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Private Labels and Retail Assortment Planning: A Differential Evolution Approach

Abstract

Despite the longstanding recognition of the importance of product assortment planning (PAP), existing literature has failed to provide satisfactory solutions to a great deal of problems that reside in this area of research. The issue of optimal assortment planning in the retail sector becomes even more important in periods of economic crisis, as retailers must adapt their product portfolios to new evolving patterns of consumer buying behaviour and reduced levels of consumer's purchasing power. Private labels (PLs) typically experience significant growth in times of recession, due to their low prices, and the reduced disposable income of households. In this direction, the present paper introduces Differential Evolution (DE) to assist retailers in adapting their product portfolios in periods of economic recession and facilitate strategic PAP decisions, related to a) optimal variety of PL product categories, b) optimal service level of PL merchandise within a product category, and hence, c) optimal balance between PLs and National Brands (NBs) in a retailer's product portfolio. The interrelated issue of assortment adaptation across different store formats is also considered. Economic recessions contribute to the prolonged upward evolution in PL share, and hence, our mechanism facilitates decisions that are nowadays more important than ever before. The proposed mechanism is illustrated through an implementation to an empirical dataset derived from a random sample of 1,928 consumers who participated in a large-scale computer assisted telephone survey during the current economic crisis period.

Keywords: Product assortment planning, Differential evolution, Private label, Economic crisis

1. Introduction

One of the basic strategic decisions a retailer must make involves the determination of the product assortment to offer. Product assortment planning (PAP) involves important decisions related to the determination of variety (i.e., number of different product categories), depth (i.e., number of stock-keeping units/distinct items for sale, within a product category) and service level (i.e., amount of merchandise inventory within a product category) in a retailer's product portfolio (Mantrala et al., 2009; Hübner and Kuhn, 2012). These decisions become even more crucial in periods of economic downturn, as retailers are expected to adapt their product portfolios to changing economic conditions. By making optimal PAP decisions, retailers hope to satisfy customers' changing needs by providing the right merchandise in the right store at the right time (Nogales and Gómez-Suarez, 2005). If the retailer fails to provide the expected assortment, customers defect, causing losses in both current and future sales.

At the same time, economic crisis hits consumers' disposable income very hard and makes them more prone to switch to Private Labels (PLs) at the expense of National Brands (NBs). PLs are products which are typically manufactured by one company for offer under another company's brand. Simply put, they are products that retail stores put their own names or brands on. They may also be called store brands, own brands, or retailer brands, but they all have one thing in common – they are manufactured and brought to market in much the same way as the familiar National Brands they sit next to on store shelves. PLs can be available in almost every food and non-food product category (e.g., fresh, frozen and refrigerated food, canned and dry foods, snacks, pet foods, health and beauty care, household and laundry products, stationery and housewares, etc). For many shoppers, PLs represent better value and savings. As a result, a closer look at the role of PLs in a retailer's product portfolio is more relevant nowadays than ever before. Several academics have theorized the growing importance of PLs in periods of economic downturn (e.g., Lamey et al., 2007; Quelch and Harding, 1996; Nandan and Dickinson, 1994). For example, Quelch and Harding (1996) suggest that PL market share goes up when the economy is suffering and down in stronger economic periods. Likewise, Nandan and Dickinson (1994) state that during difficult economic times, the popularity of PLs tend to increase, whereas in periods of relative economic prosperity, the share of NBs increases. Unlike other drivers of PL success, the general economic conditions are largely beyond the retailer's control.

In the present study we extend the PAP problem in an attempt to also consider the crucial role of PLs in a retailer's product portfolio, an issue which has been neglected in the existing PAP literature. In this direction, this paper conceptualizes the optimal Private Label-Product Assortment Planning (PL-PAP) problem, and subsequently, introduces a new mechanism, namely Differential Evolution (DE), which facilitates simultaneously relevant important decisions. More specifically, we attempt to optimize the variety and service levels of PLs in a retailer's product portfolio. A closer look at the role of PLs in a retailer's product portfolio is more relevant nowadays than ever before, due to the severe economic recession.

The rest of the article is organized as follows. The next section presents the theoretical background of our study and the optimal PL-PAP problem. Subsequently, we introduce our proposed mechanism, which is followed by a section that illustrates its implementation to our empirical dataset. Finally, results are presented, while a concluding section summarizes the article.

2. Theoretical background

2.1 Category management and assortment planning

Existing literature on retail category management attempts to develop efficient support systems as a means to facilitate decision making that focuses on two broad areas, namely shelf space planning and assortment planning. Shelf space planning considers facing and replenishment decisions (see e.g., Corstjens and Doyle, 1981), while assortment planning considers the question of which and how many different products to offer (Mantrala et al., 2009). In the past two decades, numerous models and analytical solutions have been proposed to deal with both areas of research (e.g., Anderson and Amato, 1974; Borin and Farris, 1995; Borin et al., 1994; Brijs et al., 2000; Brijs et al., 1999; Bultez and Naert, 1988; Bultez et al., 1989; Corstjens and Doyle, 1981; Corstjens and Doyle, 1983; Fadıloğlu et al., 2010; Hansen and Heinsbroek, 1979; Russell and Urban, 2010; Urban, 1998; Yang, 2001). In the shelf space planning literature, researchers traditionally apply the individual space elasticity and cross-elasticity between products to determine which products to stock and how much shelf space to display these products, whereas, the main body of literature on assortment planning models is based on the estimation of substitution effects and develops optimization algorithms to define inventory levels by stochastic demand.

