

# Ultimate Limit State design to Eurocode 7 using numerical methods Part II: proposed design procedure and application

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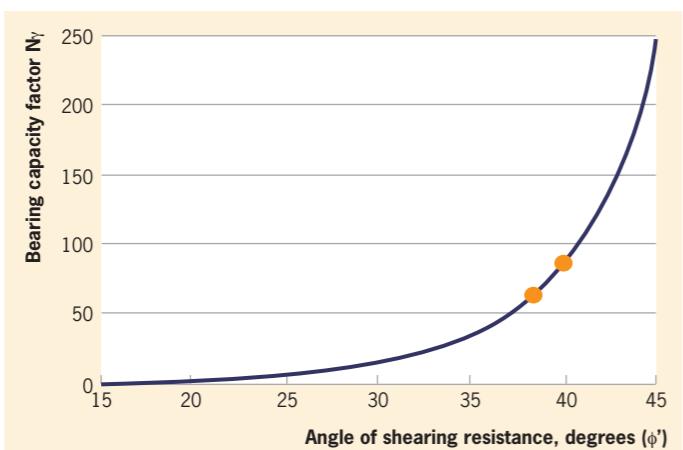
## Summary

In the UK, assessment of the Ultimate Limit State (ULS) is to be carried out using Eurocode 7 Design Approach 1. In most cases this involves two calculations, one which primarily involves an "action factor" approach (Design Approach 1, Combination 1), the other which primarily involves a "material factor" approach (Design Approach 1, Combination 2). In Part I of this paper a general methodology for undertaking ULS design using numerical methods for both material factor and action factor (including action/resistance factor) approaches was presented.

$$E_d \leq R_d \quad (1)$$

Where  $E_d$  represents the design (ie factored) actions, or action effects, and  $R_d$  represents the design resistance. In this approach the relevant parameters involved are *post-factored* following the mechanical analysis. Therefore this approach cannot be used to automatically identify the critical collapse mode. Instead, each mode must be examined separately, as has been standard practice for many years.

Internal structural stability may be assessed directly using the action/resistance factor design



**Figure 1:** The variation in bearing capacity factor ( $N_y$ ) with angle of shearing resistance  $\phi'$  for a strip footing illustrates a highly non-linear relationship between the bearing resistance and the material property ( $\phi'$ ). Markers indicate values for  $\phi' = 38.4^\circ$  and  $\phi' = 40^\circ$ .

In a material factor approach, the material parameters involved are *pre-factored* prior to performing a mechanical analysis, and the approach can therefore readily be used in conjunction with a general

numerical analysis procedure to automatically identify the critical collapse mode.

■ In an action/resistance factor approach, a numerical analysis must be carried out in a pre-specified "equilibrium direction" (appropriate to a given collapse mode, for example horizontal to replicate sliding failure of a retaining wall), where the ULS collapse state is induced by applying an additional disturbing force in the equilibrium direction. It is in this direction that the fundamental Eurocode stability equation is evaluated:

procedure developed addresses the possibility that "non-linearity" exists in the system. Appropriate examples are then used to illustrate its application.

Finally, it is assumed that the reader is familiar with the core principles and notation used in the Eurocodes, as summarised in Part I of this paper. The following abbreviated forms are also used throughout: DA1 (Design Approach 1), DA1/1 (Design Approach 1, Design Combination 1) etc. As in Part I of the paper, only Eurocode limit states GEO and STR are considered.

## The challenge of non-linearity

### Non-linearity and partial factors

The use of partial factors, as employed in the Eurocodes, is sometimes criticised because it can lead to apparent problems when the relationship between actions and/or material properties and their corresponding action effects and/or resistances, prior to the application of partial factors. These stages correspond to the following analysis levels (ie levels at which partial factors are applied in the calculations):

Having established a general methodology which allows material factor and action/resistance factor design approaches to be used in conjunction with general numerical analysis procedures, in Part II a consistent design procedure designed to ensure a "safe" overall design of a geotechnical construction in accordance with Eurocode 7 is described. Specifically, the design

a margin of safety of 1.00 on the predicted design shear stresses, whereas DA1/1 partial factor values stipulate a margin of safety of 1.35.

