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Rendering the CIE 1931 Chromaticity Diagram

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ABSTRACT

Widely used in scientific and educational applications, this iconic colour chart consists of a two-dimensional projection of colour coordinates specified by CIE tristimulus values. It is frequently to be found in text books and on posters promoting colour-measuring instruments and related products. Despite its ubiquity, little attention seems to have been paid to correctness of reproduction. On the contrary, most versions simply strive to create a "pleasing" representation which both precludes the diagram being employed for colour identification tasks and also gives a misleading impression of colour relationships.

In this paper, an approach is described through which an improved rendering of the chromaticity diagram may be created in print or other media. The goal is to create a colorimetrically accurate reproduction (given the colour gamut constraints of the medium) whilst preserving hue and chroma smoothness and continuity. In addition, the chart needs to include both white point and also the technically correct depiction of colours close to the spectrum locus.

1. INTRODUCTION

In 1931, the CIE published their system of colorimetry (CIE 1986) which enabled a colour stimulus to be defined by just three numbers, *XYZ*, for a given set of viewing conditions. The system is still in widespread use today for industrial and scientific colour-matching tasks. It had long been recognised that three dimensions were cumbersome, and so a two-dimensional projection was created which ignored the brightness (*Y*) dimension thereby enabling colours to be plotted on an *xy* chromaticity diagram (Wright 2007). On this diagram, additive mixing can be predicted via straight lines connecting the points representing the lights' chromaticities. Chromaticity coordinates are used in numerous applications such as international standards for signal lighting. The diagram is also helpful for examining the dominant wavelength and excitation purity of stimuli.

While many attempts have been made to reproduce the CIE chromaticity diagram, this is typically done of the basis of aesthetic appeal rather than accuracy. As a result, users are presented with a distorted view of CIE colour space which precludes its use for visual matching of a specimen's chromaticity. To produce a "better" diagram, the two most significant obstacles are colour fidelity and colour gamut. Approaches such was the industry-standard ICC system for colour management (Green 2010) can help address the former, however dealing with gamut is rather less straightforward.

2. COLOUR GAMUT

The CIE system of colorimetry was designed to encompass the entire range of visible colour, whereas any practical printing or display system is only capable of reproducing a subset. Furthermore, colour gamut is a device-dependent property, meaning that it varies according to the specific technology and its components.

2.1 Medium Gamut

The range of colours that are reproducible on an imaging medium (e.g. printer and paper) can be described in terms of a gamut boundary through colour measurement. Only colours within this boundary are reproducible. An example of this for the printer used in this study is shown in Figure 1. Similarly, image-acquisition devices (cameras and scanners) have their own colour gamuts which limit how bright or colourful stimuli may be. In addition, images themselves contain a certain range of colours. Gamut mismatch is typically addressed by colour management. With the ICC system, for example, users need to select one of four separate rendering intents according to their application (Green 2010).

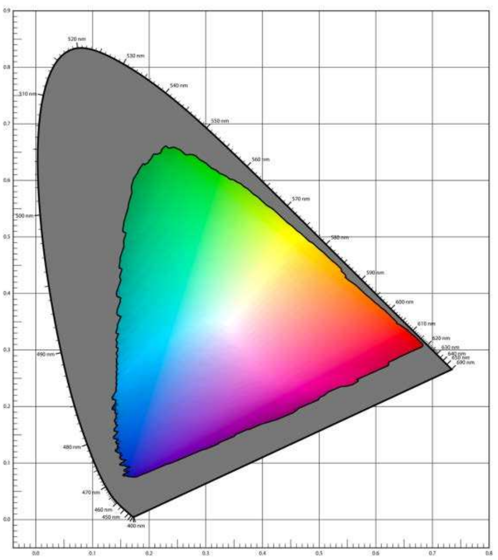


Figure 1: colour gamut of the printer used in this study.

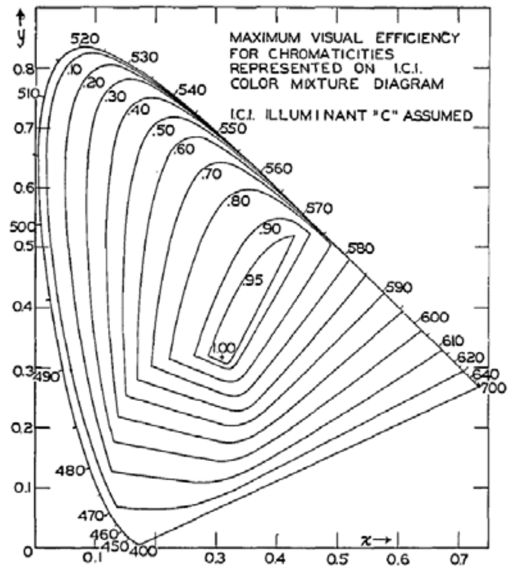


Figure 2: chromatic limits of real colours (MacAdam 1935).

2.2 CIE Gamut

The CIE chromaticity diagram is somewhat misleading in that the chromatic domain does not extend to the spectrum locus at every luminance. Calculations by MacAdam (1935) have shown that lighter colours have a much more restricted range of chromaticities. Only when the luminance factor is zero do colours reach the extremities. This means that a chromaticity diagram becomes darker as distance from the white point increases.

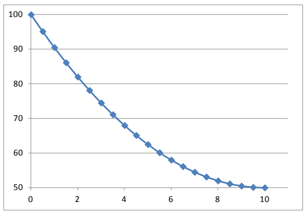
2.3 Gamut Mapping

There have been numerous algorithms proposed for mapping colours from one medium's gamut to another – for a survey of some of these, see Morovič (2008). These are generally aimed at producing pleasing images and are thus unsuitable to the task of faithfully reproducing the CIE diagram.