The present study is concerned with the latter area of research. Kök and Fisher (2007) define retail product assortment planning (PAP) as the process used to find the optimal combination of products to be carried and set the inventory levels of each product. Hübner and Kuhn (2012) define variety as the number and combination of product categories in a retailer's product portfolio, depth as the number of stock-keeping units within a category and service level as the amount of merchandise inventory within a category. Evidently, retailers want to identify the optimal balance among variety, depth, and service levels, but at the same time they are also constrained by the amount of money they can invest in inventory and/or by their physical space. For example, offering more variety may limit the depth within categories and the service level, or both.

Empirical findings of existing assortment optimization algorithms suggest that variety levels have become so excessive that sales can increase by reducing variety significantly

(Boatwright and Nunes, 2001; Dhar et al., 2001; Sloot and Verhoef, 2008). In the same direction, Iyengar and Lepper (2000) show that consumers are more willing to purchase consumer goods when offered a limited array of choices only. However, even if retailers could determine the optimal assortment mix for all individual customers, it may be unprofitable to stock such an assortment. Therefore, out-of-shelf situations are inevitable. Literature suggests that between 45% and 84% of demand can be substituted (Campo et al., 2003; Xin et al., 2009). The average potential for substitution depends on product-, situation- and consumer-specific characteristics (Fitzsimons, 2000; Xin et al., 2009).

Despite the longstanding recognition of the importance of the PAP problem, several limitations and gaps can be found in existing literature. First, existing research tends to examine analytical solutions that deal almost exclusively with questions of depth, whilst it completely fails to address issues related to variety and service levels (see for example, Mantrala et al., 2009). Second, current literature focuses on a single category or subcategory of products or services and fails to examine the interplay among various categories that are offered by a retailer. Third, although in reality a retailer might have a different assortment at each store format, the academic literature has focused on determining a single assortment for a retailer, which could be viewed as either a common assortment to be carried at all stores or the solution to the PAP problem for a single store (Kök et al., 2005). Finally, PLs have been widely neglected in existing PAP literature, despite the fact that PLs are considered as a powerful competitive tool, especially in periods of economic downturn, as they allow retailers to improve their service offering and store image, obtain greater margins and profits (Nogales and Gómez-Suárez, 2005), while they also have significant marketing potential for improving service quality (Herstein and Gamliel, 2006). The growing penetration of PLs in a number of product categories makes PAP decisions even more complicated. For extensive reviews of the assortment planning literature, the reader is advised to see the work of Mahajan and van Ryzin (1998) and Kök et al. (2005).

Against this background, the present paper attempts to correct for the omissions of existing PAP research by introducing Differential Evolution (DE). To the best of our knowledge the proposed mechanism is introduced for the first time in the broader area of marketing and service research. Therefore, the novelty of this study is two-fold and resides in both the managerial problem and the research methodology. More specifically, we show how this innovative approach can facilitate strategic PAP decisions, related to the determination of a) optimal variety of PL product categories, b) optimal service level of PL merchandise within each product category, and hence, c) optimal balance between PLs and NBs in a retailer's product portfolio.

At the same time, it is widely accepted that the heterogeneity among marketplaces requires that retailers tailor their assortments to local tastes rather than making national-level product assortment planning (PAP) decisions. In this direction, retailers have realized that a "one size/style fits all" strategy is not adequate, and are moving toward tailoring at least 15% of the merchandise in each of their store to local tastes (O'Connell 2008). In the light of this shift, the interrelated issue of assortment adaptation across different store formats is also considered in this paper.

2.2 Formulation of the assortment planning problem

The difficulty of Product Assortment Planning as a task for retailers has been widely studied in the literature. As noted earlier, PAP models are based on substitution effects and focus on developing algorithms to define inventory levels by stochastic demand. The most popular approach for estimating demand substitution in assortment planning is multinomial logit models (e.g., van Ryzin and Mahajan, 1999; Mahajan and van Ryzin, 2001; Cachon et al., 2005; Li, 2007; Hopp and Xu, 2008) and exogenous substitution models (e.g., Smith and Agrawal, 2000; Rajaram and Tang, 2001; Kok andFisher, 2007; Shah and Avittathur, 2007; Yucel et al., 2009). The multinomial logit model is a discrete consumer choice model assuming that consumers are rational utility maximizers, while exogenous demand models directly specify the consumer reaction and are mostly used in inventory models.

