wage, and when they buy Goods - also from a central Firm - and must pay a delivery cost. Distance costs are based on the simple Euclidean distance between the Person and the central Firm.

Proximity costs take one of two forms in any particular model run: either 'Landlord' agents charge rent for a plot of land where the Person is currently located, or People face a 'density cost', which is higher if more People are close by (the algorithms for both are explained in their own sections below). If Landlord agents are being used, they are set at even gridded locations

[5] The order of all People's decisions is randomised on each iteration.

covering the whole plane, representing individual land plots. Their cost of land is endogenous: each Landlord seeks a clearing price that will rent all of their plot on each iteration.

Whichever promixity cost is being used, it enters into a Person's utility maximisation along with the good they buy. The density cost is bought at a 'price per unit' just as land has a price per unit (this is discussed more below). Land and density costs differ in that land is a finite good: there is a limited stock of land that Landlords attempt to 'clear', whereas a Person's consumption of density is limited only by its cost: as more People crowd into tighter proximity, density cost rises, so less is bought. (This is important for one model outcome; see section 4.2.)

If people incur density costs, their random samples of location are completely continuous: they can look anywhere in the model space. If People are renting land, they must rent only from one Landlord - so their location choice becomes discrete and they must reside on their selected Landlord's plot. While the supply of land is limited, there is no imposed limit on the number of People who can rent land from a single Landlord. Each Person may rent as large or small a portion as they want and still locate there - as long as, in the long run, the supply of land is kept to its limit.

There are two points to make about a Person's buying and location decision. First, it is designed to be objectively correct *at their point of decision*. They can select their optimal sampled location knowing that nothing else in the model will change in the meantime: no other Person will move, good prices and wages are static and Landlord prices remain unchanged until their next decision point. Practically, this means static prices can be entered into their constrained utility function (see next section) and the result will be valid. It also avoids any consideration of collusion, strategy or 'El Farol Bar'-like issues of simultaneous decision-making (Arthur 1994; Edmonds 1999 p.12) that rob agents of any effective strategy. As Oeffner points out, agents with 'rational expectations' in these situations can do no better than myopic ones because the uncertainty is rooted in the decision others may make simultaneously (Oeffner 2008 p.25-6). However, it does introduce a problem for Landlords that needs solving - this is discussed in section 3.3.

Secondly, People choose only a small number of randomly sampled locations on their turn - usually ten. They are thus 'boundedly rational' (Stefano 2005): whilst they can know each sampled location is objectively optimal, they are highly unlikely to find the objectively optimum location in the whole space on any one decision. Over model time, however, this random sampling quickly converges on a stable optimum that is identical regardless of the ordering of agent decisions (see results). The primary impact of the number of samples per turn is simply the speed this optimum is arrived at and which Person ends up where. The resulting stable outcomes are similar to Nash equilibria, in that once stability is reached no agent can take an action that will unilaterally improve their utility[6].

The parameters used in the model are as follows (these are also listed in table form at the

[6]Given that agents have no ability to strategise, these outcomes cannot be considered game-theoretically robust (cf. Vives 2001 pp.13). Over model time, there is convergence on stable equilibria, but they are still at root probabilistic.

end of this section). The number of People is fixed at one thousand for all model runs, chosen to balance meaningful aggregate outcomes in the given size of model space against computational expense and speed[7]. For basic model runs, there is only one firm; sections examining the impact of a spread of costs use four Firms. When in use, Landlords are spread evenly across the whole space in a square grid: there are $31^2 = 961$ Landlords in total. These numbers function well to allow spatial patterns to be clearly visible. Their stock of land is set at 0.1 each. (As in Lemoy *et al.*'s papers, this stock amount is in different units to distance - that is, land stock is not the square of each plot's side. This is to allow calibration of the stock value independently of model distance.) Landlords have a 'reservation price' below which they cannot drop, set to 3.5. As in the AMM model, this serves to mark the boundary of the settlement: rents too high to offset the cost of distance are found unaffordable. Again, this value is selected solely to keep agent interaction visible within model space. Parameter values for density cost are given in its own section below.

Default parameter values, when not parameter-sweeping or examining a spread of values, are as follows. Wages and good cost are set to 1. In practice, only one of the distance costs is non-zero at a time because, as the results show, they have the same effect. The model thus investigates each separately to keep that effect clear - on any particular model run, it is assumed either that goods are acquired without delivery charge (as in the AMM model) or that wages can be acquired without commute costs to the centre. For most models, delivery cost is 0.25 and commute cost is zero.

3.2 Utility and cost functions

People use a Constant Elasticity of Substitution (CES) utility function into which goods (G) and the proximity cost, referred to below as 'land/density' (L) enter as:

$$U = (G^\rho + L^\rho)^{1/\rho} \quad 0 < \rho < 1 \tag{1}$$

- where U is utility. The parameter ρ controls 'love of variety' (Brakman *et al.* 2009 pp.94): as $\rho \to 0$, a mix of goods and land/density is increasingly preferred to either alone (high love of variety or low elasticity of substitution). The more indifferent they are between goods and land/density (as $\rho \to 1$) the lower their love of variety[8]. Changing the price of either goods or land/density has an impact on demand for the other. This is in contrast to the Cobb Douglas function, where price changes for one good balance exactly against demand for the other, leaving utility (and demand for the other good) unchanged; the effect this has on elasticity between goods and land/density is discussed below. In the models presented here, ρ is kept at

[7] The number of People does impact on the model via its effect on land costs and settlement size, which both increase with larger numbers. Otherwise their impact is neutral as goods are exogenously supplied; their labour does not affect available good quantity.

[8] Dixit and Stiglitz describe the basic principle of love of variety: "a consumer who is indifferent between the quantities $(1,0)$ and $(0,1)$ of two commodities prefers the mix $(\frac{1}{2}, \frac{1}{2})$ to either extreme." (Dixit and Stiglitz 1977 p.297)

0.5 (except where People are given heterogeneous preferences; see section 4.3).

Equations (2) and (3) are the utility optimisations derived from the CES function (1), constrained by a Person's wage. They give the optimal amounts of good (including its delivery cost, see below) and land/density that each Person uses[9]:

$$G = \frac{P_G^{\frac{1}{\rho-1}} Y}{P_L^{\frac{\rho}{\rho-1}}} \tag{2}$$

$$L = \frac{P_L^{\frac{1}{\rho-1}} Y}{P_G^{\frac{\rho}{\rho-1}}} \tag{3}$$

- where P_G is the price of goods and P_L is the price of land/density. Y is the budget: this is a Person's wage after any commute cost has been subtracted. For example, if commute cost is 0.1, a Person is a distance of 0.5 from the Firm, and the wage is 1 per unit of time worked, they are left with time of $1-(0.1*0.5)=0.95$ to exchange for the wage, leaving them with a budget $Y=0.95$.

The optimal amounts of G and L can then be plugged back into (1) to get the utility amount. People do this for each of their sample of potential locations, and move to the highest-utility result. (The cost of land/density - P_L - is worked out differently for Landlords and the density cost approach; these are explained in the following two sections.)

