Proceedings Paper:

Schiffer, JF orcid.org/0000-0001-5639-4326, Anta, A, Trung, TD et al. (2 more authors) (2012) On Power Sharing and Stability in Autonomous Inverter-Based Microgrids. In: 2012 IEEE 51st IEEE Conference on Decision and Control (CDC). 2012 IEEE 51st IEEE Conference on Decision and Control (CDC), 10-13 Dec 2012 IEEE , pp. 1105-1110. ISBN 9781467320658

https://doi.org/10.1109/CDC.2012.6426704

51st IEEE Conference on Decision and Control
December 10-13, 2012. Maui, Hawaii, USA

On power sharing and stability in autonomous inverter-based microgrids

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Abstract— **We consider the problem of voltage and frequency stability for an autonomous inverter-based microgrid. An LMI-based decentralized feedback control design is derived that stabilizes the system under the consideration of droop-like controllers aiming to achieve power sharing among the different generation units. We provide a design procedure that accounts for uncertainties in line impedances and loads while guaranteeing zero steady-state frequency deviation.**

I. INTRODUCTION

An increasing amount of renewable energy sources is present in the electrical grid, of which a large share are small-scale distributed generation units connected at the low (LV) and medium voltage (MV) levels via inverters. The physical characteristics of such power electronic devices largely differ from the characteristics of conventional electrical generators (e.g. synchronous generators (SGs)), and therefore different control strategies are needed [1]. Moreover, since these generation units are intermittent by nature, more flexible operation and control strategies are needed to balance consumption and generation. Microgrids represent one promising solution that has received increasing attention in recent years [2]. It addresses these issues by gathering a combination of generation units, loads and energy storage elements at distribution level into a locally controllable system, which can be operated in a decentralized and even completely isolated manner from the main transmission system. Microgrids have been identified as a key component in future electrical networks [3]. Many new problems arise for this type of networks. In this paper we focus on the problem of guaranteeing voltage and frequency stability for a microgrid under droop-like control (see below) by providing additional decentralized feedback. The problem of power sharing mainly addresses the following question: upon load changes in the system, how should the different generation units in the network adjust their output power in order to fulfill the demand while satisfying a desired power distribution. It is a requirement to achieve these objectives in a decentralized way without communication among units and allowing for a plug-and-play-like operation [2].

A control solution widely used to tackle this problem in large power systems is droop control [4]. Under this approach, the current value of the frequency in the network is monitored to derive how much active power each generating unit needs to provide. In this way, the frequency (present everywhere in the network) serves as an implicit communication signal. From a control perspective, droop control can be regarded as a proportional controller where the control gain (known as droop gain) specifies the steady-state power distribution in the network. Since performance under droop control is satisfactory for transmission systems, researchers have first tried to apply this technique to microgrids [5], [6], [7], [8]. Stability analysis is usually carried out by means of detailed small-signal analysis as well as extensive simulations and experimental studies aiming to characterize a range for the droop gains guaranteeing system stability. In this regard, several articles [9], [10] propose to make inverters resemble the input/output behavior of SGs, so that the widely existing knowledge and expertise on SGs can be directly applied to inverter-based networks.

However, microgrids exhibit some characteristics that considerably differ from large power systems and therefore complicate a direct implementation of droop-like control methods. Examples of such characteristics are low inertia, no inherent physical relation between network frequency and power balance, possibly large R/X ratio, etc. [11], [12]. Several modifications to droop control have been proposed [13], [14], where the stability problem is either ignored or simplified by dealing with a linear model. While droop control provides a satisfactory performance in terms of power sharing, it has been observed that droop control can result in poorly damped or even unstable systems [6], [8], [15], [16]. Recently, conditions for proportional power sharing and synchronization of a microgrid under frequency droop control have been derived in [17] by applying results of the theory of coupled oscillators. The considered model represents a lossless strongly-connected autonomous inverter-based network with constant bus voltages.

To the best of the authors' knowledge, there is no published work so far that provides a control design for inverter-based networks guaranteeing overall network stability for the nonlinear model considering variable voltages and arbitrary R/X ratios of the lines while accounting for power sharing. Our main contributions in this sense are twofold. First, we provide a decentralized control design for robust stability of the nonlinear model of an inverter-based network formed by an arbitrary number of inverters. Second, the design allows to specify a range for the droop gains, rather than a fixed value. In this way, the power sharing characteristics could then be adjusted e.g., by market mechanisms similar to the

This work was supported by Siemens AG, the Alexander von Humboldt Foundation and the Ministry of Education of Vietnam.

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978-1-4673-2066-5/12/$31.00 ©2012 IEEE

present implementation in many large power systems [18], [19], while preserving stability. Opposed to standard droop control our approach does guarantee zero steady-state frequency deviation. The proposed design merges ideas from decentralized control for large power systems of SGs [20], [21], [22] (where stabilizing control laws have been derived for nonlinear models of power systems) and droop control ideas (that target the requirement of power sharing). At the technical level, this is achieved by solving a decentralized output feedback problem for a desired range of the power sharing gains (to be defined by the user). Additionally, we account for line and load uncertainties by considering multiplicative uncertainties in the admittance elements. The control synthesis is also decentralized and formulated as a linear matrix inequality (LMI) optimization problem, which allows for efficient off-line computation. Moreover, this robust decentralized synthesis enables easy plug-and-play-like integration of new generation units in the network without recomputing controller gains of existing units, if the latter have been determined accounting for sufficient robustness with respect to uncertainties in loads and line impedances.