The exogenous demand and multinomial logit models have different origins, but allow the optimization of assortments. As a result, retail assortment studies have so far introduced various heuristic techniques for consumer-driven substitution, which have been employed to solve such complex formulations. For example, Urban (1998) introduced Genetic Algorithms, Borin et al. (1994) and Bai and Kendall (2005) implemented Simulated Annealing, while Smith and Agrawal (2000) addressed the problem using Lagrange relaxation. In these studies, the assortment planning problem is usually formulated with the use of mixed integer nonlinear objective functions in an attempt to maximize the expected product profit. Our approach differs in that we formulate the problem using a continuous real-valued function in an attempt to maximize retailer's sales volume.

The global economic recession has significantly affected consumers' purchasing behaviour. Consumers' brand preferences for NBs and PLs have dramatically changed. The global economic slump has accelerated the growth of PLs at the expense of NBs. Our proposed mechanism also considers the interplay between PLs and NBs in the retailer's product portfolio. More specifically, we assume that a retailer carries m different categories (variety), within which, a merchandize inventory of both NBs and PLs may be carried (service levels). The optimal PL-PAP problem for the retailer is to decide, in terms of customer demand, on the optimal configuration of PL categories that must be carried (i.e., PL variety), and the optimal amount of PL merchandize inventory within each category (i.e., PL service level), as a percentage of the total merchandize carried in the given category.

A number of criteria to optimize can be selected, such as profit maximization, cost minimization etc. In this study, we choose to maximize the retailer's sales volume; however, our approach can be easily adapted to any other criterion. The optimization of the PL-PAP problem is based on consumer preferences for PL product categories. We assume that each customer has made his decision whether to buy PL or NB from a given category before visiting the retail store. The probability of purchase depends on the service levels (amount of merchandize inventories) that a retailer offers in a given category. For example, a customer who generally prefers PLs in the alcoholic beverages category, is more likely to buy alcoholic beverages from a particular retailer if this retailer has an extensive PL service level within this product category. Different approaches can be adopted for modeling the relationship between probability of purchase and service level within a category. Also, we assume that the amount of money a customer spends on a certain category is linearly proportional to the respective service level of that category. In particular, we assume that the monthly expenditure of a customer in a specific PL category of a given retailer, equals the amount of PL merchandize inventory (PL service level) offered in that category by the retailer times the total budget spent by the customer on that product category per month. In this manner, the optimal PL-PAP problem is formulated as follows:

Find pl_r and b_r , for r=1,…, m that

$$\text{maximize } f=\sum_{c=1}^{n}\sum_{r=1}^{m}(pl_r * apl_{cr} + b_r * ab_{cr}) \tag{1}$$

$$\text{under } pl_r + b_r = 1 \ \forall \ r \tag{2}$$

$$pl_r, b_r \in [0, 1] \tag{3}$$

$$\frac{\sum_{r=1}^{m} pl_r}{r} \leq 0.27 \tag{4}$$

where m is the number of categories carried by the retailer, n is the average number of customers visiting the retailer per month, pl_r and b_r are the percentages (service levels) of PLs and NBs, respectively, that the retailer carries in category r, αpl_{cr} and αb_{cr} are the total monthly expenditures that customer c spends on PLs and NBs, respectively, in category r. In line with the existing literature, constraint (4) requires the average percentage of PLs that the retailer carries across all categories to be equal or less than 27% (see e.g., Gómez-Suárez, 2005; Nogales and Gómez-Suárez, 2005). For example, Gómez-Suárez (2005) suggests that in a retail grocery store the percentage of space occupied by private labels is on average up to 27%. The author observed the total shelf space occupied by private labels across a set of 40 product categories based on a large sample of retail superstores. Constraint (4), which is a fixed rate for the percentage of PLs, can be easily adapted to match any percentage assigned by the retailer, so as to address the particular characteristics of the given market conditions, geographic location, store size, and/or cultural differences. Constraint (4), along with the fact that pl_r and b_r can take any real value in the range [0, 1] makes the problem very complex. For example, if we allow pl_r to take only 10 different values *0, 0.1, …, 0.9* (i.e., a 10% step), the number of possible solutions for a retailer that carries 10 different categories is 10^{10}. If we decrease the step to 1%, the size of the solution space becomes 10^{20}! Even the fastest computer will require more than a week to completely enumerate the whole solution space (i.e., exhaustive search), if it does not run out of memory. In order to find a good approximation of the global optimal solution in tractable time, we introduce the Differential Evolution algorithm to the PL-PAP problem. We selected Differential Evolution because it is a state of the art technique for global optimization over continuous spaces and one of the most powerful stochastic real parameter optimizers of current interest. It has also displayed excellent performance in constrained optimization problems (Mohamed and Sabry, 2012). The application of other optimization algorithms to the problem (e.g., Particle Swarm

Optimization, Ant Colony Optimization, etc) may constitute an interesting area of future research.