The total cost of the good P_G is a function of its non-spatial base cost p_g, the distance it needs to move (d), and the delivery cost to ship it over a unit of distance (c). The specific form for Goods means it will be increasingly expensive to acquire over distance, but some demand will always be present:

$$P_G = p_g + (c \cdot d) \tag{4}$$

3.3 Landlords

As with People, each Landlord is required to act once per iteration. The decision sets of People and Landlords are kept separate: all People will have taken a decision within that iteration before Landlords come to take theirs[10]. Landlords' goal is to clear their set stock of land on every iteration by finding the correct clearing price. This is done by responding to the amount of land rented between actions. The rationale for this as follows.

One cannot consider spatially varied trade as 'one market' (Isard 1956 p.43). As Ladley and Bullock say (2007 p.83-4), spatial markets are not centralised. This means, as Leijonhufvud

[9]This single-good optimum was derived from Brakman *et al.*'s constraint of a CES function for a single good by substituting the price index back in and rearranging for two rather than n goods. See Brakman *et al*. 2009 pp.94

[10]This separation is to allow Landlords to respond 'knowing' all demand each 'day' is accounted for; randomly mixing timings requires a different approach to price-setting which, if not done correctly, causes instability; see Olner 2013 pp.132.

Model element	Description
People	Number: 1000 CES utility: $\rho = 0.5$ 10 sample points from random locations, including current One with maximum utility chosen; Person moves there
Firms	1 Firm for most model runs 4 Firms when examining People with wealth and preference differences
1 Firm values	When not parameter-sweeping, exogenously set to: Wage: 1 Good cost: 1 Delivery cost for Good: 0.25 Commute cost: 0
Landlords	Number: $31 * 31 = 961$ on a grid Stock of land per landlord: 0.1 Reservation price: 3.5 Land cost: endogenous
Density cost	Radius: $1/32$ of model space Density calculation: 1 if all People exactly on my spot 0 if they are on or beyond radius Between 0 and 1 otherwise Multiplication factor: density calculation $*10$ Controls density strength Add result to 1 for final cost People buy

Table 1: Model elements

says (quoting Hicks) -

> “transactors are not all brought together in a single location and at the same time. Without centralisation and synchronisation, the supply-equals-demand condition ‘cannot be used to determine price, in Walras’ or Marshall’s manner’.” (Leijonhufvud 2006 p.1633)

This is a problem many auction approaches in ACE attempt to address, but these tend to use some variant of a non-spatial clearing point. Lemoy *et al.* use a form of bidding, while allowing landlords’ costs to ‘deflate’ if demand is too low. Their approach also requires buyers to know existing occupation levels (Lemoy *et al*. 2012 pp.4).

An idea of Leijonhufvud’s offers a route into a simple distributed market adjustment process that can respond over time to swings in demand caused by any source (including the external effects of many other providers changing prices) while allowing consumers to respond simply to prices they face at the point they decide. Using a Marshallian 'laws of motion' approach (Leijonhufvud 2006 p.1633) a Landlord's price adjustment process is as follows:

- Landlords aim for a price that keeps net stock ('amount of land rented per day' minus 'amount of land I own') at zero.
- If net stock is more than zero, demand is lower than available land, and so the price is lowered to try and sell more. If net stock is less than zero, demand took too much land, and so the price is increased to sell less.
- The change in price is proportional to the amount that net stock deviates from zero: $\triangle p_g = f(stock)/x$, where x controls the magnitude of the price change[11].

In seeking the correct price, stock must be allowed to fluctuate around its target value of zero. This means on any given turn, more or less land than available may be rented. Allowing available land stock to be negative as well as positive is a convenient assumption that means Peoples' demand can be objectively met at the point they need, while also making the above stock target process able to return it to zero, achieving a supply/demand balance across space.

As Alonso notes, the traditional spatial equilibrium approach requires that "(1) no user of land can increase his... satisfaction by moving to some other location or by buying more or less land and (2) no landlord can increase his revenue by changing the price of his land" (Alonso 1964 p.77). So why do Landlords target net zero stock, not revenue? Targetting both optimal revenue and stock clearance is possible, but tests of this produce a more unstable model. Assuming that competition is strong enough that there are no gains from oligopoly (Brakman *et al.* 2009 pp.100), stock clearance is still Landlords' revenue-maximising choice.

3.4 Density cost

This paper also tests the use of a 'density cost' in place of land rent, as a different way to think about the effect of 'being here'. It allows People to react to 'second nature' relative location effects (in contrast to 'first nature' geographical features with absolute location like land plots; O'Sullivan and Unwin 2002, p.79). Second nature effects thus come about through interaction between agents with different locations. The density cost can be considered a proxy for 'congestion' - a word describing a generic loss of amenity caused by density (Glaeser 2008 pp.133), with traffic congestion one element of this (though not an insignificant one; see e.g. Glaeser and Kohlhase 2004 p.209). The overall result from both land and congestion costs, however, is the same: higher density is more expensive.

The density cost itself enters into People's utility in exactly the same way that land cost does: at a given spot, density has a particular unit cost and People will buy whatever amount

[11] This approach targets only net stock levels; in some circumstances it is possible to apply the same approach using oscillation damping methods (see e.g. Taylor 2005 pp.172) able to equilibriate production and supply; Olner 2013 pp.209.

maximises their utility. If a Person buys more density, it can be thought of as them preferring to keep a particular location over buying a certain quantity of good, while also driving up the cost of that location for others. It is thus a simple way to represent each Person's trade-off between distance and proximity costs.

In contrast to land, there is no stock limit. The density cost is in proportion to the number and proximity of others within a set radius from that agent's current location. The density cost uses three parameters. First, a radius around each Person incurring the cost, which is kept to $1/32$ of the width of model space. Second, a variable capturing the density of People within that radius, returning a normalised value between zero and one. The resulting value would be one if all actors are on exactly the same spot as 'me' (this never happens in practice as it would be too expensive), zero if none are within the density cost radius, and between zero and one otherwise. It is worked out for each Person as follows:

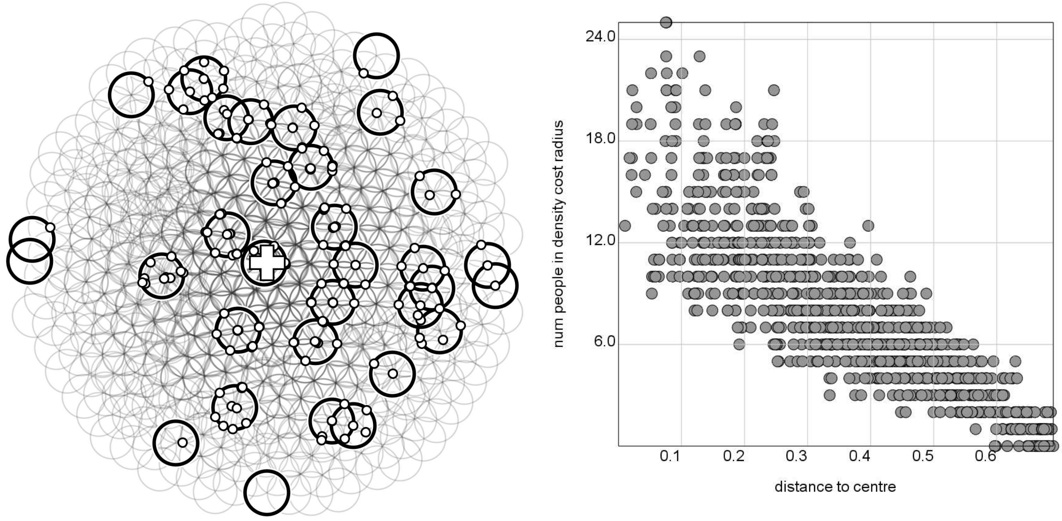
- Using the density cost radius, normalise each other Person's distance from 'me' to between one and zero, where one is the same location as 'me', and zero is on or beyond the radius.

