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On Rician statistical assumptions for geographic routing in WSNs

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Abstract

Geographic routing algorithms for wireless sensor networks (WSNs) need to be resilient to the inherent location errors of positioning systems. Proposed forwarding algorithms in the literature make use of the statistical assumption of Gaussian distributed location error and Ricianly distributed distance estimates between sensor nodes. This letter analyses the validity of the Rician hypothesis when realistic localisation is employed and simulated. We consider received signal strength based (RSS) localisation through the linear least square method (LLS). Anchor nodes estimate the position of the target sensor nodes as well as their error characteristics, which are no longer assumed Gaussian. Both theoretical and simulation results confirm that the Rician assumption is not true when realistic localisation scenarios are used, affecting the performance of geographic routing algorithms based on this statistical supposition.

Index Terms

geographic routing algorithm, Rician assumptions, wireless sensor networks, LLS-RSS localisation

I. INTRODUCTION

A very high number of position based routing algorithms are not designed to cope with location error and consider the location of nodes to be exactly known. In realistic wireless sensor networks (WSNs) with hundreds of nodes, this assumption is not valid, as these nodes are localised using local positioning systems [1], [2], [3]. Localised nodes have a specific error associated with them, whose magnitude differs depending on the employed ranging technique. Location error values are random in reality and their variance for the x and y coordinates may or may not be the same. No physical environmental factors are considered here that may affect their values (i.e. wind currents), but differences may still exist, because of the network geometry (number and position of the anchor nodes). Position errors influence the decisions

of the routing procedure [4]. Thus, efficient geographic routing algorithms have to be able to cope with location errors [5], [6], [7], [8].

The widely used mathematical error model considers location errors normally distributed [5], [6], [7], [8], which facilitates the assumption that the distances between nodes have a Rice distribution. The Rice statistical statement is possible only if a simplifying assumption is made i.e. the variance of the estimated x and y coordinates is equal. However, this is not necessarily always true and can affect the forwarding algorithms if based on Rician assumptions. This work presumes the Rician hypothesis true and verifies it via MATLAB simulations, when employing a realistic localisation simulation (here based on received signal strength (RSS) ranging).

The paper is organized as follows: Section 2 briefly lists some of the most significant sensor network localisation methods used in WSNs and presents error-coping geographic routing algorithms which suffer from the Rician distribution assumption. Section 3 discusses the problem of the Rician assumption in the design of geographic routing algorithms. Section 4 describes three simulation tests performed as part of a preliminary analysis to confirm the existence of the theoretical issue. Non-Rician geographic routing solutions are presented in Section 5 and comparatively analysed via simulations in Section 6. Conclusions are drawn in Section 7.

II. RELATED WORK

Geographic routing is an attractive option for large scale WSNs, because of its low overhead and energy expenditure, but is inefficient in realistic localisation conditions which are inevitably imprecise. Inexact range measurements and location errors can lead to low throughput and to node power being wasted. Accurate localisation is therefore essential in position-based routing for WSNs. According to the position knowledge, sensor nodes can be divided into two categories: anchors (or beacon nodes) and targets (or blindfolded nodes). Anchor positions are usually known with accuracy (either through the global positioning system (GPS) or installation measurements). GPS use is however economically unjustifiable for each node in a large or inaccessible network. The coordinates of target nodes can alternatively be estimated using localisation algorithms, the absolute positions of the anchors and inter-sensor node range measurements. Measurement techniques can be classified as based on angle (angle of arrival (AoA)), distance (one way or two way time of arrival (OW-ToA or TW-ToA), time difference of arrival (TDoA), received signal strength (RSS)) or RSS profiling techniques [9]. In this paper the RSS method of ranging

is chosen over others because it is suitable for smaller networks and allows fast simulation processing. However, possible future work can make use of other methods of localisation.

While, the most forward within range (MFR) algorithm [10] is an example of a geographic routing algorithm which does not cope with location errors, recent literature does provide a few solutions to cope with inaccurate location errors in an energy efficient way. MFR is also considered an energy efficient forwarding strategy when using a fixed transmission power, because it minimizes the hop count [5], but it has been proven less efficient than other routing algorithms in realistic WSN conditions.

The least expected distance algorithm (LED) is proposed in [6]. It is presented as an error-robust routing scheme, whose main aim is to preserve the power saving features of basic geographic forwarding. It is proven in [6] that whichever approach the position-based routing may have, either to optimize the energy spent per hop or for the overall chosen path, there is a unique energy-optimal forwarding position which can be used in the forwarding process. LED determines this theoretical optimum and chooses as a next hop the neighbour whose real position is closest to it. The algorithm strategically incorporates location error into the forwarding objective function. It is assumed that the estimated coordinates of each node are affected by a Gaussian error of a given variance. As a consequence, the erroneous distances between nodes are random variables characterized by the Rice distribution. LED calculates the expectation of the considered distances and chooses the node with the minimum expectation.

