C. Bagni et alii, Frattura ed Integrità Strutturale, 33 (2015) 105-110; DOI: 10.3221/IGF-ESIS.33.14

Focussed on characterization of crack tip fields

Gradient-enriched linear-elastic tip stresses to perform the high-cycle fatigue assessment of notched plain concrete

C. Bagni, H. Askes, L. Susmel
Department of Civil and Structural Engineering, The University of Sheffield, Sheffield S1 3JD, United Kingdom
c.bagni@sheffield.ac.uk, h.askes@sheffield.ac.uk, l.susmel@sheffield.ac.uk

ABSTRACT. Gradient Elasticity (GE) allows the stress analysis to be performed by taking into account the size of the dominant source of microstructural heterogeneity via a suitable length scale parameter. This is done by simply assuming that the material under investigation obeys a linear-elastic constitutive law, albeit equipped with additional spatial strain gradients. From a practical point of view, the most important implication of this *modus operandi* is that gradient-enriched linear-elastic stresses at the notch tips are always finite, this holding true also in the presence of sharp stress risers (such as cracks). In the present investigation, the accuracy of two different GE based design strategies was checked against a number of experimental results generated by testing, under cyclic four-point bending, plain concrete samples containing different geometrical features. The high level of accuracy which was obtained by directly using gradient-enriched linear-elastic notch stresses strongly supports the idea that GE is a powerful tool suitable for designing notched concrete components against high-cycle fatigue. This result is very promising also because the required stress analysis can directly be performed by using standard Finite Element (FE) solvers.

KEYWORDS. Concrete; Fatigue; Notch; Gradient elasticity.

INTRODUCTION

In the civil infrastructure sector, concrete is the most commonly used material. This explains the reason why the problem of optimising the static assessment of concrete has been studied by the international scientific community for decades. The outcomes from this enormous amount of work allow modern concrete structures to be efficiently designed against static loading via the adoption of low safety factors. By so doing, slender concrete structures can safely be built by remarkably reducing the usage of natural resources, this having positive effects not only on sustainability, but also on carbon emissions. Owing to the fact that the magnitude of in-service local stresses increases as the size of concrete structural components decreases, slender concrete structures are obviously more susceptible to fatigue. Examination of the state of the art shows that, since the beginning of the 1900s [1, 2], much research work has been done to formalise efficient design methods suitable for estimating the fatigue damage extent in concrete components subjected to time-variable loading. Unfortunately, the experimental work carried out so far has resulted in design curves suitable for estimating the fatigue lifetime solely for those specific concrete mixtures that were tested. It has to be highlighted also that, apart from three isolated investigations [3-5], no effort has been made so far to devise and validate (through appropriate experimental investigations) specific techniques capable of modelling the detrimental effect of notches on the overall fatigue strength of plain concrete.
In this setting, the aim of the present investigation is to check whether linear-elastic gradient-enriched notch tip stresses are successful in performing the high-cycle fatigue assessment of notched plain concrete.

FUNDAMENTALS OF GRADIENT ELASTICITY

In about the middle of the last century, Mindlin [6] investigated the most important features of gradient enriched elasticity by proposing an alternative continuum theory based on the use of a number of non-conventional constitutive parameters. Even if this theory is certainly very elegant, its implementation is not straightforward at all due to its intrinsic complexity. In light of the doubtless peculiarities of this approach, since the pioneering work done by Mindlin, the international scientific community has made a tremendous effort to try to simplify this theory to make it suitable for being used in situations of practical interest. Amongst the different formalisations which have been proposed so far, one of the most appealing solutions is the one proposed by Aifantis and co-workers [7-9]. This approach takes as its starting point the assumption that the enriched stress vs. strain relationship can be reformulated by using the Laplacian of the strain, i.e.:

$$\sigma = C\left(\varepsilon - \ell^2 \nabla^2 \varepsilon\right) \tag{1}$$

In constitutive law (1), σ and ε are second-order tensors with the stress and strain components, respectively, C is a fourth-order tensor containing the material elastic moduli, and ℓ is the length scale parameter which is employed to describe the underlying microstructural features of the material being modelled.
Thanks to the specific features of the above formalisation of GE, constitutive law (1) can efficiently be implemented numerically, the stress and strain analysis being directly performed by using standard FE solvers [10-12]. In more detail, initially the following conventional FE equation has to be solved:

$$K\underline{u} = \underline{f} \tag{2}$$

where $\underline{u}$ is the vector with the nodal displacements, $\underline{f}$ is the vector containing the nodal forces, and K is the linear-elastic stiffness matrix. The displacements determined from Eq. (2) allow the gradient-enriched nodal stresses $\underline{\sigma}$ to be determined as follows:

$$\int_{\Omega}\left(N^T S N + \sum_{\xi} \frac{\partial N^T}{\partial \xi} S \ell^2 \frac{\partial N}{\partial \xi}\right) d\Omega\, \underline{\sigma} = \int_{\Omega} N^T B\, d\Omega\, \underline{u} \tag{3}$$

where N is the matrix containing the shape functions, B is the matrix containing the derivatives of the displacement, and S is the elastic compliance matrix.
It is possible to conclude by highlighting that if constitutive law (1) is adopted to model the stress vs. strain behaviour of engineering materials, the use of material length scale parameter ℓ leads to linear-elastic stress fields which are never singular, this holding true also in the presence of cracks and sharp notches [13].

EXPERIMENTAL INVESTIGATION

To assess the accuracy of GE in modelling the high-cycle fatigue behaviour of notched plain concrete, 100mm x 100mm square section beams weakened by different types of notches were tested under cyclic four-point bending. The specimen length was equal to 500 mm and the nominal notch depth to 50 mm. The tested notched specimens contained U-notches with root radius, r_n, equal to 25 mm, 12.5 mm, and 1.4 mm, resulting in a net stress concentration factors, K_t, equal to 1.47, 1.84, and 4.32, respectively. The un-notched specimens used to determine the reference endurance limits had gauge length width equal to 50mm and thickness equal to 100 mm.
The concrete mix used to manufacture the tested samples contained Portland cement (strength class equal to 32.5 N/mm^2), natural round gravel (10 mm grading), and grade M concrete sand. In order to manufacture specimens having the same microstructural features, but different strength, two values for the water-to-cement ratio were investigated: Batch A was manufactured by setting the water-to-cement ratio equal to 0.5, whereas Batch B was cast by taking the water-to-cement ratio equal to 0.4. The samples were removed from the moulds 24 hours after casting, being subsequently stored for 28 days in a moist room at 23°C.
The static strength of the investigated concrete mixes was determined under three-point bending. The bending strength of Batch A was seen to be equal to 4.9 MPa, whereas the bending strength of Batch B to 6.5 MPa.

The un-notched specimens as well as the notched samples were loaded in cyclic four-point bending at a frequency of 10 Hz. For the force-controlled fatigue tests the adopted failure criterion was the complete breakage of the specimens.
The experimental results generated according to the above experimental procedure are shown in Fig. 1 together with the corresponding endurance limits. In this figure $\sigma_{0,MAX}$ indicates the un-notched material endurance limits, whilst σ_{MAX} denotes the notch endurance limits referred to the nominal net area. The symbol R_m is used to indicate the average value of the load ratio characterising any data sets. The above endurance limits were all estimated at $2 \cdot 10^6$ cycles to failure by post-processing the fatigue data according to the up-and-down method proposed by Dixon [14].
To conclude, it is worth observing that the determined endurance limits were estimated in terms of $\sigma_{0,MAX}$ and σ_{MAX} because, under either cyclic axial loading or bending, the maximum stress in the cycle is seen to be a stress quantity capable of efficiently taking into account the mean stress effect in concrete fatigue [15].

