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Population-driven urban road evolution dynamic model

Fangxia Zhao · Jianjun Wu · Huijun Sun · Ziyou Gao · Ronghui Liu

Abstract: In this paper**,** we propose a road evolution model by considering the interaction between population distribution and urban road network. In the model, new roads need to be constructed when new zones are built, and existing zones with higher population density have higher probability to connect with new roads. The relative neighborhood graph and a Fermat-Weber location problem are introduced as the connection mechanism to capture the characteristics of road evolution. The simulation experiment is conducted to demonstrate the effects of population on road evolution. Moreover, the topological attributes for the urban road network is evaluated using degree distribution, betweenness centrality, coverage, circuitness and treeness in the experiment. Simulation results show that the distribution of population in the city has a significant influence on the shape of road network, leading to a growing heterogeneous topology.

Keywords Road evolution · Population distribution · Relative neighborhood · Fermat-Webber location problem

1. Introduction

With the increase of population, traffic demand is growing in cities. Continuous growth of traffic demand leads to serious traffic congestion and has become one of the most challenging and important issues for the decision makers. To alleviate traffic congestion, governments worldwide have been making huge investments in transportation infrastructure. But where best to invest the infrastructure, known as

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the network design problem (NDP), has long been a great interest to decision-makers, as well as been recognized to be one of the most difficult and challenging mathematical problem to solve. For the detailed reviews of NDP, the readers are referred to Boyce (1984), Magnanti and Wong (1984), Friesz (1985), Migdalas (1995), Yang and Bell (1998), Chen et al. (2011) and Zanjirani et al. (2013). Construction of urban road network leads to changes of network topology. The question on which urban road network topology is more suitable to satisfy the traffic demand has only recently become an important problem. In fact, the evolution of urban road network has many features. Firstly, it is a complex evolution in time and space. The evolution of urban road network presents a gradual formation process of urban road system. An in-depth study of road network evolution can not only reveal the processes of urban formation and evolution, but also provide the theoretical foundation for analyzing the traffic problems. As such, the evolution of urban road network has attracted research attention in recent years. A comprehensive review of urban road evolution models can be found in Xie and Levinson (2011). The general modeling approaches can be classified as, optimization (Schweitzer et al. 1997), dynamics (Courtat and Gloaguen 2011), data mining (Levinson 2008) and simulation (Alberti and Waddell 2000; Yamins et al. 2003; Xie and Levinson 2007, 2009; Figueiredo and Machado 2007).

Over last few years, some mechanisms have been considered to develop urban road evolution models. These include land use (Levinson and Yerra 2006; Levinson et al. 2007), population density (Barthélemy and Flammini 2009), social-economics (Yang and Huang 1998; Levinson and Yerra 2006), environment (Handy et al. 2005). Among which, population distribution is the most frequently considered mechanism and is shown to have the most significant effect on the evolution of urban road network. For example, Barthélemy and Flammini (2009) proposed a model that described the impact of economical mechanisms on the evolution of the population distribution and the topology of the road network. To reflect how the distribution of population and employment responds to the accessibility patterns, Levinson et al. (2007) proposed a co-evolution model of land use and transportation network. Levinson (2008) further developed a spatial co-development model of rail networks and population distribution in London during the 19th and 20th centuries, and found that there is a positive feedback effect between population density and network density. It shows that the distribution of population plays a significant role in the transportation network evolution. It is natural to assume that an urban road network would evolve to better serve the changing distribution of population. The structure of an urban road network influences the accessibility and governs the attractiveness of different zones of the network (Barthélemy and Flammini 2009). In turn, the changes in the attractiveness of different zones can lead to changes in population distribution. In this paper, we explicitly model the dynamic interactive process between road network evolution and population distribution.

In the urban road network evolution, the accessibility of zones (or zones) is an important factor to consider. Minimal spanning tree (MST) (Karger et al. 1995) is generally used as a mechanism to describe the accessibility of a network. However, the accessibility of road network based on MST is very poor, in that MST has no cyclical paths (Toussaint, 1980). To improve network accessibility and to avoid too many cyclical paths in the network, a relative neighbor graph (RNG) (Toussaint 1980; Jaromczyk and Toussaint 1992) was applied as the mechanism in the building of new roads. In fact, MST is a subgraph of RNG, which means that RNG has higher accessibility and smaller number of circles in the network than MST (Supowit 1983). In addition, the construction cost of road based on MST is always not the minimal (Hwang and Richards, 1992). To minimize the road construction cost, Minimum Steiner tree (Hwang and Richards 1992; Chlebik and Chlebikova 2002) has been proposed to develop the urban road. Minimum Steiner tree is a graph that connects the known points by lines of minimum total length in such a way that any two points may be interconnected by line segments either directly or via the new added points (Hwang and Richards 1992). However, the limitation in the application of Steiner minimum tree to the study of urban road evolution and population distribution is that the weight of each zone is assumed to be equal (Hwang and Richards 1992; Chlebik and Chlebikova 2002). In reality, the population of each zone is not same. Therefore, the impact of different zone on the urban road topology is not same, which implies that the weight of each zone should not be the same. The Fermat-Webber location problem is to find a point in the Euclidean space that minimizes the sum of the costs from this point to the given destination points, where different given points are associated with different costs per unit distance. Therefore, based on the above discussion, RNG and the Fermat-Webber location problem (Weiszfeld 1937; Vardi and Zhang 2001) are introduced to develop the urban road with consideration of population distribution in this paper.

This paper attempts to model the evolutionary growth process of road networks with explicit consideration of the interaction between population distribution and road network growth. The remainder of this paper is structured as follows. In Section 2, we present the population-driven urban road network evolution dynamic model. Section 3 provides measures to evaluate the topological attributes for the network. The simulation results are analyzed in Section 4. Section 5 concludes the paper and proposes the future research directions.

2. The dynamical model

In this section, we present a population-driven urban road evolution dynamical model. For clarity, we list in Section 2.1 below all the notations used in this paper. The definitions of some of the variables are given in the subsequent sections when they are first used. Also presented in Section 2.1 are the assumptions used in our model.

2. 1 Notation and Assumption

Consider a connected road network $G(V, A)$. V denotes the set of distinct zones whereas A denotes the adjacent matrix of urban road network. The following notations are adopted throughout this paper:

A: the adjacent matrix of urban road network, $A=\{a_{ij}\}$, where $a_{ij}=1$ if there is a link between zone i and zone j and $a_{ij}=0$ otherwise;

$B(y)$: the numbers of lattice that all links pass through, where y denotes the side length of lattice that can cover the network;

C_{ij}: the construction cost between zones i and j;

C_i: the construction cost between zone i and optimal point of the Fermat-Weber location problem;

$d(p, q)$: the Euclidean distance between zones p and q;

$D(u)$: the degree distribution of network, where u denotes the node degree;

$g(e)$: the betweeness centrality of edge e ;

$g(p)$: the betweeness centrality of zone p ;

H_i : the population of i -th community;

L_{pq} : the Euclidean distance between zones p and q ;

n : the numbers of distinct zone in $\mathbb{R}^m$;

p : a zone in $\mathbb{R}^m$;

p_i : the i -th distinct zone in $\mathbb{R}^m, i = 1, 2.., n$;

p_0 : the solution of the Fermat-Webber location problem;

q : a zone in $\mathbb{R}^m$;

r : a zone in $\mathbb{R}^m$;

$\mathbb{R}^m$: an m-dimension Euclidean space;

$\mathbb{R}^m_+$: an m-dimension non-negative Euclidean space;

u_i : the degree of community i ;

$\bar{U}$: the average degree of network;

V : the set of distinct communities, $V = \{p_1, p_2, ... p_n\}$;

w_i : the weight of i th $(i = 1, ..., n)$ point in $\mathbb{R}^m$;

$\phi_{circuit}$: the circuitness of network;

ϕ_{tree} : the treeness of network;

Assumption (1): The total network population increases with time due to net migration from other cities and natural growth (Zhao et al. 2015).