The mean square error ratio (MSER) and the conditioned mean square error ratio (CMSER) algorithms are proposed in [8] as alternative methods for geographic routing resilient to location errors. MSER represents the basic forwarding strategy, while CMSER represents a better developed version of the algorithm. Both MSER and CMSER make use of the notion of maximum advance to destination (just as MFR), but consider the probability of a successful packet transmission when coordinates are affected by location error. The algorithms are analysed via simulation using different network sizes, error characteristics and communication ranges and all results confirm the expected performance of CMSER in terms of both throughput and energy savings. As does LED, MSER and CMSER make use of initial Gaussian error assumptions and of Ricianly distributed distances.

Geographic routing algorithms which cope with inaccurate position knowledge of the nodes are based on a chain of statistical assumptions, amongst which, one is that of equality of the location error variance for the x and y coordinates of the same node. When RSS ranging is used for the localisation, the error statistics for the x and y coordinates of each node are assumed to be the same in order for the Rician

assumption of distance estimates to be valid. However, if the theoretical calculations or the simulation results of these statistics are not the same (or not assumed to be the same) then node distances may also not be Ricianly distributed. Consequently, the routing algorithms using this assumption may not perform optimally either. This is studied further in the following sections.

III. PROBLEM STATEMENT

Efficient geographic routing algorithms for WSNs are designed to cope with inaccurate localisation [5], [6], [7], [8]. The widely used mathematical error model considers the distance estimate between two localised nodes as a random variable (RV) with a Rician distribution. The estimated distance between node i and j is given by

$$\hat{d}_{ij} = \sqrt{X^2 + Y^2},$$

where X and Y are represented by $(\hat{x}_i - \hat{x}_j)$ and $(\hat{y}_i - \hat{y}_j)$ and where (\hat{x}_i, \hat{y}_i) and (\hat{x}_j, \hat{y}_j) are the estimated location coordinates of nodes i and j respectively. The distance \hat{d}_{ij} Ricianly distributed i.e.

$$f(\hat{d}_{ij}) = \left(\frac{\hat{d}_{ij}}{\sigma_{ij}^2}\right) \exp\left(-\frac{\hat{d}_{ij}^2 + d_{ij}^2}{2\sigma_{ij}^2}\right) I_0\left(\frac{\hat{d}_{ij}d_{ij}}{\sigma_{ij}^2}\right). \quad (1)$$

The distribution of the difference of two normally distributed variates, $(\hat{x}_i - \hat{x}_j)$ or $(\hat{y}_i - \hat{y}_j)$, is also Gaussian with mean $\mu_{ij} = \mu_i - \mu_j$ and variance $\sigma_{ij}^2 = \sigma_i^2 + \sigma_j^2$. \hat{d}_{ij} can be a Rician RV only if X and Y have the same variance σ_{ij}^2 . This is the equivalent of the variance in any node i or j being the same on the x and y axes: $\sigma_{ix}^2 = \sigma_{iy}^2$ (referred to as σ_i^2) and $\sigma_{jx}^2 = \sigma_{jy}^2$ (referred to as σ_j^2). Such a statistical presumption is a simplification, which in reality is not always true and can affect the forwarding algorithms based on Rician assumptions.

It is considered that the target sensor nodes (TN) are randomly distributed in the network having a location error model as described above. Anchor nodes (AN) estimate the position of the target sensor nodes and their error characteristics. Received signal strength (RSS) ranging is used for localisation and simulated using the linear least square method (LLS) as in [1]. Following [1], the error variance associated with each node i is theoretically estimated using the trace of the covariance matrix:

$$MSE(\hat{\theta}_i) = Tr\{\text{Cov}(\hat{\theta}_i)\}, \quad (2)$$

where $\hat{\boldsymbol{\theta}}_i = \begin{bmatrix} \hat{x}_i \\ \hat{y}_i \end{bmatrix}$ represents the estimated location via LLS, $\boldsymbol{\theta}_i = \begin{bmatrix} x_i \\ y_i \end{bmatrix}$ represents the true location coordinates and $\text{Cov}(\hat{\boldsymbol{\theta}}_i)$ is the covariance matrix:

$$\begin{aligned} \text{Cov}(\hat{\boldsymbol{\theta}}_i) &= E \left[(\hat{\boldsymbol{\theta}}_i - \boldsymbol{\theta}_i) (\hat{\boldsymbol{\theta}}_i - \boldsymbol{\theta}_i)^T \right] \quad (3) \\ &= E \left[\begin{bmatrix} \hat{x}_i - x_i \\ \hat{y}_i - y_i \end{bmatrix} \begin{bmatrix} \hat{x}_i - x_i & \hat{y}_i - y_i \end{bmatrix} \right] \\ &= E \begin{bmatrix} (\hat{x}_i - x_i)^2 & (\hat{x}_i - x_i)(\hat{y}_i - y_i) \\ (\hat{y}_i - y_i)(\hat{x}_i - x_i) & (\hat{y}_i - y_i)^2 \end{bmatrix} \\ &= \begin{bmatrix} E [(\hat{x}_i - x_i)^2] & E [(\hat{x}_i - x_i)(\hat{y}_i - y_i)] \\ E [(\hat{y}_i - y_i)(\hat{x}_i - x_i)] & E [(\hat{y}_i - y_i)^2] \end{bmatrix}. \end{aligned}$$

