



Estimation of fretting fatigue life using a multiaxial stress-based critical distance methodology

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ABSTRACT. This work presents a methodology for life estimation of mechanical couplings subjected to fretting fatigue. In this approach, a stress-based multiaxial fatigue parameter is evaluated at a critical distance below the contact surface. The fatigue parameter is based on an improved formulation of the Modified Wöhler Curve Method, in which the shear stress amplitude is measured via the Maximum Rectangular Hull method. To apply the Theory of Critical Distances in the medium-cycle fatigue regime, the critical distance is assumed to depend on the number of cycles to failure. Available fretting fatigue data, conducted on a cylinder-plane contact configuration made of Al alloy 4% Cu, were used to assess the methodology. Most of the life estimates were within an error band given by a factor of 2.

KEYWORDS. Fretting fatigue; multiaxial fatigue, life estimation; Theory of Critical Distances.

INTRODUCTION

Fretting fatigue refers to the conjoint action of a small oscillatory motion between contacting bodies and a cyclic remote loading. The oscillatory motion often leads to surface damage phenomena that may speed up the formation of micro-cracks. Due to the remote loading, these cracks may propagate until catastrophic failure occurs. Many engineering assemblies are prone to fretting fatigue problems as, for instance, the blade/disk interfaces of gas turbine engines, wires of overhead conductors and riveted joints.

Fretting fatigue usually displays high stress gradients that affect observed lives [1, 2]. Among the different formulations that account for the stress gradient in fretting, non-local approaches have received considerable attention in the literature [3]. The appeal of such approaches derives from their relative simplicity and from satisfactory correlations with experimental data [2, 4-7].

The Theory of Critical Distances (TCD) [8-10] has been one of the most used non-local approaches in the last ten years. In this approach, the size of the fatigue process zone is related to fatigue thresholds of cracked or sharply notched specimens. The success of the TCD in correlating fatigue thresholds of notched members has been widely reported in the

literature (see [9, 10] and references therein). Due to the similarities between notch and fretting fatigue, an attempt to estimate fretting fatigue thresholds using the TCD has been carried out by Araújo et al. [6]. In that paper, the Modified Wöhler Curve Method (MWCM) [11] was applied at a point located at a critical distance below the trailing edge of the contact. Estimates of fretting fatigue thresholds fell within an error interval of $\pm 20\%$ when compared to experimental data.

The methodology proposed by Araújo et al. [6] is only applicable to design situations involving threshold conditions. In this paper, an extension of this approach to the medium-cycle fatigue regime is presented. A first attempt to assess the new methodology is carried out based on available fretting fatigue tests [1].

MULTIAXIAL FATIGUE LIFE ESTIMATION

The MWCM [10, 11, 12] is a multiaxial stress-based critical plane approach where the driving parameters for crack nucleation are the maximum shear stress amplitude, τ_a , and the maximum normal stress acting on the maximum shear stress plane, $\sigma_{n,max}$. Once the values of τ_a and $\sigma_{n,max}$ have been evaluated, the stress ratio ρ is defined as

$$\rho = \frac{\sigma_{n,max}}{\tau_a} \quad (1)$$

It is noteworthy that ρ is sensitive not only to mean stresses, but also to the degree of multiaxiality and non-proportionality of the stress path [10]. Also, it is worth recalling that for an unnotched specimen under fully reversed uniaxial loading the ρ ratio is equal to unity, whereas under fully reversed torsional loading ρ equals zero [10].

