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Owens, D.H. (1977) Optimization of Control Margins in Nuclear Reactor Systems. Research Report. ACSE Research Report 53 . Department of Automatic Control and Systems Engineering

Optimization of Control Margins in Nuclear Reactor Systems

by

D. H. Owens, B.Sc.,A.R.C.S.,Ph.D.,A.F.I.M.A.

Lecturer in Department of Control Engineering
University of Sheffield
Mappin Street
Sheffield S1 3JD

January 1977

SRC Vacation School on Systems Modelling and Optimization 1977

Optimization of Industrial Processes I

Optimization of Control Margins in Nuclear Reactor Systems

Dr. D. H. Owens, University of Sheffield

1. Introduction

In many applications of optimal control theory to industrial processes it can be difficult to choose an appropriate form of performance criterion to guarantee the form of system response required. In most cases the desired structure of the performance criterion is known but the individual control and state weightings can only be 'guessed' and the optimal design achieved by iterating on available parameters. In rough terms, the typical design exercize can be undertaken as follows,

STEP ONE: Formulate the system model, control constraints and a
             trial performance criterion.

STEP TWO: Use a numerical technique to solve the resulting optimal
             control problem.

STEP THREE: Examine the resulting state and control trajectories.   If
             satisfactory, stop.   If unsatisfactory, return to step two
             with a performance criterion, modified in such a manner
             as to improve the system response.

In this sense, optimal control theory does not, in general, remove the trial and error aspect of classical approaches to control. It does however enable the inclusion of terminal state constraints and hard controller constraints, and provides an explicit solution to many problems of practical interest where the performance criterion is well-defined in terms of a known control target and the physical nature of the system under consideration. It is the purpose of this lecture to describe the application of optimal control to such a situation arising in the analysis and control of thermal nuclear power reactors.

2. Nuclear Reactor Dynamics and Xenon Poisoning

A thermal nuclear reactor can be regarded, for our purposes, as a finite volume of space containing uranium fuel, a neutron moderation medium, coolant and structural elements required to contain radioactive products and separate the various components. The reactor can be modelled[1] either by a lumped or point reactor model describing only those variables directly relevant to the reactors engineering environment (eg. net coolant flow, total power output, average core temperature etc.) or by a more detailed distributed model describing the reactor dynamics at points within the core. Each type of model has applications to different situations e.g. a point model would be used when the dynamics and control of the reactor-boiler-turbine combination is under consideration whereas a distributed model is required if the control of peak fuel temperature is under consideration. Whatever the form of model used[2], it will contain the basic physical laws relating reactor power or reactor power distribution to fuel temperature, moderator temperature, coolant density and temperature, fission cross-sections and absor ption cross-sections etc.

Of particular relevance to the long term (one hour to twenty-four hour) dynamics of the reactor is the effect of xenon-135 poisoning[1-2]. Xenon is produced via a radioactive chain of events initiated by the fission of the uranium nucleus by a free neutron producing as a fission fragment iodine-135 which decays by emission of an electron to xenon-135 with a half-life of approximately seven hours. Xenon itself decays by an emission of an electron with a half life of approximately nine hours but, more importantly, xenon can itself absorb neutrons. In other words the fission reaction produces the 'poison' Xenon-135 which tends to inhibit the reaction and suitable control action must be included within the reactor core to offset its effect.

The dynamic equations governing the behaviour of the iodine $I(\underline{r},t)$ and xenon $X(\underline{r},t)$ concentrations at a point $\underline{r}$ within the reactor core are[1-2]

$$\gamma_I \Sigma_f(\underline{r}) P(\underline{r},t) - \lambda_I I(\underline{r},t) = \frac{\partial I}{\partial t}(\underline{r},t) \qquad \dots(1)$$

$$\gamma_X \Sigma_f(\underline{r}) P(\underline{r},t) + \lambda_I I(\underline{r},t) - (\lambda_X + \sigma_X(\underline{r}) P(\underline{r},t)) X(\underline{r},t) = \frac{\partial X}{\partial t}(\underline{r},t) \qquad \dots(2)$$

where
γ_I = fractional yield of iodine-135 in fission = 0.056

$\Sigma_f(\underline{r})$ = macroscopic fission cross-section at the point $\underline{r}$. (cm^{-1})