3. Method

Differential Evolution (DE) is an evolutionary, population-based algorithm, for global optimization over continuous spaces. It was first introduced by Storn and Price (1997), and has been extensively applied to a wide domain of optimization problems due to its ability to efficiently handle non-differentiable, nonlinear and multimodal cost functions (for a state of the art survey see Das and Suganthan, 2011). Recently, DE has been implemented to solve the lot size problem in stochastic supply chain management systems (e.g., Lieckens and Vandaele, 2015), and also identify optimal groups of assets in active portfolio management (e.g., Krink et al., 2009).

DE's great popularity comes from its good convergence properties, as well as its parallelizability that enables the successful handling of computation intensive cost functions. DE is based on the Darwinian theory of Evolution (Engelbrecht, 2007): In a world with limited resources and stable populations, each individual competes with others for survival. The individuals with the best characteristics will more probably survive and reproduce. Those desirable characteristics (a) are passed on to their offspring, (b) are inherited by the subsequent generations, and (c) over time will become dominant among the population. During the production process of a child organism, random events may cause random changes to its characteristics. If these altered characteristics benefit the organism, then the likelihood of survival for the organism is increased.

In accordance to this, DE works with a group (population) of candidate solutions to the problem (individuals). The algorithm searches for the global optimum through an iterative process, as described below.

3.1 Initialization

DE begins with the random creation of a number of individuals. Each individual i corresponds to a candidate solution of the problem, and is represented by a vector $\vec{x}_i \in \Re^d$:

$\vec{x}_i$ =(x_{i1}, x_{i2},..., x_{id}), i = 1, 2,..., NP, $x \in \Re$,

where d is the number of problem's dimensions, and NP is the population size. The individuals of the initial population are randomly created and usually follow a uniform probability distribution. The initial population should cover the entire range of parameter values, or at least the domain space that may contain the global optimum.

3.2 Mutation

Once the initialization is completed, the mutation process follows, whereas other evolutionary algorithms first apply crossover. The mutation operator of DE generates new vectors of individuals by adding to a base vector the weighted difference between two difference vectors. For each individual i (represented by a target vector x_i) three vectors are randomly chosen from the population: a base vector x_b, and two differentials x_{d1}, x_{d2}, ($i \neq b \neq d1 \neq d2$). The mutant vector is then produced as follows: $u_i = x_b + F * (x_{d1} - x_{d2})$, where the scale factor F is a positive real number in [0, 2] that controls the amplification of the differential variation, which in turn controls the rate at which the population evolves. At each algorithm's iteration (generation) every individual i serves once as the target vector. This is known as "classic DE" or DE/rand/1, where the word rand denotes that the base vector is randomly chosen, and the number shows how many vector differences are considered for the perturbation (one in this case). The five most frequently referred mutation strategies are (Islam et al., 2012):

1. DE/rand/1: $u_i = x_b + F * (x_{d1} - x_{d2})$
2. DE/best/1: $u_i = x_{best} + F * (x_{d1} - x_{d2})$

3. DE/current-to-best/1: $u_i = x_b + F * (x_{best} - x_b) + F * (x_{d1} - x_{d2})$
4. DE/best/2: $u_i = x_{best} + F * (x_{d1} - x_{d2}) + F * (x_{d3} - x_{d4})$
5. DE/rand/2: $u_i = x_b + F * (x_{d1} - x_{d2}) + F * (x_{d3} - x_{d4})$

The indices d1, d2, d3, d4 are mutually exclusive integers randomly chosen from the range [1, NP], and are all different from the index b, while x_{best} is the best individual vector in the current population t.

3.3 Crossover

The crossover process follows, which produces an offspring x_i' (trial vector) through implementing a discrete recombination of the target vector x_i and the newly produced mutant vector u_i (Price et al., 2005):

$$\mathbf{x}'_{ij} = \begin{cases} \mathbf{u}_{ij}, & \textbf{if } \left(\textbf{rand}_j\ (\mathbf{0},\mathbf{1}) \leq Cr \textbf{ or } j = j_{rand}\right) \\ \mathbf{x}_{ij}, & \textbf{otherwise} \end{cases}$$

The crossover probability, Cr $\in$ [0,1], is defined by the user, and controls the fraction of parameter values that are copied from the mutant. In uniform crossover the value of Cr is compared to a random generated number from a uniform distribution in (0, 1). If the random number is less than or equal to Cr, the trial parameter is copied from the mutant, otherwise the parameter is inherited from the target vector. Furthermore, the trial parameter with randomly selected index j_{rand} is taken from the mutant, in order to ensure that the trial vector does not duplicate the target vector.

3.4 Selection

If the trial vector performs better than the target vector with regard to the problem's objective function, then the trial vector replaces the target vector in the population of the subsequent generation. Otherwise the target vector survives intact into the next generation. This constitutes the selection process of DE.