- Sum these normalised distances, then divide by the total number of People for that model run.

Thirdly, this normalised density cost variable is then multiplied by a factor that determines its overall effect. This is set to 10 in the models presented here. This final value is added to 1, so that 1 is the density costs' lower bound. As well as being equivalent to the reservation price, this is to avoid the price of density potentially dropping to zero (which would make demand infinite).

Figure 1 shows how People arrange themselves in reaction to the density cost, given a 'monocentric' setup with a single Firm (the white cross) at the centre. Light circles indicate the density cost range ($1/32$) for each Person. Darker circles show a random selection of People, including the position of others they are incurring a density cost from (the smaller white circles). The resulting hexagonal patterning is a striking feature of People's self-organising behaviour, resulting from minimising density cost by finding interstitial points. However, for the purposes of this paper, the economically interesting aspect is People's ability to optimise location choice, trading off against distance to the centre. Sub-figure 1b shows the discrete number of People entering into density cost calculations correlated to how far each Person is from the central Firm. Each circle represents an individual Person; they are willing to incur higher density costs towards the centre.

Table 1 summarises the key elements of the model and the selected parameter values. The values given are used in all models unless otherwise stated in the text.



(a) **(b)**

Figure 1: People optimise in response to delivery and Density cost, figures from a single stable model timepoint. **(a)**: Model space visualisation. All People's density cost radii (grey circles); random selection of People's density cost radii (black circles) showing location of detected agents entering into density cost calculation (small white circles). **(b)**: discrete number of People entering into density cost calculation increases with proximity to centre.

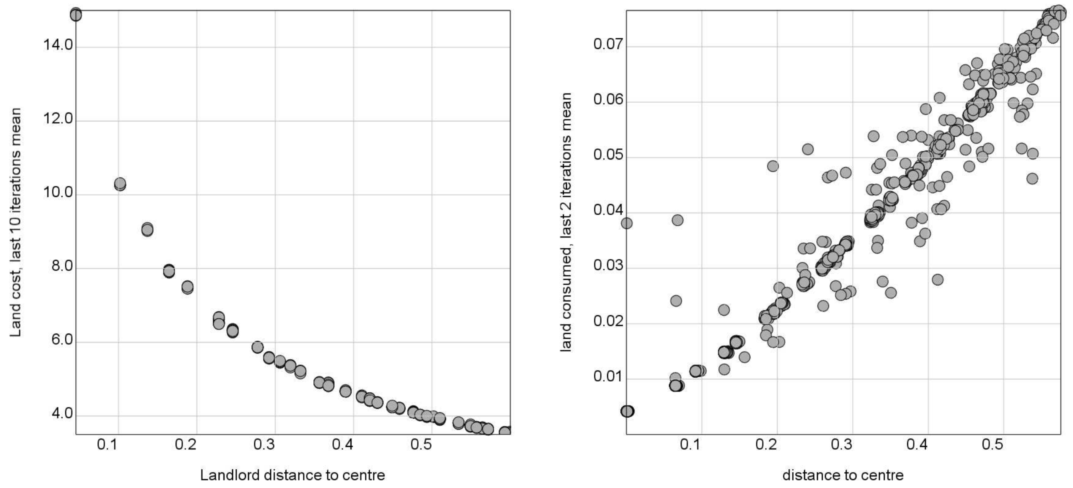
4 Results

4.1 Spatial equilibrium from density cost and Landlords

These first results show agents replicating the most essential elements of the classical monocentric model: spatial equilibrium emerges through both the density cost and Landlord approach (figure 2). A settlement of People forms around the fixed location of the Firm as they trade off the delivery cost of goods against proximity costs. Sub-figure 2a shows the price for a unit of land that Landlord agents arrive at after 250 iterations, against their distance from the centre. Sub-figure 2b shows People's consumption of land dropping towards the centre: they are finding the utility-maximising option is to squeeze into less land per Person; land costs rise towards the centre of the settlement. People rent more land the further out they are, trading off consumption of land against the extra spatial costs. This matches the effect of endogenising land area in the AMM model (Glaeser 2008 p.27).

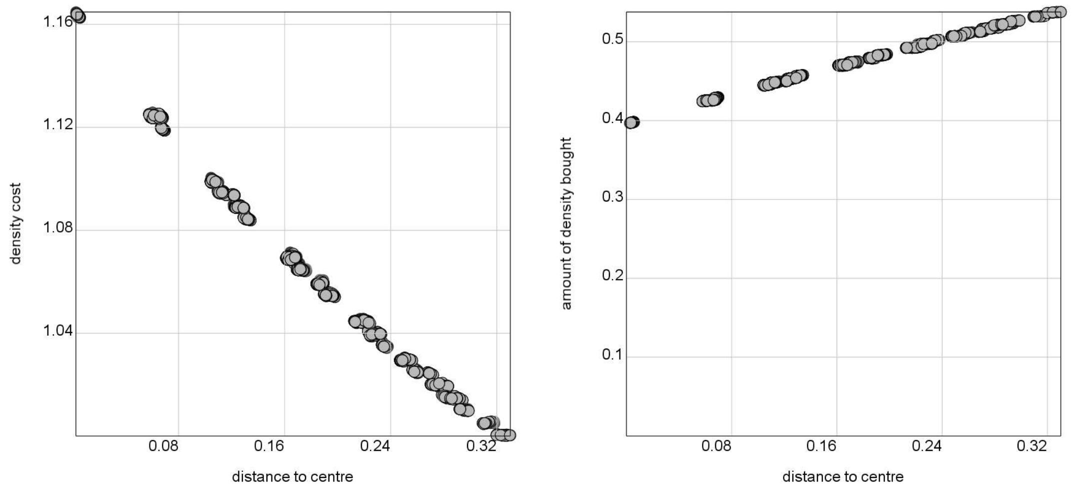
A similar (though not identical) pattern emerges via density cost also, purely through the choices made by People responding to density. Density cost is higher towards the centre (sub-figure 2c) while People buy less of it as they draw closer in (sub-figure 2d). The larger variety of land quantities when buying from Landlords reflects People's land-buying having a Bertrand-competition dynamic (see e.g. Vives 2001 pp.117) where they will select the single utility-maximising option; Landlords' price adjustments thus tend to be more volatile.