λ_I = iodine decay constant = 0.29×10^{-4} sec^{-1}

γ_X = fractional yield of xenon-135 in fission = 0.003

λ_X = xenon decay constant = 0.21×10^{-4} sec^{-1}

$\sigma_X(\underline{r})$ = microscopic absorbtion cross-section of xenon at $\underline{r}$. (cm^2)

$P(\underline{r},t)$ = power density at the point $\underline{r}$ and time t $(neutrons/sec/cm^2)$

Note that, although (1)-(2) are partial differential equations they contain no spatial derivatives; a useful feature when numerical discretization becomes necessary.

Given a steady state power distribution $P_o(\underline{r})$ it is easily demonstrated that the iodine and xenon concentrations have steady state distributions,

$$I_o(\underline{r}) = \frac{\gamma_I \Sigma_f(\underline{r}) P_o(\underline{r})}{\lambda_I} , \quad X_o(\underline{r}) = \frac{(\gamma_I + \gamma_X) \Sigma_f(\underline{r}) P_o(\underline{r})}{\lambda_I + \lambda_X + \sigma_X(\underline{r}) P_o(\underline{r})} \qquad \dots(3)$$

The rate of neutron absorbtion $(neutrons/sec/cm^3)$ can be expressed in the form $\Sigma_a(\underline{r},t) P(\underline{r},t)$ where $\Sigma_a(\underline{r},t)$ is the macroscopic absorbtion cross-section, taking the form

$$\Sigma_a(\underline{r},t) = \Sigma_a^{(o)}(\underline{r},t) + \sigma_X(\underline{r}) X(\underline{r},t) + \Sigma_c(\underline{r},t) + \Sigma_c^{(o)}(t)$$
$$\dots(4)$$

where $\quad \Sigma_a^{(o)}(\underline{r},t)$ = power dependent absorbtion cross-section

$\quad\quad\quad \Sigma_c(\underline{r},t)$ = spatially dependent trimming control action

$\quad\quad\quad \Sigma_c^{(o)}(t)$ = spatially independent bulk control action

3.   Spatial Control Margins and Power Manoeuvres

Consider the situation of a load-following nuclear station during the 'quiet period' overnight when power demand drops.   For theoretical purposes it is supposed that the reactor maintains a 100% steady state power distribution $P_o(\underline{r})$ during the daytime period and at time $t = 0$ the total power output drops to f% of the 100% value and is maintained at this value until the time $t = T$ (known terminal time) when the total power moves rapidly back to the 100% power condition (a new day begins!). This situation is represented schematically in Fig.1 together with a typical xenon concentration transient at a sample point in the core. It is noted that the xenon concentration rises during the interval $0 \leqslant t \leqslant T$ so that the absorbtion cross-section $\Sigma_a(\underline{r},t)$ will rise throughout the core in the absence of control action.   More particularly, in general, the xenon concentration will rise by differing amounts at different points within the core.

The initial control problem is to choose the overall control effect $\Sigma_c(\underline{r},T) + \Sigma_c^{(o)}(T)$ to ensure that $P(\underline{r},T) = P_o(\underline{r})$ ie the power distribution at the terminal time T is equal to that at $t = 0$.   This is desirable as any discrepancy here automatically implies higher fuel temperatures at some point within the core which must be avoided to preserve the integrity of the fuel elements.   This condition can be achieved by choosing control so that

$$\Sigma_a(\underline{r},o) \equiv \Sigma_a(\underline{r},T) \qquad\qquad ...(5)$$

or

$$\Sigma_c(\underline{r},T) + \Sigma_c^{(o)}(T) = \Sigma_c(\underline{r},o) + \Sigma_c^{(o)}(o)$$

$$+ \sigma_X(\underline{r}) \{ X(\underline{r},o) - X(\underline{r},T) \} \qquad \ldots(6)$$

which relates the desired control action to the terminal xenon
distribution. This is an idealized situation which can only be
approximated in practice.