3. PROPOSED SOLUTION

Although it is evidently not possible to render colours beyond the target device's gamut, the out-of-gamut regions should at least be expected to preserve the correct hue if not the chroma. In order to include the white point without further reducing available colour gamut, it is necessary to reduce lightness as chroma is increased. A function was therefore defined which accomplished this as follows:

$$L' = L_{max} + \alpha \bullet C^2 + \beta \bullet C \quad \text{if } C < C_{max}$$
$$L' = L_0 \quad \text{otherwise}$$
$$\text{where } \alpha = (L_{max} - L_0) / C_{max}^2$$
$$\text{and } \beta = -2 \bullet \alpha \bullet C_{max}$$



In the above, L' represents the new CIELAB lightness value to be rendered based on the hue-dependent maximum CIELAB chroma, C_{max} at a CIELAB lightness of L_0. L_{max} is the lightness of the white point (i.e. when $a^* = b^* = 0$). In this case, constant values for L_0, α and β of 50, 0.5 and -10 respectively were chosen in order to make lightness flatten out once the gamut threshold at $L^* = L_0$ and $C^* = C_{max}$ has been crossed. Beyond this boundary, lightness could either be preserved or alternatively – at the risk of changing hue – diminished to better reflect the MacAdam limits.

3.1 Gamut Smoothing

A further complication arises at the interface between in- and out-of-gamut regions. Gamut boundaries differ significantly between devices not only in volume but also shape. The irregularities and practical uncertainties of this boundary can lead to unacceptable discontinuities and so a smoothness constraint needs to be applied. This was accomplished using a combination of averaging of the maximum chroma values at each hue together with a slight relative reduction. The specific parameters used were determined empirically in order to produce visually acceptable results.

3.2 Creating the Diagram

For this study, an HP Designjet Z3200 pigmented inkjet printer was used since it offered a relatively wide colour gamut and also included built-in spectrophotometer that could be used to both calibrate (ensuring repeatability) and characterise (profile) the device. The printer is capable of printing on a wide variety of substrates, however a semi-gloss instant-dry paper was chosen since this was found to deliver superior colour gamut compared to matt paper and also had good lightfastness and stabilisation properties.

The basis for computation was an ICC profile for the printer. Given this, gamut checking calculations were performed to determine the gamut boundary before creating a composite image of the final chart. This image was stored as a 16-bit CIELAB TIFF file to ensure colorimetric fidelity. Other elements of the diagram such as the spectrum locus, axes, grid lines, labels and black body curve were created as vector images using the Postscript language and these were subsequently rasterised as layers within the final TIFF image. These components are illustrated in Figure 3. (It should be pointed out that the colours shown are only approximate: they are meant to be rendered onto a specific printer and so are device dependent.)

4. CONCLUSIONS

Clearly any attempt to render the CIE chromaticity diagram has to involve a compromise due to real-world gamut limitations, however this work has succeeded in creating a much more faithful representation of the chart. The same technique could be applied to the reduction of other – more perceptually uniform – chromaticity diagrams such as the CIE 1976 UCS or $L^*a^*b^*$ which are more approprate for tasks such as comparing colour gamuts.

The entire method has been implemented using a 12-channel wide-format printer which, in conjunction with an enhanced colour-management algorithm, is capable of reproducing individual colours to an accuracy of around 1 ΔE. The resultant charts are now being considered for adoption by the CIE as their official version of the chromaticity diagram.

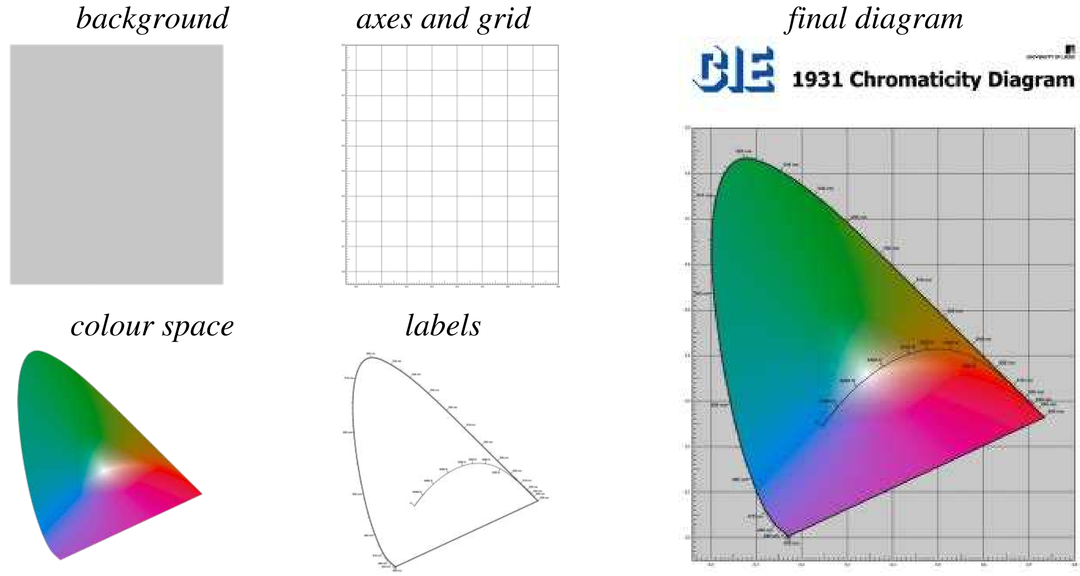


Figure 3: constructing the CIE 1931 chromaticity diagram.

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