The main diagonal terms of of the matrix in Eq. 3 represent the theoretical variance of the location error on the x and y axes:

$$\sigma_{ix}^2 = E [(\hat{x}_i - x_i)^2], \quad (4)$$

$$\sigma_{iy}^2 = E [(\hat{y}_i - y_i)^2], \quad (5)$$

and the terms in the off-diagonal represent the covariance between the x and y location error (for reference σ_{xy}) and, if the RVs are independent,

$$E [(\hat{x}_i - x_i)(\hat{y}_i - y_i)] = E [(\hat{y}_i - y_i)(\hat{x}_i - x_i)] = 0.$$

The calculation of the theoretical MSE from Eq. 2 is presented in detail in [1]. The simulation of the LLS-RSS localisation takes place with a prescribed noise value s_{rss} [dB] reflected in the distance variance [m^2]. Environmental interference is taken into account only by the localisation simulations (whose ranging is affected by assumed noise values). It is not considered in the simulation of the routing process. Although this is a simplification, it does not affect the scope of the work. The localisation simulation results in the erroneous coordinates \hat{x}_i and \hat{y}_i and the value of the variance for each TN i based on Eq.

2, $\sigma_{RSS}^2 = \sigma_{ix}^2 + \sigma_{iy}^2$. If the distances are assumed Ricianly distributed, then $\sigma_{ix}^2 = \sigma_{iy}^2 = \frac{\sigma_{RSS}^2}{2}$. The aim is to show that $\sigma_{ix}^2 \neq \sigma_{iy}^2$, to analyse the impact of this inaccurate assumption on geographic routing performance and to propose more accurate forwarding alternatives.

IV. PRELIMINARY ANALYSIS

To verify whether the location variance of random nodes i is actually the same for the x and y coordinates, for the RSS technique, three tests are used. The randomly deployed TNs are localised through LLS-RSS by the ANs, placed at the edge of the square network (with side $l = 50$ [m]). The communication range $R = 10$ [m] is the same for all the TNs, while the transmission range of the ANs covers the entire network surface. The localisation process is simulated assuming different values for the path loss exponent (PLE) α [dB] and η iterations are considered for averaging.

The variance of the location error of the nodes is influenced only by the number and position of the anchors nodes. The number of target nodes is not relevant, but their position in regards to the anchor nodes is. For example, the coordinates of a centrally placed target node, which is equally far from all anchor nodes, will be estimated with more precision than a target node which is closer only to few anchor nodes.

- **Test 1: Comparison via simulation samples**

A network of $TN = 30$ and $AN = 9$ is used for the simulation (ANs are placed on the edges, in the corners and in the center of the network) ($\alpha = 2.5$). Random nodes i are selected and a comparison is made between the average variance value resulting from the RSS localisation process, using $\frac{\sigma_{RSS}^2}{2}$ (averaged over $\eta = 100$), and the estimated values of σ_{ix}^2 and σ_{iy}^2 , calculated using $Var(X) = E[(X - E(X))^2]$, where X is represented by the erroneous coordinates of node i whose values are different for each iteration and $E(X)$ is the mean of X , which should be the actual coordinates of node i .

Table I: Results for Test 1

TN	l	s_{rss}	$\frac{\sigma_{RSS}^2}{2}$	σ_{ix}^2	σ_{iy}^2
4	100	0.6	37.69	27.82	23.93
10	100	0.6	44.00	32.71	33.89
4	100	1	63.25	46.10	40.02
25	50	1	7.84	5.01	6.71
2	50	0.6	9.61	8.87	9.62
27	50	0.6	6.72	5.25	6.43
3	50	1.5	12.48	15.89	8.88
10	50	1.5	27.92	20.71	22.02

The results of this initial test show $\sigma_{ix}^2 \neq \sigma_{iy}^2$ and that the RSS calculated variance is an approximation (an example is presented in Table I). As the $\frac{\sigma_{RSS}^2}{2}$ value is an estimation, its accuracy depends on the TN position referenced to the ANs and on the network size (RSS ranging is not suitable for large networks [1], [11]). The values for σ_{ix}^2 and σ_{iy}^2 which are obtained through theoretical calculations are similar, but not equal.

- **Test 2: Network visualization comparison**

This test aims to illustrate the location error of the nodes, when estimated with equal or different variance for the x and y coordinates. The simulations consider $TN = 10$ and $\eta = 100$ and the employed scenarios are listed in Table II. Using the LLS-RSS localisation, the variations are made for α , the number and positions of the ANs and for the noise value s_{RSS} .

