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# Sensorless Flux-Weakening Control of Permanent-Magnet Brushless Machines Using Third Harmonic Back EMF

J. X. Shen, *Senior Member, IEEE*, Z. Q. Zhu, *Senior Member, IEEE*, and David Howe

**Abstract**—The sensorless control of brushless machines by detecting the third harmonic back electromotive force is a relatively simple and potentially low-cost technique. However, its application has been reported only for brushless dc motors operating under normal commutation. In this paper, the utility of the method for the sensorless control of both brushless dc and ac motors, including operation in the flux-weakening mode, is demonstrated.

**Index Terms**—Back electromotive force (EMF), brushless drives, permanent-magnet machines, sensorless control.

## I. INTRODUCTION

NUMEROUS sensorless control techniques have been developed for brushless dc (BLDC) and brushless ac (BLAC) machines. For example, methods based on the detection of the zero crossing of the back-electromotive-force (EMF) waveforms [1], [2] are simple, and various commercial ICs are available for BLDC drives. However, they are not applicable to BLAC operation, or even to BLDC operation if the commutation advance or the current decay in the freewheeling diodes is greater than  $30^\circ$  electrical, since the zero crossing of the back EMFs cannot then be detected. Theoretically, back-EMF reconstruction and integration methods [3] are suitable for surface-mounted permanent-magnet (SPM) BLDC and BLAC motors, but not for interior permanent-magnet (IPM) motors, since the stator winding inductance is simulated by a transformer or appropriate electronic components, and cannot vary with rotor position. Moreover, the hardware is relatively complex and implementation is difficult. In all back-EMF-based sensorless techniques, the low-speed performance is limited, and an open-loop starting strategy is required [1]–[3]. In most other sensorless control strategies, such as flux observers [4], model-reference adaptive control [5], [6], extended Kalman filters [7], and adaptive sliding observers [8], a mathematical model of the machine is required. Further, phase or terminal

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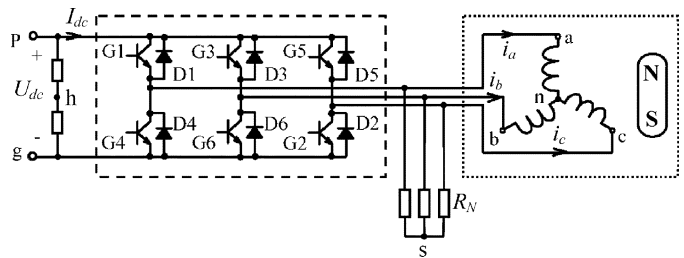


Fig. 1. Schematic of brushless motor with Y-connected resistors.

voltages often have to be measured and A/D converters are required. In order to reduce the number of voltage transducers, voltages can be deduced from the inverter switching status and the measured dc-link voltage [9]. Rotor saliency detection methods [10]–[12] are suitable for low-speed operation, and applicable to IPM BLDC and BLAC drives. Methods which utilize the third harmonic component of the back EMF are attractive since they are relatively simple and potentially low cost. To date, however, their performance has been reported only for BLDC motors operating under normal commutation, i.e., without commutation advance or flux weakening [3], [13]. In this paper, the utility of the third harmonic back-EMF method is demonstrated for the sensorless operation of both BLDC and BLAC drives.

Three methods have been proposed to extract the third harmonic component of the back EMF [3], [13]–[15]. These are evaluated and the most appropriate is identified and implemented for the sensorless control of both BLDC and BLAC drives, including operation in the flux-weakening mode. Finally, experimental results are given to further verify the utility of the method.

## II. EVALUATION OF METHODS FOR EXTRACTING THIRD HARMONIC BACK EMF

Fig. 1 shows a schematic of a permanent-magnet brushless drive with Y-connected resistors to enable the third harmonic component of the back EMF to be sensed. In the literature, three methods for obtaining the third harmonic voltage have been proposed: 1) from the voltage  $u_{sn}$  between the star point “s” of the resistor network and the neutral point “n” of the stator windings [3], [13], [14]; 2) from the voltage  $u_{sh}$  between “s” and the midpoint “h” of the dc bus [13]; and 3) from the voltage  $u_{nh}$  between “n” and “h” [15]. These are evaluated in detail as follows.

### A. Voltage $u_{sn}$

Assuming the inductances of the brushless motor are constant and that the three phases are symmetrical, from Fig. 1

$$u_{sn} = \frac{(u_{an} + u_{bn} + u_{cn})}{3} = (e_3 + e_9 + e_{15} + \dots) \approx e_3 = -E_{m3} \sin 3\theta_r \quad (1)$$

where  $e_3$ ,  $e_9$ , and  $e_{15}$  are the triplen harmonics of the back EMF. Since the derivation does not depend on the motor operating mode, the third harmonic back EMF can be extracted from the voltage  $u_{sn}$  irrespective of whether the motor is operated in BLDC or BLAC mode, provided, of course, that the phase back-EMF waveform contains a third harmonic component. Fig. 2 shows measured waveforms of  $u_{sn}$  when a surface-mounted magnet motor was run as BLAC with pulsewidth modulation (PWM), and as a BLDC motor both with and without PWM. Although, theoretically,  $u_{sn}$  should have a smooth and clean waveform, as will be seen, in practice high-frequency noise exists as a result of commutation and PWM switching events. However, this can be easily eliminated by using either a low-pass filter (LPF) or a bandpass filter (BPF).