II. Modeling of a microgrid

Whenever a grid is mainly formed by inverters, the latter are normally operated in voltage source mode. A voltage source inverter (VSI) behaves as a voltage source with controllable magnitude and frequency of the output voltage [23]. To address the stability problem, the inverter is modeled as an ideal voltage source where all internal control loops but the power control loop are neglected [6], [8]. Given a particular network, we work with the Kron-reduced admittance matrix of the network, a standard method in power systems to eliminate passive nodes [4]. Based on these assumptions, the active and reactive power flow exchange at node i is given by:

$$\begin{aligned} P_i &= \sum_{j=1}^{n} \alpha_{ij}|Y_{ij}|V_iV_j\cos(\delta_i-\delta_j-\phi_{ij}) \\ Q_i &= \sum_{j=1}^{n} \alpha_{ij}|Y_{ij}|V_iV_j\sin(\delta_i-\delta_j-\phi_{ij}), \end{aligned} \tag{II.1}$$

where δ_i and δ_j are the phase angles at node i and j, $\dot{\delta}_i$ and $\dot{\delta}_j$ their corresponding frequencies, V_i and V_j are the voltage magnitudes, $|Y_{ij}|$ represents the expected magnitude of the admittance Y_{ij} between node i and j, ϕ_{ij} is the admittance angle of Y_{ij}, $(\alpha_{ij}-1)$ represents a multiplicative uncertainty in $|Y_{ij}|$ and n is the number of nodes in the network. All phase angles δ_i are expressed with respect to an arbitrary rotating reference frame with angular velocity ω_{nom} [7].

The active and reactive power flows are measured through a low pass filter with time constant τ_{P_i}:

$$\dot{\tilde{P}}_i = \frac{1}{\tau_{P_i}}\left(-\tilde{P}_i+P_i\right), \quad \dot{\tilde{Q}}_i = \frac{1}{\tau_{P_i}}\left(-\tilde{Q}_i+Q_i\right), \tag{II.2}$$

where $\tilde{P}_i$ and $\tilde{Q}_i$ denote the measured active and reactive power. In most power control approaches, e.g., droop control, the inverter output frequency $\dot{\delta}_i$ is controlled instead of directly controlling the phase angle δ_i [7]. Further, we consider the input delay in the voltage via another low-pass filter with time constant $\tau_{V_i} \ll \tau_{P_i}$, hence

$$\dot{\delta}_i = u_i^{\text{a}}, \quad \dot{V}_i = \frac{1}{\tau_{V_i}}(-V_i+V_i^{\text{d}}+u_i^{\text{b}}), \tag{II.3}$$

where V_i^{d} denotes the desired (nominal) operating voltage. In that way, the model resembles the typical droop control structure.

Based on (II.1), (II.2) and (II.3), we now build a nonlinear multi-inverter network (Fig. 1). Notice that in the here considered case of an autonomously operated network the isolation switch is open. We denote by $y_i^{\text{e}} = [\delta_i^{\text{e}}\ V_i^{\text{e}}\ \tilde{P}_i^{\text{e}}\ \tilde{Q}_i^{\text{e}}]^T,\ i=1,\ldots,n$ an equilibrium point of the network, characterized by equations (II.1), (II.2) together with (II.3) and $u_i^{\text{a,e}}=0,\ u_i^{\text{b,e}}=const$. This equilibrium point y_i^{e} is usually not completely known explicitly in power systems, as it depends on the network topologies and load conditions (that are represented in the model through the uncertainty coefficients α_{ij}). In order to derive control laws guaranteeing stability with respect to y_i^{e}, we define our state variables as:

$$\begin{aligned} x_{i_1} &= \delta_i-\delta_i^{\text{e}}, & x_{i_2} &= V_i-V_i^{\text{e}}, \\ x_{i_3} &= \tilde{P}_i-\tilde{P}_i^{\text{e}}, & x_{i_4} &= \tilde{Q}_i-\tilde{Q}_i^{\text{e}}, \quad i=1,\ldots n. \end{aligned} \tag{II.4}$$

Further we define:

$$\Delta u_i = \begin{bmatrix}\Delta u_i^{\text{a}} \\ \Delta u_i^{\text{b}}\end{bmatrix} = \begin{bmatrix} u_i^{\text{a}}-u_i^{\text{a,e}} \\ u_i^{\text{b}}-u_i^{\text{b,e}}\end{bmatrix}, \quad i=1,\ldots,n. \tag{II.5}$$