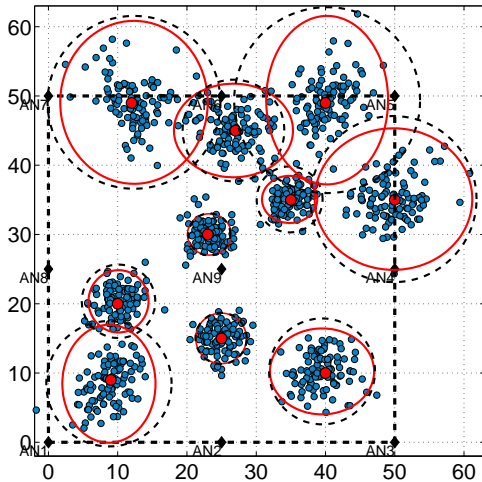
Table II: Scenarios for test 2

Scenario	α	AN	s_{RSS}
1	2	9	0.6
2	2.5	9	0.6
3	3	9	0.6
4	3.5	9	0.6
5	2.5	5	0.6
6	2.5	6	0.6
7	3	6	0.6
8	3	5	1

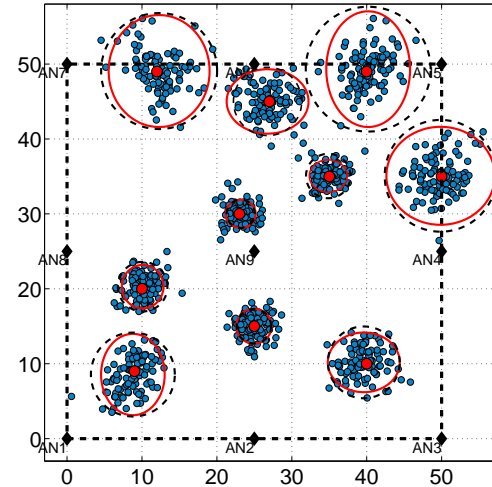
The values of σ_{ix}^2 and σ_{iy}^2 are calculated for each node i and compared with the values $\frac{\sigma_{RSS}^2}{2}$ obtained from the simulation. The black dashed circles in the figures represent the area where the estimated positions of the nodes are considered to be when $\sigma_{ix}^2 = \sigma_{iy}^2$ (centered on the accurate location of node i , of radius $\frac{\sigma_{RSS}^2}{2}$). The red ellipses in the figures represent the areas of the estimated positions when $\sigma_{ix}^2 \neq \sigma_{iy}^2$ (centred in the accurate location of node i , with ellipse axes $\sigma_{ix}^2, \sigma_{iy}^2$).

Aside from facilitating the possibility to observe the difference in the area covered by the circles and that of the ellipses, the scenarios 1, 2, 3 and 4 in Fig. 1 illustrate how a larger α makes the location estimation more accurate (notice the decreased area of both the ellipses and the circles). Fig. 2 shows the influence of the anchor positions on the localisation process. By reducing the number of the anchors and removing them from the middle of the edge (scenarios 5 and 8), the localisation loses from its overall accuracy, but not as much as when eliminating anchors from key positions, such as all the ones on the north side of the network, affecting the localisation especially in this region (scenarios 6 and 7). While all the error ellipses of the nodes in scenarios 5 and 8 become larger and flatter, in scenarios 6 and 7 it is

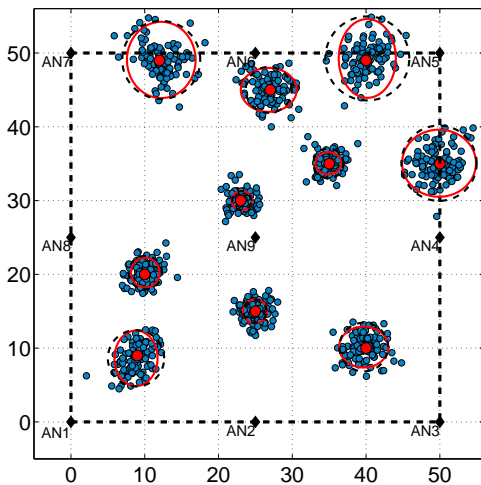
mostly the nodes on the north side that are affected by the change and their error ellipses are particularly larger and more elongated.