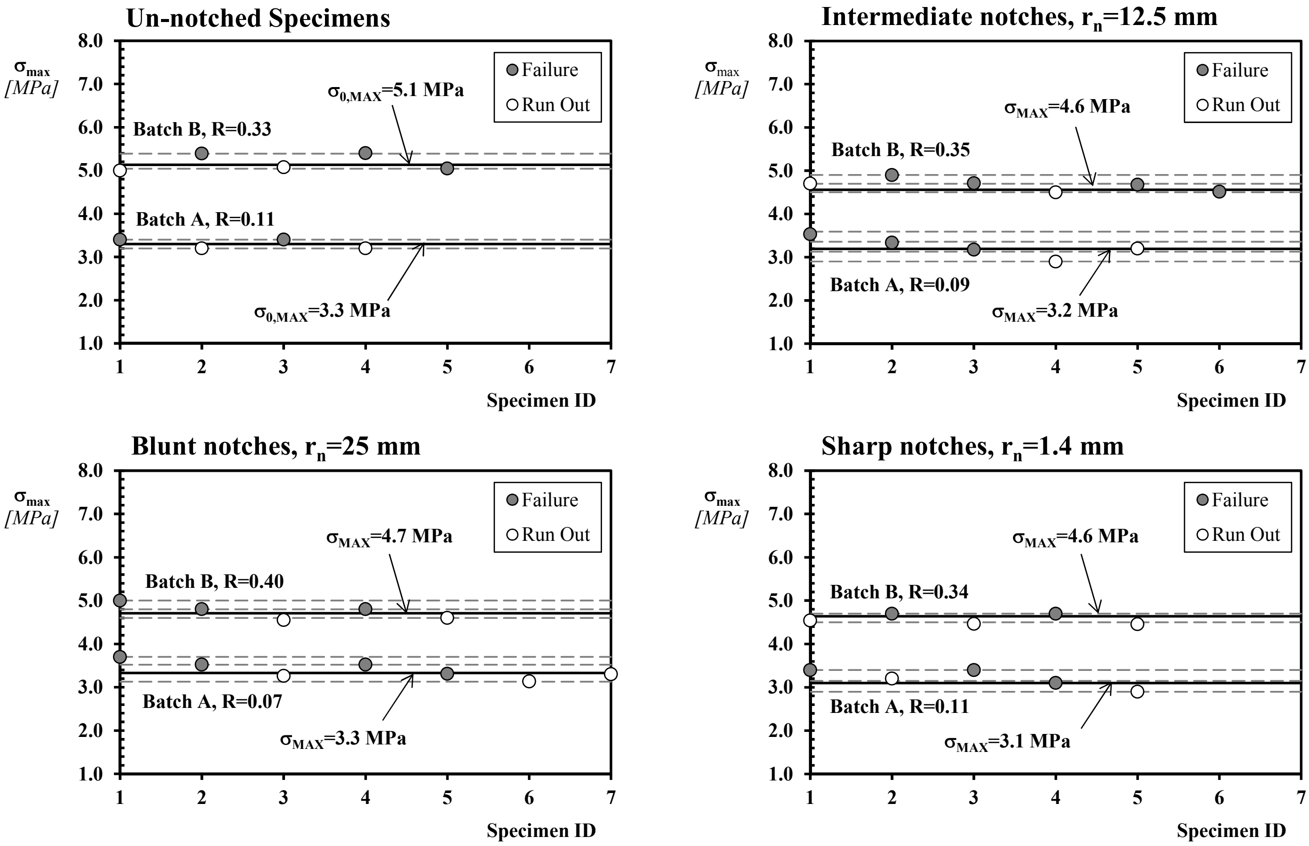

Figure 1: Summary of the generated fatigue results and endurance limits estimated according to Dixon's technique [14].