### B. Voltage $u_{sh}$

From Fig. 1

$$u_{sh} = \frac{(u_{ag} + u_{bg} + u_{cg})}{3} - \frac{U_{dc}}{2} \quad (2)$$

In BLAC mode, the terminal voltages ( $u_{ag}$ ,  $u_{bg}$ , and  $u_{cg}$ ) are either 0 or  $U_{dc}$ . Therefore, the voltage  $u_{sh}$  comprises a series of pulses, the amplitude being either  $U_{dc}/2$  or  $U_{dc}/6$ , as shown in Fig. 3. Clearly,  $u_{sh}$  cannot be used for the sensorless control of a BLAC drive.

In BLDC mode without PWM

$$u_{sh} = \frac{e_k}{2} - \frac{(e_a + e_b + e_c)}{6} \quad (3)$$

where the subscript  $k$  represents the nonenergized phase. It was assumed in [13] that the back-EMF waveforms of the two energized phases were identical and instantaneously sum to zero. Thus, (3) was rewritten as

$$u_{sh} = \frac{(e_a + e_b + e_c)}{2} - \frac{(e_a + e_b + e_c)}{6} = e_3 + e_9 + e_{15} + \dots \approx e_3 \quad (4)$$

It would appear, therefore, that the third harmonic component of the back EMF could be extracted from  $u_{sh}$ . However, this assumption is only valid when the back-EMF waveform is trapezoidal with a flat top =  $120^\circ$  electrical, which is not generally the case in practice. In general, (3) can be further derived as

$$u_{sh} = \frac{e_k}{2} - \frac{(e_3 + e_9 + e_{15} + \dots)}{2} = \frac{(e_{k1} + e_{k5} + e_{k7} + e_{k11} + \dots)}{2} \approx \frac{e_{k1}}{2} \quad (5)$$

Clearly,  $u_{sh}$  contains information on nonzero-sequence components in the back EMF of the nonenergized phase. Moreover, if the current in the freewheeling diode has not decayed to zero or if PWM is employed, pulse or noise is introduced in  $u_{sh}$ , as

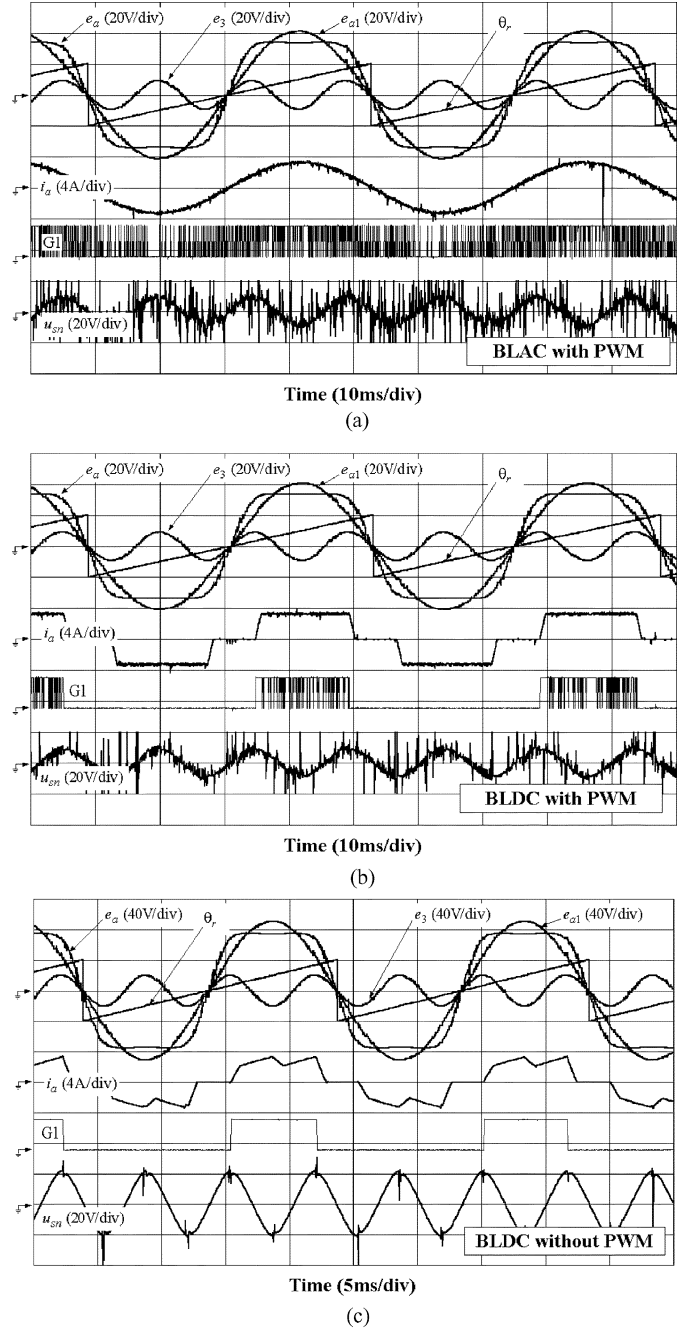


Fig. 2. Measured waveforms of  $u_{sn}$  ( $U_{dc} = 200$  V).  $\theta_r$ : measured rotor position;  $e_a$ ,  $e_{a1}$ ,  $e_3$ : predicted phase back EMF and its fundamental and third harmonic components, which are predicted according to  $\theta_r$  and EMF constant for each harmonic;  $i_a$ : measured phase current; G1: measured gate drive signal. (a) BLAC operation with PWM. (b) BLDC operation with PWM. (c) BLDC operation without PWM.

evidenced in Fig. 3. Therefore, in general, it is inappropriate to use  $u_{sh}$  for the sensorless control of a BLDC drive.