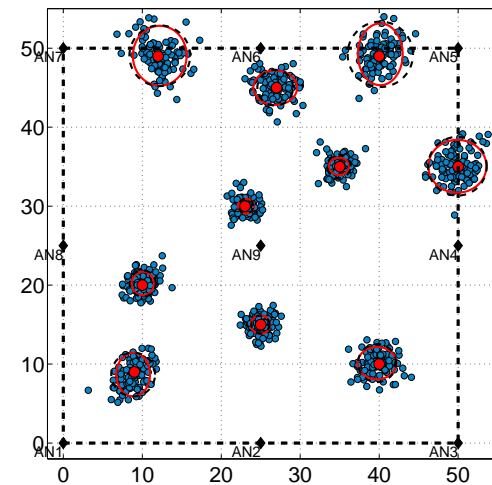
(a) Scenario 1, $\alpha = 2$



(b) Scenario 2, $\alpha = 2.5$



(c) Scenario 3, $\alpha = 3$



(d) Scenario 4, $\alpha = 3.5$

Figure 1: Varying α

In this study it has been assumed that the location errors on the x and y axes are independent and therefore uncorrelated. Consequently their covariance is zero and the minor and major axes of the error-ellipses are parallel to the x and y axes. However, in reality, the error on the two axes may be correlated to a degree (as the LLS-RSS localisation process shows) and this would imply a rotation of the error ellipses with an angle whose calculation is based on the correlation matrix [11].

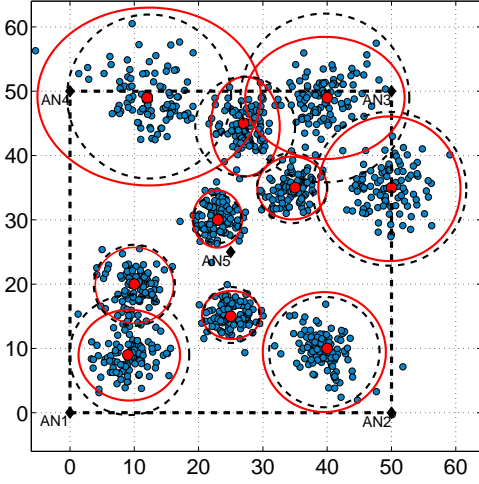
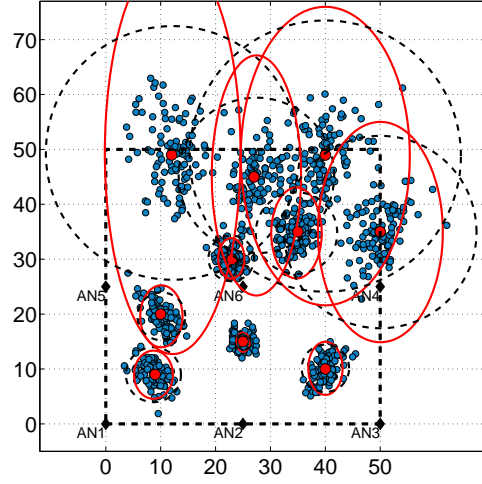
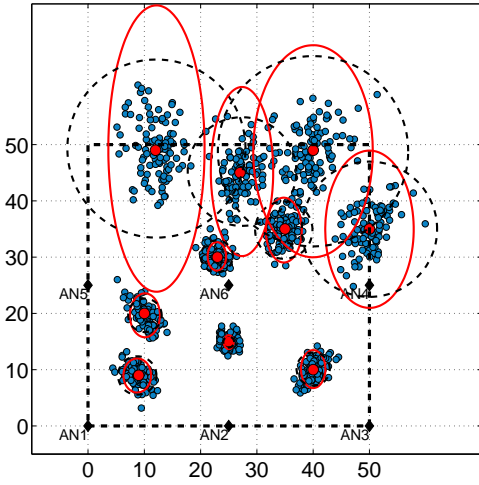
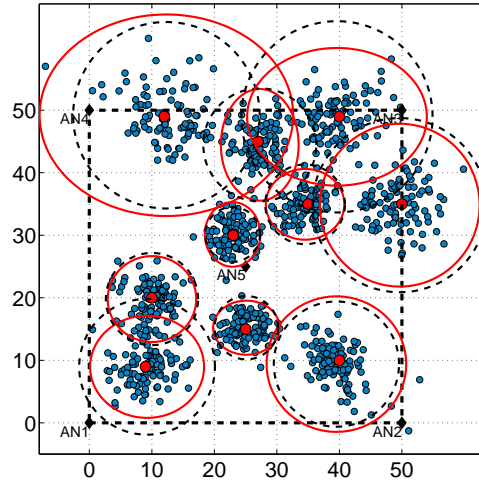
(a) Scenario 5, $\alpha = 2.5$ (b) Scenario 6, $\alpha = 2.5$ (c) Scenario 7, $\alpha = 3$ (d) Scenario 8, $\alpha = 3$

Figure 2: Varying the anchor node placement and the noise values

In Fig. 3, where $\alpha = 3$ and $s_{rss} = 0.6$, the lengths of the ellipse axes are equal to the square roots of the eigenvalues λ of the covariance matrix $\text{Cov}(\hat{\theta}_i)$ in Eq. 3. The eigenvalues are calculated using $\text{Cov}(\hat{\theta}_i) * v = \lambda * v$, where v is the eigenvector of $\text{Cov}(\hat{\theta}_i)$. The counter-clock angle of rotation of the ellipses is determined with

$$\phi = \frac{1}{2} \arctan \left(\frac{2\sigma_{xy}}{(\hat{x}_i - x_i)^2 - (\hat{y}_i - y_i)^2} \right),$$

where $(\hat{x}_i - x_i)^2 - (\hat{y}_i - y_i)^2 \neq 0$. Fig. 3 illustrates another example of an unrealistic assumption in algorithmic design.