Most importantly, utility equilibriates: this is illustrated in figure 3 for People's response over model time to both Landlords and density cost by showing their average utility and a band



(a) Landlords' equilibrium land costs vs distance to centre

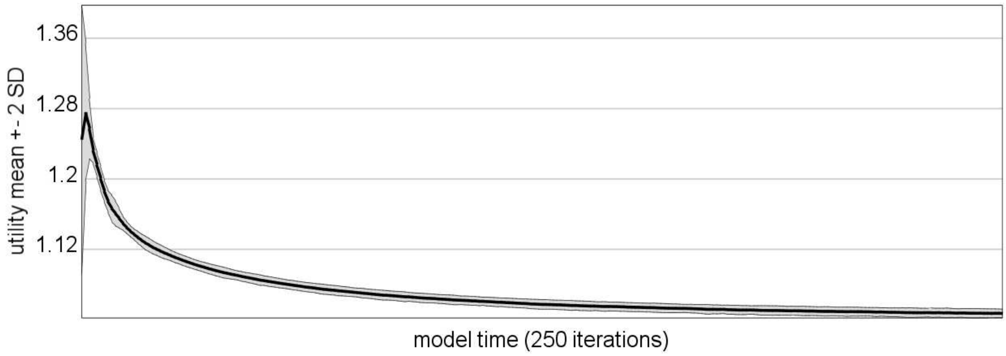
(b) People's consumption of land in response to Landlords' prices



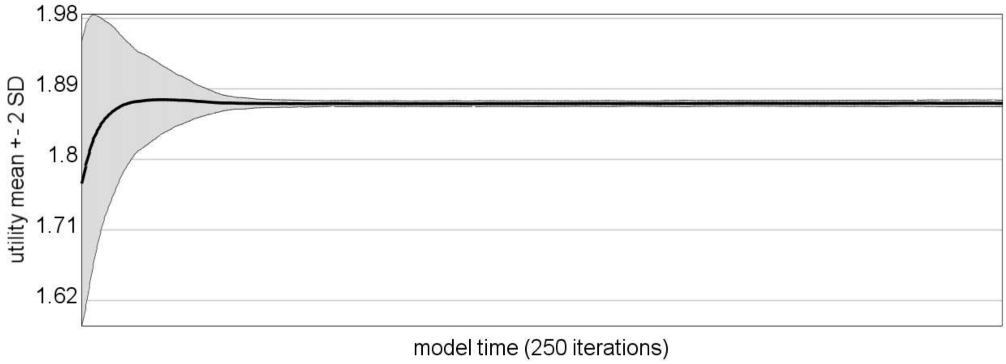
(c) Equilibrium per-unit density cost

(d) People's consumption of density cost

Figure 2: Comparison of spatial equilibrium for Landlords versus density cost. **(a)** and **(b)**: Landlords set rents. **(c)** and **(d)**: land costs from density.



(a) Landlords



(b) Density cost

Figure 3: Model time diagrams showing the process of utility equilbrium being reached for **(a)** Landlord model and **(b)** density cost model over 250 iterations of a single model run for each. People's mean utility is shown in each with two standard deviations each side.

of two standard deviations either side of it. The utility of all People converges to close to the same value, though settling on a stable common value takes longer for Landlords adjusting prices - and also has a slightly higher variation due to price fluctuations - but the same dynamic is present[12].

4.2 Spatial morphology: reaction to cost changes

What drives agent location choice to produce stable emergent equilibria? Section 4.4 digs into that question in more detail. To set the context, this section looks at how People in the model respond to changes in costs, as reflected in settlement size. Figure 4 shows how settlement morphology is affected by People's choices as costs are changed. Each of these costs can be considered in terms of a Person's real wage: if good costs, delivery costs or commute costs

[12]In sub-figure 3b, People are given only one random sampled location per turn (on top of checking their current one) to slow down the equilibrium process in response to density costs, in order to make it visible. They usually use ten sample points; this leads to a very rapid equilibrium. In contrast, Landlords' price-seeking behaviour slows the equilibrium process down.

increase, the real wage is decreasing - but the spatial outcomes are not always the same. Changes in base good cost, delivery cost for that good, and commute costs are shown, both in situations where People incur density cost (sub-figures 4a, 4b, 4c) and where Landlords determine land prices (sub-figures 4d, 4e, 4f). (Base wage changes are examined separately below).

In figure 4 each datapoint is an average taken from *all People* for each point of a parameter sweep, after the model has stabilised. Holding all other parameters fixed, the cost parameter on the y-axis is incremented in small steps after each stabilisation[13]. To avoid any 'lock-in', each time an increment is made, the model is fully restarted - People's locations are randomised and (if Landlords are used) Landlords' price is reset to the reservation value. The x-axis for each sub-figure is 'mean distance to centre': each datapoint shows the average distance of *all People* from the centre at each stabilisation point. This average distance provides a single metric to measure overall settlement size shrinking or growing in response to a cost change.

Three things are revealed. People facing density costs or Landlords make the same choices given a change in cost (though see below regarding base wages). Those choices vary depending on the cost type. Base good cost and its delivery cost produce opposite results: cheaper base-cost goods means denser settlements as People can afford to buy more goods, trading off against land or density bought. Decreasing either delivery or commuting costs expands the settlement: the effect of varying either type of distance cost is the same.

Morphological reaction to *wage* change, however, does not fit this pattern: it varies depending on the type of proximity cost. Wage change in the presence of density costs has *no effect* on settlement size, due to the CES function's homothetic form and the fact that there is no limited stock of density at a given price point. Doubling the wage simply means doubling the consumption of both goods and density, leaving all People's relative utility levels unaltered (thus causing no disequilibrium).

If land is of limited stock, however, and Landlord agents must seek a price to clear that stock, the story is different: rising wages expands the settlement. The basic mechanism is clear from AMM models where endogenous land consumption enters into utility: where land is a normal good, higher wages means higher consumption. Thus more land is consumed overall and the settlement must grow. In the model presented here, the mechanism that drives this process has the following steps: wages rise; People rent more land (their demand is objectively met); Landlords increase prices to keep stock cleared. As prices increase over time, some People find that their current location becomes a worse utility option than land further out; they move and the settlement expands. (Section 4.4 examines this externality interaction effect in detail.)

In order to make wage change comparable to good cost change, figure 5 reframes the wage

[13]For density cost, 'stability' comes directly from People's utility equilibrium: an average is taken over ten days of the change in all People's mean utility, and of the standard deviation of utility; these must both drop below 0.005. A low standard deviation indicates all People having very similar utility; the daily change in mean value staying consistent implies that global utility has stabilised. Once stability is reached, ten iterations are allowed to pass before changing the value again and checking for People's location choices settling down. For landlords, 'stability' instead targets land prices: when mean land price change, averaged over 30 model days, drops below 0.001, with a ten day buffer.