### C. Voltage $u_{nh}$

From Fig. 1

$$u_{nh} = u_{sh} - u_{sn} \quad (6)$$

In BLAC mode,  $u_{sh}$  is essentially noise while  $u_{sn}$  represents the zero-sequence back EMF of a nonenergized phase. Therefore, while  $u_{nh}$  contains the third harmonic component of

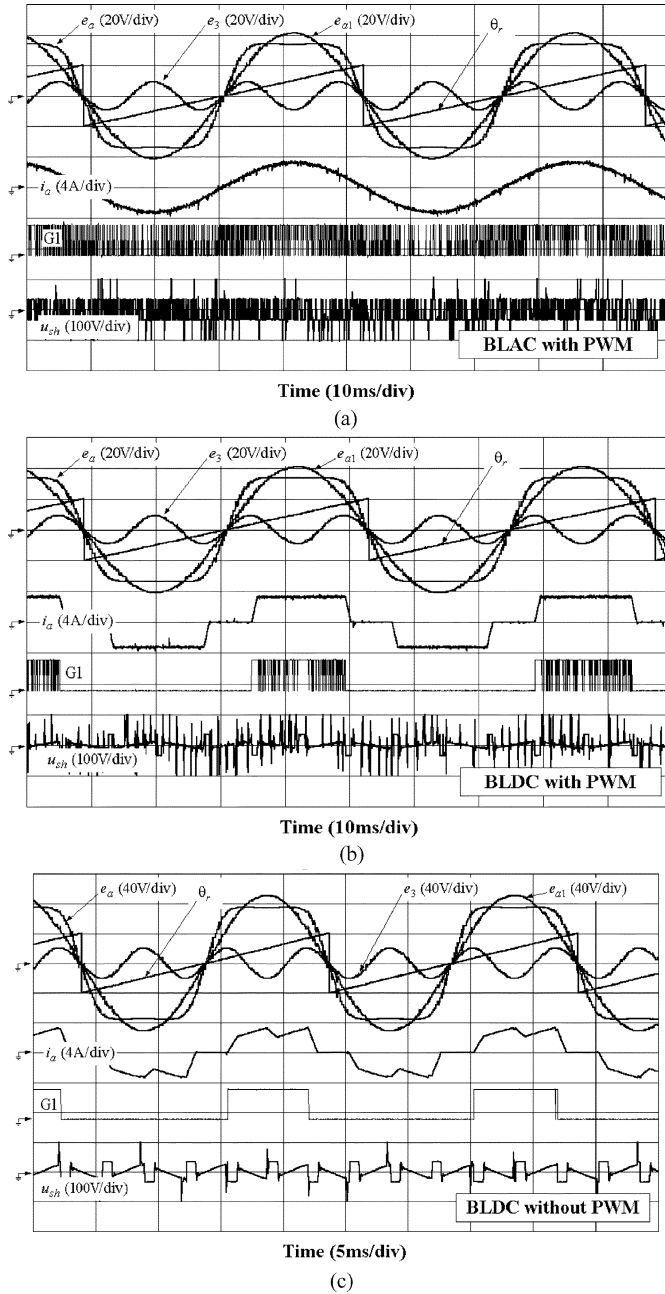


Fig. 3. Measured waveforms of  $u_{sh}$  ( $U_{dc} = 200$  V). (a) BLAC operation with PWM. (b) BLDC operation with PWM. (c) BLDC operation without PWM.

back EMF, it also contains significant noise, as shown in Fig. 4. Clearly, it is more difficult to extract the third harmonic component of back EMF from  $u_{nh}$  than from  $u_{sn}$ . Moreover, both methods require access to the winding neutral point.

In BLDC mode,  $u_{nh}$  can be derived by substituting (1) and (5) into (6), to give

$$\begin{aligned}
 u_{nh} &= \frac{[e_k - (e_3 + e_9 + \dots)]}{2} - (e_3 + e_9 + \dots) \\
 &= \frac{e_k}{2} - \frac{3(e_3 + e_9 + \dots)}{2} \\
 &= \frac{e_k}{2} - \frac{(e_a + e_b + e_c)}{2} \\
 &= -\frac{(e_m + e_n)}{2}
 \end{aligned} \tag{7}$$