Assumption (2): The human dynamics process can be classified into two mechanisms: accessibility-seeking and space-seeking. Accessibility-seeking refers to the behavior that humans prefer to move to the zones which have more people, while space-seeking represents that humans randomly explore other zones (Zhao et al. 2015).

Assumption (3): Not all regions of an urban network can be developed as zone.

Land use can be divided into three classes: built area (BA), non-built-up area (NBA) and reserved area (RA). BA has been explored for people to live in. NBA has not been explored, but can be explored in the future. RA is reserved for special purposes (such as park, roads, place of interest and so on) and cannot be explored for zone use (Zhao et al. 2015).

Assumption (4): A city is represented as a lattice which contains many cells. Each cell is randomly assigned as BA, NBA, or RA, based on a given probability. A zone is represented by a cell where people is currently living in (i.e., the cell is a BA).

Assumption (5): New zones are built randomly based on the current condition. Some population will be assigned into new zones according to the zones' capacities. When the population of a cell exceeds its capacity, new zones will be built to house the remaining population.

Fig. 1 illustrates the initialization of urban road network and population distribution, based on the above assumptions, where black points and color points represent undeveloped areas (such as parks, rivers and lakes, etc.) and zones (dark color means that the zones have a high population density), respectively.

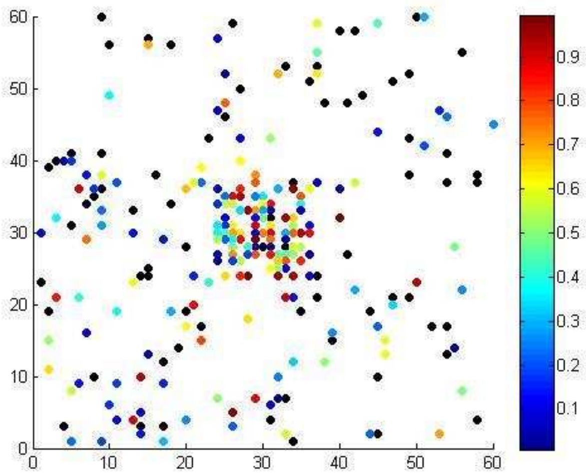


Fig. 1 Result of urban spatial evolution. The X-Y axes correspond to the spatial coordination of various zones on the lattice lattice, while the color represents the population size of these zones

2.2 Relevant Concepts

This section introduces the relevant concepts and their definitions used in this paper.

(1) Relative neighborhood graph (RNG)

RNG was first proposed by Toussaint (1980) in the studies of computational geometry. RNG of a finite set V in the Euclidean space $\mathbb{R}^m$ is defined as an undirected graph with a set of distinct points V and set of edges $RNG(V)$ which are exactly those pairs (p,q) of points for which $d(p,q) \le \max_{z \in V \setminus \{p,q\}} \{d(p,z), d(q,z)\}$ (Barthélemy and Flammini 2008; Toussaint 1980). The MST is a subgraph of RNG. This implies that the network constructed according to RNG will have higher accessibility than that constructed according to MST. For further reading on RNG, the readers are referred to Supowit (1983), and Jaromczyk and Toussaint (1992).

Given a set V of n distinct points on the Euclidean space, i.e., $V = \{p_1, ..., p_n\}$, how to find $RNG(V)$. The following is the procedure of RNG algorithm:

Step 1. Calculate the Euclidean distance of all pairs $d(p_i, p_j)$, $i, j = 1, ..., n, i \ne j$.

Step 2. For each pair of the distinct points $k = 1, ..., n, k \ne i, k \ne j$ (p_i, p_j), compute $d_{\max}^k = \max \{d(p_k, p_i), d(p_k, p_j)\}$.

Step 3. If $d_{\max}^k \ge d(p_i, p_j)$, then the points p_i and p_j are connected by an edge, otherwise, they cannot be connected.

Step 4. Return to Step 2 until all points are searched.

(2) The Fermat-Weber location problem

The Fermat-Weber location problem is one of the most famous problems in location theory. It is used to find a point in $\mathbb{R}^m$ that minimizes the sum of weighted Euclidean distances from this point to n given points in $\mathbb{R}^m$. If all weights are equal, the Fermat-Weber location problem reduces to Euclidean minimum Steiner tree problem (Hwang and Richards 1992; Chlebik and Chlebikova 2002). Specifically, considering an m-dimensional Euclidean space, we let $V = \{p_1, ..., p_n\}$ denote n distinct points in $\mathbb{R}^m$. The Fermat-Weber location problem is to determine an optimal point $p_0 = \left(x_1^*, ..., x_m^*\right)$ in the Euclidean space to satisfy the following condition (Weiszfeld 1937; Vardi and Zhang, 2001):

$$f(p_0) = \min f(x) = \min \sum_{i=1}^{n} w_i \|x - p_i\|_2 \quad (1)$$

where $w_i (i = 1,....,n)$ denotes the positive weight of i-th $(i = 1,....,n)$ point in $\mathbb{R}^m$.

Weissfeld (1937) proved that if p_0 is the optimal solution of Eq. (1), the optimal point p_0 is one of n distinct points or a new added point which satisfies the following conditions:

$$p_0 = \frac{1}{\sum_{i=1}^{n} \frac{w_i}{\|p_0 - p_i\|_2}} \sum_{i=1}^{n} \frac{w_i}{\|p_0 - p_i\|_2} p_i \quad (2)$$

Then, the following heuristic algorithm for solving the Fermat-Weber location problem was proposed (Weissfeld 1937):

$$T : p_0^k \to p_0^{k+1} = T(p_0^k),$$
$$T(p_0^k) = \frac{1}{\sum_{i=1}^{n} \frac{w_i}{\|p_0^k - p_i\|_2}} \sum_{i=1}^{n} \frac{w_i}{\|p_0^k - p_i\|_2} p_i \quad (3)$$

where T denotes a mapping. For an arbitrarily initial point p_0^1 which is different to p_i, the point p_0^{k+1} is closest to the point p_0 when k approaches infinite.