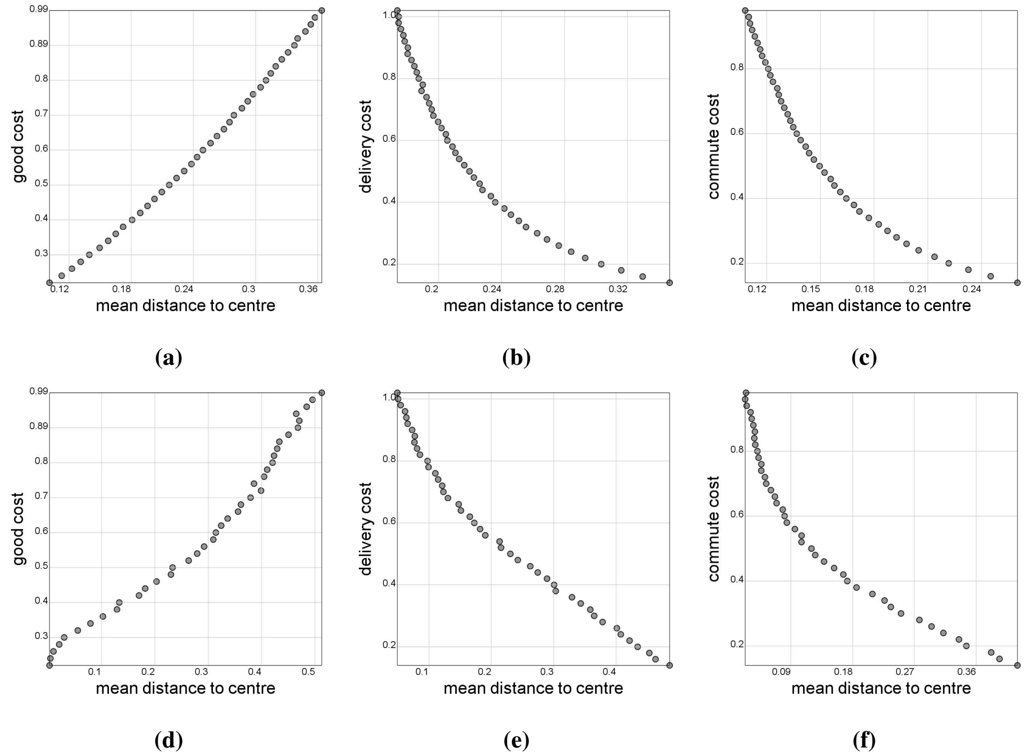


Figure 4: Settlement size response to changing good cost, delivery cost and commute cost. Each datapoint is an average of all People's distance to the centre, used as a measure of settlement size change. The cost parameter on the y-axis is incremented in small steps and the model reset each time; a datapoint 'snapshot' is taken when model re-stabilises. **(a)**, **(b)** and **(c)** are response to density cost; **(d)**, **(e)** and **(f)** are response to Landords' prices.

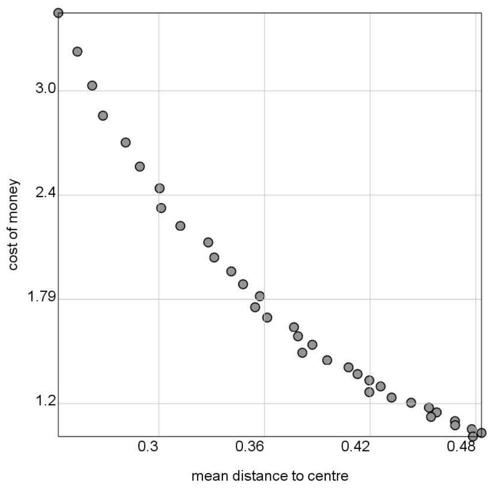


Figure 5: Settlement size response to changing the 'cost of money' (inverse of the wage) in the presence of Landlords. When cost of money is cheaper (wage higher), settlement expands as People rent more land. Same parameter-sweep setup as for Landlords in figure 4.

as the 'cost of money'; a drop in the cost of money is thus - like a drop in good cost - a real wage increase. For example, if the cost of money is 4, a Person must pay one day's time to get 0.25. Where the cost of money is cheaper, People are able to rent more land.

4.3 The impact of differences in wealth and preferences

Giving People exogenous differences in wealth very clearly illustrates the different effect of spatial versus non-spatial costs. Lemoy *et al.* (2013) explore this issue for the effect of commute versus wage differences, in line with the related AMM model (Glaeser 2008 pp.33). They also use a particular form of CES function from Brueckner *et al.* (1999, p.106) to explore "the marginal value of amenities rising faster with income than housing consumption" (Lemoy *et al.* 2013 p.7). This section, alternatively, looks at changes in base good cost and delivery cost. The impact of variable wages is also examined, with a different outcome to similar models, before finishing the section with a look at how heterogeneous 'love of variety' preferences can affect settlement patterns. Landlord agents are used in these models. The same polarity of outcomes results from the density cost approach, but Landlords create a steeper cost curve towards the centre.

A small number of wealth bands are used in order to clearly observe the impact: four are used here. These values are exogenously set, and spread evenly so that the richest People are four times wealthier than the poorest. Each Person is assigned to one of four Firms, from whom they must buy their Good; the shaded shapes indicate which Firm each Person is attached to. A key for the cost attached to each of these can be seen in sub-figures 6a and 6b.

Figure 6 shows the result at equilibrium[14] of varying base good cost in the first column and

[14]In this instance, 'equilibrium' is stable land prices - because of the Bertrand-competition nature of those prices,