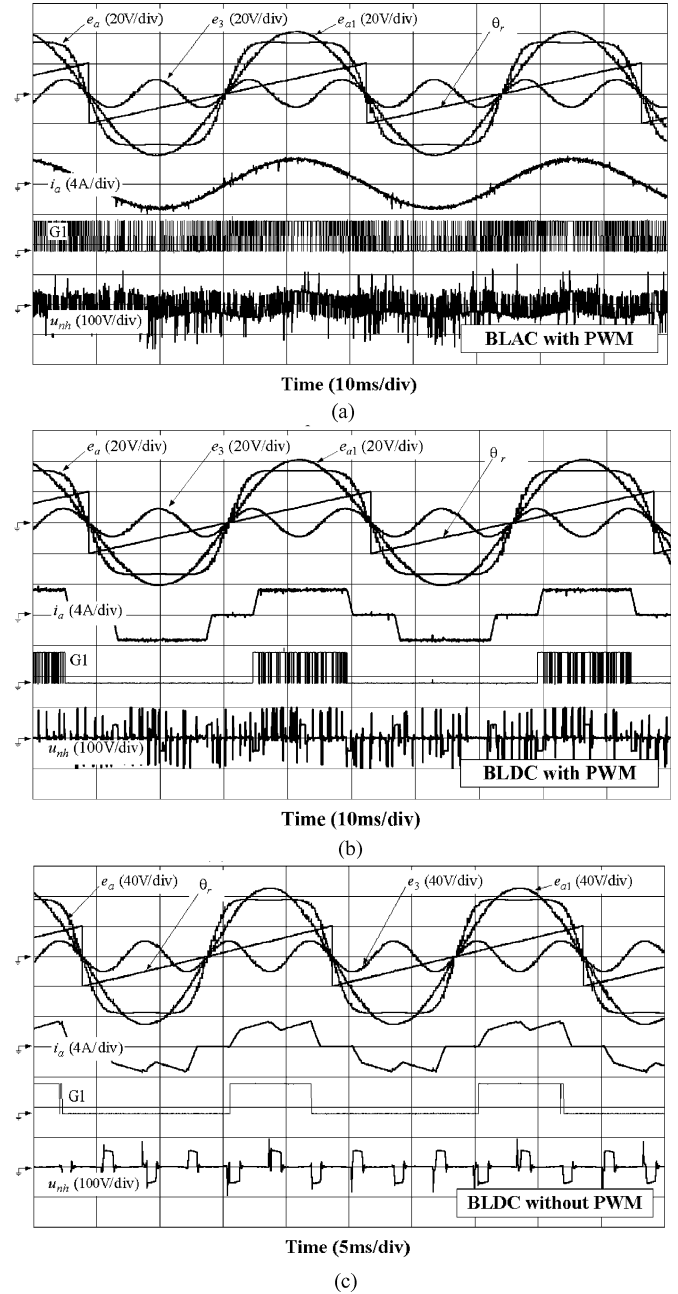


Fig. 4. Measured waveforms of  $u_{nh}$  ( $U_{dc} = 200$  V). (a) BLAC operation with PWM. (b) BLDC operation with PWM. (c) BLDC operation without PWM.

where the subscripts  $m$  and  $n$  represent two energized phases. Clearly,  $u_{nh}$  does not contain any effective information that can be used to deduce the rotor position. Moreover, if the phase back EMF has an ideal trapezoidal waveform, the instantaneous sum of the back EMFs of the energized phases is effectively zero, as shown in Fig. 4, the waveform of  $u_{nh}$  comprising pulses due to current decaying in the freewheeling diodes and noise due to PWM switching.

From the foregoing, only the voltage  $u_{sn}$  is suitable for the third harmonic back-EMF sensorless operation of both BLDC and BLAC SPM motors, as summarized in Table I. Therefore, although it requires access to the neutral point of the stator winding,  $u_{sn}$  can be used for sensorless control in both the constant-torque and flux-weakening modes.

TABLE I  
SUMMARY OF ALTERNATIVE SENSING METHODS

| Voltage  | BLDC mode                                                                                                       | BLAC mode                    |
|----------|-----------------------------------------------------------------------------------------------------------------|------------------------------|
| $u_{sn}$ | 3rd-harmonic emf                                                                                                | 3rd-harmonic emf             |
| $u_{sh}$ | 50% of fundamental phase emf + PWM noise + distortion due to free-wheel diode conduction                        | PWM noise                    |
| $u_{nh}$ | sum of fundamental phase emfs in 2 energised phases + PWM noise + distortion due to free-wheel diode conduction | 3rd-harmonic emf + PWM noise |

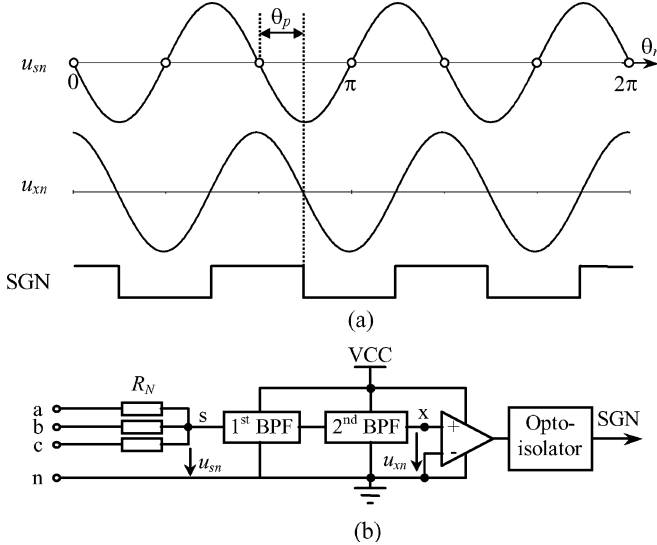


Fig. 5. Zero-crossing detection of third harmonic voltage. (a) Typical waveforms. (b) Detection hardware.

### III. SENSORLESS OPERATION: NONFLUX-WEAKENING MODE

#### A. Rotor Position Estimation

The zero crossings in the third harmonic voltage  $u_{sn}$  correspond to rotor positions  $0, \pi/3, 2\pi/3, \dots, 2\pi$  rad., respectively, as illustrated in Fig. 5(a). Generally, other zero-sequence EMF harmonics in SPM motors will be relatively small, as is the case for the experimental motor, for which details are given in the Appendix. In practice, however,  $u_{sn}$  will contain noise due to switching events and asymmetries between phases. Therefore, a BPF is employed before the zero crossings of the third harmonic voltage  $u_{xn}$  are detected, as illustrated in Fig. 5. Since the operating speed range of the experimental six-pole motor varies from 150 to 1600 r/min, the frequency of third harmonic EMF varies from 22.5 to 240 Hz. Hence, the low and high cutoff frequencies of the BPF were chosen to be 2.2 Hz and 2.4 kHz, which are an order of magnitude different to the minimum and maximum frequencies.  $u_{xn}$  is then processed with a voltage comparator and opto-isolator to generate a digital signal  $SGN$ . The BPF introduces a phase delay  $\theta_p$  between  $u_{sn}$  and  $u_{xn}$ , as illustrated in Figs. 5(a) and 6, which can be calculated from the filter parameters and the motor speed  $\omega_{rm}$ ,  $\omega_{rm}$  being obtained from the frequency of  $SGN$ .