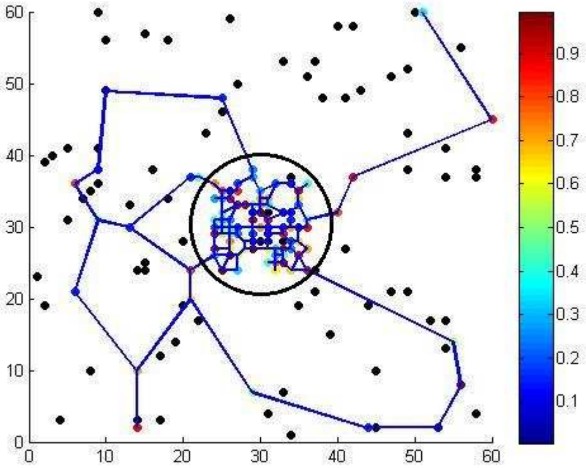
2. 3 Urban road network evolution

(1) Model 1: road network evolution without consideration of population

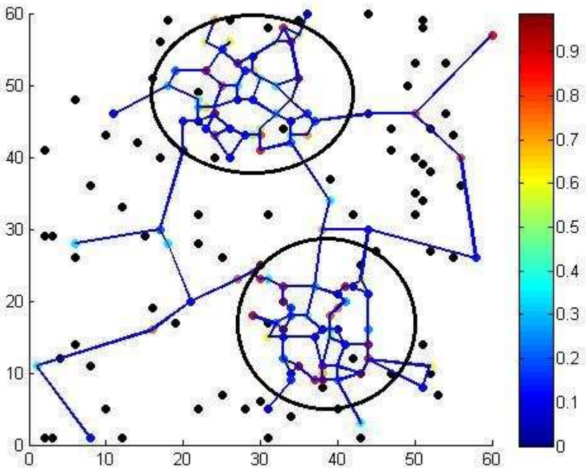
Generally, when new zones are generated, we need to build new roads to connect them to the existing road network. If the population distribution is not taken into account, the road construction cost will depend only on the road length. We assume therefore the construction cost to link zone i with zone j is $C_{ij} = \alpha L_{ij}$, where L_{ij} denotes the Euclidean distance between zones i and j, and α is a parameter corresponding to the unit cost by road length. Without loss of generality, the parameter α is assumed to be 1. To facilitate the presentation of the main process of road network evolution, we introduce the concepts of the lune of two points and the "relative neighbor" of a point. The lune of points p and q is defined as the set of points that satisfy $\{z \in \mathbb{R}^m : d(p,z) < d(p,q), d(q,z) < d(p,q)\}$. If there exists no such point r in the lune of the points p and q ($q \neq r$), point q is called as the "relative neighbor" of point p.

With the above definitions, the modeling process of road network evolution without consideration of population distribution can be described as follows:

Step1. Initialization. Set the total number of iterations K and the initial iteration counter $k = 1$. We implement the above procedure of RNG algorithm using initial road network in Fig. 1. Fig. 2a and 2b display the initial network with one center and two centers respectively by using RNG.



(a) RNG with one center.



(b) RNG with two centers.

Fig. 2 Initial road network. The X-Y axes correspond to the spatial coordinates of the network zones on the lattice lattice

Step2. Generating zones and building roads. New zones are randomly generated at each iteration according to the given probability. Besides, each new zone is connected to the existing road network, according to the following rules:

(i) If a newly generated zone has only one relative neighbor, two different costs are calculated. One is the cost between the new zone and its relative neighbor, denoted as C^1. The other is the cost between the new zone and the nearest link which is denoted by C^2.

(ii) If the newly added zones p_i and p_j have different relative neighbors, calculate two costs for each new zone. If $C^1 > C^2$ for either or both zones p_i and p_j, the new zones p_i and p_j are connected to their nearest link. Otherwise, they are connected to their own relative neighbor.

(iii) If the newly added zones p_i and p_j have the same relative neighbor q, calculate two costs C^1 and C^2 for each new zone. If $C^1 > C^2$ for either or both new zones, the new zone p_i and p_j is connected to their nearest link. If $C^1 \leq C^2$ for both new zones p_i and p_j, we implement the Eq. (3)for the Fermat-Webber location problem, the optimal point p_0 that satisfies the following condition can be founded

$$f(p_0) = \min f(x) = \min \sum_{i=1}^{n} \|x - p_i\| \tag{4}$$

Then, we connect the new zones p_i, p_j and their relative neighbor q to the point p_0.

Step 3: If $k > K$, stop. Otherwise, set $k = k+1$ and return to step 2.

The flowchart of the road network evolution without consideration of population distribution is illustrated in Fig. 3.

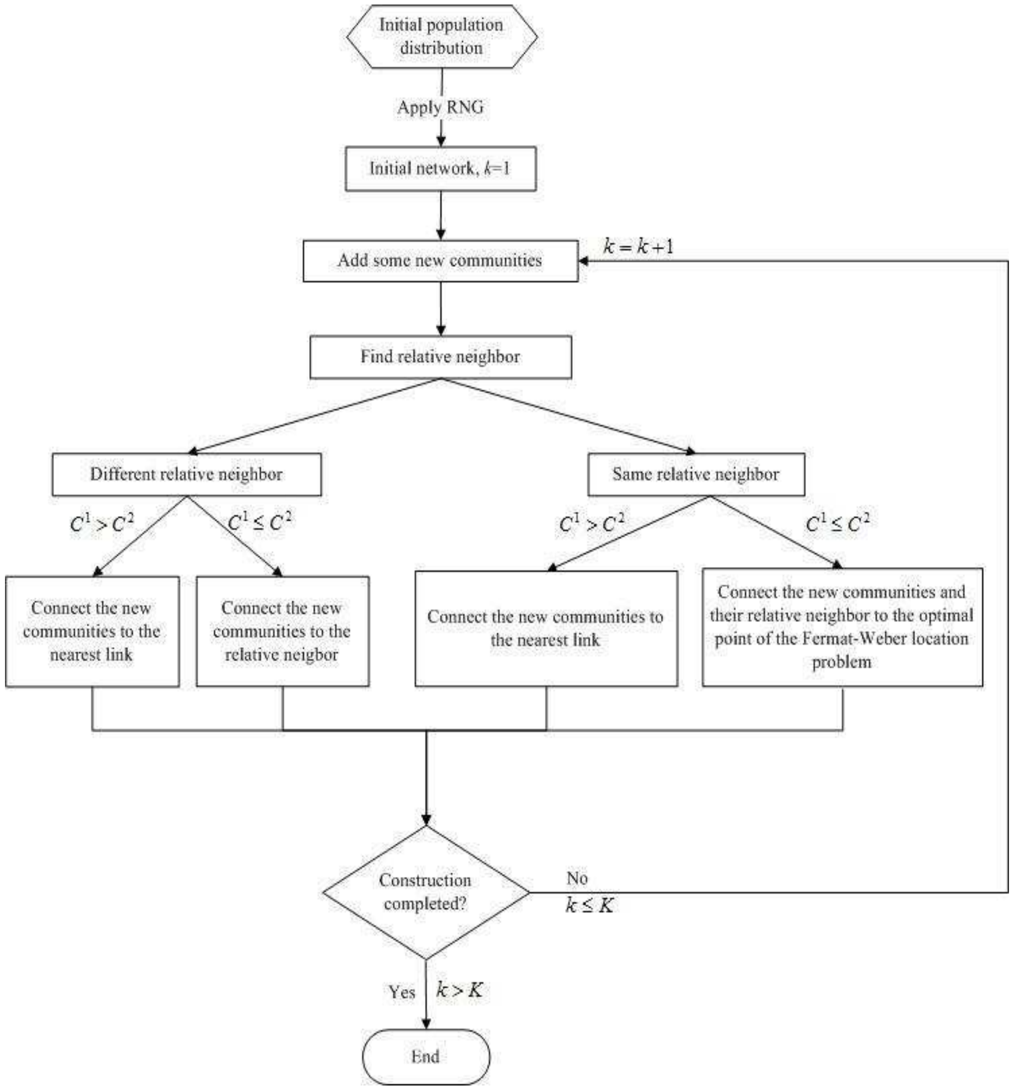


Fig.3 Flowchart of the road network evolution without consideration of population

(2) Model 2: population-driven road network evolution

In reality, population distribution has an important influence on the road network evolution. For example, the densely populated zones tend to have more convenient traffic conditions, thus the accessibility of these zones are higher. In this paper, we assume that the more populated zone is more likely to be connected to the new zone than less populated zone. Take a simple example shown in Fig. 4, points p and q are two existing zones, and point r is a new zone. Assume L_{pr} and L_{qr} are the same and equal to L. Assume the population of zone p is higher than that of q, i.e., $H_p > H_q$. Then, we choose zone p to be connected to the new zone r.