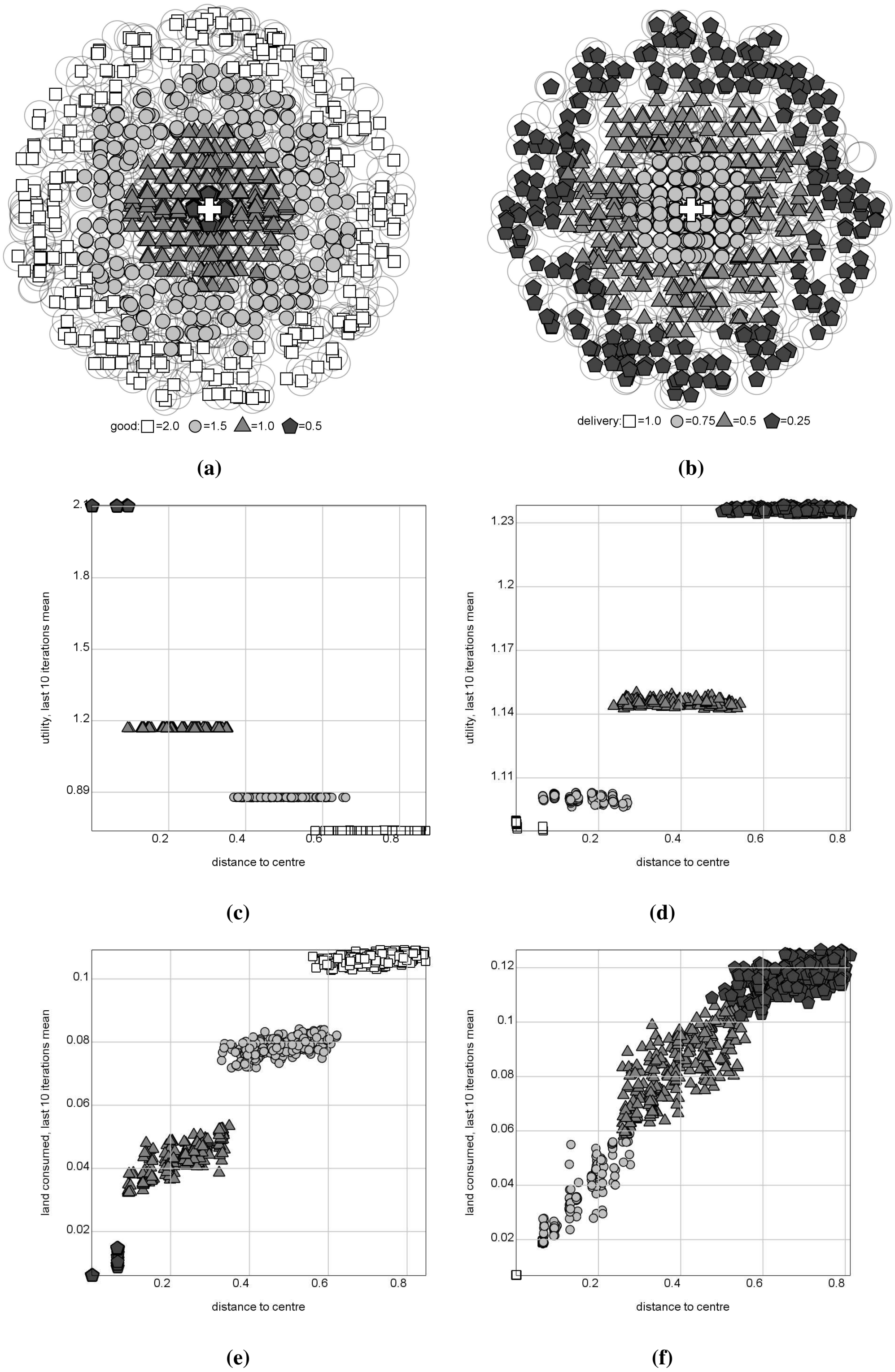
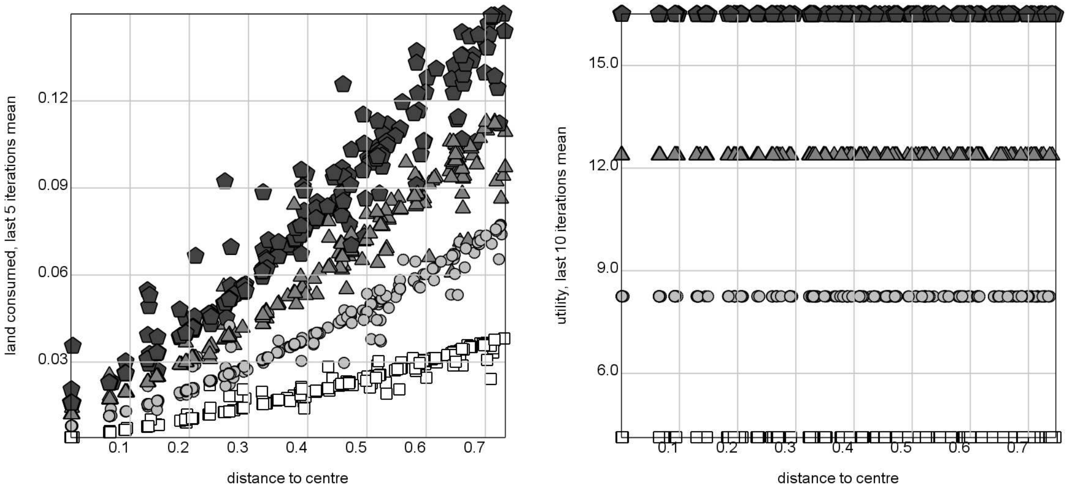


Figure 6: People given a range of wealth. In **(a)**, **(c)** and **(e)** People have a range of good costs (key in **(a)**). In **(b)**, **(d)** and **(f)**, they have a range of delivery costs (key in **(b)**). Top row shows morphology of settlement; second row shows utility equilibriating in wealth bands, at opposite settlement locations for spatial versus non-spatial cost; bottom row is land consumption for each band versus distance from centre.



(a) **(b)**

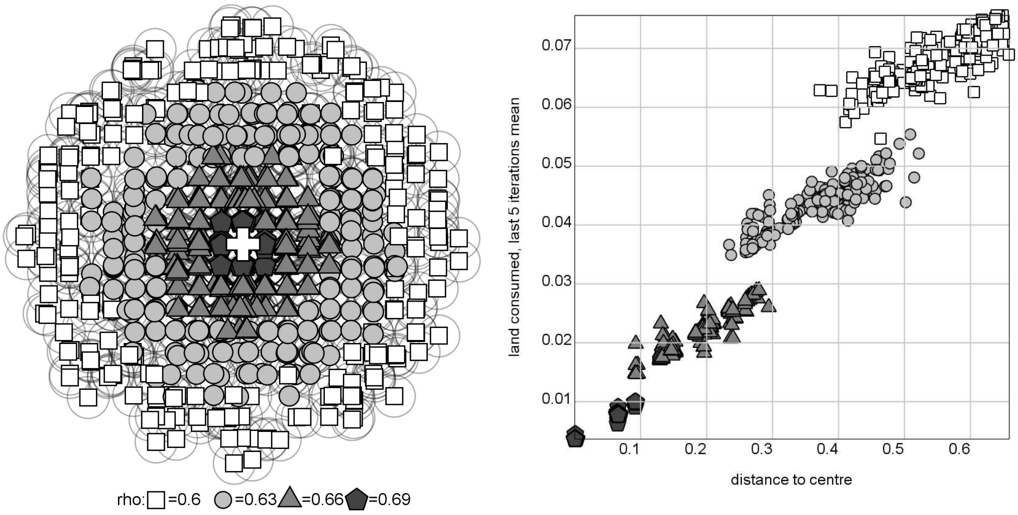
Figure 7: Land consumed and utility versus distance to centre for People in four wage bands, ranging from 1 (white squares) to 4 (dark pentagons)

varying delivery cost in the second (while holding the base cost constant at 1). In each case, the poorer People must pay a higher delivery cost, thus having a lower real wage. The top row shows the spatial outcome: if People are 'base-cost rich' (sub-figure 6a) they maximise utility by clustering at the centre; the poorest are on the outskirts. Conversely, transport-rich People (paying lower shipping costs, sub-figure 6b) locate at the edges.

Sub-figures 6c and 6d confirm these are the utility-maximising options for each wealth group. Richer groupings have higher utility, of course, but each group equilibriates with its own. Finally, sub-figures 6e and 6f show the amount of land being bought by each group. In both cases, the more expensive central land is much denser while the outskirts see People buying larger quantities - but the utility outcome is determined by the cost type. Base-cost rich People use their wealth to rent a smaller quantity of expensive land, minimising the Good's shipping cost. Poorer People facing the same good-shipping costs are forced to the edges. The more modern outcome of rich People living in outer suburbs (that Glaeser concentrates on, ibid.) is the result of differences in transport costs, as shown above.

Homothetic utility determines the outcome, however, if People are given a spread of wages. Because utility is homothetic, all People arrive at equilibrium at the same point, regardless of their income, and they are indifferent to location within each band. The pattern is the same whether People face Landlords or density costs; figure 7 shows the outcome with Landlords. This suggests that direct wage differences alone cannot create segregation. With land as a normal continuous good, people can buy into any location regardless of wealth - the poorest Person's demand will drive up rent slightly for the richest, and vice versa, but they can both live

People continue to move around, especially on the outskirts, but land prices keep to a stable pattern as People keep prices moving around their equilibria.