It is noted that the control action can be shared between the bulk
and trimming controls and that the total effect (equation (6)) is
governed by the terminal xenon distribution. By (1) and (2) the
terminal xenon distribution is governed by the spatial power dynamics
$P(\underline{r},t)$ during the low power period, $0 \leqslant t \leqslant T$ ie the total spatial control
distribution at $t = T$ is dictated by the spatial power dynamics $P(\underline{r},t)$
during the low power period. This observation makes possible the
secondary control problem of choosing $P(\underline{r},t)$, $0 \leqslant t \leqslant T$, such that the
terminal control action defined by (6) has desired characteristics.

The problem considered in this lecture is the maximization of the
spatial control margins or equivalently the choice of low power profile
to minimize the required trimming action. In principle we require that

$$\Sigma_c(\underline{r},T) \equiv 0 \qquad \ldots(7)$$

as, in practice, the effect of trimming control is limited in capacity
and the trimming control also has to deal with disturbance regulation.
There is no reason to suppose that (7) can be satisfied exactly but
it is possible to obtain a least squares approximation to this condition
by the application of optimal control methods.

4. Formulation of the Optimal Control Problem

The distributed system of (1)-(2) is approximated by a lumped state vector model by the division of the reactor core into N zones of volume ΔV_j, $1 \leqslant j \leqslant N$, and the iodine and xenon dynamics restricted to single points $\underline{r}_j$, $1 \leqslant j \leqslant N$, within each zone. The state equations take the form

$$\underline{\dot{x}}(t) = Ax(t) + B\underline{p}(t) + C(x(t), p(t))$$

$$\underline{x}(o) = \underline{x}_o \qquad \qquad \ldots(8)$$

where

$$A \triangleq \begin{bmatrix} -\lambda_I & 0 & 0 & 0 & \cdots & \cdots & 0 \\ \lambda_I & -\lambda_X & 0 & 0 & & & \vdots \\ 0 & 0 & -\lambda_I & 0 & & & \vdots \\ 0 & 0 & \lambda_I & -\lambda_X & & & \vdots \\ \vdots & & & & \ddots & & \vdots \\ \vdots & & & & & -\lambda_I & 0 \\ 0 & \cdots & \cdots & \cdots & 0 & \lambda_I & -\lambda_X \end{bmatrix} \quad (2N \times 2N) \qquad \ldots(9)$$

(block-diagonal)

$$B \triangleq \begin{bmatrix} \gamma_I \Sigma_f(\underline{r}_1) & 0 & 0 & \cdots & 0 \\ \gamma_X \Sigma_f(\underline{r}_1) & 0 & 0 & & \vdots \\ 0 & \gamma_I \Sigma_f(\underline{r}_2) & 0 & & \vdots \\ 0 & \gamma_X \Sigma_f(\underline{r}_2) & 0 & & \vdots \\ \vdots & & & & \vdots \\ \vdots & & & & 0 \\ \vdots & & & 0 & \gamma_I \Sigma_f(\underline{r}_N) \\ 0 & \cdots & \cdots & 0 & \gamma_X \Sigma_f(\underline{r}_N) \end{bmatrix} \quad (2N \times N) \quad \ldots(10)$$

$$C(\underline{x},\underline{p}) \overset{\Delta}{=} \begin{pmatrix} 0 \\ -\sigma_X(\underline{r}_1)x_2p_1 \\ -\sigma_X(\underline{r}_2)x_4p_2 \\ 0 \\ \cdot \\ \cdot \\ \circ \\ \cdot \\ \cdot \\ 0 \\ -\sigma_X(\underline{r}_N)x_{2N}p_N \end{pmatrix} \qquad \dots(11)$$