GRADIENT-ENRICHED TIP STRESSES TO DESIGN NOTCHED CONCRETE AGAINST HIGH-CYCLE FATIGUE

The accuracy of GE in estimating high-cycle fatigue strength of notched concrete was checked against the generated experimental data by using this approach according to two different strategies as described in what follows. Initially GE was applied along with the so-called Theory of Critical Distance (TCD) [16, 17]. The TCD postulates that the high-cycle fatigue strength of a notched component can be estimated via critical distance L which takes on the following value [16]:

$$L = \frac{1}{\pi}\left(\frac{\Delta K_{th}}{\Delta \sigma_0}\right)^2 \tag{4}$$

In the above definition ΔK_{th} is the range of the threshold value of the stress intensity factor, whereas $\Delta\sigma_0$ is the range of the un-notched material endurance limit.

By following an articulated reasoning based on non-local mechanics [18], it is possible to prove that characteristic length ℓ can directly be estimated from the TCD's critical distance as follows:

$$\ell \approx \frac{L}{2\sqrt{2}} \tag{5}$$

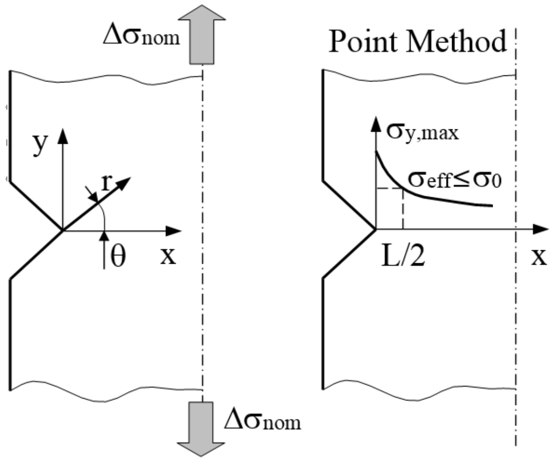

Figure 2: Effective stress, σ_{eff}, determined according to the PM.

The simplest formalisation of the TCD is known as the Point Method (PM) [17]. As shown in Fig. 2, the PM postulates that a notched/cracked engineering material is in the endurance limit condition as long as the stress at a distance from the assessed geometrical feature equal to L/2 is lower than (or, at least, equal to) the un-notched material endurance limit, σ_0.
The PM can also be used to determine critical distance L when the range of the threshold value of the stress intensity factor is not available [19, 20]. In particular, assume that a notch endurance limit experimentally determined by testing samples containing a known geometrical feature is available for the material being investigated. According to the PM, at the endurance limit, L/2 is equal to the distance from the notch tip at which the local linear-elastic stress equals the un-notched material endurance limit.
As shown in the charts of Fig. 3, this alternative strategy was adopted to determine the TCD critical distance for the investigated concrete. This simplified procedure resulted in an L value equal to 5.8 mm for both batches. The fact that the two tested batches of concrete had the same microstructural features with different strength strongly supports the idea that the TCD critical distance value is mainly related to the morphology of the material being assessed. The linear-elastic stress-distance curves plotted in Fig. 3 were determined from bi-dimensional FE models solved by using commercial software ANSYS®. Further, these curves were estimated, at the endurance limit, under the maximum stress in the cycle in order to correctly take into account the presence of non-zero mean stresses [15]. Fig. 4 shows the linear-elastic gradient-enriched stress-distance curves determined, in the endurance limit condition, by taking, according to Eq. (5), the material characteristic length, ℓ, equal to:

$$\ell = \frac{L}{2\sqrt{2}} = \frac{5.8}{2\sqrt{2}} = 2.05 \ mm \tag{6}$$

As done when estimating the TCD critical distance L (Fig. 3), these curves were determined, by using an in-house FE code, under the maximum loading in the cycle to model the mean stress effect in concrete fatigue [15].
The above charts make it evident that the use of the linear-elastic gradient-enriched notch tip stresses [21] resulted in estimates mainly falling, on the conservative side, within an error interval equal to ±20%. It is worth observing here that, as postulated by the TCD [16, 17], such a satisfactory level of accuracy was reached by using the conventional un-notched material endurance limit as reference fatigue strength.
As briefly recalled above, the length scale parameter ℓ allows the underlying material microstructure to be directly incorporated into the stress analysis. A close inspection of the fracture surfaces of the broken samples revealed that in the tested concrete fatigue cracks tended to initiate at the interface between matrix and aggregates, the subsequent propagation mainly occurring in the cement paste. Both for Batch A and Batch B, the average distance between the aggregates was measured to be equal to about 4 mm. Accordingly, one may argue that length scale parameter ℓ should

directly be taken equal to the average inter-aggregate distance, the results generated by testing the un-notched specimens being used to determine a suitable value for the reference strength.

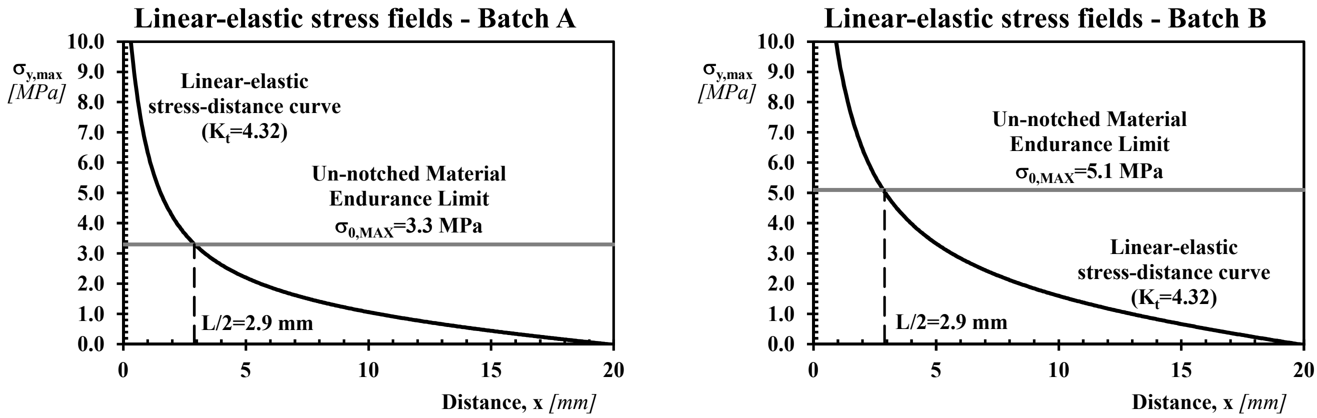

Figure 3: Critical distance L determined for Batch A and Batch B according to the PM.