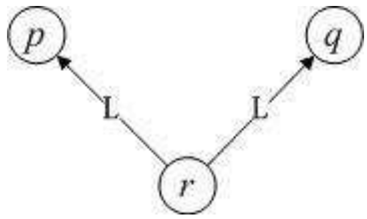


Fig. 4 Diagrammatic sketch of new road

In the model of road evolution without population distribution, the construction cost linking two zones i and j is related only to the Euclidean distance between the two, i.e. $C_{ij} = \alpha L_{ij}$. With the consideration of population distribution, the following linear function is defined as the cost between zones i and j.

$$C_{ij} = \beta_1 L_{ij} - \beta_2 (H_i + H_j),\ \beta_1 > 0, \beta_2 > 0, \frac{\beta_1}{\beta_2} > \frac{H_i + H_j}{L_{ij}} \tag{5}$$

where β_1 and β_2 are two positive parameters, H_i and H_j represents the population size in the zone i and j, respectively. According to Eq. 5, it is clearly that C_{ij} is an increasing function of the road length, whilst it decreases with population sizes.

A special case to Eq. (5) is where there is only one zone on the link. There, Eq.5 can be formulated as $C_i = \beta_1 L_i - \beta_2 H_i$, where C_i and L_i represent the cost and Euclidean distance between the zone i and the point p_0, respectively. It can be seen that the population-driven road network evolution model, with just one zone on the link, reduces to the road network evolution model without accounting for the population distribution when β_2 approaches zero.

In the case of road evolution with consideration of population distribution, the optimal point p_0 satisfies as the following condition:

$$f(p_0) = \min\ f(x) = \min \sum_{i=1}^{n} C_i \left\| x - p_i \right\|_2 \tag{6}$$

where $\left\| x - p_i \right\|_2$ denotes the Euclidean distance between points x and p_i. According to the following heuristic algorithm, the optimal point p_0 can be found (Weissfeld 1937):

$$p_0^{k+1} = T(p_0^k) = \frac{1}{\sum_{i=1}^{n} \frac{C_i}{\left\| p_0^k - p_i \right\|_2}} \sum_{i=1}^{m} \frac{C_i}{\left\| p_0^k - p_i \right\|_2} p_i \tag{7}$$

Substitute $C_i = \beta_1 L_i - \beta_2 H_i$ and $L_i = \left\| p_0^k - p_i \right\|_2$ into (7):

$$p_0^{k+1} = T(p_0^k) = \frac{\beta_1 \sum_{i=1}^{n} p_i + \beta_2 \sum_{i=1}^{n} \frac{H_i\, p_i}{\left\| p_0^k - p_i \right\|_2}}{\beta_1 n + \beta_2 \sum_{i=1}^{n} \frac{H_i}{\left\| p_0^k - p_i \right\|_2}} \tag{8}$$

As above-mentioned, the point $p_0^k \to p_0$ when $k \to \infty$. Then, Eq. (8) can be formulated as:

$$p_0 = T(p_0) = \frac{\beta_1 \sum_{i=1}^{n} p_i + \beta_2 \sum_{i=1}^{n} \frac{H_i\, p_i}{\left\| p_0 - p_i \right\|_2}}{\beta_1 n + \beta_2 \sum_{i=1}^{n} \frac{H_i}{\left\| p_0 - p_i \right\|_2}},\ p_0 \in \mathbb{R}_{+}^{m} \tag{9}$$

Now Eq. (9) is a fixed point problem. Since the set $\mathbb{R}_{+}^{m}$ is a compact set and $T(p_0)$ is a continuous function with respect to point p_0, according to Brouwer's fixed-point theory there exists at least one solution to the fixed point problem (9) (Facchinei and Pang, 2003). Then, we can obtain the solution p_0 by solving the fixed point problem Eq. (9). Note that the uniqueness of the solution cannot be guaranteed because the Jacobain Matrix $J_{p_0} T(p_0)$ of the mapping T is not always definite for any point p_0 in $\mathbb{R}_{n}^{+}$. Therefore, we cannot expect the mapping T appearing in the fixed-point problem (9) to be strictly monotone. It is clearly that the evolution process in Model 1 can also be used in Model 2. The only difference is that the RNG and relative neighbors in Model 1 are based on Euclidean distance, while in Model 2, they are based on cost of Eq. (5).

3. Measures of network evolution

In this section, we introduce some commonly used measures of network evolution.

3. 1 Degree distribution

The degree of a zone in the network is defined as the number of links or edges that the zone has. In terms of the adjacency matrix A of a network, the degree of node i is just the ith row of A (Dorogovtsev et al. 2001), i.e.

$$u_i = \sum_j a_{\ i} \tag{10}$$

Then, the degree distribution $D(u)$ of a network is defined as the fraction of nodes in the network with degree u. For example, for a network of n nodes, if n_u nodes have degree u, we can get $D(u) = \frac{n_u}{n}$. A network's degree distribution is sometimes described in cumulative form, as the fraction of nodes with degree greater than or equal to u:

$$D_u(U \geq u) = \sum_{u' \geq u}^{\infty} D(u') \tag{11}$$

The average degree of the entire network is the simple arithmetic mean of all the node degree:

$$\overline{U} = \frac{1}{n}\sum_{i=1}^{n} u_i \tag{12}$$

3. 2 Betweenness centrality

Betweenness centrality is a common measure in the research of complex network. It is defined as the number of shortest paths from all nodes to all others that pass through the specific node or edge. Betweenness centrality is a more useful measure of the load placed on the given node or edge, hence the node's or the edge's importance to the network, than accessibility (Barthélemy 2003).

To justify the existence of such a hierarchy in the proposed model, edge betweenness centrality is used as a simple proxy for the traffic on the urban road network. For a generic graph, the edge betweenness centrality $g(e)$ is defined as the fraction of shortest paths between any pairs of nodes in the network that go through the edge e (Barthélemy 2003; Freeman 1977). In reality, there could be multiple shortest paths between any two points. In this paper, we allow multiple shortest paths as and define the edge betweenness centrality as follows:

$$g(e) = \sum_{q,r,q \neq r} \frac{\sigma_{qr}(e)}{\sigma_{qr}} \tag{13}$$

where σ_{qr} is the number of shortest paths going from node q to node r. $\sigma_{qr}(e)$ is the number of shortest paths going from the zone q to zone r and passing through the edge e. Therefore, central edges are those that are most frequently visited if shortest paths are chosen to move from and to arbitrary node. Analogously, the node betweenness $g(p)$ can be defined as the fraction of shortest paths between all other nodes which go through the node p. Mathematically, it can be formulated as:

$$g(p) = \frac{1}{n(n-1)} \sum_{q,r,q \neq r} \frac{\sigma_{qr}(p)}{\sigma_{qr}} \tag{14}$$

where σ_{qr} is the number of shortest paths going from zone q to zone r and $\sigma_{qr}(p)$ is the number of shortest paths going from zone q to zone r and passing through zone p, n is the total number of zones in the network. By its definition in Eq. (14), $g(p)$ is normalized and reaches the highest value of 1 when every shortest path involves node p.