3.5 Pseudocode

A pseudocode of the Differential Evolution algorithm is presented below:

Initialization

Select the values of the control parameters F and Cr , and the population size NP

Select the maximum number of iterations t_{max} and set the iteration counter t=0

Generate the initial population Pop(0) of NP individuals

Evaluate the fitness $f(x_i(t))$ of each individual i of the initial population

Main phase

do until $t=t_{max}$

    for each individual $x_i(t) \in Pop(t)$ do

        Generate the mutant vector $u_i(t)$ through mutation

        Create the trial vector $x_i^{'}(t)$ through crossover

        Evaluate the fitness $f(x_i^{'}(t))$

        If $f(x_i^{'}(t))$ is better than $f(x_i(t))$ then

            Replace x_i with $x_i^{'}$ in Pop(t+1)

        else

            Add x_i to Pop(t+1)

        end if

    end for

t=t+1

end do

return the best solution

As Lampinen and Storn (2004) state, DE is self-adjusting because, in contrast to classical Evolutionary Strategies, it deduces the perturbation information from the distances between the vectors that comprise the population. This feature automatically yields reasonably large vector perturbations at the first phase of the optimization (exploratory stage). At the later stages, when the algorithm is approaching the optimum, the distances between the vectors automatically get smaller. These smaller perturbations allow DE to conduct a fine-grained search for the optimal solution. This self-adjusting property of DE uses fewer control mechanisms than other algorithms, making DE both easy to use and effective.

4. Implementing DE to the PL-PAP Optimization Problem

4.1 Data and variables

The proposed mechanism is implemented to empirical data that have been collected for the purposes of a large-scale telephone survey research examining consumer buying behaviour and preferences in the grocery market of a European metropolitan area. A highly structured questionnaire was developed and data were collected from a random sample of 1,928 supermarket customers. The telephone survey was conducted by the Computer Assisted Telephone Interviewing (CATI) facilities of a local university. Respondents, among others, were asked to state their average expenditure per supermarket visit, the number of supermarket visits per month, the supermarket store format they usually prefer for their main shopping, the amount of money they usually spend on PLs, and the PL categories they mostly prefer. In total we examined consumer preferences for a set of twelve product categories that are usually available in a typical supermarket.

In Table 1, some basic descriptive statistics of our sample are presented in a condensed form. As is shown, we recruited customers from three distinct supermarket store formats, which differ significantly in terms of the assortment they offer: small local supermarket chains (7.6%), discount supermarket chains (5.2%), and large mainstream supermarket chains (87.1%). In terms of buying behaviour, our sample spends approximately 67.7 euros per supermarket visit and pays 6.9 supermarket visits per month, while 21.3% of its budget is spent on PL products. In Table 1 the percentage of customers who buy PLs per product category is also presented. It can be inferred that the majority of respondents buys PLs from categories such as disposable paper products (70.7%), packaged foods (57.1%), and household cleaning products (40.5%), while PLs in product categories such as clothing products (10.2%), tea/coffee (12.5%) and alcoholic beverages (13.0%) are the least successful in terms of customer demand.

Take in Table 1.

4.2 Solution representation

Since the number of categories is m=12, we represent a potential solution i to the problem with a vector: $\vec{x}_i = (x_{i1}, x_{i2},..., x_{ir},\dots, x_{i12})$, $x \in [0, 1]$, where x_{ir} is the percentage of PLs (PL service level) that the retailer carries in category r (pl_r). The parameter x_{ir} is allowed to take any real value in the range [0, 1], which corresponds to a 0-100% percentage range. The percentage of NBs (NB service level) in the same category (b_r) is easily derived from constraint (2). We set the number of customers visiting the retailer per month equal to n=1,928. Also, the monthly amount that customer c spends on PLs (αpl_{cr}) and NBs (αb_{cr}) are known. Hence, we are looking for the optimal $\vec{x}$ that maximizes the objective function (1). An issue that arises is the handling of constraint (3). The reproduction process of DE (mutation and crossover) is possible to extend the search outside of the range of the search space ([0, 1]). In order to ensure that parameter values lay inside their allowed ranges after reproduction we adopt the approach for boundary constrained problems by Lampinen and Zelinka (1999) (i.e., the parameter values that violate boundary constraints are replaced with random values generated within the feasible range).

4.3 Selection of DE Parameters

We fine-tuned the parameters of DE to select the best configuration for the optimal PL-PAP problem. Based on suggested general guidelines from previous research (Mezura-Montes et al., 2006) we tested six different values for the Population Size (NP), seven different values for the maximum number of iterations (t_{max}), seven different values for the Scaling Factor (F), and four different values for the Crossover rate (CR). Table 2 illustrates the values for each of the four parameters in the 5x7x7x4 full factorial design that was implemented.

Take in Table 2.

We performed 25 replications (5 for each of the five mutation strategies) for each of the 980 combinations of the four parameters, resulting in a total of 24,500 runs of the algorithm. The results indicate that for more than 100 iterations, and for NP>80 there is no gain in performance, while the best performance was achieved for F=0.3 and CR=0.9.