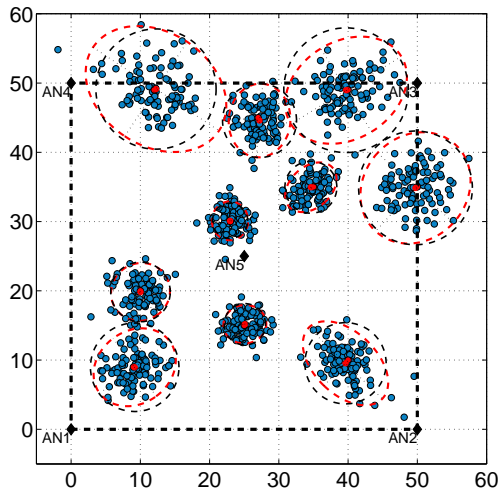


Figure 3: Network example with correlated errors for the x and y coordinates

• Test 3: Cumulative distribution function comparison

It is considered that $TN = 2$, $AN = 9$ and $\alpha = 2.5$. The distance between TNs is a multivariate RV (depending on the σ^2 of both coordinates of two TNs). So, the cumulative distribution function (CDF), $F_X(x) = 1 - Q_1\left(\frac{d_{ij}}{\sigma_{ij}}, \frac{\hat{d}_{ij}}{\sigma_{ij}}\right)$, where Q_1 is the Marcum Q-function, can be used to verify the non-Rician hypothesis for the LSS-RSS localisation resulted errors.

Firstly, the location error of two TN is considered Gaussianly distributed, so $\sigma_i^2 \neq \sigma_j^2$ and $\sigma_{ix}^2 = \sigma_{iy}^2$ and $\sigma_{jx}^2 = \sigma_{jy}^2$. Both the theoretical as well as the empirical Rician CDFs are computed, the empirical CDF using simulation-obtained estimated distances \hat{d}_{ij} , for $\eta = 1000$.

Secondly, the theoretical and empirical Rician CDFs are calculated for the same nodes, when these are located through LLS-RSS. The estimated \hat{d}_{ij} are taken from the ANs, which perform the ranging over the same η as before. To calculate the scale parameter σ_{ij} , the theoretical Rician CDF (for the RSS case) assumes $\sigma_{ix}^2 = \sigma_{iy}^2$ and $\sigma_{jx}^2 = \sigma_{jy}^2$. The CDFs can be seen in Fig. 4.

For the TNs with Gaussian errors and an equal variance on both x and y axes, the theoretical and empirical CDFs overlap as a confirmation that the distances are Ricianly distributed. For RSS estimated node coordinates and variances, it is not accurate to assume the error variance is equal on the x and y axes. The difference in the CDF curves shows that such an assumption would implicitly lead to a suboptimal routing performance when the forwarding decisions are based on Rician statistics.

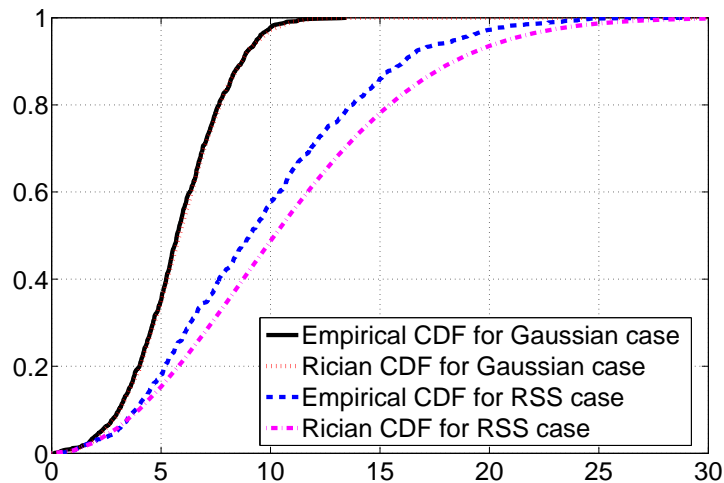


Figure 4: CDF Analysis

V. NON-RICIAN GEOGRAPHIC ROUTING SOLUTION

Two new algorithms are proposed, non-Rician mean square error ratio (NR-MSER) and non-Rician conditioned mean square error ratio (NR-CMSER), adaptations of the MSER and CMSER algorithms proposed in [8]. The forwarding is made on the same principles as before, but the algorithms are adapted to cope with the difference in the x and y location error; they no longer use the Rician expectation and variance in their calculations.