The MWCM is based on a modified Wöhler diagram where τ_a is plotted against the number of cycles to failure, N_f (Fig. 1). This diagram is made of different fatigue curves, each one corresponding to a certain value of ρ ratio and being unambiguously described by its negative inverse slope κ and by a reference shear stress amplitude, $\tau_{A,Ref}$, corresponding to an appropriate number of cycles to failure, N_A . To obtain the diagram, one should properly define the κ vs. ρ and $\tau_{A,Ref}$ vs. ρ relationships, and correctly calibrate them by running appropriate experiments. The following linear relationships have been found by Lazzarin and Susmel [12] to correlate a wide range of experimental data:

$$\kappa(\rho) = a\rho + b \quad (2)$$

$$\tau_{A,Ref}(\rho) = c\rho + d \quad (3)$$

where a , b , c and d are material constants. When these constants are obtained from fully-reversed uniaxial and torsional tests on plain specimens, Eqs. (2) and (3) are stated as

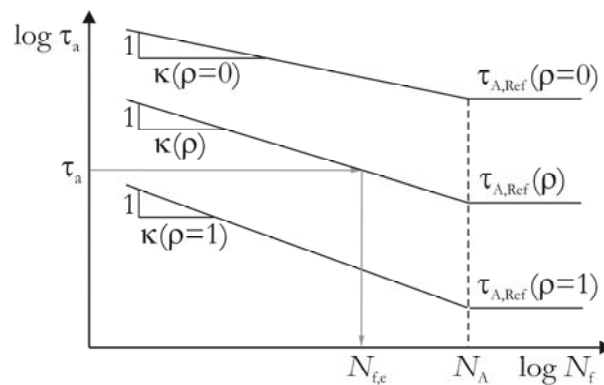


Figure 1: Modified Wöhler diagram.



$$\kappa(\rho) = [\kappa(\rho = 1) - \kappa(\rho = 0)]\rho + \kappa(\rho = 0) \quad (4)$$

$$\tau_{A,Ref}(\rho) = \left[\frac{\sigma_0}{2} - \tau_0 \right] \rho + \tau_0 \quad (5)$$

where $\kappa(\rho=1)$ and σ_0 are, respectively, the inverse slope of the modified Wöhler curve and the fatigue limit under uniaxial loading condition, whereas $\kappa(\rho=0)$ and τ_0 are the corresponding quantities for torsional loading. It is noteworthy that, for materials that do not exhibit a fatigue limit, σ_0 and τ_0 must be defined as endurance limits corresponding to an appropriate number of cycles to failure. After a proper calibration of Eqs. (2) and (3), any curve of the modified Wöhler diagram can be obtained. Hence, the number of cycles to failure can be estimated as

$$N_{f,c} = N_A \left[\frac{\tau_{A,Ref}(\rho)}{\tau_a} \right]^{\kappa(\rho)} \quad (6)$$

Usually, the value of τ_a in the MWCM is determined via the Minimum Circumscribed Circle (MCC) method [13]. Here, the Maximum Rectangular Hull (MRH) approach is adopted, since fatigue estimates based on the MWCM are improved when τ_a is measured by the MRH rather than by the MCC [14]. The MRH method is schematically represented in Fig. 2. The halves of the sides of the rectangular hull with orientation φ are calculated as

$$a_i(\varphi) = \frac{1}{2} \left[\max_t \tau_i(\varphi, t) - \min_t \tau_i(\varphi, t) \right], \quad i = 1, 2 \quad (7)$$

where $\tau_i(\varphi, t)$ ($i=1,2$) are the components of the shear stress vector $\boldsymbol{\tau}(t)$ with respect to the φ -oriented frame. The amplitude of each φ -oriented rectangular hull can be evaluated as

$$\tau_a(\varphi) = \sqrt{a_1^2(\varphi) + a_2^2(\varphi)} \quad (8)$$

The shear stress amplitude is then defined as the maximum value of Eq. (8) among all φ -oriented rectangular hulls:

$$\tau_{a,MRH} = \max_{0 \leq \varphi < 90^\circ} \sqrt{a_1^2(\varphi) + a_2^2(\varphi)} \quad (9)$$

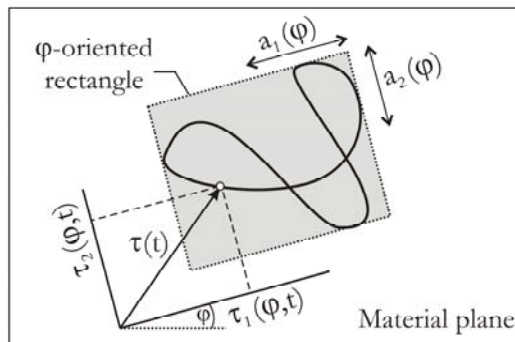


Figure 2: Amplitudes of the φ -oriented rectangular hull bounding the shear stress path.