Sensorless control has been implemented with a TMS320C31 digital signal processor (DSP), the rotor position being estimated at time increments  $\Delta t_c = 50 \mu s$ , as set by the DSP timer, the flowchart of the timer interrupt service routine (*ISR*)

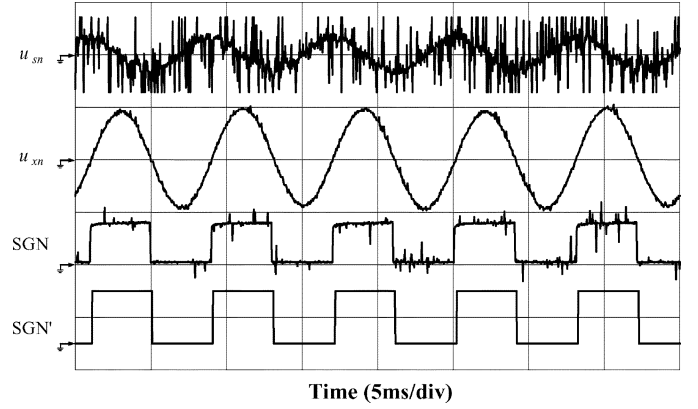


Fig. 6.  $u_{sn}$ ,  $u_{xn}$ , and detection circuit output signal, without ( $SGN$ ) and with ( $SGN'$ ) noise suppression, measured at 200 Vdc, 2.86 Adc, 829 r/min, BLAC operation with PWM and without flux-weakening ( $u_{sn}$ : 50 V/div;  $u_{xn}$ ,  $SGN$ ,  $SGN'$ : 5 V/div).

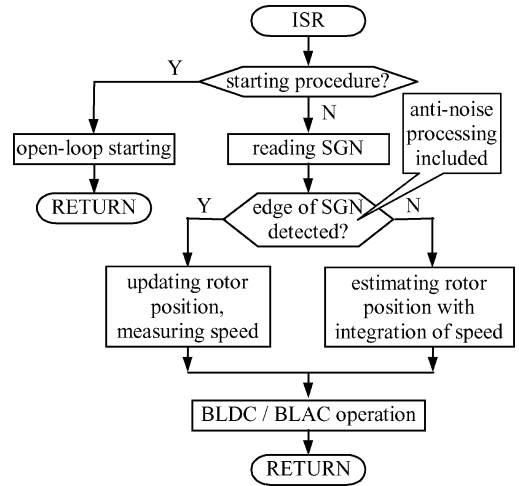


Fig. 7. DSP timer interrupt service routine for sensorless control.

being shown in Fig. 7. The DSP scans the signal  $SGN$  in the timer *ISR*. If a falling edge is detected, the estimated rotor position  $\theta_{re}$  is updated incrementally by either  $\theta_p$ ,  $(2\pi/3 + \theta_p)$  or  $(4\pi/3 + \theta_p)$ . For example, if the updated value on the occurrence of the last falling edge was  $\theta_p$ , the updated value should be  $(2\pi/3 + \theta_p)$ , while the next will be  $(4\pi/3 + \theta_p)$ , and so on. In order to avoid erroneous updating, two requirements have to be satisfied: 1) noise in the signal  $SGN$  must be distinguishable and 2) the first update of  $\theta_{re}$  must be correct. Details of how to achieve these requirements are given in the following sections. The rising edge of  $SGN$  could be similarly utilized to enable  $\theta_{re}$  and, therefore,  $\omega_{rm}$ , to be updated six times per electrical cycle. However, in order to simplify the software, updating is implemented only three times each cycle, viz., on  $SGN$  falling edges, since no discernible performance deterioration was observed. If a falling edge is not detected during the execution of the DSP timer *ISR*, the rotor position is simply estimated by integration of the motor speed, i.e.,

$$\theta_{re}(K) = \theta_{re}(K-1) + \omega_{rm} \cdot \Delta t_c. \quad (8)$$

Once the rotor position has been estimated, the instantaneous current in the windings can be controlled to facilitate either BLAC or BLDC operation, and this is also implemented in the

TABLE II  
BLDC CURRENTS

| $(\theta_{re} + \theta_{ad})$ | $0 \sim \pi/6$ | $\pi/6 \sim \pi/2$ | $\pi/2 \sim 5\pi/6$ | $5\pi/6 \sim 7\pi/6$ | $7\pi/6 \sim 3\pi/2$ | $3\pi/2 \sim 11\pi/6$ | $11\pi/6 \sim 2\pi$ |
|-------------------------------|----------------|--------------------|---------------------|----------------------|----------------------|-----------------------|---------------------|
| $\tilde{i}_a$                 | 0              | $-\tilde{I}_m$     | $-\tilde{I}_m$      | 0                    | $\tilde{I}_m$        | $\tilde{I}_m$         | 0                   |
| $\tilde{i}_b$                 | $\tilde{I}_m$  | $\tilde{I}_m$      | 0                   | $-\tilde{I}_m$       | $-\tilde{I}_m$       | 0                     | $\tilde{I}_m$       |
| $\tilde{i}_c$                 | $-\tilde{I}_m$ | 0                  | $\tilde{I}_m$       | $\tilde{I}_m$        | 0                    | $-\tilde{I}_m$        | $-\tilde{I}_m$      |