with the state and control vectors

$$\underline{p}(t) \overset{\Delta}{=} \left[P(\underline{r}_1,t), P(\underline{r}_2,t), \dots, P(\underline{r}_N,t) \right]^T$$

$$\underline{x}(t) \overset{\Delta}{=} \left[I(\underline{r}_1,t), X(\underline{r}_1,t), I(r_2,t), X(\underline{r}_2,t), \dots, X(\underline{r}_N,t) \right]^T$$

$$\dots(12)$$

with initial condition

$$\underline{x}_o \overset{\Delta}{=} \left[I_o(\underline{r}_1), X_o(\underline{r}_1), \dots, X_o(\underline{r}_N) \right]^T \qquad \dots(13)$$

The vector function $C(\underline{x},\underline{p})$ is continuous and bilinear ie in particular,

$$\lambda C(\underline{x},\underline{p}) = C(\lambda\underline{x},\underline{p}) = C(\underline{x},\lambda\underline{p}) \qquad \dots(14)$$

for all scalars λ and vectors $\underline{x},\underline{p}$. Also

$$C(\underline{x}_1+\underline{x}_2, \underline{p}) = C(\underline{x}_1, \underline{p}) + C(\underline{x}_2, \underline{p})$$

$$C(\underline{x},\underline{p}_1+\underline{p}_2) = C(\underline{x},\underline{p}_1) + C(\underline{x},\underline{p}_2) \qquad \dots(15)$$

for all vectors $\underline{x}$, $\underline{p}_1$, $\underline{p}_2$.

During the low power period the total reactor power must be $f\%$ of the steady state full power condition ie

$$\sum_{j=1}^{N} P(\underline{r}_j,t)\Sigma_f(\underline{r}_j)\Delta V_j = \frac{f}{100} \sum_{j=1}^{N} P_o(\underline{r}_j)\Sigma_f(\underline{r}_j)\Delta V_j \qquad \dots(16)$$

or, defining

$$\underline{\beta} \triangleq \left[\Sigma_f(\underline{r}_1)\Delta V_1, \ \Sigma_f(\underline{r}_2)\Delta V_2, \ \dots, \ \Sigma_f(\underline{r}_N)\Delta V_N\right]^T \qquad \dots(17)$$

$$\underline{p}_o \triangleq \left[P_o(\underline{r}_1), \ P_o(\underline{r}_2), \ \dots, \ P_o(\underline{r}_N)\right]^T \ \frac{f}{100} \qquad \dots(18)$$

then the control restraint set Ω is defined by the relation

$$\Omega \triangleq \{ \ \underline{p} \in R^N : \ \underline{\beta}^T \{\underline{p} - \underline{p}_o\} = 0 \ \} \qquad \dots(19)$$

To formulate the required performance criterion note that, without loss of generality, we can always suppose that (equation (6))

$$\sum_{j=1}^{N} \Sigma_c(\underline{r}_j, t) = 0 \ , \qquad 0 \leqslant t \leqslant T \qquad \dots(20)$$

by suitable sharing the control between trimming and bulk action. Using this relation in (6) yields the equation

$$N \Sigma_c^{(o)}(T) = N \Sigma_c^{(o)}(o) + \sum_{j=1}^{N} \sigma_X(\underline{r}_j)\{X(\underline{r}_j, o) - X(\underline{r}_j, T)\} \dots(21)$$

Assuming for simplicity that $\Sigma_c(\underline{r}, o) \equiv 0$ then, substituting (21) into (6) yields, $1 \leqslant j \leqslant N$,

$$\Sigma_c(\underline{r}_j, T) = \sigma_X(\underline{r}_j) \{X(\underline{r}_j, o) - X(\underline{r}_j, T)\}$$

$$- \frac{1}{N} \sum_{i=1}^{N} \sigma_X(\underline{r}_j)\{X(\underline{r}_i, o) - X(\underline{r}_i, T)\} \qquad \dots(22)$$

Defining

$$J(\underline{p}) \triangleq \sum_{j=1}^{N} (\Sigma_c(\underline{r}_j, T))^2 \qquad \dots(23)$$

then the problem of maximization of control margins is formulated[3] as the search for the input trajectory $p^*(t) \in \Omega$, $0 \leqslant t \leqslant T$, generating a solution of the state equations (8) and minimizing the performance criterion (23).