Driscoll & Simpson (2001) note that EN1990 (BSI 2002) does address this issue by recommending that the following simple rules are followed when a single action dominates [EN1990, 6.3.2(4)]:

- When the *action effect* increases more than the *action*, the partial factor  $\gamma_F$  should be applied to the representative value of the *action*.
- When the *action effect* increases less than the *action*, the partial factor  $\gamma_F$  should be applied to the *action effect* of the representative value of the *action*.

Methods for undertaking the appropriate calculation type for each analysis level were described in Part I of the paper (though without specific reference to the concept of analysis levels).

It should also be noted that although these "analysis levels" are often implied in current literature on Eurocode 7, it is useful to identify them explicitly, as this should ensure that factors appropriate to different levels are not inadvertently applied in a single calculation (this can sometimes lead to fundamental mechanical principles being violated in the problem being analysed).

In this context, it is also necessary to consider an appropriate methodology for handling variable actions at Analysis Level 2 or Analysis Level 3. This is addressed in Appendix I.

It is proposed that a general design procedure should, in principle, involve design checks at all three analysis levels, using three separate calculations to address the potential issue of non-linearity. It should also be noted that Analysis Level 1 checks will, in many cases, also implicitly cover checks subsequently carried out at Analysis Level 2 and Analysis Level 3 (for example the bending moments in a sheet pile wall can be checked during an Analysis Level 1 calculation, as well as in a subsequent Analysis Level 3 calculation).

Similarly, checks carried out at Analysis Level 2 will often implicitly also cover checks subsequently carried out at Analysis Level 3. However, because of non-linearity and the application of differing partial factor values, this does not necessarily mean that the

■ Analysis Level 1: At source ie applied to the originating *action* such as soil self weight or external load. This permits global stability to be assessed.

■ Analysis Level 2: At a structure

and at soil/structure interfaces ie applied to an *action effect* or

resistance such as an earth pressure. This permits the overall stability of a structure to be assessed.

■ Analysis Level 3: Within the structure ie applied to an *action effect* such as a bending moment. This permits internal structural stability to be assessed. (This is sometimes referred to as the \* approach in the literature, for example DA1/\* or DA2\*).

Methods for undertaking the appropriate calculation type for each analysis level were described in Part I of the paper (though without specific reference to the concept of analysis levels).

It is suggested that a small number of checks at Analysis Level 2 are also carried out, typically using the critical collapse modes identified at Analysis Level 1. With the increasing availability of general purpose numerical analysis procedures, such checks are not too onerous to undertake. Finally, if it is obvious that design checks at one particular level are unnecessary, then these do not need to be undertaken.

Illustrative examples of the design checks to be undertaken at each analysis level will now be presented.

## Worked examples

### Analysis Level 1 example: strip footing design (DA1/1 pre-factoring approach)

#### Problem statement

It is required that Analysis Level 1 DA1/1 checks of the vertical bearing stability of a strip footing are undertaken; full details of the problem are given in Figure 3 (see overleaf).

This sort of simple problem does not generally involve *action effects*, and is therefore amenable to the DA1/1 "pre-factoring approach", in which factors are applied to *actions* but not *resistances*.

In this case, the actions are: the external load applied to the footing, the weight of the footing itself and the surcharge applied to the surrounding soil. The only *resistance* is that provided by the soil itself, experienced by the footing at the soil/footing interface.

As part of the design check, an overdesign factor with respect to external applied load will be sought. When using the DLO-based numerical limit analysis software LimitState:GEO (LimitState 2009) this means that the adequacy factor  $\lambda_A$  applied to the external applied load is computed, together with an associated collapse mechanism (for example, as shown in Figure 3).

#### Numerical analysis

The values of the parameters used in the pre-factoring analysis are given in Table 2 (using the unfactored soil properties given in Figure 3), together with results from a numerical limit analysis model of the problem undertaken using LimitState:GEO and a "fine" numerical discretisation comprising approx. 1,000 nodes (LimitState 2009). In this problem the soil self-weight has no effect (ie is neutral). The result indicates that the design is safe and is overdesigned over and above the Eurocode requirements by a factor of 1.05 on the applied load.