DSP timer *ISR*. By way of example, the phase currents for BLAC operation are

$$\begin{cases} \tilde{i}_a = -\tilde{I}_m \sin(\theta_{re} + \theta_{ad}) \\ \tilde{i}_b = -\tilde{I}_m \sin(\theta_{re} + \theta_{ad} - \frac{2\pi}{3}) \\ \tilde{i}_c = -\tilde{I}_m \sin(\theta_{re} + \theta_{ad} + \frac{2\pi}{3}) \\ \tilde{I}_m = \sqrt{\tilde{i}_d^2 + \tilde{i}_q^2} \\ \theta_{ad} = \arctan\left(-\frac{\tilde{i}_d}{\tilde{i}_q}\right) \end{cases} \quad (9)$$

while those for BLDC operation are specified in Table II, where  $\theta_{ad}$  is the commutation-advance angle.

### B. Noise Suppression

In a practical system, the digital signal *SGN* may contain noise which could cause an erroneous update of the estimated rotor position. Hence, signal edges which may arise due to noise must be distinguishable from true falling edges using the DSP software. Therefore, the values of *SGN* in three successive timer interrupts are recorded and compared. Only if they are identical is their value accepted as the actual value of *SGN*. Usually, the duration of the noise is much less than  $\Delta t_c$ , experimental results showing that it usually ranges from  $10 \sim 20 \mu s$ . Practically, such noise is highly unlikely to occur three times in succession within a constant time interval of  $\Delta t_c = 50 \mu s$ . Therefore, noise can generally be recognized and discarded. However the foregoing procedure results in an actual falling edge in *SGN* being confirmed after it occurs, the time delay varying from  $2\Delta t_c$  to  $3\Delta t_c$ , i.e., the average delay is  $2.5\Delta t_c$ . The equivalent phase delay  $\theta_d$  can be calculated from the motor speed, and while it may be negligible at low speed, it may be considerable at high speed. Therefore,  $\theta_p$  in the previously described rotor position update procedure should be replaced by  $(\theta_p + \theta_d)$ . On the other hand, the additional time delay which is caused by noise processing has little influence on the estimated speed measurement, as its variation is much shorter than the cycle time of *SGN*.

The number of recorded and compared values of *SGN* is determined experimentally according to the nature of the noise, more values being required as the noise becomes more prominent. In a practical system, however, the noise is effectively eliminated by this noise processing procedure when the average of three readings is used, as shown in Fig. 7, which compares the signal of *SGN* with and without noise suppression.

## IV. SENSORLESS OPERATION: FLUX-WEAKENING MODE

The effectiveness of third harmonic back-EMF sensorless control has been demonstrated on the experimental motor for which details are given in the Appendix and whose base speed is 830 r/min. As will be seen from Figs. 2–4, the phase

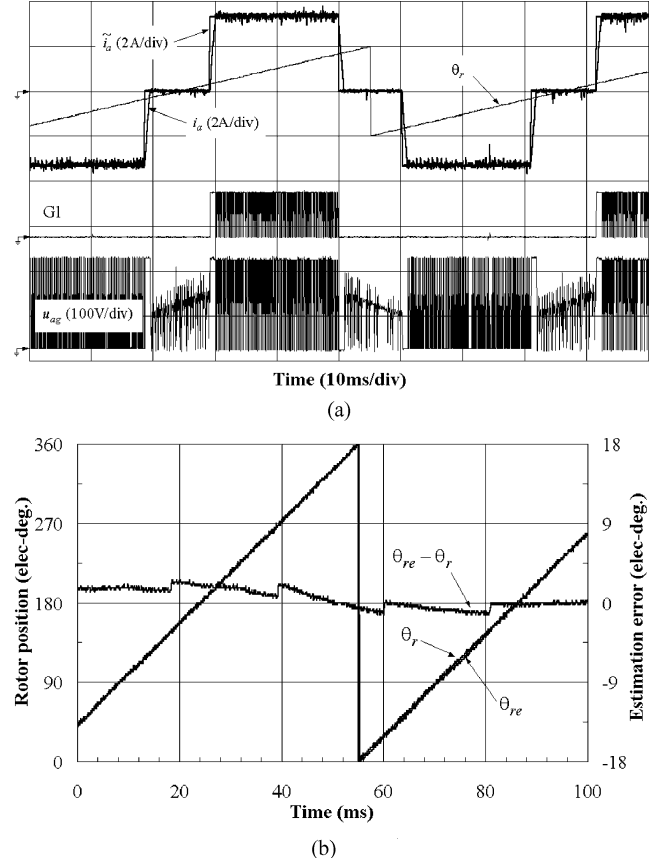


Fig. 8. BLDC operation without commutation advance ( $U_{dc} = 200$  V, 320 r/min, 4.62 N·m).  $\tilde{i}_a$ : current reference;  $i_a$ : actual current; G1: gate drive signal;  $u_{ag}$ : terminal voltage;  $\theta_r$ : actual rotor position;  $\theta_{re}$ : estimated rotor position;  $\theta_{re} - \theta_r$ : position estimation error. (a) Measured waveforms. (b) Rotor position and error.

back-EMF waveform is not a pure sine wave. It contains a sufficiently high third harmonic component to facilitate implementation of the proposed sensorless control strategy in both BLDC and BLAC modes. However, in order to achieve the same rated torque, the amplitude of the phase current reference for BLDC operation is set to 3.30 A while that for BLAC operation is set to 3.56 A.