Hence, we can rewrite the dynamics for the deviations from the equilibrium point at node i as follows:

$$\dot{x}_i(t) = A_ix_i(t)+B_i\Delta u_i(t)+\sum_{j=1}^{n}\kappa_{ij}G_if_{ij}(x_i,x_j) \tag{II.6}$$

with $B_i = \begin{bmatrix}B_{i_1} & 0\end{bmatrix}^T$, $G_i = \begin{bmatrix}0 & \frac{1}{\tau_{P_i}}I\end{bmatrix}^T$,

$$A_i = \begin{bmatrix}A_{i_1} & 0 \\ 0 & -\frac{1}{\tau_{P_i}}I\end{bmatrix},\ A_{i_1} = \begin{bmatrix}0 & 0 \\ 0 & -\frac{1}{\tau_{V_i}}\end{bmatrix},\ B_{i_1} = \begin{bmatrix}1 & 0 \\ 0 & \frac{1}{\tau_{V_i}}\end{bmatrix}.$$

and κ_{ij} are constants with values either 1 or 0 ($\kappa_{ij}=0$ means that the jth subsystem is not connected with the ith subsystem). The nonlinear interconnections are given by:

$$f_{ij}(x_i,x_j)=\begin{bmatrix} \alpha_{ij}|Y_{ij}|\Big((x_{i_2}+V_i^{\text{e}})(x_{j_2}+V_j^{\text{e}}) \\ \quad\cos(x_{i_1}-x_{j_1}+\tilde{\phi}_{ij})-V_i^{\text{e}}V_j^{\text{e}}\cos(\tilde{\phi}_{ij})\Big) \\ \alpha_{ij}|Y_{ij}|\Big((x_{i_2}+V_i^{\text{e}})(x_{j_2}+V_j^{\text{e}}) \\ \quad\sin(x_{i_1}-x_{j_1}+\tilde{\phi}_{ij})-V_i^{\text{e}}V_j^{\text{e}}\sin(\tilde{\phi}_{ij})\Big) \end{bmatrix},$$

where $\tilde{\phi}_{ij} = \delta_i^{\text{e}}-\delta_j^{\text{e}}-\phi_{ij}$.

III. A decentralized control solution for voltage and frequency stability

In this paper we propose a decentralized control design procedure that achieves stability (with respect to y_i^{e}) of system (II.6) for a given set of droop-like controllers aiming to achieve power sharing among the different nodes in the network. From a control perspective, the power sharing requirement relates to the design of the control law so that the equilibrium point y_i^{e} lies in a desired manifold,

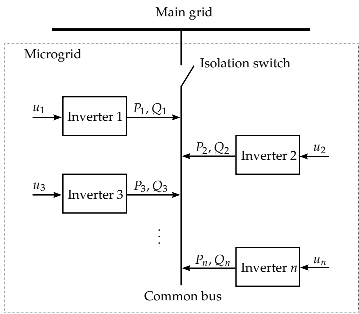

Fig. 1. Schematic representation of a multi-inverter network.

regardless of the network conditions (*i.e.*, regardless of the current value of α_{ij}.) Indeed, looking at the state equations in (II.1), (II.2) and (II.3) it can be concluded that the equilibrium point $y_i^{\rm e}$ is just *partially* determined by the control laws. In particular, the difference between the current equilibrium point $y_i^{\rm e}$ and a so-called nominal equilibrium point $y_i^{\rm d} = [\delta_i^{\rm d}\ V_i^{\rm d}\ \tilde{P}_i^{\rm d}\ \tilde{Q}_i^{\rm d}]^T,\ i = 1, \ldots, n$ (that typically corresponds to the nominal operating conditions) is of interest. For that matter, we define the deviations of the system variables with respect to their desired values as

$$\begin{aligned} z_{i_1} &= \delta_i - \delta_i^{\rm d}, \qquad z_{i_2} = V_i - V_i^{\rm d}, \\ z_{i_3} &= \tilde{P}_i - \tilde{P}_i^{\rm d}, \qquad z_{i_4} = \tilde{Q}_i - \tilde{Q}_i^{\rm d}, \quad i = 1, \ldots n. \end{aligned} \tag{III.1}$$

We would like to point out that the choice of the nominal operation point does not affect the stability of the system and can thus be made arbitrarily, but will affect the steady state $y_i^{\rm e}$ as will be shown in the following. We further divide the control input $u_i = [u_i^{\rm a}\ \ u_i^{\rm b}]^T$ in two components:

$$u_i(t) = u_{i_1}(t) + u_{i_2}(t) = -K_{i_1} z_i(t) - K_{i_2} z_i(t). \tag{III.2}$$

Notice that the control law depends on z_i and not on x_i, since $y_i^{\rm e}$ is unknown and therefore x_i is not available. The first input u_{i_1} imitates the effect of droop control and is thus responsible for power sharing, whereas u_{i_2} stabilizes the system once K_{i_1} has been selected. We include this second component u_{i_2} since, to the best of our knowledge, it has not been proven that droop control can stabilize an inverter-based network (considering a nonlinear model). The role played by u_{i_1} is to modify the voltage magnitude and angle according to the active and reactive power in the spirit of droop control:

$$u_{i_1}(t) = -K_{i_1} z_i(t), \quad K_{i_1} = \begin{bmatrix} 0 & K_{PQ_i} \end{bmatrix}. \tag{III.3}$$