3. 3 Circuitness and Treeness

The basic structures of a planar transportation network can be classified into two groups: circuit networks and branching networks (Haggett and Chorley 1969; Ding and Lou 1998). Circuit networks are regional networks structured with closed circuits, where a circuit is defined as a closed path (with no less than three links) with the same vertex as start and end. Branching networks are characterized by tree structures with multiple connected links without any circuits. Specifically, a graph without cycles is called as a forest and a connected forest is called as a tree. There are some typical connection patterns emerging in circuit and branching urban road network. A circuit block is defined in this study as a block that contains at least one circuit and contains neither bridges nor articulation points. If a circuit block contains only one circuit, it is defined as a ring; if it contains more than one circuit, it is defined as a web (Xie and Levinson 2007). For example, ring and web are typical circuit networks, whilst star and hub-and-spoke are typical branching networks (Xie and Levinson 2007).

The circuitness and treeness for a general network are defined as (Xie and Levsion 2009):

$$\phi_{circuit} = \phi_{ring} + \phi_{web} \tag{15a}$$

$$\phi_{ring} = \frac{\text{Total length of links on rings}}{\text{Total length of links}} \tag{15b}$$

$$\phi_{web} = \frac{\text{Total length of links on webs}}{\text{Total length of links}} \tag{15c}$$

$$\phi_{tree} = 1 - \phi_{circuit} \tag{15d}$$

Clearly, these ratios vary from 0 to 1 and they indicate the extent to which the entire network is connected as circuits or trees. A higher treeness (ϕ_{tree}) means that a branching structure while a higher circuitness ratio ($\phi_{circuit}$) implies a circuit network. These measures provide a consistent and easily computable way to examine the topology for an entire network. Xie and Levinson (2007) presented an algorithm to determine whether a link is on the tree or on a circle for a given connected network, and we adopt the same algorithm here in this paper.

3. 4 Coverage

Coverage (also known as c) is a basic index used to measure the structure of the road networks, and it can be used to represent the uniformity degree of road network.

To define the coverage of urban road network, we consider a lattice of equal length y on two sides which covers the entire network. Next, this lattice is divided into four equal parts each with length $\frac{y}{2}$ on both sides. Similarly, this lattice may be divided further. Then, the coverage can be defined as (Ding and Lou 1998):

$$\mathrm{Coverage}(y_i) = \frac{\ln(B(y_{i-1})) - \ln(B(y_i))}{\ln(y_i) - \ln(y_{i-1})} \tag{16}$$

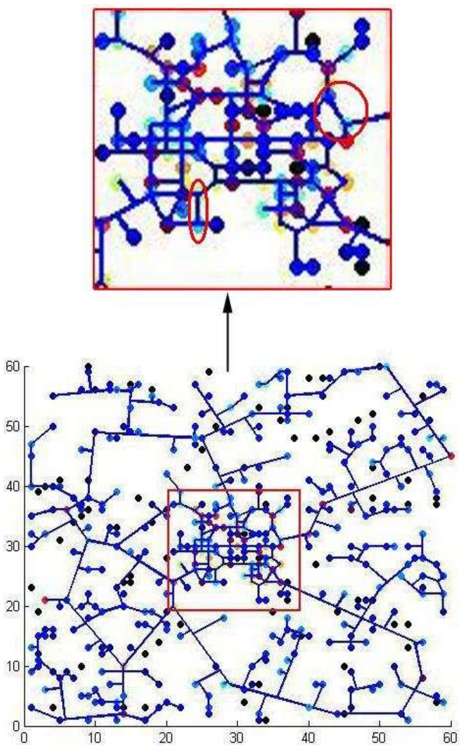
where y_i is the length of lattice after i th subdivision, and $B(y_i)$ denote the number of lattices that the network links pass through at the i th subdivision. This measure reflects the covering form of an urban road network. A greater coverage measure indicates that more lattices are passed by network links, and the covering form of the network is higher.

4. Results and analysis

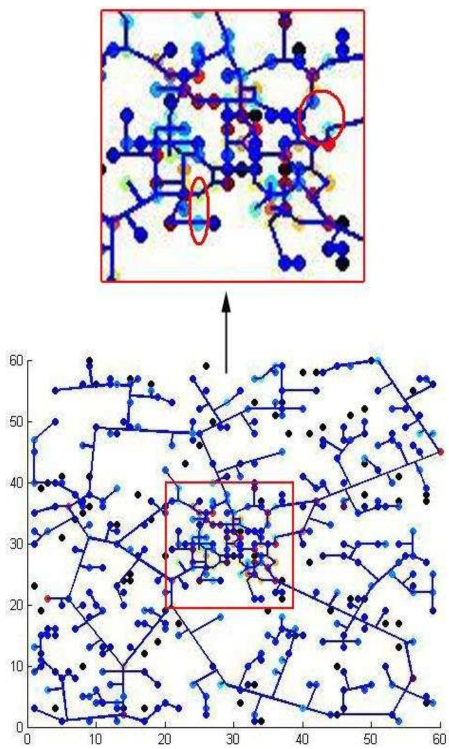
The purpose of presenting the simulated experiments in this section is two folds: 1) to demonstrate the effectiveness of the proposed models with and without population distribution; and 2) to illustrate the advantage of Model 2 with a population-driven urban road network evaluation over Model 1 without population-driven. The test city is divided into 60*60 zones, where the number of BA, NBA and RA zones are 100, 2780 and 720, respectively. Thus, 20% (=60*60/720) of the city are RA. Initial population in the city is 50000 ($H_i(0) = 50000$) and the population growth include natural growth and migrate in (Zhao et al. 2015). The total number of iterations is set to be $K = 20$ in our experiment. The simulation results presented below are the averages of the total 12 iterations.

4. 1 The evolution of population-driven road network

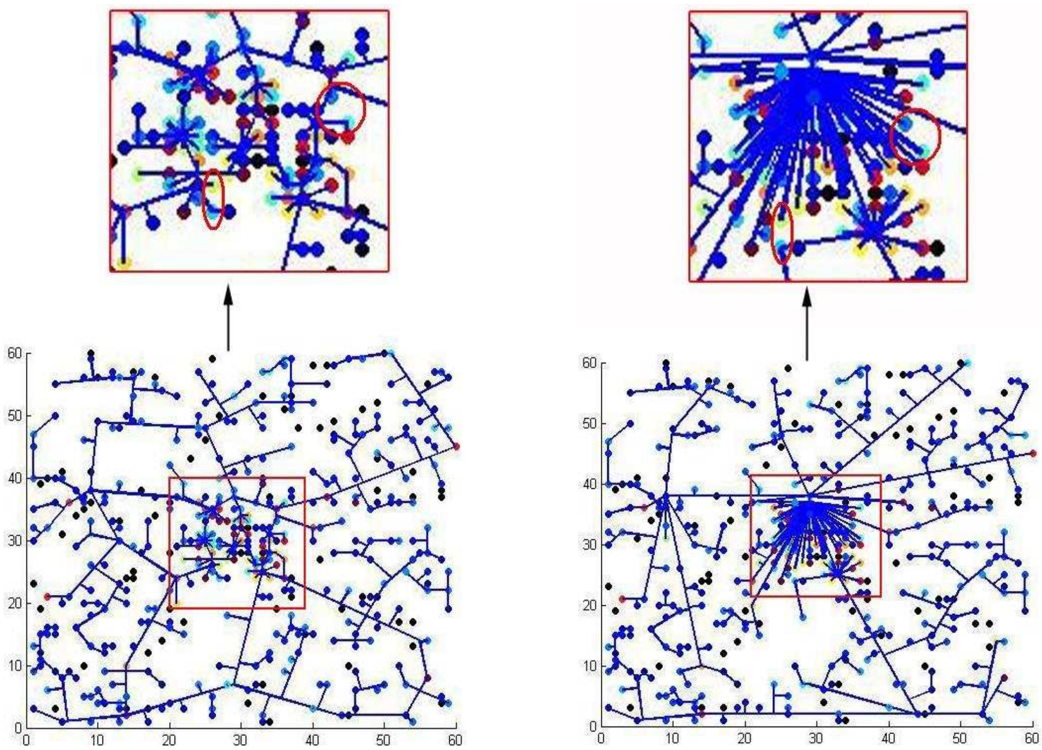
Fig. 5 depicts the resulting topological structures of urban road network with and without the effect of population distribution. Fig.5 (a) shows the road network without considering population distribution. In this case, the construction cost for urban road network only takes into account the Euclidean distance. Fig. 5 (b)-(d) display the road network evolution results under population-driven, where both population distribution and Euclidean distance are considered in the cost. It can be seen from Fig.5 (b)-(d) that the topologies of urban road networks change with increasing values of the parameter β_2. With small β_2 value, the result (in Fig.5 (b)) is similar to that without population-driven and there is no clear center. This implies that the population-driven road network will reduce to the road network without accounting for population when the parameter β_2 approaches 0. As the β_2 value increases, we begin to see a small center in Fig. 5 (c), while a major center is emerged with a higher β_2 value in Fig. 5 (d). According to Eq. (5), as the parameter β_2 increases, more will connect to zones with high population.