4.4 Performance Evaluation

We implement the DE algorithms to find optimal solutions (i.e., PL service level per category) in the entire dataset and each of the three store-formats separately. The parameter values used for all the five versions of the algorithm (different mutation strategies) are t_{max}=100, NP=80, F=0.3, and CR=0.9. Without loss of generality we assume that customers equally divide the PL monthly budget to each category. In order to evaluate the performance of our approach we compare the results of the five DE versions to that of a Simulated Annealing (SA) algorithm, which has been applied in the assortment planning problem in the past. SA is an optimization algorithm introduced by Kirkpatrick et al. (1983). It is a local search algorithm inspired from the physical process of annealing in metallurgy. The algorithm begins with the generation of a random initial solution (PL service level per category), which is evaluated using the problem's objective function (eq. 1 - retailer's sales volume). An iterative process follows, where a single parameter x_{ir} of the potential solution vector $\vec{x}_i$ is randomly altered at each iteration. If the change improves the value of the objective then the change is accepted, else it is accepted with a probability P as follows:

$$P = e^{\left(f - f' / T\right)},$$

where f is the retailer's sales volume after the change, *f'* is the retailer's sales volume before the change, and T is the "temperature", a control parameter of the algorithm. T takes a relatively large value in the initial stages of the algorithm, which allows the acceptance of a high percentage of changes that worsen the objective function. This enables the algorithm to escape from possible local optima, and favors global search. The initial value of T is gradually decreased during the process, and the algorithm accepts fewer worsening changes, favoring local search at the final stages. We fine-tuned the parameters of SA to select the best configuration for the optimal PL-PAP problem. We tested several values for the initial T in the range [10, 100], as well as for the number of parameter x_{ir} changes per temperature level in the range [1000, 10000]. After calibration, we set a number of 8000 parameter x_{ir} changes and an initial T=37. We decrease T by multiplying the current temperature level by 0.75 in each iteration. This results in 27 different temperature levels and a total of 216000 changes. Throughout the algorithm's iterations the best solution so far is stored, in order to be protected from a worsening change.

5. Results

All algorithms were developed and executed in Matlab. We performed 30 independent runs for each algorithm on a 2.8GHz-i7 PC with 8GB RAM. Table 3 provides statistical results for the performance of the SA and the five DE implementations. The results show the average values across the 30 runs, and are calculated as a percentage of the best solution for each problem (entire dataset and each of the three store-formats) found by any algorithm in a single run.

Take in Table 3.

On average, the DE/best implementations exhibit the highest performance, followed by the DE/rand implementations. The superiority of the DE/best implementations to Simulated Annealing, as well as DE/current-to-best is statistically significant ($p<0.05$). With regard to computational time the DE implementations terminate after 11.2sec on average, and SA terminates after 89.5sec.

Table 4 illustrates the best solution (% of merchandize inventory allocated for PLs in each category) found in each case.

Take in Table 4.

The retailer can also decide on the threshold below which the introduction of PLs in a specific category is not desirable. This threshold may vary across retailers depending on the respective inventory and handling costs. If for example, a retailer sets this threshold to 5%, the derived optimal solution for the entire dataset in Table 4 suggests that this retailer should mainly focus its efforts on providing extensive PL service levels in product categories such as disposable paper products and packaged foods, and also maintain a decent PL presence in categories such as bakery, laundry, household cleaning products, tea-coffee, and non-alcoholic beverages. The derived percentages are well below the threshold of 5% in categories such as frozen foods, personal hygiene products and clothing products; a finding which implies that the introduction of PLs in these categories would not be advisable. These results are also graphically depicted in Figure 1, which also outlines the optimal balance between PL and NB service levels per category, based on constraint (2).

Regarding the adaptation of PL service levels across store formats, interesting conclusions can be drawn from Table 4. For example, managers of large mainstream supermarket chains must offer extensive PL service levels in categories such as disposable paper products and packaged foods, whilst they should also maintain a decent PL presence in categories such as laundry and dairy products. In line with our expectations, discount retailers are expected to provide broader varieties of PLs, because in addition to the PL categories offered by mainstream supermarkets, discounters must also provide extensive PL service levels in household cleaning products. Finally, the derived optimal percentages in most product categories of local supermarket chains are extremely low. This finding indicates that local grocery stores should concentrate their efforts in providing a narrow variety of PLs, by focusing on few categories, such as packaged food and laundry products. A cross-format comparison of the optimal PL service levels per category is graphically depicted in Figure 2.

It can be seen that discount supermarkets not only must offer broader varieties of PLs, but also more extensive service levels within those varieties (the derived percentages are higher compared to the respective percentages of the other two store formats).

Take in Figure 1.

Take in Figure 2.

6. Conclusions-Discussion

The present paper introduces evolutionary analysis to strategic assortment planning. We have shown how Differential Evolution algorithms can address assortment management problems and identify optimal PL varieties and service levels in a retailer's product portfolio. The performance of five different DE implementations was benchmarked against Simulated Annealing. The interrelated issue of assortment adaptation across different retail store formats is also taken into consideration.