Equation (23) can be transformed into a more conventional form by using (22) to write

$$J(\underline{p}) = \{\underline{x}(T) - \underline{\alpha}\}^T F \{\underline{x}(T) - \alpha\} \qquad \qquad \ldots(24)$$

where F is symmetric and positive semi-definite and α is a constant vector. In this form the optimization problem is recognized[4] as a quadratic terminal optimization problem with bilinear state equations and equality control constraints.

5. A Numerical Optimization Technique

The numerical solution of the optimal control problem posed in section (5) can be attempted by application of techniques such as the steepest descent algorithm. An improved algorithm[4] can be obtained in this case however which also provides a simple illustration of Pontriagins Principle.

Theorem[3,4]

If $\underline{p}_1(t)$ and $\underline{p}_2(t)$ are allowable power trajectories, $0 \leqslant t \leqslant T$, generating state trajectories $\underline{x}_1(t)$, $\underline{x}_2(t)$ respectively satisfying (8) then

$$J(\underline{p}_2) - J(\underline{p}_1) = \int_o^T \{H[\underline{x}_1(t),\underline{p}_2(t),\underline{\lambda}(t),t] - H[\underline{x}_1(t),\underline{p}_1(t),\underline{\lambda}(t),t]\}dt$$

$$+ \{\underline{x}_2(T) - \underline{x}_1(T)\}^T F\{\underline{x}_2(T) - \underline{x}_1(T)\} \qquad \ldots(25)$$

where the Hamiltonian

$$H[\underline{x},\underline{p},\underline{\lambda},t] \triangleq \{\{B + \frac{\partial C}{\partial \underline{p}}\bigg|_{\underline{x}}\}^T \underline{\lambda}\}^T \underline{p} \qquad \ldots(26)$$

and the costate equations are

$$\dot{\underline{\lambda}}(t) = -A^T \underline{\lambda}(t) - \frac{\partial C}{\partial \underline{x}}\bigg|^T_{\underline{p}_2} \underline{\lambda}(t)$$

$$\underline{\lambda}(T) = 2F\{\underline{x}_1(T) - \underline{\alpha}\} \qquad \qquad \ldots(27)$$

<u>Proof</u>

Using integration by parts

$$\lambda(T)^T\{\underline{x}_2(T) - \underline{x}_1(T)\}$$

$$= \int_0^T \{ \underline{\dot{\lambda}}(t)^T\{\underline{x}_2(t) - \underline{x}_1(t)\} + \underline{\lambda}(t)^T\{\underline{\dot{x}}_2(t) - \underline{\dot{x}}_1(t)\}\}dt \quad \ldots(28)$$

It follows directly from the definitions that

$$\lambda(T)^T\{\underline{x}_2(T) - \underline{x}_1(T)\} = J(\underline{p}_2) - J(\underline{p}_1)$$

$$- \{\underline{x}_2(T) - \underline{x}_1(T)\}^T F \{\underline{x}_2(T) - \underline{x}_1(T)\} \quad \ldots(29)$$