#### Analytical check

The collapse soil resistance »

Analysis Level	Usage – from the literature <sup>1</sup> (i)	Usage – from the literature <sup>1</sup> (ii)	Proposed usage (iii)	Proposed usage (iv)	Typical purpose (eg for footing design)
1	DA1/2	–	DA1/2	DA3	sizing of footing <sup>3</sup>
2	DA1/1 <sup>2</sup>	DA2	DA1/1	DA2	sizing of footing <sup>3</sup>
3	–	–	DA1/1	DA2	internal footing design

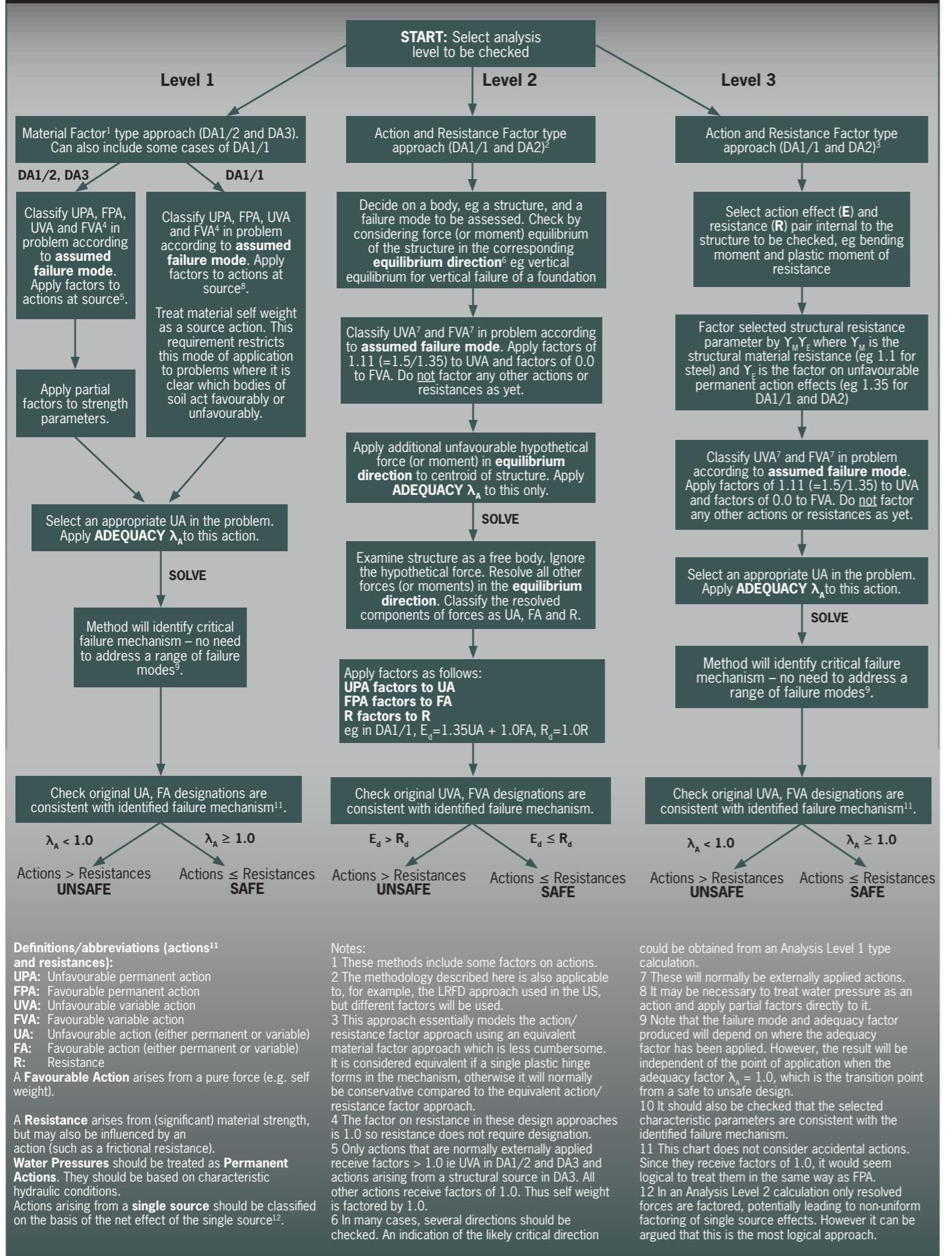
<sup>1</sup>Analysis Level 3 checks are sometimes implicitly performed as part of Analysis Level 1 or Analysis Level 2 calculations.