It is widely recognized that economic recessions contribute to the prolonged upward evolution in PL share, leaving scars on NBs performance levels. As a result, the proposed mechanism facilitates PL-PAP decisions that are nowadays more important than ever before. Evolutionary notions such as selection and variation shed new light on retail and assortment management, as they can facilitate, in a unified framework, important decisions related to optimal variety of PL product categories, optimal service level of PL merchandise within a product category, and optimal balance between PLs and NBs in a retailer's product portfolio.

Nowadays most retailers gather huge amounts of data each day. Sales transactions are recorded with the use of scanner panels, or online systems in case of Internet sales. Due to developments in scanning and electronic point of sale technologies, it is increasingly possible to monitor and model contributions of certain items within product categories to sales. Decisions about assortment reductions, therefore, seem straightforward. Such large amounts of data are hard to be manipulated, requiring sophisticated methods such as optimization algorithms. Implementation of algorithms as the one presented here to big scanner data will give retailers the opportunity to track sales effects and allow timely adjustments of merchandise and other marketing mix elements. The effective processing of such data transforms them into valuable information that can provide a company with sustainable competitive advantage. DE can assist retailers in determining the optimal level of inventory and variety to carry. Those with experience in optimization methods and computer programming can find several DE implementations on the Internet (e.g., see http://www1.icsi.berkeley.edu/~storn/code.html). Retailers not familiar with software customization may consult specialized professionals. Ultimately, DE can be integrated into a retailer's existing information system, or even constitute the foundation for the development of new system or standalone application.

In conclusion, we believe that evolutionary analysis can open new avenues and reveal exciting opportunities not merely for new research, but for novel, revolutionary views of market behavior. Differential evolution algorithms can be applied to several similar marketing problems such as advertising scheduling, service design and diversification, product line management and innovation. We hope that the ideas presented here will motivate research in new ways to view and innovative methods to address marketing problems.

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Table 1. Descriptive statistics

Variable	Mean (Stand. Dev.)
Number of supermarket visits per month	6.9 (5.2)
Expenditure per supermarket visit (in euros)	67.7 (66.9)
Percentage of budget spent on PLs	21.3 (19.3)
Store format most frequently visited for main shopping	**Total sample (in %)**
Small local supermarket chain	7.6
Discount supermarket chain	5.2
Large mainstream supermarket chain	87.1
Product category	**Percent of customers buying PLs - Yes (No)**
Frozen Foods	20.1 (79.9)
Packaged Foods	57.1 (42.9)
Laundry Products (e.g., detergents)	35.3 (64.7)
Household Cleaning Products	40.5 (59.5)
Personal Hygiene Products	14.7 (85.3)
Disposable Paper Products	70.7 (29.3)
Non-Alcoholic Beverages (e.g., soft drinks, juices, bottled water)	17.7 (82.3)
Dairy Products	24.2 (75.8)
Bakery Products	29.3 (70.7)
Clothing Products	10.2 (89.8)
Tea and Coffee	12.5 (87.5)
Alcoholic Beverages (e.g., wines, beers)	13.0 (87.0)

Table 2. Parameters and Values used in Full Factorial Design Experiment

Parameters	Values						
Population Size (NP)	40	60	80	100	120		
Number of iterations (t_{max})	50	60	70	80	90	100	110
Scaling Factor (F)	0.3	0.4	0.5	0.6	0.7	0.8	0.9
Crossover rate (CR)	0.2	0.4	0.7	0.9			

Table 3. Performance comparison results

	DE/rand/1	DE/best/1	DE/current-to-best/1	DE/best/2	DE/rand/2	Simulated Annealing
Best	0.9932	0.9968	0.9907	0.9973	0.9940	0.9915
Mean	0.9683	0.9736	0.9662	0.9791	0.9704	0.9566
Worst	0.9261	0.9285	0.9109	0.9327	0.9283	0.8947
Standard Deviation	0.0085	0.0047	0.0088	0.0021	0.0057	0.0097

Table 4. Optimal PL Service Levels per Category*

FF	PF	LP	HCP	PHP	DPP	NAB	DP	BP	CP	TC	AB
Entire data set											
0.4	47.29	7.18	6.71	1.48	82.83	5.2	4.62	10.76	1.78	6.12	3.94
Large mainstream supermarket chains											
2.23	68.2	7.75	2.49	0.5	83.94	0.49	6.11	2	0.82	1.37	0.3
Discount supermarket chains											
0.42	87.45	51.06	83.89	1.42	86.42	1.32	5.2	1.58	1.02	0.64	2.86
Small local supermarket chains											
0.7	6.75	6.46	4.02	2.98	3.39	0.27	0.05	0.23	1.63	0.88	2.58

*FF: Frozen foods, PF: Packaged foods, LP: Laundry products (e.g., detergents), HCP: Household cleaning products, PHP: Personal hygiene products, DPP: Disposable paper products, NAB: Non-alcoholic beverages (e.g., soft drinks, juices, bottled water), DP: Dairy products, BP: Bakery products, CP: Clothing products, TC: Tea and coffee, AB: Alcoholic beverages (e.g., wines, beers).