In [8], the MSER and CMSER algorithms use the mean square error (MSE):

$$MSE_{ij} = E(\hat{d}_{ij} - d_{ij})^2 = E(\hat{d}_{ij}^2) - 2d_{ij}E(\hat{d}_{ij}) + d_{ij}^2, \quad (6)$$

where d_{ij} and \hat{d}_{ij} are the real and the estimated distance between two nodes i and j . The calculation of the MSE_{ij} is made with the help of the Rician mean $E(\hat{d}_{ij})$. Here, the mean is assumed equal to the actual distance, $E(\hat{d}_{ij}) = d_{ij}$, and the mathematical expression of the MSE in Eq. 6 changes into:

$$NRMSE_{ij} = E(\hat{d}_{ij}^2) - d_{ij}^2, \quad (7)$$

$E(\hat{d}_{ij}^2)$ is calculated using the Euclidean distance equation, $\hat{d}_{ij}^2 = (\hat{x}_i - \hat{x}_j)^2 + (\hat{y}_i - \hat{y}_j)^2$, and the second moments which are: $E(\hat{x}_i^2) = x_i^2 + \sigma_{ix}^2$, $E(\hat{y}_i^2) = y_i^2 + \sigma_{iy}^2$, $E(\hat{x}_j^2) = x_j^2 + \sigma_{jx}^2$ and $E(\hat{y}_j^2) = y_j^2 + \sigma_{jy}^2$.

Therefore,

$$E(\hat{d}_{ij}^2) = E(\hat{x}_i^2 - 2\hat{x}_i\hat{x}_j + \hat{x}_j^2 + \hat{y}_i^2 - 2\hat{y}_i\hat{y}_j + \hat{y}_j^2)$$

$$\begin{aligned}
&= E(\hat{x}_i^2) + E(\hat{x}_j^2) + E(\hat{y}_i^2) + E(\hat{y}_j^2) - 2E(\hat{x}_i\hat{x}_j) - 2E(\hat{y}_i\hat{y}_j) \\
&= \sigma_{ix}^2 + \sigma_{iy}^2 + \sigma_{jx}^2 + \sigma_{jy}^2 + x_i^2 + x_j^2 + y_i^2 + y_j^2 - 2x_ix_j - 2y_iy_j.
\end{aligned} \tag{8}$$

In Eq. 7, because d_{ij} is not known, \hat{d}_{ij} is used in simulations instead. Similarly to [8], the mean square error ratio (MSER) is calculated using Eq. 9. For a balanced selection of the forwarding node, NR-MSER makes its decision by minimizing the MSE and maximizing the distance between i and j (the maximization of \hat{d}_{ij} is also used by the most forward within range algorithm (MFR) [10], which is used in Section V for comparison):

$$NRMSER_{ij} = NRMSE_{ij}/\hat{d}_{ij}. \tag{9}$$

$$F_j = \arg \min (NRMSER_{ij}). \tag{10}$$

The NR-CMSER also makes use of an additional, modified condition. With CMSER the routing selection was refined through: $(R - \hat{d}_{ij})^2 > Var(\hat{d}_{ij})$, where $Var(\hat{d}_{ij})$ is the Rician variance of the estimated distance. However, because the Rician assumption is considered incorrect, $Var(\hat{d}_{ij})$ is replaced for the NR-CMSER with the sum of the average variance in the x and y coordinates of the two nodes i and j ,

$$(R - \hat{d}_{ij})^2 > \frac{\sigma_{ix}^2 + \sigma_{iy}^2}{2} + \frac{\sigma_{jx}^2 + \sigma_{jy}^2}{2}. \tag{11}$$

VI. SIMULATIONS AND RESULTS

The PDR of NR-MSER and NR-CMSER is compared with their Rician-based algorithmic counterparts. The behavior of the MFR algorithm is included in the comparison because its distance metric is used by the other algorithms. The MFR throughput, with and without an assumed location error, is shown as a lower and upper bound on the routing performance. All simulations are based on LLS-RSS localisation. MSER and CMSER make use of inaccurate Rician assumptions and, unaware of a difference between the x and y error of node i , use $\sigma_{ix}^2 = \sigma_{iy}^2 = \frac{\sigma_{RSS}^2}{2}$ in their decisions. The provided solutions, NR-MSER and NR-CMSER, are designed to cope with $\sigma_{ix}^2 \neq \sigma_{iy}^2$ and are expected to perform the same or better than MSER and CMSER. The packet delivery ratio (PDR) is analysed for $\eta = 1000$, $l = 50$, $TN = 200$, $AN = 5$ (placed in the corners and center of the network), $\alpha = 3$, $R = 10$, $SE = 10$, $pkts = 1$ and

increasing values of the s_{RSS} .