CRITICAL DISTANCE APPROACH

The stress gradient effect is a crucial aspect in the design of stress raisers such as notches and mechanical contacts. Among the formulations that account for this effect, the TCD [8-10] has been recognized as one of the most attractive

due to its simplicity and good results for a number of notch configurations. The central idea of the TCD is the definition of an effective stress, σ_{eff} , based on an averaging procedure over a volume surrounding the stress raiser. Fatigue failure is expected to occur if σ_{eff} exceeds a reference material strength, σ_{ref} . Simplified methods can also be formulated by considering averages over an area or a line (Area and Line Methods, respectively) or the effective stress of the point located at a critical distance, L , from the stress raiser (Point Method). In this paper attention is focused on the Point Method, as schematically illustrated in Fig. 3.

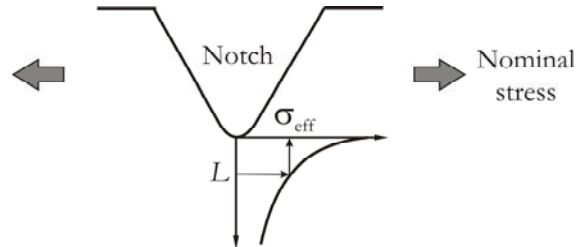


Figure 3: Schematic representation of the Point Method.

Taylor [8] has developed fitting procedures to determine the critical distance, which are based on the fatigue thresholds of cracked or sharply notched specimens. In particular, the critical distance for the Point Method has been found to be expressed as

$$L = \frac{1}{2\pi} \left(\frac{\Delta K_{\text{th}}}{\Delta \sigma_0} \right)^2 \quad (10)$$

where ΔK_{th} is the threshold stress intensity factor range and $\Delta \sigma_0$ is the uniaxial plain fatigue limit range. Further work [10, 15] has investigated the extension of the TCD to stress raisers under multiaxial loadings, concluding that the combination of the MWCM with the Point Method requires the same critical distance given by Eq. (10).

An extension of the TCD to estimate fatigue life of stress raisers has been developed in Refs. [16, 17]. To explain the formulation, it should be recalled that the appropriate critical distance to estimate static failure of notched members is given as [9, 18]:

$$L_S = \frac{1}{2\pi} \left(\frac{K_{\text{Ic}}}{\sigma_r} \right)^2 \quad (11)$$

where K_{Ic} is the plane strain material fracture toughness and σ_r is a reference material constant which can be equal or larger than the ultimate tensile strength, σ_{UTS} [9]. As the values of the critical distances at the threshold and static conditions are usually different, it can be assumed that the critical distance at the medium-cycle fatigue regime, L_M , depends on the number of cycles to failure, N_f . In particular, a power law relationship between L_M and N_f has been proposed by Susmel and Taylor as follows:

$$L_M(N_f) = AN_f^B \quad (12)$$

where A and B are material constants. Two fitting procedures have been developed to obtain these constants: one based on critical distances determined at threshold and static conditions (Eqs. (10) and (11), respectively), and another based on fatigue curves of plain and sharply notched specimens. Although the latter procedure has proved to provide more accurate notch life estimates when compared to experimental data, the former is simpler to work with as the material constants required to determine A and B are usually available or can be extracted from empirical correlations.

APPLICATION TO FRETTING FATIGUE

Several attempts to address the fretting fatigue problem using notch methodologies have been investigated in the literature [2-7] due to the similarities between notch and fretting fatigue. Indeed, the stress fields in both problems are characterized by stress gradients and multiaxial stresses. In the fretting fatigue problem, however, there is also a wear process due to the relative motion between the contacting surfaces. In this setting, notch methodologies could be applied to fretting fatigue if the surface damage could be regarded as negligible. This approximation is considered in this paper, at least for the partial slip case where the amount of wear debris is usually small [19].