<sup>2</sup>In some cases DA1/1 checks can be performed at Analysis Level 1

<sup>3</sup>for (iii) DA1/2 typically controls footing width, while for (iv) DA3 is not normally used for foundation design

**Table 1:** Design checks to be performed for soil-structure interaction problems at different analysis levels, either (i), (ii) inferred from current literature, or (iii), (iv) proposed here. (NB checks are expected to be performed at all analysis levels indicated in a given column)

Figure 2: Proposed design procedure for performing ULS design to Eurocode 7 in conjunction with numerical methods



» R beneath the footing may be calculated from the Terzaghi bearing capacity equation:

$$R/B = cN_c + qN_q + \frac{1}{2}\gamma BN_y \quad (2)$$

For an undrained failure with no surcharge,  $N_c = 5.14$  and:

$$R/B = c_u N_c \quad (3)$$

Hence, for this problem,  $R = 617\text{kN/m}$ . Details of a “hand” analysis check are summarised in Table 3. The over-design factor with respect to the applied load is  $(617 - 54)/540 = 1.04$ . The numerical result (in terms of adequacy, or over-design factor) is thus  $\approx 1\%$  different from the exact theoretical answer.

**Analysis Level 2 example: gravity wall design (DA2 post-factoring approach)**

#### Problem statement

It is required that an Analysis Level 2, DA2, design check of the short term horizontal stability of the gravity wall shown in Figure 4 is performed (ie a check against horizontal sliding failure). The problem requires a “post-factoring approach” and a hypothetical horizontal force  $H$  acting in the postulated direction of failure (ie equilibrium direction) is to be used, as shown in Figure 4. The “adequacy factor”  $\lambda_A$  is applied to this load,  $H$ , while the degree of overdesign can be determined by computing the ratio of resistance to actions.

#### Numerical analysis

The relevant Eurocode 7 design parameters are given in Table 4 (using the unfactored soil properties given in Figure 4), together with results from the analysis of the problem obtained using LimitState:GEO and a “medium” numerical discretisation comprising approximately 500 nodes (LimitState 2009). In this example a modified partial factor  $\gamma_Q/\gamma_G$  is applied to the surcharge; refer to Appendix I for justification of this. The result indicates that an additional horizontal force  $H = 38.5\text{kN/m}$  is required to cause failure.

Table 5 lists the key action effects and resistances identified after undertaking the numerical analysis. In this case the effective active pressure resultant is an *unfavourable action effect* and is therefore factored by 1.35. The water pressure is an *action* since its value is not modified by the collapse mechanism. Its net effect in the equilibrium direction is *unfavourable* and so it is also factored by 1.35.

The data indicates that the factored resistances are less than the factored action effects. The wall is therefore under-designed when