Also

$$\int_0^T \{ \underline{\dot{\lambda}}^T \{\underline{x}_2 - \underline{x}_1\} + \underline{\lambda}^T \{\underline{\dot{x}}_2 - \underline{\dot{x}}_1\}\}dt$$

$$= \int_0^T \{\{-A^T\underline{\lambda} - \frac{\partial C}{\partial \underline{x}} \Big|_{\underline{p}_2}^{\mathcal{T}} \underline{\lambda}\}^T\{\underline{x}_2 - \underline{x}_1\}$$

$$+ \underline{\lambda}^T \{A\{\underline{x}_2 - \underline{x}_1\} + B\{\underline{p}_2 - \underline{p}_1\} + C(\underline{x}_2, \underline{p}_2) - C(\underline{x}_1, \underline{p}_1)\}\}dt$$

$$= \int_0^T \{\underline{\lambda}^T B\{\underline{p}_2 - \underline{p}_1\} + \underline{\lambda}^T\{C(\underline{x}_2, \underline{p}_2) - C(\underline{x}_1, \underline{p}_1) - \frac{\partial C}{\partial \underline{x}} \Big|_{\underline{p}_2} \{\underline{x}_2 - \underline{x}_1\}\}\}dt$$

$$= \int_0^T \{H[\underline{x}_1, \underline{p}_2, \underline{\lambda}, t] - H[\underline{x}_1, \underline{p}_1, \underline{\lambda}, t]\}dt \quad \ldots(30)$$

as

$$C(\underline{x}_2, \underline{p}_2) - C(\underline{x}_1, \underline{p}_1) - \frac{\partial C}{\partial \underline{x}} \Big|_{\underline{p}_2} \{\underline{x}_2 - \underline{x}_1\}$$

$$= C(\underline{x}_1, \underline{p}_2) - C(\underline{x}_1, \underline{p}_1)$$

$$= \frac{\partial C}{\partial \underline{p}} \Big|_{\underline{x}_1} \{\underline{p}_2 - \underline{p}_1\} \quad \ldots(31)$$

The result follows by substituting (29) and (30) into (28). Q.E.D.

A necessary condition for $J(\underline{p}_2) < J(\underline{p}_1)$ is that,

$$\int_0^T H\left[\underline{x}_1(t),\underline{p}_2(t),\underline{\lambda}(t),t\right]dt < \int_0^T H\left[\underline{x}_1(t),\underline{p}_1(t),\underline{\lambda}(t),t\right]dt \quad \ldots(32)$$

A sufficient condition for (32) to hold is that

$$H\left[\underline{x}_1(t),\underline{p}_2(t),\lambda(t),t\right] < H\left[\underline{x}_1(t),\underline{p}_1(t),\lambda(t),t\right]$$
$$, \quad 0 \leqslant t \leqslant T \quad \ldots(33)$$

ie a 'descent direction' for the Hamiltonian H is a 'descent direction' for J.

The above analysis suggests the following numerical procedure for the systematic reduction of the performance criterion.

STEP 1: Choose an initial trial solution $\underline{p}_1(t)$.

STEP 2: Evaluate the state trajectory $\underline{x}_1(t)$, $0 \leqslant t \leqslant T$, from the state equations and evaluate the cost $J(\underline{p}_1)$.

STEP 3: Choose a real scalar $\varepsilon > 0$ and compute an updated power transient $\underline{p}_2(t)$ by integrating the costate equations (27) backwards in time and at each time step solve the algebraic minimization problem

$$H\left[\underline{x}_1(t), \underline{p}_2(t), \underline{\lambda}(t),t\right] - H\left[\underline{x}_1(t), \underline{p}_1(t), \underline{\lambda}(t),t\right]$$
$$+ \varepsilon \{\underline{p}_2(t) - \underline{p}_1(t)\}^T \{\underline{p}_2(t) - \underline{p}_1(t)\}$$
$$= \min_{\underline{p} \in \Omega} \{H\left[\underline{x}_1(t), \underline{p}, \underline{\lambda}(t),t\right] - H\left[\underline{x}_1(t), \underline{p}_1(t), \underline{\lambda}(t),t\right]$$
$$+ \varepsilon \{\underline{p} - \underline{p}_1(t)\}^T \{\underline{p} - \underline{p}_1(t)\} \} \quad \ldots(34)$$

The scalar $\varepsilon > 0$ is required to reduce the magnitude of $\underline{p}_2(t) - \underline{p}_1(t)$ and hence reduce the positive effect of the second order terms in (25).