(a) Without population-driven



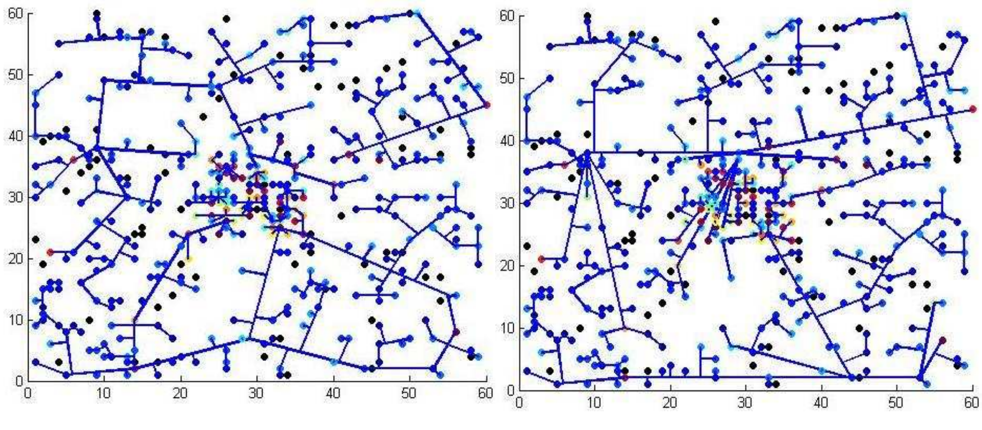
(b) With population-driven for $\beta_2 = 0.0001$



(c) With population-driven for $\beta_2 = 0.001$ (d) With population-driven for $\beta_2 = 0.01$

Fig. 5 The topology of road network with and without consideration of population. The topology for the center 20x20 lattices is enlarged for each case and presented at the top of each case

As shown in Fig.5 (c)-(d), the maximum nodal degree of road network is very large, especially in Fig. 5. (d). However, in reality, the maximum nodal degree of node on the road is no more than 4 (sometimes 6). Fig. 6 presents the resulting topological structures of urban road network with the effect of population distribution under degree constraint (maximum nodal is six). According to Fig. 6, we can see that the topological structures of urban road network are more realistic than the ones in Fig. 5 (c)-(d).

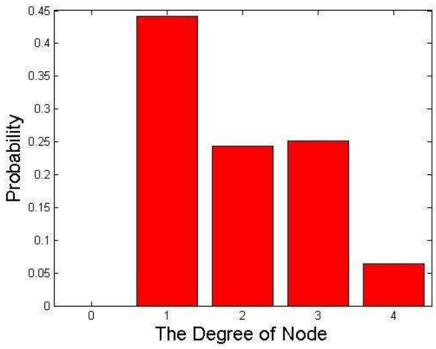


(a) With population-driven for $\beta_2 = 0.001$ (b) With population-driven for $\beta_2 = 0.01$

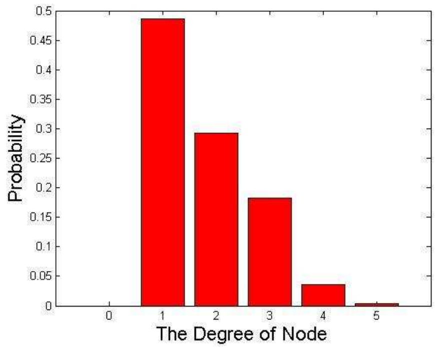
Fig. 6 The topology of road network with population distribution under degree constraint

4. 2 Degree distribution

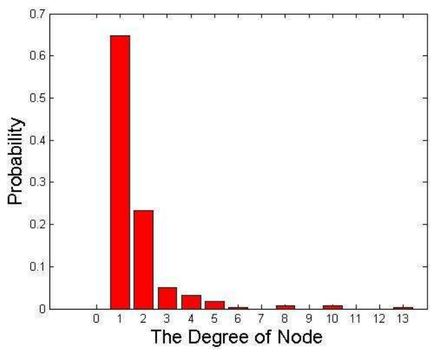
Fig.7 shows the degree distribution of road network with and without the population-driven. In Fig. 7 (a), we can see that, without population-driven, there are just four classes of node degrees (with node degree 1, 2, 3, and 4), and the percentage of the node degree 1 is the highest whilst that of node degree 4 is the lowest. On the other hand, Fig. 7 (b) - (d) clearly depict the sensitivity of node degrees to the parameter β_2, in that Figure, the node degree increases with the parameter β_2 when the population distribution is considered. A common feature is that nodes with one link have the maximum percentage. This is because that more and more centers with the different degrees (zones with different degrees) are generated with consideration of population, but the number of leaf nodes is always than that of the root nodes.



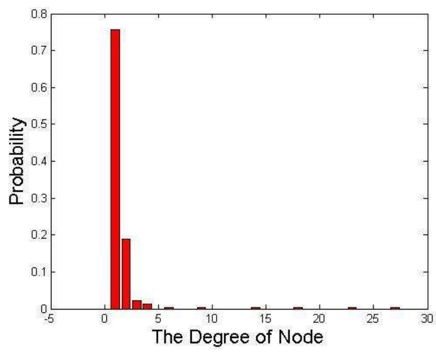
(a) Without population-driven



(b) With population-driven $\beta_2 = 0.0001$



(c) With population-driven $\beta_2 = 0.001$



(d) With population-driven $\beta_2 = 0.01$

Fig. 7 Node degree distributions without and with consideration of population

The average degree of a network is considered as a very important measure of network topology (Zhou et al. 2005). Fig.8 shows the evolution of average degree over the dynamical iterations. It can be seen that the average degrees with and without population-driven are both monotonically decreasing with iterations, while the average degree of population-driven road network is consistently lower than that without accounting for population. This is because the number of leaf nodes increases with iteration times. When population is considered, higher degree of the zones will be generated.