### A. Flux-Weakening in BLDC Mode

Below base speed, the phase current is in phase with the back EMF, and the current amplitude is PWM regulated to produce rated output torque, as shown in Fig. 8. Above base speed, commutation is progressively advanced, so as to achieve flux weakening, until the phase current waveforms ultimately become continuous, as shown in Fig. 9. Clearly, when  $\theta_{ad} > 30^\circ$  or the current waveforms are almost continuous, the zero crossing of the back EMF in a nonenergized phase is not discernable in the terminal voltage [16], and conventional sensorless operation based on detection of the zero crossings of the phase back-EMF waveforms [1], [2] is not possible. However, the proposed third harmonic back-EMF sensorless method results in high accuracy, as can be seen by comparing the measured and estimated rotor positions, and observing the estimated rotor position error.

The torque which can be realized in the flux-weakening mode depends on the commutation-advance angle  $\theta_{ad}$ , as shown by

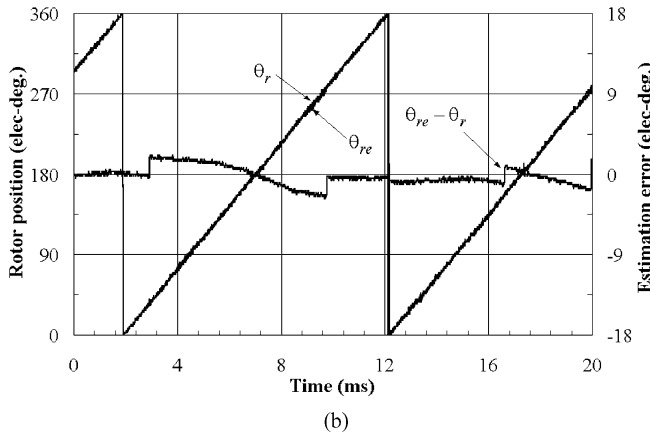
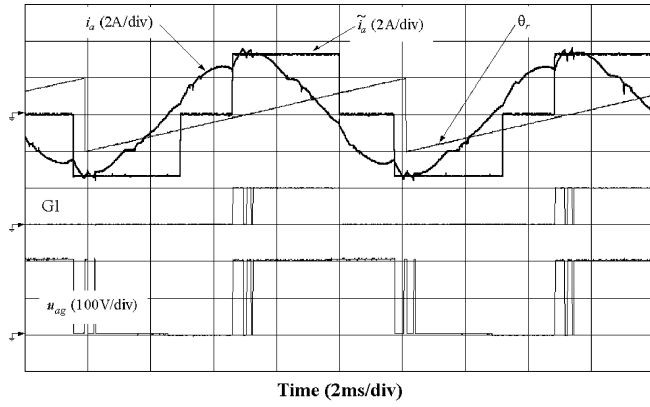


Fig. 9. BLDC operation with 45° commutation advance ( $U_{dc} = 200$  V, 1950 r/min, 0.25 N·m). (a) Measured waveforms. (b) Rotor position and error.

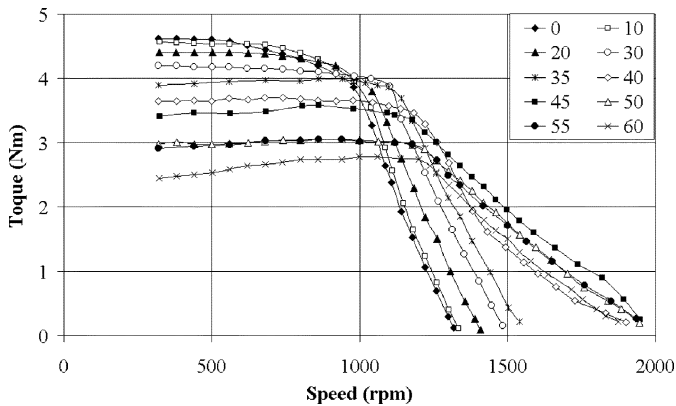


Fig. 10. Torque-speed curves of BLDC machine with different commutation-advance angles  $\theta_{ad}$ .

the experimental results in Fig. 10. The optimal values of  $\theta_{ad}$  for maximum torque at any speed can be deduced from such data, as shown in Fig. 11, which compares the torque-speed characteristics which result both with and without optimal commutation advance.

**B. Flux Weakening in BLAC Mode**

In order to maximize the output power above the base speed the windings must be supplied with the optimum value of negative  $d$ -axis current [17], [18]. With the amplitude of the current and voltage vectors set at the maximum values,  $I_{max}$  and  $U_{max}$ ,

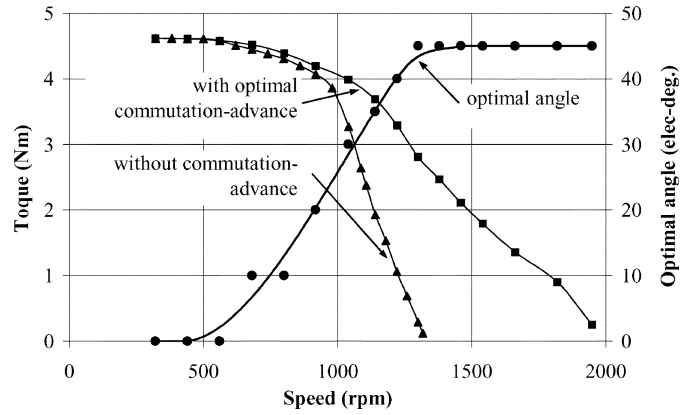


Fig. 11. Torque-speed characteristics of BLDC machine with and without commutation advance, and optimal commutation-advance angle  $\theta_{ad}$ .