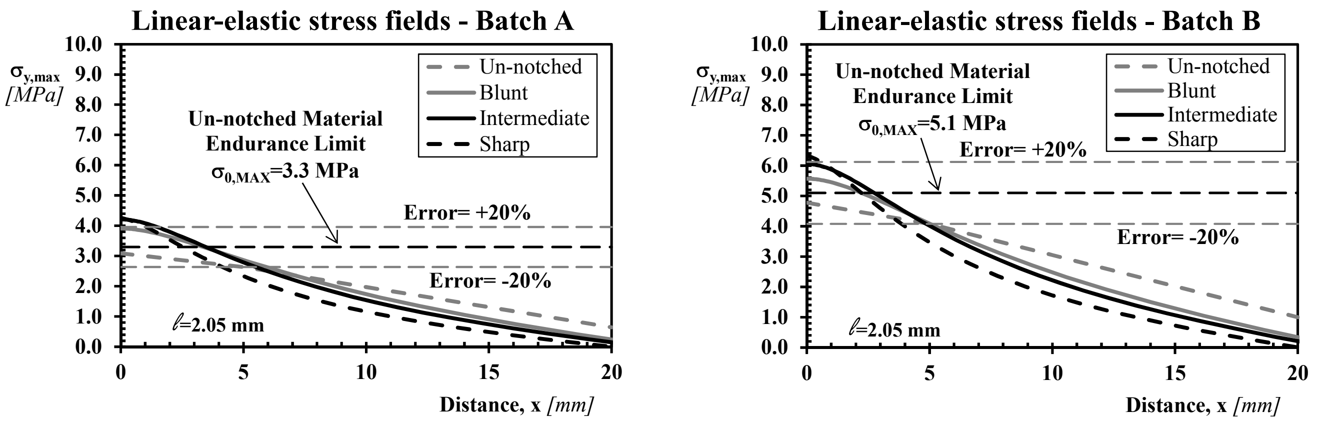

Figure 4: Accuracy of GE applied by taking ℓ = 2.05 mm, Eq. (6).

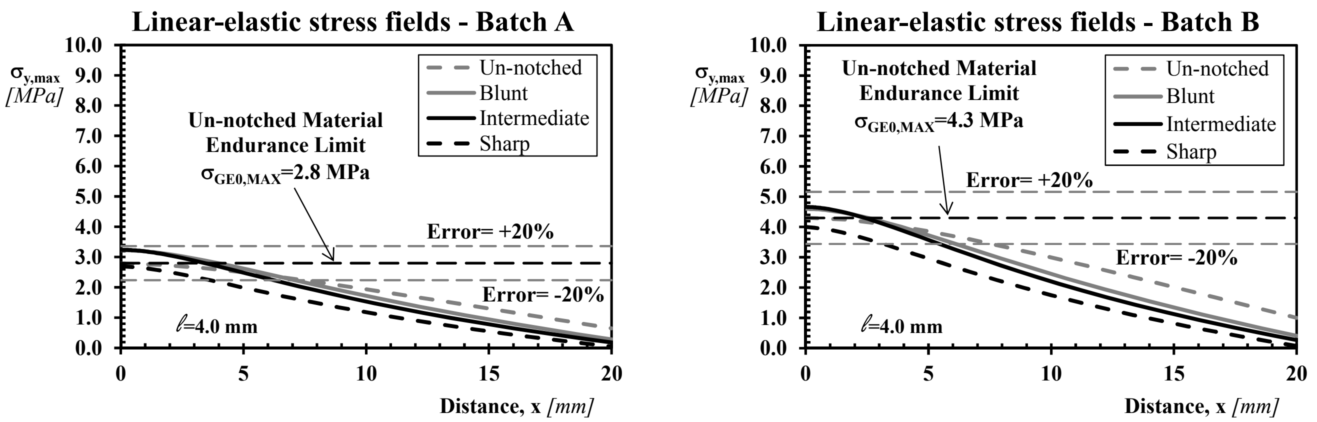

Figure 5: Accuracy of GE applied by taking ℓ equal to the average inter-aggregate distance (ℓ = 4 mm).

The charts reported in Fig. 5 show the linear-elastic gradient-enriched stress-distance curves plotted, at the endurance limit, under the maximum loading in the cycle. According to this figure, the reference strength, $\sigma_{GE0,MAX}$, determined from the results generated by testing the un-notched specimens was equal to 2.8 MPa for Batch A and to 4.3 for Batch B. These diagrams clearly prove that the use of the gradient-enriched notch tip stresses determined by taking ℓ equal to the average inter-aggregate distance resulted in highly accurate estimates. Owing to the fact that the two batches of concrete were characterised by the same morphology, this remarkable accuracy strongly supports the idea that length scale parameter ℓ in constitutive law (1) is capable of directly incorporating into the stress analysis the underlying material microstructural features.