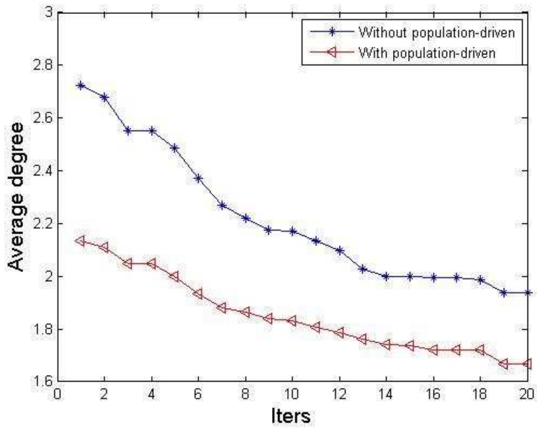
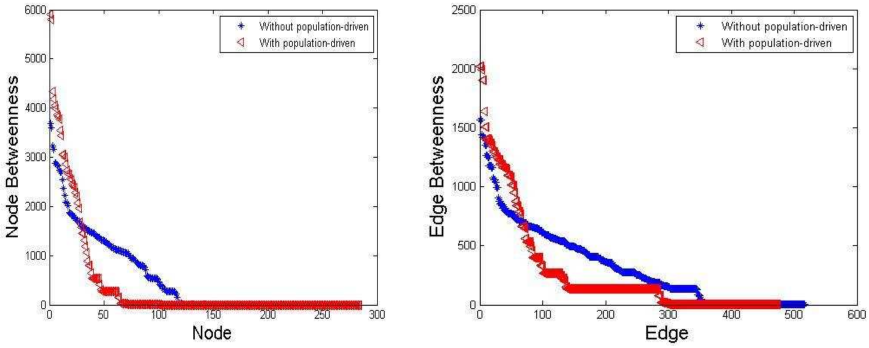


Fig. 8 Average degree distribution without population-driven, and with population driven with $\beta_2 = 0.001$

4. 3 Betweenness centrality

Fig. 9 shows the node betweenness centrality and edge betweenness centrality with and without population-driven. According to Fig. 9 both the node and edge betweenness with population-driven are initially higher than their counterparts without population-driven at lower node (and edge) values. As the nodes (and edges) increase, the betweenness values with population-driven decrease more sharply with node (and edge) numbers than those without population. This is because that when the population distribution is considered, more zones may be linked to the same zone and the center (one zone with high degree) will be generated. Therefore, some roads will be used frequently, while the rest of the roads will be rarely used, leading to small betweenness centrality when population is not considered.



(a) Node betweenness centrality (b) Edge betweenness centrality

Fig. 9 Node betweenness centrality (a) and edge betweenness centrality (b).

Next, we examine the hierarchy in the road network evolution. The edge betweeness centrality is adopted as representing the traffic volume on the road network. We calculate the betweeness centrality for all links of the road network generated by Model. For a simple representation of the hierarchy, all of edges are arbitrarily divided into five classes and are marked with different thicknesses representing the betweenness centrality. It is clear in Fig. 10 that the lines further from the center are thinner, indicating smaller centrality values. This result implies that the links nearer to the network center need to be built with larger capacity, whilst links further away from the center need relatively smaller capacity.

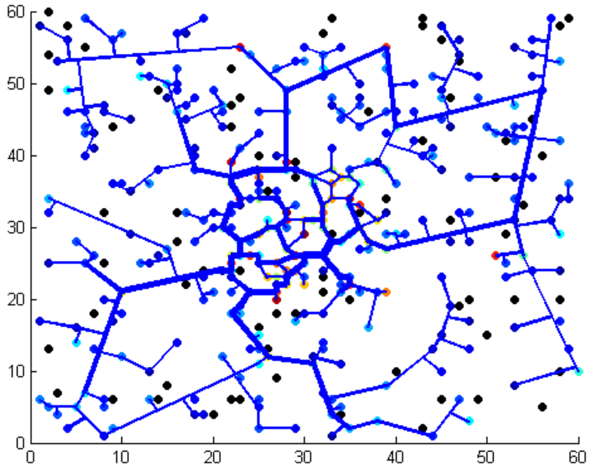
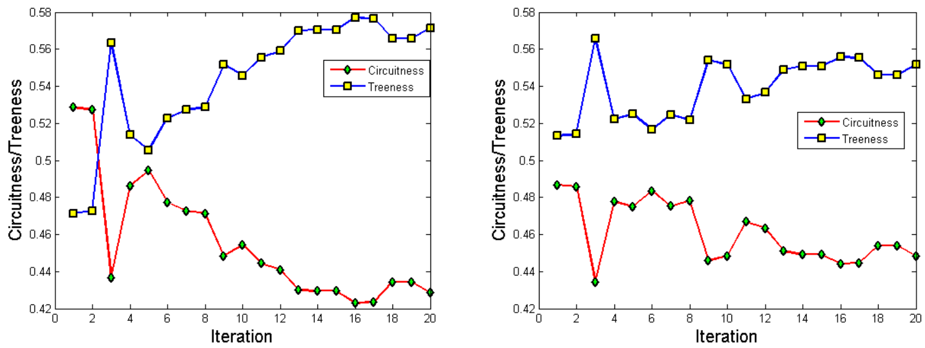


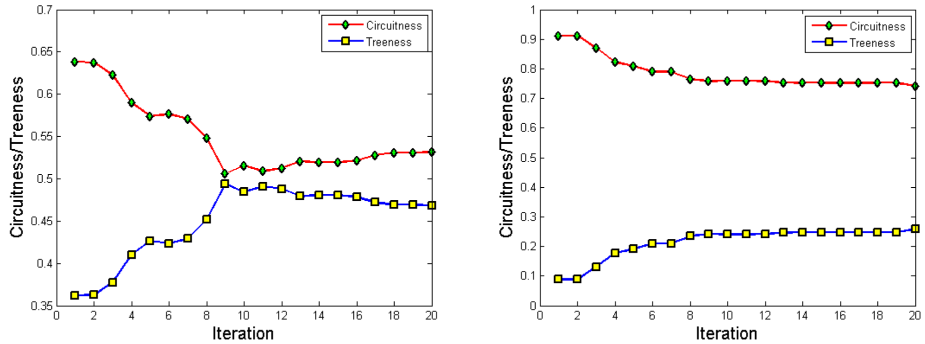
Fig. 10 The hierarchy of road network

4. 4 Circuitness and treeness

The size of the circuitness or treeness reflects the closeness of the urban road network to a circle or tree structure. Fig. 11 reports the evolution results of the circuitness and treeness with and without consideration of population.



(a) Without population-driven (b) With population-driven $\beta_2 = 0.0001$



(c) With population-driven $\beta_2 = 0.001$ (d) With population-driven $\beta_2 = 0.01$

Fig. 11 The circuitness or treeness with and without consideration of population

Fig. 11(a) shows that without consideration of population distribution and after initial oscillation, treeness is consistently greater than that of circuitness, and it increases steadily with iteration whilst circuitness decreases with iteration. Fig.11 (b) - (d) display the results with population-driven under the different β_2 values. At low β_2 value (Fig. 11(b)), the results are similar to those in Fig. 11(a). As the β_2 increases, the network evolves towards a circle structure. At high β_2 value (Fig. 11(d)), the network is of a clear circle structure. These results imply that the lowers cost between zones leads to the topology of network approaches the circle structure.

4. 5 Coverage

Coverage measures the uniformity of a road network. Fig. 12 shows the evolution of the network coverage under model with population. It can be seen that the coverage of urban road network increases with iterations, starting with the coverage of 0.6 at iteration 1 and reaching 1.3 by iteration 20. The result implies that the uniformity of road network increases as population grows.