Different structures can be considered for the matrix K_{PQ_i}. While our formal approach is valid for any structure of K_{PQ_i}, we focus for simplicity on the following diagonal matrix:

$$K_{PQ_i} = \begin{bmatrix} k_{P_i} & 0 \\ 0 & k_{Q_i} \end{bmatrix}, \tag{III.4}$$

with $k_{P_i} > 0$ and $k_{Q_i} > 0$, which is the most commonly proposed pairing [6]. Under this controller gain, the voltage angle of the inverter is modified according to the active power at the node, while the reactive power modifies the voltage magnitude. Other pairings to achieve power sharing have been proposed in the literature [14], [24], [13]. The gains k_{P_i} and k_{Q_i} are coefficients selected by the user according to the desired power distribution in the network (as in droop control). Such gains might not be known beforehand; instead, the user might adjust them according to the status of the network, number of generation units present, economic factors,...[18], [19]. To account for this case, our approach considers a set of droop-like gain matrices $K_{i_1} \in \Gamma_i : \{[0, \text{diag}(k_{P_i}, k_{Q_i})] | 0 \le k_{P_i} \le \bar{k}_{P_i}, 0 \le k_{Q_i} \le \bar{k}_{Q_i}\}$ rather than preassigned specific values. Once K_{i_1} has been specified, the second gain matrix K_{i_2} has to be designed to guarantee stability. It can be proven[1] that a feedback law of the form $u_{i_2} = [-k_{\delta_i} z_{i_1}\ -k_{V_i} z_{i_2}]^T$ can stabilize the system for sufficiently large k_{δ_i} and k_{V_i}. Under such control laws u_{i_1} and u_{i_2}, the following holds in steady state for system (II.6) in z-coordinates:

$$\begin{aligned} z_{i_1}^{\rm e} &= -\frac{k_{P_i}}{k_{\delta_i}} z_{i_3}^{\rm e}, \quad z_{i_2}^{\rm e} = -\frac{k_{Q_i}}{k_{V_i}+1} z_{i_4}^{\rm e}, \\ z_{i_3}^{\rm e} &= \sum_{j=1}^{n} \Big(\alpha_{ij} Y_{ij} (V_i^{\rm d} - \frac{k_{Q_i}}{k_{V_i}+1} z_{i_4}^{\rm e})(V_j^{\rm d} - \frac{k_{Q_j}}{k_{V_j}+1} z_{j_4}^{\rm e}) \cdot \\ &\quad \cos(\delta_i^{\rm d} - \frac{k_{P_i}}{k_{\delta_i}} z_{i_3}^{\rm e} - \delta_j^{\rm d} + \frac{k_{P_j}}{k_{\delta_j}} z_{j_3}^{\rm e} - \phi_{ij}) \Big) - \tilde{P}_i^{\rm d}, \\ z_{i_4}^{\rm e} &= \sum_{j=1}^{n} \Big(\alpha_{ij} Y_{ij} (V_i^{\rm d} - \frac{k_{Q_i}}{k_{V_i}+1} z_{i_4}^{\rm e})(V_j^{\rm d} - \frac{k_{Q_j}}{k_{V_j}+1} z_{j_4}^{\rm e}) \cdot \\ &\quad \sin(\delta_i^{\rm d} - \frac{k_{P_i}}{k_{\delta_i}} z_{i_3}^{\rm e} - \delta_j^{\rm d} + \frac{k_{P_j}}{k_{\delta_j}} z_{j_3}^{\rm e} - \phi_{ij}) \Big) - \tilde{Q}_i^{\rm d}. \end{aligned} \tag{III.5}$$

Similar to droop control, the steady-state deviations of z_{i_1} and z_{i_2} are determined by those of active and reactive power z_{i_3}, z_{i_4} via the relations k_{P_i}/k_{δ_i} and $k_{Q_i}/(k_{V_i}+1)$. The heuristic approach of droop control is that by choosing adequate relations of the droop gains at nodes i and j one obtains the desired power sharing between those inverters, which in our case is reflected by the steady-state differences in phase angles $\delta_i^{\rm e} = \delta_i^{\rm d} - k_{P_i}/k_{\delta_i} z_{i_3}^{\rm e}$, $\delta_j^{\rm e} = \delta_j^{\rm d} - k_{P_j}/k_{\delta_j} z_{j_3}^{\rm e}$ and voltage amplitudes $V_i^{\rm e} = V_i^{\rm d} - k_{Q_i}/(k_{V_i}+1) z_{i_4}^{\rm e}, V_j^{\rm e} = V_j^{\rm d} - k_{Q_j}/(k_{V_j}+1) z_{j_4}^{\rm e}$. The role of the stabilizing gains k_{δ_i}, k_{V_i} can be interpreted as a restriction in magnitude of the user-selected droop gains k_{P_i}, k_{Q_i} in order to guarantee stability. That overall network stability requires constraints on the droop gains has been reported by several authors [6], [8]. To maximize the effect of the user-selected gains k_{P_i}, k_{Q_i} to achieve a desired power distribution, the control design proposed in Section III-B attempts at minimizing the gains k_{δ_i} and k_{V_i} for given upper bounds of the droop gains specified in Γ_i.

While our design method does guarantee overall network stability, we can not make any claims regarding the power sharing performance. Given the complex structure of the interconnected network, such claims are difficult to derive in general and are part of our on-going investigations. Analysis for more specific cases have been presented in [13], [17].