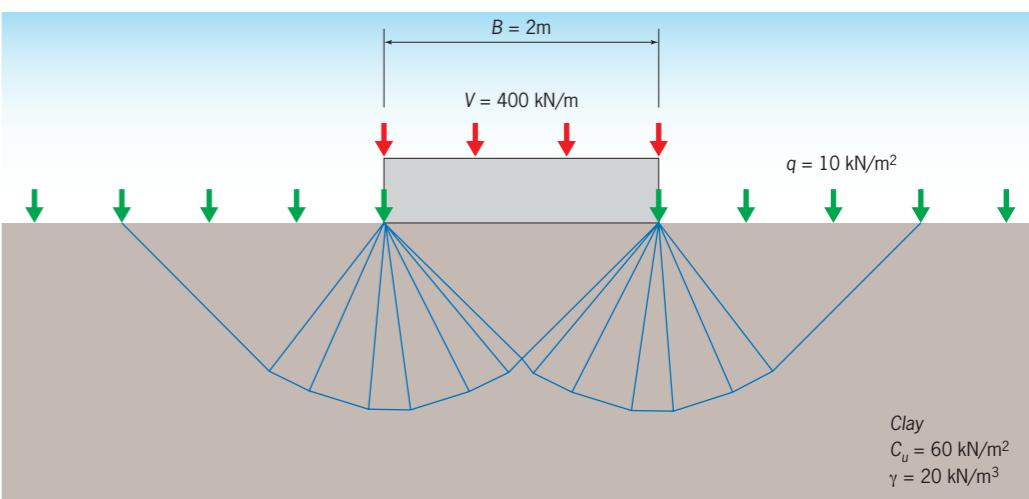


Figure 3: Analysis Level 1 (strip footing) design example: details of problem, also showing predicted collapse mechanism. (Additional parameters: footing weight (exerted at soil-footing interface) = 40kN/m. Surcharge  $q$  is taken as variable.)

$$U_a = \frac{1}{2}\gamma_w A^2 = 78.5\text{kN/m} \quad (5)$$

The passive force  $P$  (short term total stress), based on the undrained shear strength  $c_u$  and the soil unit weight  $\gamma_w$  is given by:

$$P = \frac{1}{2}\gamma_w l^2 + 2c_u = 110\text{kN/m} \quad (6)$$

which matches the result obtained using the numerical analysis procedure (see Table 5).

Since the walls are smooth, the normal force on the base of the wall is equal to the wall weight  $W$  less the upthrust  $U_a$ . The base friction  $T$  is therefore given by:

$$(W - U_a) \tan\phi'_d = 1.5 \times 4 \times (24 - 9.81) \times \tan 30 = 49.2\text{kN/m} \quad (7)$$

Once partial factors are applied, the over-design factor is therefore the same as computed previously, ie  $(110.1/1.4 + 49.2/1.1)/(120.4 \times 1.35) = 0.76$ .

Note that in this case the calculated shear resistance at the base of the wall is based on the unfactored weight of the wall, not because the wall weight is classed as a *favourable action*, but because the calculation is carried out using characteristic values, with the factors subsequently being applied. This is an important distinction.

For similar reasons the shear resistance at the base of the wall and the active earth pressures are based on the unfactored water pressure, despite the fact that the effect of the water pressure in the equilibrium direction is *unfavourable*.

**Analysis Level 3 example: embedded cantilever design (DA1/1 inverse-factor approach)**

#### Problem statement

It is required that an Analysis Level 3, DA1/1 check on the bending

Table 2: Analysis Level 1 (strip footing) design example: Eurocode 7 parameters and outcome

Quantity	Type	Partial factor
Pre-analysis		
Applied load ( $V$ )	Permanent unfavourable	1.35
Footing weight	Permanent unfavourable	1.35
Surcharge ( $q$ )	Variable favourable	0.0
Soil unit weight ( $\gamma$ )	(Neutral)	1.0
Adequacy factor $\lambda_A$ on:	applied load	
Numerical analysis		
Adequacy factor $\lambda_A$ , $\lambda_A \geq 1.0 \rightarrow E_d \leq R_d$ ?	true $\rightarrow$ safe	1.05

Table 3: Analysis Level 1 (strip footing) design example: analytical determination of actions and resistances (all actions and resistances in kN/m)

Quantity	Characteristic value	Partial factor	Design value
Soil properties			
Undrained shear strength $c_u$ ( $\text{kN/m}^2$ )	60	1.0	60
Actions ( $F$ )			
Applied load	400	1.35	540
Footing weight	40	1.35	54
Sum	440		594
Resistances ( $R$ )			
Applied load	617	(1.0)	617
$F \leq R$ ?	true	true	safe

necessarily correct in all situations. In a DA2 check soil strengths are unfactored and on the active side the design angle of drained shearing resistance  $\Phi'_d = 30^\circ$ . Thus the active force  $A'$  due to the soil only (ie based on the effective stresses) is given by:

$$A' = \frac{1}{2}K_a(\gamma_{a,sat} - \gamma_w)A^2 + 1.11qK_a4 = 41.9\text{kN/m} \quad (4)$$

The resultant water force  $U_a$  on the retained side is given by:

Quantity	Type	Partial factor
<i>Pre-analysis</i>		
Retained soil	(Neutral)	1.0
Soil on excavated side	(Neutral)	1.0
Wall weight	(Neutral)	1.0
Water pressure resultants	(Neutral)	1.0
Soil surcharge	Variable unfavourable	1.11
Unit load H	(Neutral)	1.0
Adequacy Factor $\lambda_A$ on:	<i>load H</i>	
<i>Numerical analysis</i>		38.8
Adequacy factor $\lambda_A$		
<i>Post-analysis stability check (horizontal equilibrium)</i>		
Active force from soil	Unfavourable permanent	1.35
Active force from water	Unfavourable permanent	1.35
Passive force	Resistance (passive)	1.4
Base friction	Resistance (sliding)	1.1
Load H	ignore	0.0

Table 4: Analysis Level 2 (gravity wall) design example: Eurocode 7 parameters and outcome.

Quantity	Characteristic value	Partial factor	Design value
<i>Action effects (E)</i>			
Active force (soil)	42.0	1.35	56.7
Active force (water)	78.5	1.35	105.9
Total	120.4		162.6
<i>Resistances (R)</i>			
Passive force	110.1	1.4	78.6
Base Friction	49.2	1.1	44.7
Total	159.2		123.3
<i>Outcome</i>		$E_d \leq R_d$ <b>safe</b>	$E_d > R_d$ <b>unsafe</b>

Table 5: Analysis Level 2 (gravity wall) design example: post analysis stability check (horizontal equilibrium). Characteristic values taken from numerical analysis (all forces in kN/m).

» strength of the embedded wall problem shown in Figure 5 is undertaken. In this example the action effect ( $E$ ) is considered to be the bending moment in the wall, while the resistance ( $R$ ) is considered to be the plastic moment of resistance of the wall. This problem is most conveniently analysed using a pre-factoring ("inverse factoring") approach (Smith & Gilbert 2011). All parameters enter the calculation unfactored, apart from variable loads.

The resistance is factored down by the product of the action effect factor and the wall strength factor,  $\gamma_G \gamma_M$ . The adequacy factor is applied to any unfavourable action – in this case the surcharge applied to the surface of the retained soil is chosen. The numerical analysis procedure will automatically find the point in the wall at which bending failure occurs.

The method should provide equivalent results to using a direct action/resistance factor approach (as typically used at Analysis Level 2) for problems which involve formation of a single plastic hinge.

#### Numerical analysis

The values of the parameters used in the pre-factoring analysis are given in Table 6 (unfactored soil properties are shown in Figure 5), together with results from the LimitState:GEO analysis of the problem obtained using a "medium" numerical discretisation comprising approximately 500 nodes, with the nodes concentrated along the wall and the soil surface. A one-dimensional "Engineered Element" (LimitState 2009) was used to model the wall, with locations of potential plastic hinges spaced at 0.1m intervals along the embedded portion.

#### Analytical check

In a conventional factored load and resistance approach it is implicitly assumed that the soil is yielding either side of the wall. In this case active and passive Rankine pressure distributions are typically taken to act on each side of the wall. This is valid in this case but not necessarily correct in all situations. For a DA1/1 analysis soil strengths are unfactored and the design drained angle of shearing resistance  $\Phi'_d = 30^\circ$ . Let the depth to the hinge be  $d$ , and thus the resultant active force  $A$  is given by:

$$A = A_1 + A_2 = \frac{1}{2}K_a\gamma(4 + d)^2 + 1.11qK_a(4 + d) \quad (8)$$

and the resultant passive force  $P$  is given by:

$$P = \frac{1}{2}K_p\gamma d^2 \quad (9)$$

Taking moments about the hinge

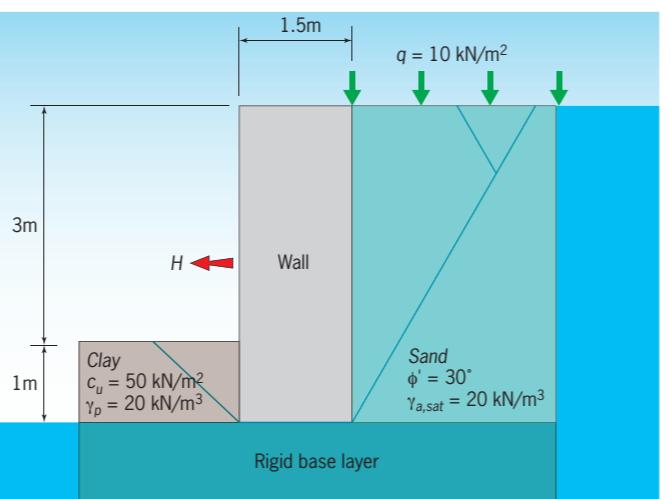
$$M = A_1 \frac{4 + d}{3} + A_2 \frac{4 + d - Pd}{2} \quad (10)$$