STEP 4: Evaluate the state trajectory $\underline{x}_2(t)$, $0 \leqslant t \leqslant T$, and $J(\underline{p}_2)$.
If $J(\underline{p}_2) > J(\underline{p}_1)$ increase the value of ε (typically, double it!) and return to step three.   If $J(\underline{p}_2) < J(\underline{p}_1)$ replace $\underline{p}_1$ by $\underline{p}_2$ and return to step two.

In practice the algorithm is terminated by a convergence criterion which could be based on the magnitude of the changes in power transients, but is more conveniently based on a maximum number of iterations coupled with the requirement that the performance criterion is reduced by a required fraction of the original guess.

6. A Numerical Example

Dividing the reactor into three zones (ie N = 3) with the data

$$P_o(\underline{r_1}) = 0.8 \times 10^{14} \quad , \quad P_o(\underline{r_2}) = 1.0 \times 10^{14} \quad , \quad P_o(\underline{r_3}) = 0.8 \times 10^{14}$$

$$f = 20 \quad , \quad T = 7 \text{ hours}$$

$$\Sigma_f(\underline{r_1}) = 0.21 \times 10^{-2} \quad , \quad \Sigma_f(\underline{r_2}) = 0.2 \times 10^{-2} \quad , \quad \Sigma_f(\underline{r_3}) = 0.21 \times 10^{-2}$$

$$\sigma_X(\underline{r_1}) = 0.1 \times 10^{-17} \quad , \quad \sigma_X(\underline{r_2}) = 0.1 \times 10^{-17} \quad , \quad \sigma_X(\underline{r_3}) = 0.1 \times 10^{-17}$$

$$\Delta V_1 = 1.0 \quad , \quad \Delta V_2 = 2.0 \quad , \quad \Delta V_3 = 1.0$$

$$\Sigma_c^{(o)} = 0.167 \times 10^{-6}$$

$$\Sigma_c(\underline{r_1}, o) = 0.184 \times 10^{-5} \quad , \quad \Sigma_c(\underline{r_2}, o) = 0.34 \times 10^{-6} \quad , \quad \Sigma_c(\underline{r_3}, o) = -0.216 \times 10^{-5}$$

with the initial guess obtained by choosing the low power profile to be f% of the 100% steady state power distribution. The following numerical results were obtained

Iteration	J
0	0.102×10^{-9}
1	0.250×10^{-10}
2	0.742×10^{-11}
3	0.247×10^{-11}
4	0.981×10^{-12}
5	0.412×10^{-12}
6	0.184×10^{-12}
7	0.829×10^{-13}
8	0.380×10^{-13}
9	0.174×10^{-13}
10	0.802×10^{-14}

ie, in ten iterations, the performance criterion has been reduced by a factor of 10^4 and the algorithm was terminated at this point producing the power transients shown in Fig.2.   In many cases the power transients are difficult to achieve in practice, but the solution can provide useful insight into the general structure of the optimal control policy which can be used to guide the solution of an extended optimization problem including hard power constraints.