Fig. 2 displays the proposed control structure. Only the output voltage and current need to be measured, from which the active and reactive power can be computed. Notice that the control of active power is done based on the voltage angle

[1]Not shown here due to space constraints. This fact can be easily concluded using $V = \sum_{i=1}^{n} V_i = \sum_{i=1}^{n} x_i^T x_i$ as a control Lyapunov function for the system and considering (II.4).

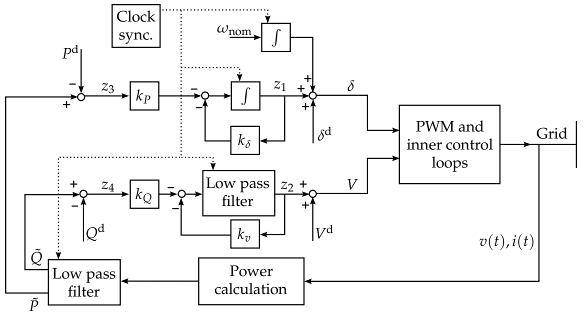

Fig. 2. Decentralized control structure with clock synchronization signal for referencing.

and not voltage frequency, as in traditional droop control. A similar option has been previously explored in [15]. As mentioned in the introduction, this solution guarantees zero steady-state deviation for the frequency. It is clear that this property, among other benefits, removes the need for secondary control in charge of frequency restoration. The proposed method requires referencing for clock synchronization, that can be achieved for instance via GPS [15]. We now formally define the problem addressed in this paper:

Problem 3.1: Given a desired droop-like control law $u_{i_1} = -K_{i_1} z_i$ and a set of desired droop-gain matrices $K_{i_1} \in \Gamma_i : \{[0, \text{diag}(k_{P_i}, k_{Q_i})] | 0 \leq k_{P_i} \leq \bar{k}_{P_i}, 0 \leq k_{Q_i} \leq \bar{k}_{Q_i}\}$, design a decentralized control $u_{i_2} = -K_{i_2} z_i$ that stabilizes the system (II.6) (for $i = 1, \ldots, n$).

Remark 3.2: Note that for ease of explanation, we assume the particular structure of K_{PQ_i} given in (III.4) throughout the paper. Our approach always guarantees stability (for appropiate values of K_{i_2}) independently of the chosen structure of K_{PQ_i}, although the quality of power sharing might deteriorate if the system characteristics assumed during the design process do not match the real system.

A. Preliminaries

1) Bounds for nonlinearities: Before proposing a design procedure for K_{i_2}, we analyze the nonlinearities present in the network model in (II.6). Given an operating region $\Omega \subset \mathbb{R}^n$ such that $0 \in \Omega$, we can derive a quadratic bound for the nonlinearities in (II.6). In particular, we select a set $\Omega : \{(x_1, x_2, x_3, x_4) \in \mathbb{R}^4 | \, x_2 < V_{\text{max}} - V^{\text{e}}\}$, where V_{max} represents the maximum operating voltage of the considered power system. Using the Hölder inequality $2ab \leq \frac{1}{\mu} a^2 + \mu b^2$ together with standard trigonometric identities, we obtain:

$$
\begin{aligned}
\sum_{j=1}^{n} f_{ij}^T f_{ij} &\leq \sum_{j=1, j\neq i}^{n} 2\alpha_{ij}^2 Y_{ij}^2 \Big((1+\mu) V_{\text{max}}^2 (x_{i_2}^2 + x_{j_2}^2) \\
&+ (1 + \frac{1}{\mu}) V_{\text{max}}^4 (x_{i_1}^2 + x_{j_1}^2) \Big) + 4\alpha_{ii}^2 Y_{ii}^2 V_{\text{max}}^2 x_{i_2}^2 \\
&= \sum_{j=1}^{n} x_i^T F_{i_{ij}}^T F_{i_{ij}} x_i + \sum_{j=1}^{n} x_j^T F_{j_{ij}}^T F_{j_{ij}} x_j \qquad \text{(III.6)}
\end{aligned}
$$

for appropriate values of $F_{i_{ij}}$ and $F_{j_{ij}}$. We further define for convenience $F_i^T F_i = \sum_{j=1}^{n} (F_{i_{ij}}^T F_{i_{ij}} + F_{i_{ji}}^T F_{i_{ji}})$.

Remark 3.3: These bounds are valid provided that the state trajectories stay in the set Ω. A fairly simple, though conservative, estimate of Ω for initial conditions satisfying $|x_0| \leq r$ is given by:

$$
\Omega : \{x \in \mathbb{R}^4 | \, |x(t)| \leq \sqrt{\frac{\lambda_{\text{max}}(\Phi)}{\lambda_{\text{min}}(\Phi)}} r, \quad t \geq 0\} \qquad \text{(III.7)}
$$

where λ_{max} and λ_{min} are respectively the maximum and minimum eigenvalues of $\Phi = \text{blockdiag}[\Phi_1, \ldots, \Phi_n] = \text{blockdiag}[W_1^{-1}, \ldots, W_n^{-1}]$, as defined in the proof of Theorem 3.4. Equation (III.7) allows to identify the values of admissible initial conditions.