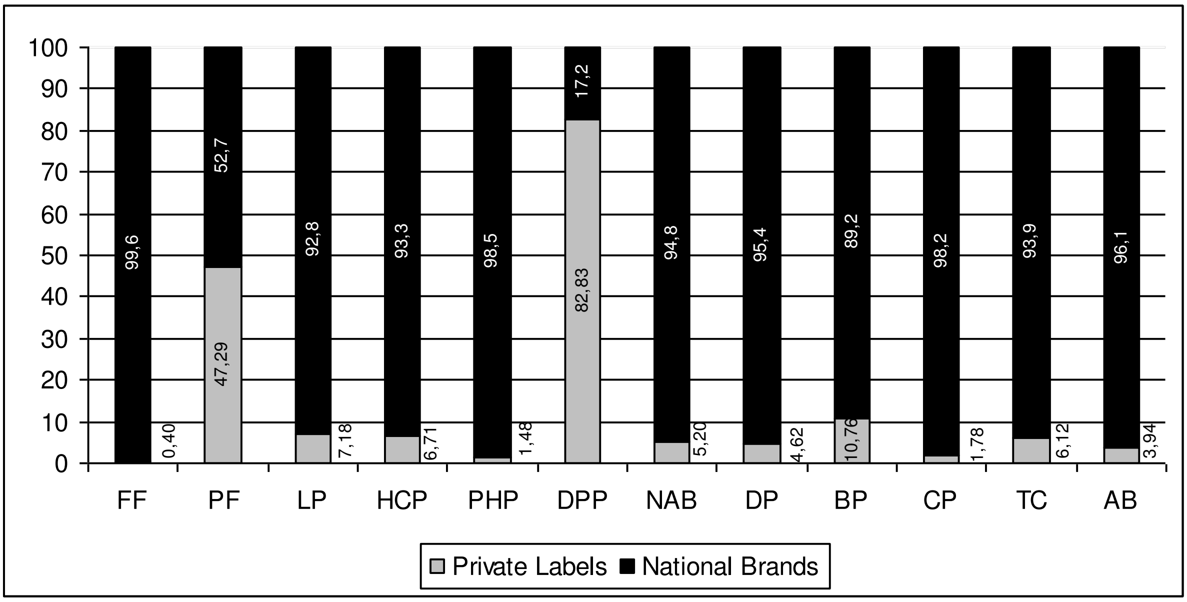

Figure 1. Optimal Balance Between PLs and NBs

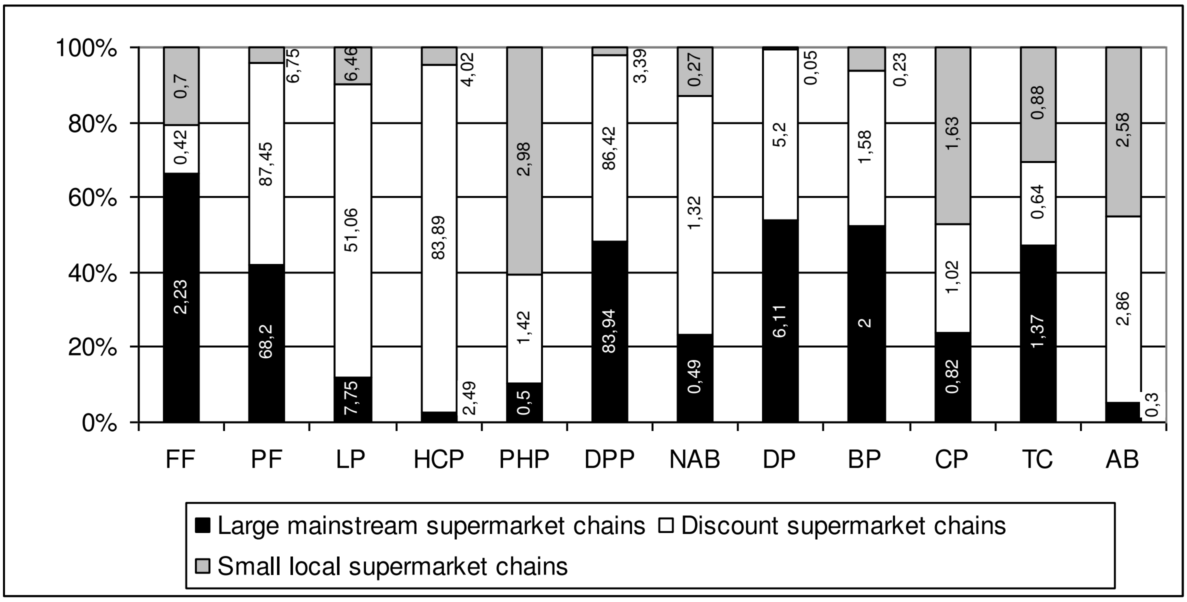

Figure 2. Optimal PL Service Levels across Store formats