CONCLUSIONS

- GE applied along with the TCD was seen to result in conservative estimates with the level of conservatism decreasing as the sharpness of the notch decreases.
- The use of GE with length scale parameter ℓ equal to the average inter-aggregate distance resulted in highly accurate estimates for the fatigue strength.

REFERENCES

[1] Von Ornum, J.L., Fatigue of cement products, ASCE Trans., 51 (1903) 443-451.
[2] Von Ornum, J.L., Fatigue of concrete, ASCE Trans., 58 (1907) 294-320.
[3] Ohlsson, U., Daerga, P.A., Elfgren, L., Fracture energy and fatigue strength of unreinforced concrete beams at normal and low temperatures, Engng Frac. Mech., 35 l/2/3 (1990) 195-203.
[4] Plizzari, G.A., Cangiano, S., Alleruzzo, S., The fatigue behaviour of cracked concrete, Fatigue Fract. Engng Mater. Struct., 20 8 (1997) 1195-1206.
[5] Thun, H., Ohlsson, U., Elfgren, L., A deformation criterion for fatigue of concrete in tension, Structural Concrete, 12 3 (2011) 187-197.
[6] Mindlin, R.D., Micro-structure in linear elasticity. Arch. Rat. Mech. Analysis, 16 (1964) 52–78.
[7] Aifantis, E.C., On the role of gradients in the localization of deformation and fracture, Int. J. Engng. Sci., 30 (1992) 1279–1299.
[8] Altan, S.B., Aifantis, E.C., On the structure of the mode III crack-tip in gradient elasticity, Scripta Metall. Mater. 26 (1992) 319–324.
[9] Ru, C.Q., Aifantis, E.C., A simple approach to solve boundary-value problems in gradient elasticity, Acta Mech. 101 (1993) 59–68.
[10] Askes, H., Gutiérrez, M.A., Implicit gradient elasticity, Int J Numer Meth Engng, 67 (2006) 400-416.
[11] Askes, H., Morata I., Aifantis E.C., Finite element analysis with staggered gradient elasticity, Comput Struct, 86 (2008) 1266-1279.
[12] Askes, H., Gitman, I.M., Non-Singular Stresses in Gradient Elasticity at Bi-Material Interface with Transverse Crack, Int J Fract, 156 (2009) 217-222.
[13] Askes, H., Aifantis, E.C., Gradient elasticity in statics and dynamics: An overview of formulations, length scale identification procedures, finite element implementations and new results, Int. J. Solids Struct., 48 (2011) 1962–1990.
[14] Dixon, W.J., The up-and-down method for small samples. Journal of the American Statistical Association, 60 (1965) 967-978.
[15] Susmel, L., A unifying methodology to design un-notched plain and short-fibre/particle reinforced concretes against fatigue, Int. J. Fatigue, 61 (2014) 226–243.
[16] Taylor, D., Geometrical effects in fatigue: a unifying theoretical model, Int J Fatigue, 21 (1999) 413-420.
[17] Taylor, D., The Theory of Critical Distances: A new perspective in fracture mechanics, Elsevier, Oxford, UK (2007).
[18] Susmel, L., Askes, H., Bennett, T., Taylor, D., Theory of Critical Distances vs. Gradient Mechanics in modelling the transition from the short- to long-crack regime at the fatigue limit, Fatigue Fract Engng Mater Struct., 36 9 (2013) 861-869.
[19] Susmel, L., Taylor, D., A novel formulation of the Theory of Critical Distances to estimate Lifetime of Notched Components in the Medium-Cycle Fatigue Regime, Fatigue Fract. Engng. Mater. Struct., 30 7 (2007) 567-581.
[20] Susmel, L., Multiaxial Notch Fatigue: from nominal to local stress-strain quantities, Woodhead & CRC, Cambridge, UK, (2009).
[21] Askes, H., Susmel, L., Understanding cracked materials: is Linear Elastic Fracture Mechanics obsolete?, Fatigue Fract. Engng. Mater. Struct., 38 (2015) 154–160.