B. Stabilizing decentralized LMI control design

The following theorem provides a solution to Problem 3.1.

Theorem 3.4: The nonlinear multi-inverter system (II.6) is stabilizable via decentralized linear feedback control for any W_i, M_i, N_i, $i = 1, \ldots, n$ satisfying the following conditions $\forall K_{i_1} \in \Gamma_i$:

$$
\begin{bmatrix}
\hat{A}_i & W_i \bar{K}_{i_1}^T & B_i & G_i & W_i F_i^T \\
\bar{K}_{i_1} W_i & -\frac{1}{\epsilon} I & 0 & 0 & 0 \\
B_i^T & 0 & -\epsilon I & 0 & 0 \\
G_i^T & 0 & 0 & -\tilde{\rho} I & 0 \\
F_i W_i & 0 & 0 & 0 & -\rho I
\end{bmatrix} < 0,
$$

$$
M_i C_i = C_i W_i, \quad W_i > 0, \qquad \text{(III.8)}
$$

where $\hat{A}_i = W_i A_i^T + A_i W_i - B_i N_i C_i - C_i^T N_i^T B_i^T$, $C_i = \begin{bmatrix} I & 0 \end{bmatrix}$, $\bar{K}_{i1} = \max_{K_{i1}}(\Gamma_i)$, and $\tilde{\rho} = (\rho \ \sum_{j=1}^{n} \kappa_{ij})^{-1}$. Moreover, the control law is then given by: $u_i = -K_i z_i = -K_{i_1} z_i - K_{i_2} z_i = -\begin{bmatrix} N_i M_i^{-1} & K_{PQ_i} \end{bmatrix} z_i$.

Proof: The proof draws inspiration from [20], [21] and [22]. Analyzing (II.5) under control (III.2) and considering (II.4) together with (III.1) gives:

$$
\Delta u_i = \begin{bmatrix} \Delta u_i^{\text{a}} \\ \Delta u_i^{\text{b}} \end{bmatrix} = -K_i z_i + K_i z_i^{\text{e}} = -K_i x_i.
$$

Thus, stability of system (II.6) under control (III.2), is equivalent to stability of

$$
\dot{x}_i = A_i x_i - B_i K_i x_i + \sum_{j=1}^{n} \kappa_{ij} G_i f_{ij}(x_i, x_j)
$$

for any z_i. We then define a Lyapunov function of the following form:

$$
V = \sum_{i=1}^{n} V_i = \sum_{i=1}^{n} x_i^T \Phi_i x_i. \qquad \text{(III.9)}
$$

Defining $\tilde{A}_i = A_i - B_i K_{i_2}$ and making use of

$$
X^T Y + Y^T X \leq \frac{1}{\epsilon} X^T X + \epsilon Y^T Y \qquad \text{(III.10)}
$$

for $\epsilon > 0$, the time derivative of V_i along the trajectories of (II.6) with the controller given in (III.2) becomes:

$$
\begin{aligned}
\dot{V}_i &= \dot{x}_i^T \Phi_i x_i + x_i^T \Phi_i \dot{x}_i \\
&= x_i^T \left((\tilde{A}_i - B_i \bar{K}_{i_1})^T \Phi_i + \Phi_i (\tilde{A}_i - B_i \bar{K}_{i_1}) \right) x_i \\
&+ \sum_{j=1}^{n} \kappa_{ij} f_{ij}^T G_i^T \Phi_i x_i + x_i^T \Phi_i \sum_{j=1}^{n} \kappa_{ij} G_i f_{ij} \\
&\leq x_i^T \left(\tilde{A}_i^T \Phi_i + \Phi_i \tilde{A}_i + \frac{1}{\epsilon} \Phi_i B_i B_i^T \Phi_i + \epsilon \bar{K}_{i_1}^T \bar{K}_{i_1} \right) x_i \\
&+ \sum_{j=1}^{n} \kappa_{ij} f_{ij}^T G_i^T \Phi_i x_i + x_i^T \Phi_i \sum_{j=1}^{n} \kappa_{ij} G_i f_{ij}.
\end{aligned}
$$

Recalling $V = \sum_{i=1}^n V_i$, using the fact that $\sum_{i=1}^n \sum_{j=1}^n a_{ij} = \sum_{i=1}^n \sum_{j=1}^n a_{ji}$ and applying (III.10) with $\rho > 0$ together with the bounds in (III.6) we have

$$\begin{aligned}\dot{V} \leq &\sum_{i=1}^{n}\Big(x_i^T\big(\tilde{A}_i^T\Phi_i+\Phi_i\tilde{A}_i+\frac{1}{\epsilon}\Phi_iB_iB_i^T\Phi_i+\epsilon\bar{K}_{i_1}^T\bar{K}_{i_1}\big)x_i\\ &+x_i^T\rho\sum_{j=1}^{n}\kappa_{ij}\Phi_iG_iG_i^T\Phi_ix_i+\frac{1}{\rho}\sum_{j=1}^{n}f_{ij}^Tf_{ij}\Big)\\ \leq &\sum_{i=1}^{n}\Big(x_i^T\big(\tilde{A}_i^T\Phi_i+\Phi_i\tilde{A}_i+\epsilon\bar{K}_{i_1}^T\bar{K}_{i_1}+\frac{1}{\epsilon}\Phi_iB_iB_i^T\Phi_i\\ &+\rho\sum_{j=1}^{n}\kappa_{ij}\Phi_iG_iG_i^T\Phi_i+\frac{1}{\rho}F_i^TF_i\big)x_i\Big).\end{aligned}$$