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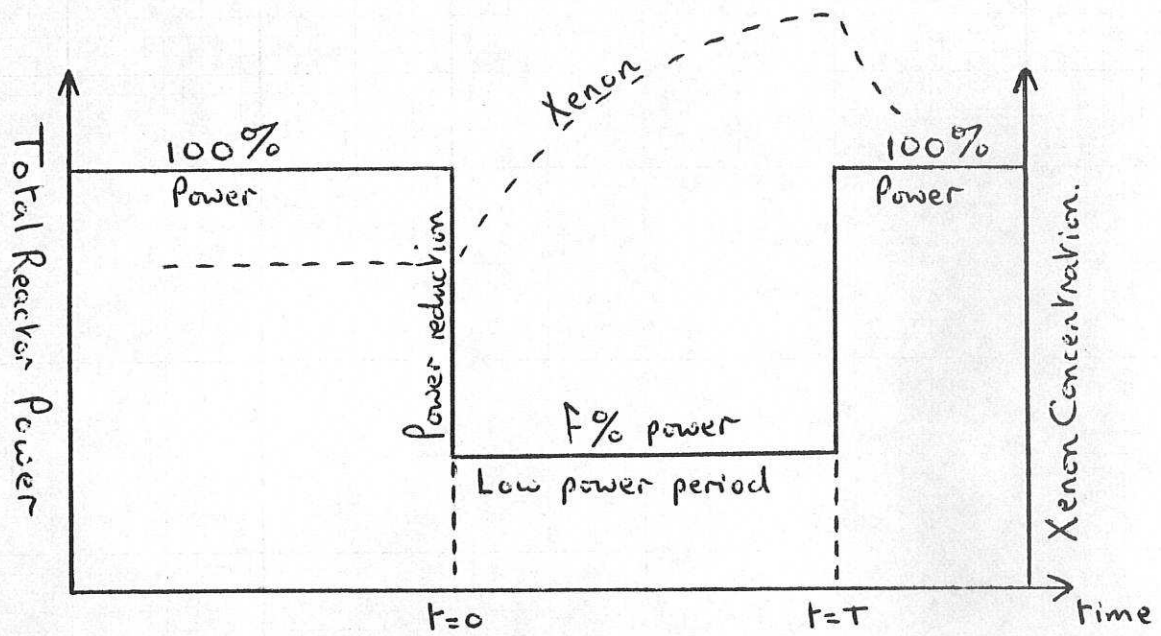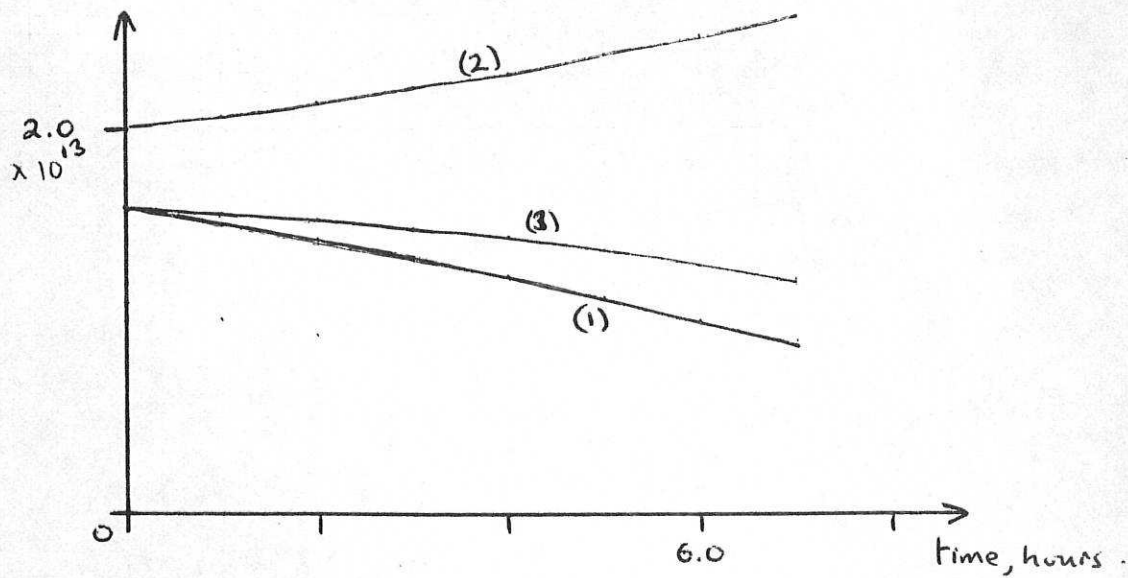
Fig. 1. Overnight Power Manoeuvre.

Fig. 2. Optimal Low Power Transients.
(Curve (i) represents the optimal power profile in zone i)