The use of the methodology for a typical fretting fatigue problem is shown in Fig. 4. The contact configuration involves a normal force P , a cyclic tangential loading $Q(t)$, and a cyclic remote stress $\sigma(t)$. As a first step, the crack initiation point must be determined. This task can be accomplished, for instance, by searching the point where a given fatigue parameter achieves its maximum value. Normally, the crack initiation point occurs at the trailing edge of the contact, as illustrated in Fig. 4a. Subsequent analysis is carried out on the straight line that emanates from the crack initiation point and is perpendicular to the contact surface. In order to obtain the number of cycles to failure, the critical distance corresponding to a trial number of cycles to failure, N , is determined as follows:

$$L_M(N) = AN^B \quad (13)$$

The stress quantities τ_a , $\sigma_{n,max}$, and ρ are then evaluated at this critical distance, and the number of cycles to failure, $N_{f,e}$, is calculated using Eq. (6). If the calculated $N_{f,e}$ is different from the trial value N , a new iteration is started considering $N = N_{f,e}$. This process is repeated until convergence between the trial and calculated fatigue lives has been attained.

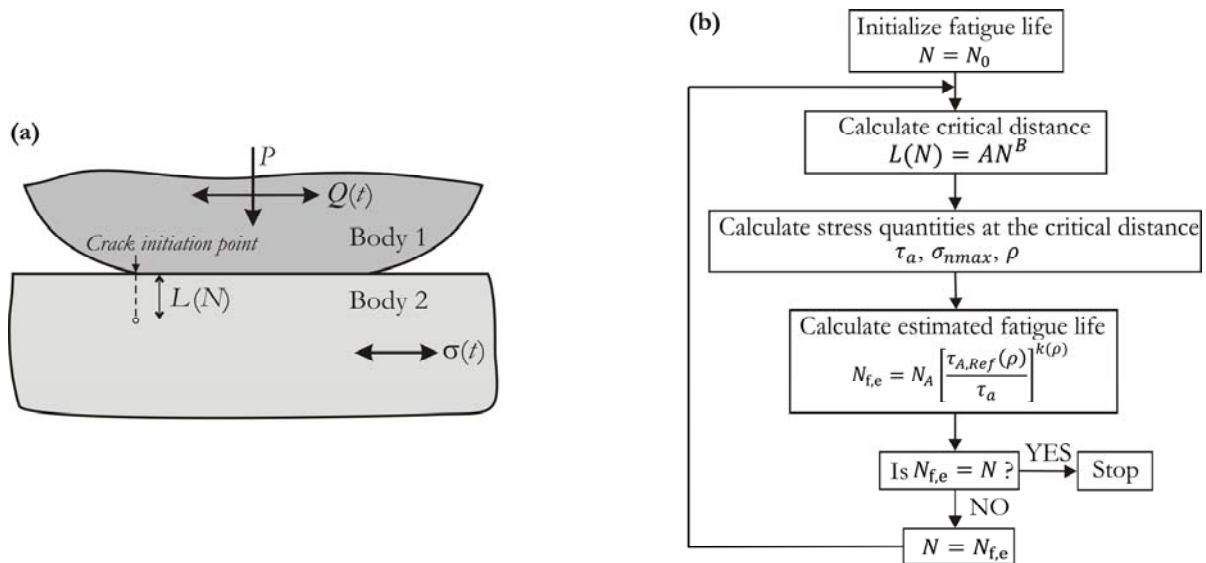


Figure 4: Procedure to estimate fretting fatigue life: (a) location of the critical distance and (b) flowchart of the iterative procedure for life estimation.

COMPARISON WITH EXPERIMENTAL DATA

Available fretting fatigue data [1] were used to assess the methodology. Such tests were carried out with a pair of cylindrical pads pressed against a flat dog-bone specimen, both made of an Al alloy 4% Cu. In each series of tests, the tests were run (each one with a different pad radius) at constant values of peak contact pressure, tangential force and remote stress. Hence, the magnitude of the stress field of each test was the same, but different stress gradients were induced by the contact.