For the smallest value of $s_{RSS} = 0.1$, the highest PDR is reached, 54% for MSER and NR-MSER and 85% for CMSE and NR-CMSE. As s_{RSS} is increased, reaching the value of 1, the routing performance for all algorithms degrades to such a level that the PDR becomes 15% for MSER and NR-MSER or 30% for CMSE and NR-CMSE. In Fig. 5 the PDR of the non-Rician algorithms remains approximately the same as that of the Rician ones. Because one would expect a bigger difference in routing performance, it is considered that the routing performance may be affected only by a large difference in σ_{ix}^2 and σ_{iy}^2 . Such a scenario may exist and the large difference in the x and y variance may be undetected by the localisation system, depending on its method of estimation, accuracy, number of anchors and network area. This possibility is tested in Fig. 6.

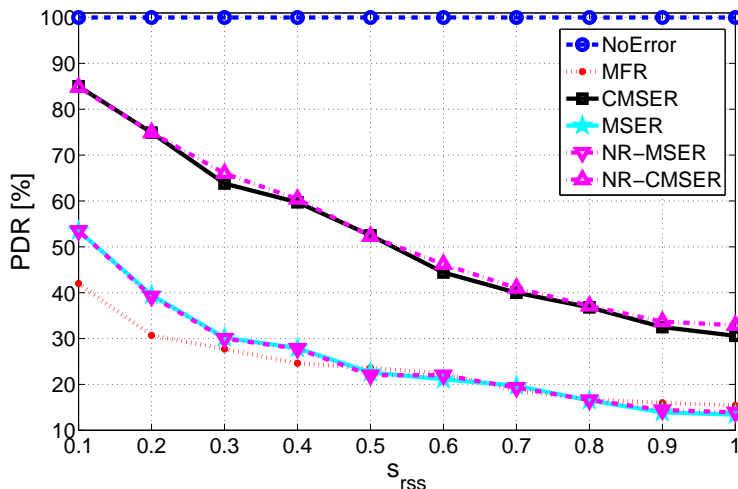


Figure 5: Performance based on RSS localisation

The difference in the x and y error is increased “artificially” by considering $\sigma_{ix}^2 = \frac{\sigma_{RSS}^2}{2}$ as obtained from the LLS-RSS localisation process and $\sigma_{iy}^2 = 3\sigma_{ix}^2$. MSER and CMSE are unaware of this difference (all according to the hypothesis that the localisation method does not reflect accurately the difference in the actual error on the x and y axes) and still use $\sigma_{ix}^2 = \sigma_{iy}^2 = \frac{\sigma_{RSS}^2}{2}$. In this new scenario (from Fig. 6), although the PDR for all algorithms is smaller, the NR-CMSE algorithm performs better. The test illustrates that only large differences in σ_{ix}^2 and σ_{iy}^2 reveal the improvement of the new algorithms.

Overall, the performance of the new algorithms is either the same or better than the unadjusted counterparts and most importantly, NR-MSER and NR-CMSE are formulated correctly and cope with realistic location error differences in x and y location.

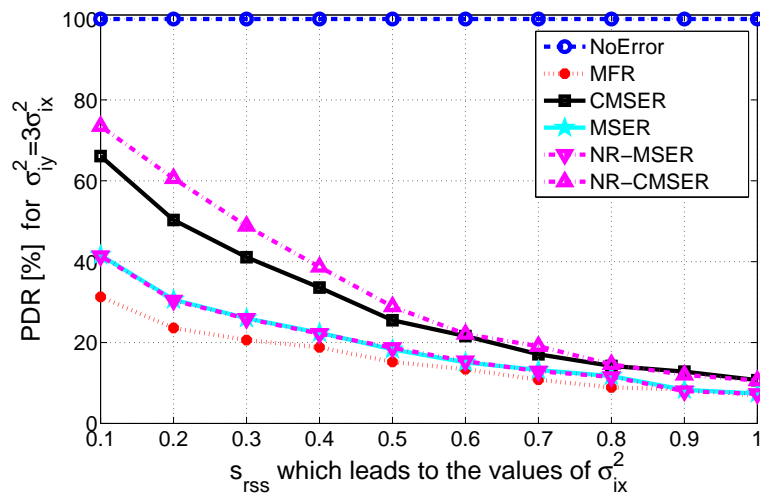


Figure 6: Performance based on RSS resulted σ_{ix}^2 , but with large σ_{iy}^2

VII. CONCLUSIONS

Analysis through the above described methods proves the Rician assumption is not valid when sensor nodes are located through realistic localisation methods (RSS or otherwise), because the error variance in the x and y coordinates of a node may not be the same (as the Rician assumption implies). While algorithms based on the Rician hypothesis can perform well in simulations, their results are not realistic and their performance can be affected by a large difference in the x and y error. The proposed geographic routing solutions NR-MSER and NR-CMSER realistically forward data, while coping with location error, without making use of the Rician assumption.

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