Defining $W_i = \Phi_i^{-1}, W_i > 0$ and $\tilde{\rho} = (\rho \sum_{j=1}^n \kappa_{ij})^{-1}$, using the Schur complement and pre- and postmultiplying with W_i [22], we can rewrite the condition on $\dot{V}$ as the following bilinear matrix inequalities (BMI) in W_i and K_{i_2} for $i = 1, \cdots, n$:

$$\begin{bmatrix} W_i\tilde{A}_i^T+\tilde{A}_iW_i & W_i\bar{K}_{i_1}^T & B_i & G_i & W_iF_i^T\\ \bar{K}_{i_1}W_i & -\frac{1}{\epsilon}I & 0 & 0 & 0\\ B_i^T & 0 & -\epsilon I & 0 & 0\\ G_i^T & 0 & 0 & -\tilde{\rho}I & 0\\ F_iW_i & 0 & 0 & 0 & -\rho I\end{bmatrix} < 0. \quad \text{(III.11)}$$

Notice that, because of the particular structure imposed on the controller, the present problem resembles the case of output feedback control. It is well known that the problem of output feedback stabilization in W_i and K_{i_2} is nonconvex and numerically very difficult to solve. There has been extensive research on finding appropriate related convex problem formulations via a change of variables [25]. Among others, one possible variable change leads to the W-Problem [26]. Applying this variable transformation to equations (III.11) leads to the proposed LMI optimization problem stated in Theorem 3.4. ■

Remark 3.5: Notice that Theorem 3.4 holds for any structure of K_{i_2}. However, as discussed at the beginning of Section III, we are interested in a diagonal structure for K_{i_2}, for the purpose of power sharing. It was also mentioned that the gains in K_{i_2} should be minimized as well. This can be achieved by enforcing N_i and M_i to be diagonal matrices and limiting the feedback gains by adding the following constraints to the set of equations (III.8) [25]:

$$N_i^TN_i < \kappa_{N_i}I_i, \quad M_i^{-1} < \kappa_{M_i}I_i,$$

which can be expressed as the LMIs

$$\begin{bmatrix}-\kappa_{N_i}I & N_i^T\\ N_i & -I\end{bmatrix} < 0, \begin{bmatrix}M_i & I\\ I & \kappa_{M_i}I\end{bmatrix} > 0, \; M_i > 0. \quad \text{(III.12)}$$

Remark 3.6: Theorem 3.4 not only provides a decentralized controller, but also represents a decentralized design: each node can design its controller without the knowledge of the controllers in the other nodes (provided that all nodes follow the control design proposed in (III.8)). Notice as well that the proposed design allows for plug-and-play-like integration of new generation units in the network without recomputing controller gains of existing units.

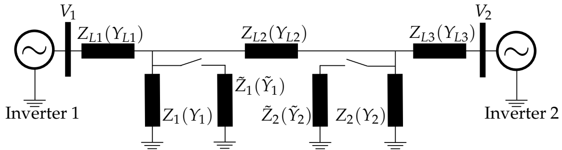

Fig. 3. Test system with two inverters represented as voltage sources.

TABLE I
TEST SYSTEM PARAMETERS

	Inverter 1	Inverter 2
Voltage magnitude and phase angle	$V_1^{\mathrm{d}} = 232\,\mathrm{V}$ $\delta_1^{\mathrm{d}} = 0\,\mathrm{rad}$	$V_2^{\mathrm{d}} = 228.5\,\mathrm{V}$ $\delta_2^{\mathrm{d}} = -10^{-4}\,\mathrm{rad}$
Active and reactive power	$\tilde{P}_1^{\mathrm{d}} = 15.68\,\mathrm{kW}$ $\tilde{Q}_1^{\mathrm{d}} = 7.73\,\mathrm{kVar}$	$\tilde{P}_2^{\mathrm{d}} = 15.6\,\mathrm{kW}$ $\tilde{Q}_2^{\mathrm{d}} = 5.16\,\mathrm{kVar}$
Time constant low pass filter	$\tau_{P_1} = 0.0265\,\mathrm{s}$ $\tau_{V_1} = 10^{-3}\,\mathrm{s}$	$\tau_{P_2} = 0.0265\,\mathrm{s}$ $\tau_{V_2} = 10^{-3}\,\mathrm{s}$
Control gains via LMI approach	$k_{\delta_1} = 17.72\,\frac{1}{\mathrm{s}}$ $k_{V_1} = 6.98$	$k_{\delta_2} = 17.72\,\frac{1}{\mathrm{s}}$ $k_{V_2} = 7.05$
User-selected droop gains	$k_{P_1} = 10^{-2}\,\frac{\mathrm{rad}}{\mathrm{skW}}$ $k_{Q_1} = 1\,\frac{\mathrm{V}}{\mathrm{kVar}}$	$k_{P_2} = 10^{-2}\,\frac{\mathrm{rad}}{\mathrm{skW}}$ $k_{Q_2} = 1\,\frac{\mathrm{V}}{\mathrm{kVar}}$
Load impedances $Z_i = R_i + j\omega L_i$	$Z_1 = (4 + j1.95)\,\Omega$ $\tilde{Z}_1 = (20 + j9.7)\,\Omega$	$Z_2 = (2.24{+}j.79)\,\Omega$ $\tilde{Z}_2 = (11.2{+}j3.7)\,\Omega$
Line impedances $Z_{\mathrm{L}i} = R_i + j\omega L_i$	$Z_{\mathrm{L1}} = (.01{+}j.05)\,\Omega$ $Z_{\mathrm{L3}} = (.01{+}j.04)\,\Omega$	$Z_{\mathrm{L2}} = (.12{+}j.03)\,\Omega$
Nominal voltage	$V_{\mathrm{nom}} = 230\,\mathrm{V}$	
Nominal frequency	$f_{\mathrm{nom}} = 50\,\mathrm{Hz}$	