Figure 4: Analysis Level 2 (gravity wall) design example: typical pattern of slip-lines at collapse. (Surcharge  $q$  is taken as variable; wall vertical faces are taken as smooth; wall base angle of drained shearing resistance is taken as  $\phi' = 30^\circ$ ; retained fill is saturated with water table at soil surface; water pressures are assumed to act beneath the wall, but in the short term not in the clay.)

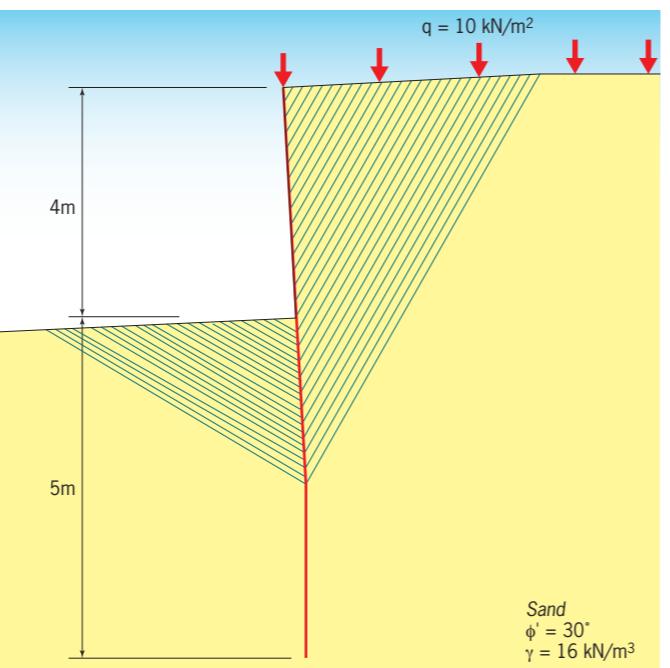


Figure 5: Analysis Level 3 (embedded wall) design example: typical pattern of slip-lines at collapse. (Surcharge  $q$  is taken as variable; wall bending strength  $M_p = 409 kNm/m$ ; wall is taken as smooth.)

This reaches a maximum value of  $M = 198 kNm/m$  when  $d = 6.3m$ . The value of the moment and its position is within ~ 1% of the value obtained using the numerical analysis ( $M = 195 kNm/m$ ;  $d = 6.3m$ ).

#### Discussion

In Part I of this paper (GE October), it was noted that Eurocode 7 provides the engineer with a significant degree of flexibility in how to apply each of the three design approaches (DA) described. This allows application of partial factors at the analysis level(s) selected by the engineer, to provide an appropriate safety margin. This also allows "non-linearities" of the sort referred to previously to be appropriately accounted for by experienced users the code. Unfortunately, this flexibility also means there is potential for the code to be applied inconsistently, in turn leading to confusion, especially amongst inexperienced users.

The goal here has been to marry a design code of general applicability (ie Eurocode 7) with analysis procedures of general applicability (for example based on numerical limit analysis), leading to a "one size fits all" methodology. By requiring all three of the analysis levels described to be addressed, it is anticipated that most non-linear effects will be handled appropriately. The advantage of general numerical analysis procedures is that otherwise repetitive checks can be automated.

If the proposed procedure is to be used with DA1, in principle, all three analysis levels should be checked using DA1/1 and DA1/2, implying six calculations in all. However, in practice, as listed in Table 1, three checks will typically suffice:

■ A DA1/2 calculation is typically viewed as a check that corresponds to Analysis Level 1. However, it implicitly covers all three analysis levels simultaneously.