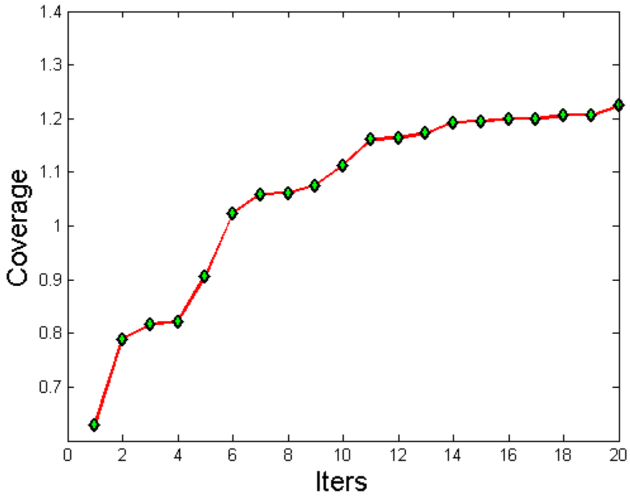


Fig. 12 Coverage of the road network

5. Conclusions

This paper proposed an urban dynamic evolution model considering the population distribution and the road network topology. In the model, the road network evolution is described as an iterative process of adding new zones for the growing population and building new roads to connect the new zones. The urban network is represented as a relative neighborhood graph and the new road-building is formulated as a Fermat-Weber location problem. A simulation experiment is presented to illustrate the key features of proposed model. More specifically, measures on degree distribution, betweenness centrality, coverage, circuitness and treeness, are used to examine the impact of population distribution on the evolution of network topology. Experimental results demonstrate that the accessibility and uniformity of road network with the consideration of population distribution is better.

This paper opens up many future research directions. In the current paper, we have not considered the travelers' route choice responses to new roads, nor have we considered the investment constraints on road building. These effects could be incorporated into the proposed framework in future studies. Second, in reality, the population growth is uncertain and the capacities of urban road network are stochastic. Extending the proposed framework to capture these uncertainties is an important future research direction. Finally, how to incorporate the other social-economic mechanisms, such as land use and environment, into the proposed framework is another direction worthy investigation.

Acknowledgements

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References

Alberti M, Waddell P (2000) An integrated urban development and ecological simulation model. Integr Assess 1(3): 215-227.

Barthélemy M (2003) Betweenness centrality in large complex networks. Eur Phys J B 38(2):163-168.

Barthélemy M, Flammini A (2008) Modeling Urban Street Patterns. Phys Rev Lett 100(13): 138702.

Barthélemy M, Flammini A (2009) Co-evolution of density and topology in a simple model of city formation. Netw Spat Econ 9(3): 401-425.

Boyce DE (1984) Urban transportation network-equilibrium and design models: recent achievements and future prospects. Environment and Planning A 16 (1):1445–1474.

Chen A, Zhou Z, Chootinan P, Ryu S, Yang C, Wong SC(2011) Transportation network design problem under uncertainty: a review and new developments. Transp Rev 31(6): 743-768

Courtat T, Gloaguen C (2011) Mathematics and morphogenesis of the cities: a geometrical approach. Phys Rev E 83(3): 036106.

Ding Y, Lou Y (1998) Application of fractal theory in the evaluation transportation network. J Shanghai Maritime Univ 19(4): 7-12.

Dorogovtsev SN, Mendes JFF, Samukhin AN (2001) Size-dependent degree distribution of a scale-free growing network. Phys Rev E 63(6): 062101.

Erath A, Löchl M, Axhausen KW (2009) Graph-Theoretical Analysis of the Swiss Road and Railway Networks Over Time. Netw Spat Econ 9(3): 379-400.

Facchinei F and Pang JS. Finite-Dimensional Variational Inequalities and Complementarity Problems, Volume I, Spring-Verlag New York Inc, 2003.

Figueiredo L, Machado JAT (2007) Simulation and dynamics of freeway traffic. Nonlinear Dynam 49 (4): 567-577.

Freeman LC (1997) A set of measures of centrality based on betweenness. Sociometry 40 (1): 35-41.

Friesz TL (1985) Transportation network equilibrium, design and aggregation: key developments and research opportunities. Transp Res A 19(5–6): 413–427.

Haggett P, Chorley R (1969) Network Analysis in Geography. London, U.K: Edward Arnold.

Handy S, Cao X, Mokhtarian P (2005) Correlation or causality between the built environment and travel behavior? Evidence from northern california. Transp Res D 10(6): 427-444.

Hwang FK, Richards DS (1992) Steiner Tree Problems. Networks 22(10): 55-89.

Jaromczyk JW, Toussaint GT (1992) Relative Neighborhood Graphs and Their Relatives. Proc of the IEEE 80 (9):1502-1517.

Karger DR, Klein PN, Tarjan RE (1995) A Randomized Linear-time Algorithm to Find Minimum Spanning Trees. J ACM 42 (2): 321-328.

Levinson D (2008) Density and dispersion: the co-development of land use and rail in London. J Econ Geogr 8(1): 55-77.

Levison D, Xie F, Zhu S (2007) The co-evolution of land use and road networks. Proc 17th Int Sympo Transp Traffic Theory 111-126 839-859.

Levinson D, Yerra B (2006) Self-organization of transportation networks . Transp Sci 40(2), 179-188.

Magnanti TL, Wong RT (1984) Network design and transportation planning: models and algorithms. Transp Sci 18 (1):1–55.

Migdalas A (1995) Bilevel programming in traffic planning: models, methods and challenge. J. Globl Optim 7 (4):381–405.

Schweitzer F, Ebeling W, Rose H, Weiss O (1997) Optimization of road networks using evolutionary strategies. Evolu Comput 5(4): 419-438.

Supowit KJ (1983) The relative neighborhood graph with an application to minimum

spanning Trees. J ACM 30(3): 428-448.
Toussaint GT (1980) The Relative Neighbourhood Graph of a Finite Planar Set. Pattern Recogn 12(4): 261-268.
Vardi Y, Zhang CH (2001) A modified Weiszfeld algorithm for the Fermat-Weber location problem. Mathem Program 90(3): 559-566.
Weiszfeld E. (1937): Sur le point pour lequel la somme des distances de n points donnés est minimum. Tôhoku Mathematics Journal **43**, 355–386
Wu JJ, Xu MT, Gao ZY (2013) Coevolution dynamics model of road surface and urban traffic Structure. Nolinear Dynam 73(3): 1327-1334.
Xie F, Levinson D (2007) Measuring the Structure of Road Networks. Geogr Anal 39(3): 336-356.
Xie F, Levinson D (2009) Topological Evolution of Surface Transportation Networks. Comput Environ & Urban Syst 33(3): 211-223.
Xie F, Levinson D (2011) Evolving Transportation Networks, Springer.
Yamins D, Rasmussen S, Fogel D (2003) Growing Urban Roads. Netw Spat Econ 3(1): 69-85.
Yang H, Huang HJ (1998) Principle of marginal-cost pricing: How does it work in a general road network? Transp Res A 32(1): 45-54.
Yang H, Bell MGH (1998) Models and algorithms for road network design: a review and some new developments. Transp Rev 18(3): 257-278.
Zanjirani FR, Miandoabchi E, Szeto WY, Rashidi H (2013) A review of urban transportation network design problems. Eur J Oper Res 229(2): 281-302.
Zhao F X, Wu J J, Sun H J, Gao Z Y (2015). Role of human moving on city spatial evolution. Phys A: 419: 642-650.
Zhou T, Yan G, Wang BH (2005) Maximal planar networks with large clustering coefficient and power-law degree distribution. Phys Rev E 71(4): 046141.