IV. AN ACADEMIC EXAMPLE

The presented approach is now implemented on a test system composed of two inverters having each a local load represented by a frequency dependent impedance and being connected via an LV line, Fig. 3. We consider the following scenario in the simulations: the system is first stabilized at the determined equilibrium point; then, at $t = 1s$ (respectively $t = 2s$) a new load $\tilde{Z}_1$ (respectively $\tilde{Z}_2$) is connected; subsequently both new loads $(\tilde{Z}_1, \tilde{Z}_2)$ are disconnected simultaneously at $t = 3s$. According to the proposed control design, it is expected that the system stabilizes in all operating conditions and that the new loads are shared among the inverters.

The system parameters and control gains are given in Table I. The control parameters for the presented design are derived using the LMIs in (III.8) with $k_{P_i} \in [0, 10^{-2}]$, $k_{Q_i} \in [0, 1]$ and $V_{\max} = 1.2V_{\mathrm{nom}}$. We account for uncertainties of the absolute values of the elements of Y of up to $\alpha_{ij} = 1.1$, $\{i, j\} \in \{1, 2\}$, so that the controllers are robust against e.g., load changes. The LMIs are solved using Yalmip 3 [27] together with Sedumi 1.3 under Matlab. The simulations are carried out in Plecs [28].

The simulation results are displayed in Fig. 4. After initialization, the system is first stabilized at the desired equilibrium point. At $t = 1s$ both inverters react to the new load by increasing their power outputs. After a short transient, the system is again stabilized. Similar behavior can be observed for the second load change at $t = 2s$. When both new loads are switched off at $t = 3s$, the inverters decrease their output powers and the system returns to the first equilibrium point. We would like to point out that the system stabilizes in each operating condition with zero

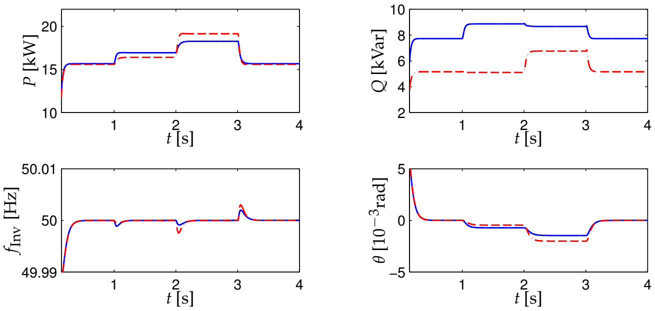

Fig. 4. Test system with proposed control, Inverter 1 '–', Inverter 2 '- -'.

steady-state frequency deviation, a behavior which cannot be achieved with conventional droop control.

In all cases, the active power sharing is improved by over a factor $3/2$ with respect to the case without control, although does not represent a share of 50% as suggested by the selected droop gains. A reduction in power sharing performance in favor of stability and zero steady state frequency deviation has also been reported in [15]. Notice as well that there is no overshoot in the active power output, which is favorable for voltage and current limitations in both the DC circuit fed by the renewable source and the inverter. The reactive power sharing is not very accurate. This behavior has been often reported related to droop control in LV networks [29]. The power sharing may be improved by considering inverse droop control or more advanced modified droop control strategies as well as appropiate scaling of the output impedances as proposed e.g., in [14], [29], [24], [13].

V. Conclusion

A decentralized feedback control design addressing the problem of voltage and frequency stability for a nonlinear inverter-based microgrid model under droop-like control by providing additional decentralized feedback has been presented. Opposed to standard droop control, our approach guarantees zero steady-state frequency deviation. The control synthesis (also decentralized) is formulated as an LMI and allows for a user-specified range for power sharing gains as well as line and load uncertainties. The presented example demonstrates the benefits of this approach in terms of zero steady-state frequency deviation and stability. Future research will consider networks that include synchronous generators and inverters as well as formal consideration of the power sharing performance.

VI. Acknowledgements

The authors would like to thank Björn Heinbokel and Raúl Mur Artal for helpful discussions on the topics of this paper.

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