Constants for the methodology are given in Table 1. The constants A and B in the L_M versus N_f relationship, Eq. (12), were determined using L values calculated at the threshold and static conditions, Eqs. (10) and (11), respectively. The values of κ and $\tau_{A,Ref}$ were extracted from fully reversed uniaxial and torsional fatigue curves. These curves were estimated by fitting fatigue data for 10^3 cycles and 10^7 cycles, using empirical relationships [20] to obtain the fatigue strengths at 10^3 cycles from the ultimate tensile strength.

σ_{UTS} (MPa)	σ_0 (MPa)	K_{IC} (MPa·m ^{0.5})	ΔK_{th} (MPa·m ^{0.5})	$\kappa(\rho=0)$	$\tau_{A,Ref}(\rho=0)$ (MPa)	$\kappa(\rho=1)$	$\tau_{A,Ref}(\rho=1)$ (MPa)
500	124	34	4.4	12.8	161	12.8	115

Table 1: Constants for the methodology for Al alloy 4% Cu.

Due to the geometry of the experimental setup and the applied loadings, analytical techniques [21] were employed to solve the elastic contact problem. The surface tractions are characterized by a Hertzian contact pressure distribution, and by shear tractions that are similar to the Mindlin-Cattaneo one except that the stick zone is not symmetrical with respect to the center of the contact zone but shifted due to the presence of an alternating remote stress. Once the surface tractions have been determined, subsurface stresses can be obtained by using a Muskhelishvili potential. The time varying elastic stress field in any material point in the specimen can be finally calculated by superposing the effects of contact pressure, shear traction and remote stress.

Estimated and observed number of cycles to failure is shown in Fig. 5. The solid diagonal line corresponds to a perfect correlation between estimated and observed fatigue lives, and the two dashed lines define the factor of 2 bandwidth. As can be clearly seen in this figure, most of the life estimates fall within a error band given by a factor of 2. Considering the well known scatter that characterizes the fatigue phenomena this can be considered a very good correlation.

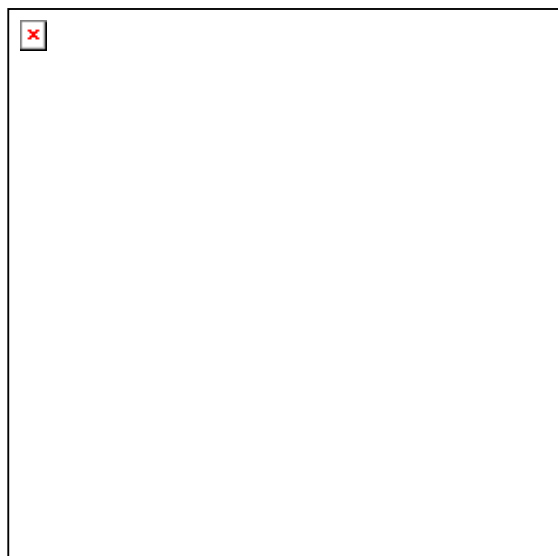


Figure 5: Observed and estimated fatigue lives for fretting fatigue tests conducted by Nowell [1].

CONCLUSIONS

A engineering methodology for fatigue life estimation of mechanical couplings subjected to fretting fatigue was presented in this paper. Constants for the methodology are relatively simple to determine, as only experimental data from conventional fatigue tests on plain and sharply notched (or cracked) specimens are required. It can also be easily incorporated in a Finite Element Method based software for fast assessment of fretting fatigue life in more realistic type of mechanical couplings.

Fatigue life estimates correlated well with the experimental results [1], falling in most cases within a factor of two bandwidth. Although some of the required material constants were obtained either from literature or from empirical



relations, the methodology was still capable of producing satisfactory life estimates. However, further assessment considering different contact configurations and other materials is still required to corroborate the proposed life methodology.

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