(a) (b)

Figure 8: People in four preference bands: ρ ranges from 0.6 (white squares, higher 'love of variety') to 0.69 (dark pentagons, higher elasticity of substitution); land consumed and utility versus distance to centre.

there. However, this relies on perfectly homothetic consumption - it is not stable, particularly given that many goods (including land, as Lemoy *et al.* note) are income-elastic. Any elasticity at all would trigger separation of wealth groups[15].

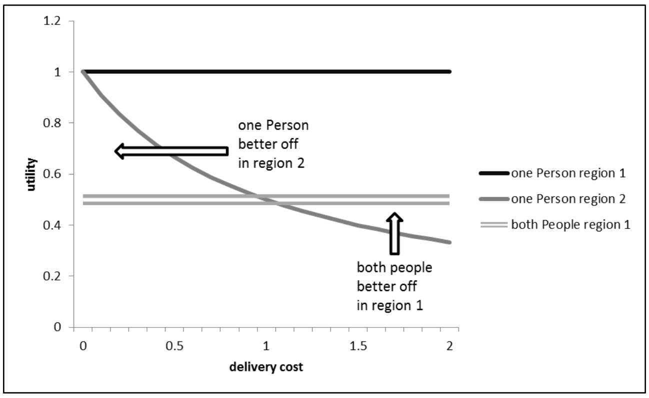
One way to break this symmetry across distance that does not rely on differences in wealth is for People to have heterogeneous preferences. Where Huang *et al.* exogenously impose these (for example by making some agents prefer being close to the centre), figure 8 shows the result of giving People a range of 'love of variety', for values of ρ ranging from 0.6 to 0.69[16]. Preferences are still homothetic but because distance imposes heterogeneous costs, People in different locations vary in their preferred mix. Some prefer more of a mix of land and goods, whereas others with lower love of variety will not mind squeezing into a small space and consuming more goods. This does not require a Zimmerman-like proportional change due to wealth (Zimmerman 1932) - people can have the same wage - but space imposes heterogeneity on costs; People with different elasticities thus buy different mixes.

4.4 Analysis of two and three Person decisions

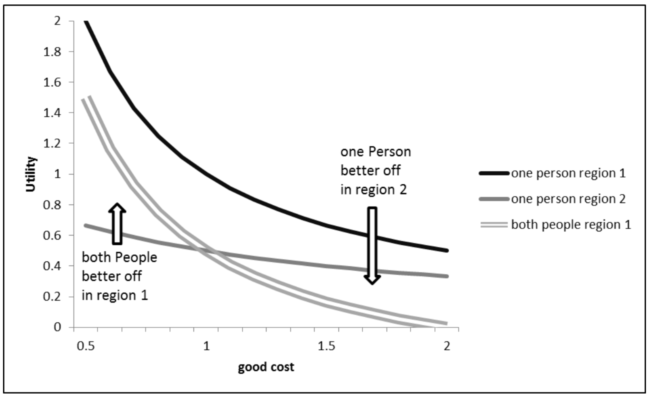
Moving away from the agent models above, this section uses a simple mathematical analysis, taking inspiration from Alonso's original 'simple game solutions' (Alonso 1964 pp.77). The aim is to show, by looking at the full choice set of two and three Person games, how decision

[15] This may also suggest that the discrete nature of land buying also may determine segregation outcomes; wealthy areas may move towards selling packets of land only to richer buyers. Wealth may therefor shape the market in ways not captured by considering land as a continuous good with no limit on the smallest buyable quantity.

[16] This approach cannot be used for comparing utility, though, since the CES function's utility output is dependent on ρ: utility for the same mix of goods increases as $\rho \to 0$.



(a) utility vs delivery cost



(b) utility vs base good cost

Figure 9: Two People, two regions. Wages good cost = 1. Density cost if two People in same region: 0.5, subtracted from wage. No delivery cost in region 1, delivery cost = 1 in region 2.

externalities lead to the emergent spatial outcomes seen in the main model results, and to illustrate the source of spatial equilibrium in those choices. To do this, one of the simplest examples from the above results is examined - People's reactions to base good cost and delivery cost in the presence of a density cost. Having shown People's reaction to proximity costs is the same in this case, whether density or land rent, using density cost removes the complications of a land market.

In the first example, there are two discrete regions and two People. Region one has no delivery costs, but in region two it costs 1 to deliver a unit of good. There is a single Firm in region one paying a wage of 1. It sells goods at a base cost of 1. As there is a delivery charge in region two, it costs 2 to buy one unit of good there.

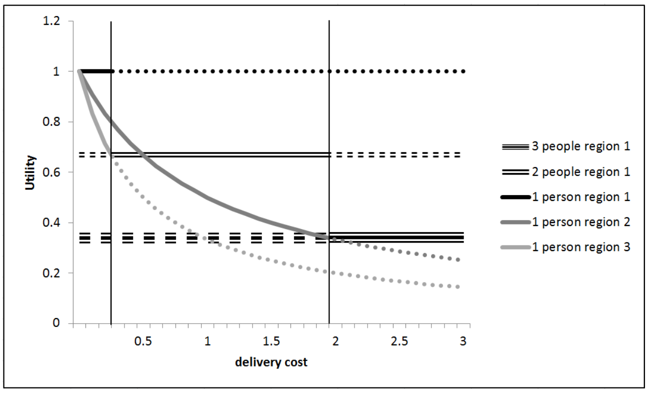
The two People must decide where to locate to maximise their utility. They can avoid delivery costs by staying in region one, but if they both reside there, density costs are incurred. Let the density cost be zero if only one Person is present, and 0.5 if both are in the same region. If so, subtract the density cost from the wage so it becomes 0.5 for each. Good quantity consumed equals utility.

Exclude the obviously poor choice of both residing in region two, where delivery and density costs would be high: at least one Person would choose to minimise delivery costs. This leaves only two possible states: both People in region one; or one Person in each region. Figure 9 shows the utility for these two states, changing base good cost and delivery cost (the spatial and non-spatial components of the total good cost).

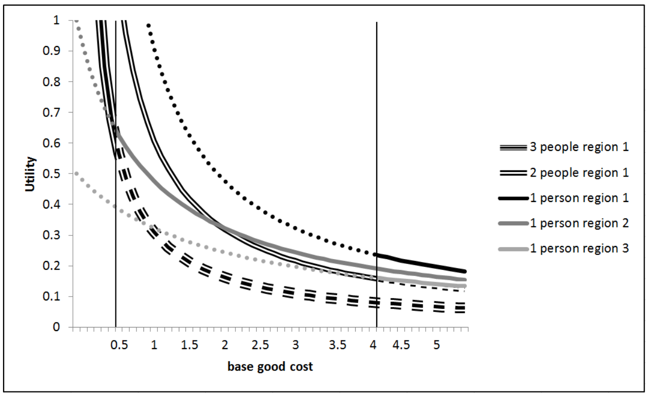
It is better for one Person to move from region one to two where the single grey line is higher than the double. Increasing delivery cost (sub-figure 9a) is agglomerating: while it is better for one person to move to region two when delivery costs are below 1, as they go beyond this, sharing region one with the other Person becomes utility-maximising. Conversely, increasing the base good cost (sub-figure 9b) is centrifugal: as it increases past 1, one Person will find moving away from the centre the best choice.