■ In the case of DA1/1, only certain problems are amenable to Analysis Level 1 checks since it is often challenging to distinguish between favourable and unfavourable actions arising from soil self weight. Therefore these checks need only be carried out at Analysis Level 2 and Analysis Level 3 (for footings, for example, Analysis Level 1 and 2 checks are essentially equivalent).

Furthermore, since DA1/2 is normally found to govern over DA1/1 for checks at Analysis Level 1 and Analysis Level 2, it could be argued that in most cases DA1/1 checks at Analysis Level 2 could be omitted, requiring a DA1/1 check only at Analysis Level 3.

This situation is ideal as far as users of general purpose numerical

Quantity	Type	Partial factor
<i>Pre-analysis</i>		
Surcharge	Variable unfavourable (Neutral)	1.11
Soil unit weight	(Neutral)	1.0
Wall bending strength	Resistance	1.35
<i>Adequacy Factor <math>\lambda_A</math> on:</i>		
$\lambda_A \geq 1.0 \rightarrow E_d \leq R_d$	$\lambda_A \geq 1.0 \rightarrow E_d \leq R_d$	2.38
Hinge location (from wall top)	6.3m	true → safe

Table 6: Analysis Level 3 (embedded cantilever) design example: Eurocode 7 parameters and outcome. (NB the factor on wall bending strength is calculated as  $1.0 \times 1.35$ , where 1.0 is the material partial factor for steel from EN 1993-5.)

analysis procedures are concerned since both cases allow the use of the much more straightforward pre-factoring approaches discussed in Smith & Gilbert (2011), which can identify automatically the collapse mechanism without any further user input.

Alternatively, if the proposed procedure is to be used with DA2, only Analysis Level 2 and Analysis Level 3 can realistically be checked, since DA2 is not generally amenable to an Analysis Level 1 check. Thus it has been proposed that DA3 is used to perform the Analysis Level 1 check, if required. Implicit in the use of DA2 with general purpose numerical analysis procedures has been proposed. Three worked examples have been provided to illustrate its application.

5. It is argued that use of Design Approach 1 will pragmatically often only require checks at Analysis Level 1 using DA1/2 and Analysis Level 3 using DA1/1. Both are amenable to pre-factoring approaches and thus benefit from automatic identification of collapse mechanisms.

#### Acknowledgements

All numerical analyses described herein were undertaken using LimitState:GEO version 2.0; see: www.limitstate.com/geo

#### Appendix I: Variable and permanent actions

Different values of partial factors are stipulated for variable and permanent actions in Eurocode 7. This presents a challenge for Analysis Level 2 and Analysis Level 3 calculations, which generally apply factors to action effects and resistances at locations internal within a problem.

These quantities are usually considered permanent, and corresponding partial factors are used.

A variable action will almost always be an externally applied action, and in general it is not straightforward to quantify directly its individual contribution to

an action effect or resistance at Analysis Level 2 and Analysis Level 3. The pragmatic solution to this, as suggested by, for example, Frank et al. (2004) and Bond & Harris (2008), is to use a modified partial factor of  $\gamma_0/\gamma_c$ , which is applied to the variable action at its source. The contribution of the effect of this factored action to an action effect (for example earth pressure on a structure) is then factored by  $\gamma_c$ . If the influence of the variable action on the action effect is linear then this process is equivalent to using a factor of  $\gamma_0$  on the source action.

Since variable actions are small relative to permanent actions in most design problems, it would seem pragmatic also to adopt generally this simple rule for cases where the relationship is non-linear. (However, if a variable action dominates then it would be necessary to adopt a more refined approach in cases where strong non-linearity exists.)

When performing a DA1/1 check the modified partial factor becomes  $1.5/1.35 = 1.11$  for unfavourable variable actions, and  $0.0/1.0 = 0.0$  for favourable variable actions. In a rare case where an unfavourable variable action contributes to the reduction of a resistance, the variable action would remain effectively factored by  $\gamma_0/\gamma_c$ . However, at Analysis Level 3 this issue does not present itself since the resistance is typically a structural material resistance which is known in advance.

Finally, whether a variable action should be classified as "favourable" or "unfavourable" will generally be obvious to the engineer, but an analysis should be carried out to confirm this.

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