When a Person decides to move to region two (single grey line), the Person left behind gains a permanent utility advantage in region one (black line) due to density dropping. The reverse is also true: if moving to region one is best, this will drag the utility of the existing resident down. These are both obvious externalities: moving from one to two increases both People's utility - but the Person staying put actually benefits more. Moving from two to one increases only the mover's utility; the other pays a price in increased density.

Extending the thought experiment to three People and three regions, with the third region twice as distant again (figure 10) produces more complex outcomes, but can be used to identify where the pressure for spatial equilibrium comes from. It is made easier to understand since there are only three rational states to be in: all three People in region one; two People in region one with another in region two; or one Person in each of the three regions. Looking at the case of changing delivery cost and base good cost again (figures 10a and 10b) shows where the change between these three options takes place.



(a) utility vs delivery cost



(b) utility vs base good cost

Figure 10: Two People, two regions. Wages good cost = 1. Density cost if two People in same region: 0.5, subtracted from wage. No delivery cost in region 1, delivery cost = 1 in region 2. Unbroken lines indicate optimal choices.

Each graph is split into three sections at two key cross-over points where People are incentivised to move. Unbroken lines indicate the utility-maximising choice in each case. Again, increasing delivery costs is agglomerating: starting at lower values, People's best option is to spread out, one per region. Region three's resident has no incentive to move to region two: if two people reside there, its utility drops. They should stay put until 'two people in region one' becomes the better option. That has a knock-on effect: the existing resident is made worse off. At the second cross-over, region two's new resident now finds region one better, increasing their utility but lowering that of the other two in the process.

For base good cost, the direction of agglomeration is different, but the outcome is the same: the pressure for spatial equilibrium comes from the external effect of one Person's optimal choice. Note, it is also possible to read off who will be incentivised to move. In the case of increasing base good cost, the utility of '2 in 1' - higher for cheaper goods - drops until a Person in region one finds moving right out to the edge the best option.

Economically, the impact of Peoples' choices on proximity costs cause *externalities* for others: if 'my' location decision is before others, it will change land or density costs for them with no compension being made (Button 2010 p.161) and this will alter how they weigh them against distance costs. The externalites will be both positive and negative, decreasing proximity costs in the place a Person moves from while increasing them in their newly chosen spot. The accumulation of these external effects for many agents over time is the source of the model's emergent outcomes.

5 Conclusions

This paper has presented an agent-based 'monocentric' model of the emergence of urban form. It has been argued that the minimum conditions for urban spatial equilibrium to emerge are two types of fundamental spatial cost - distance cost and proximity cost. Commuting costs and land markets, the most common form of spatial costs in AMM-style urban models, are special cases of each. This paper has added good delivery and density cost to make its case for the more general categories of distance and proximity.

The presented results have, firstly, confirmed the essential spatial equilibrium outcomes of previous models (both the classical AMM models and agent-based approaches such as the work of Lemoy *et al.*). Secondly, they have demonstrated how spatial morphology of the settlement is determined by agents' reaction to costs, as well as differences in wealth and preferences and whether agents are 'transport-rich' (leading to locating on the outskirts) and 'good-cost rich' (living near the centre). Direct wage differences, in the presence of Landlords, is shown to create 'cheek by jowl' rich and poor spread across the settlement, though this outcome is unstable, reliant on exactly homothetic utility. Heterogeneous preferences, in the form of variable 'love of variety', is shown to determine morphology as those with high elasticity of substitution rent less land, crowding into the centre.

These results have used a distributed market structure to show how spatially heterogeneous supply and demand can produce equilibria. Reactive Landlord agents that respond with Leijonhufvud-style 'laws of motion' to every tick of demand have been coupled with consumer agents whose consumption can input into the system at arbitrary times and places. As section 3.3 noted, spatial economic problems differ fundamentally from 'dimensionless' economics - space breaks assumptions of centralised market mechanisms, such as auction processes, that are vital for traditional supply-demand approaches. A neoclassical solution to this spatial problem was first presented by Krugman in 1991 (see section 2) but it avoids the spatial market problem. ABM has the potential to tackle this issue more directly, being especially suited to modelling distributed systems - this paper's market model is a contribution to that aim.

The addition of a density cost illustrates that spatial equilibrium does not need to be a market equilibrium; agents facing any kind of spatial cost, if they can choose their location, will push the system to equilibrium. The density cost has also provided a novel way to dig deeper into the origin of spatial equilibrium in simple two- and three-Person scenarios, identifying how opposite spatial outcomes depend on whether the cost is spatial or non-spatial. By showing the crossover points for where agents' optimal choices change, it is possible to see how proximity costs interact with distance costs through the externalities of each sequential agent choice, changing the environment on subsequent timesteps. Agents' ability to be cognisant of better utility options elsewhere in space (however limited their sampling) pushes the whole system to consistent equilibrium outcomes.

The model structure presented here is ideal for investigating a number of obvious extensions. ABM's flexibility and 'loose coupling' approach is perfect for doing this. Lemoy *et al*'s 'polycentric city' models are a step in this direction, but there are still more permutations to explore once the monocentric assumption is dropped, allowing both goods and wages to be provided from heterogeneous locations. The 'partial' approach of the monocentric model could also be opened up further with ABM to examine fully open wage markets and firms making their own price and location decisions. Some steps in this direction are taken in Olner 2013, where the distributed market mechanism presented here is used to model Krugman-style economies of scale in a more 'general' model[17].

There is further work to be done also on the dynamics of distributed spatial markets and how they differ from 'dimensionless' markets. As well as examining markets as distributed networks (see e.g. Ladley and Bullock 2008), there are a range of other spatio-temporal ideas to investigate. For instance, in this paper, while agents' decisions are independent, they are still embedded within a timing structure where all agents decide 'daily'. What are the dynamics of more heterogenous timing decisions over longer periods?

These more complex modelling problems bring their own challenges, but ABM is well-positioned to deal with them. This paper has argued, however, that ABM is often 'method-

[17]The existence of economies of scale are a necessary condition for spatial economies to exist; see e.g. Fujita 1999 p.376.

centred' at the expense of 'problem-centring' (Maslow 1966 p.15); this has kept it away from many central spatial economic questions. To shift ABM back towards 'problem-centring', emergence needs demoting from its common position as the end-goal of agent models, or even in some cases as the 'explanation' for outcomes. An analogy has been made between equilibrium and emergence as the founding assumption of analytic economics and ABM respectively. The comparison is far from perfect: static equilibrium states are a fixed foundation on which models can be built; emergent results are dynamic, often surprising and - by definition - not predictable from examination of single agents. But this is precisely the source of ABM's strength. If used to move beyond 'what happens if' to 'why does it happen?', emergence will thrive as a tool for digging into a rich seam of spatial economics.

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