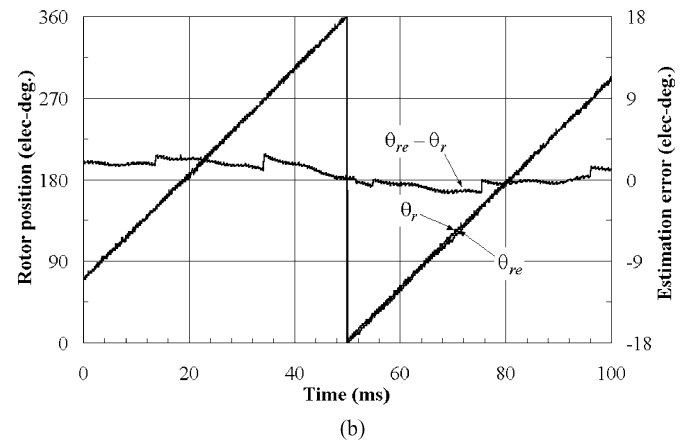
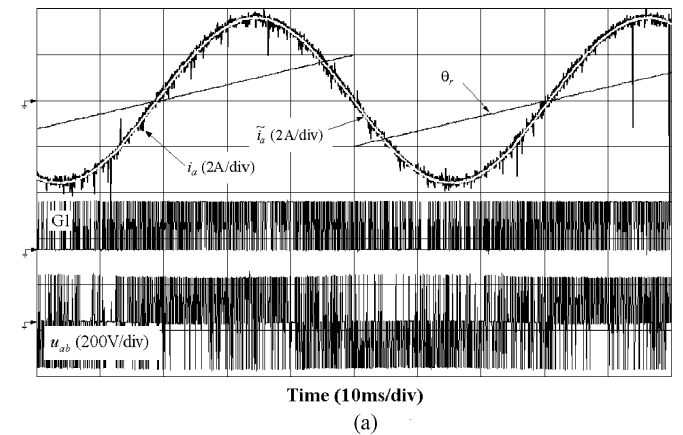


Fig. 12. BLAC operation without flux weakening ( $U_{dc} = 200$  V, 320 r/min, 4.63 N·m).  $i_a$ : current reference;  $i_a$ : actual current; G1: gate drive signal;  $u_{ab}$ : line-to-line voltage;  $\theta_r$ : actual rotor position;  $\theta_{re}$ : estimated rotor position;  $\theta_{re} - \theta_r$ : position estimation error. (a) Measured waveforms. (b) Rotor position and error.

the  $d$ -axis current reference is calculated, with due account of the winding resistance voltage drop ( $RI_{max}$ ), from [17]

$$\tilde{i}_d = \frac{(U_{max} - RI_{max})^2}{2\omega^2(L - M)\Psi_f} - \frac{[(L - M)I_{max}]^2 + \Psi_f^2}{2(L - M)\Psi_f} \quad (10)$$

where  $\Psi_f$  is the permanent-magnet excitation flux linkage, and  $U_{max}$  is usually set at  $2U_{dc}/\pi$  [17].  $\tilde{i}_d$  is set to 0 if the calculated value is  $> 0$ , or set to  $-I_{max}$  if the calculated value is  $<$

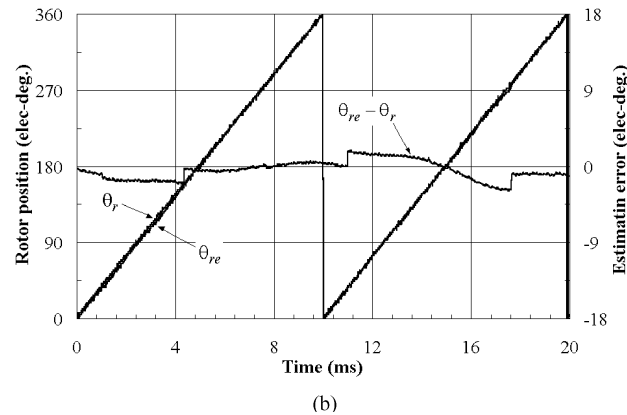
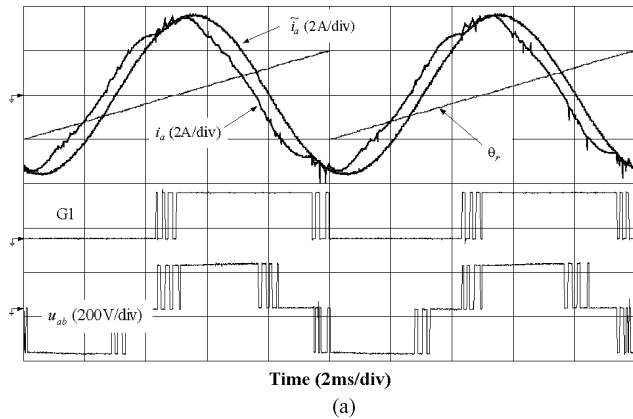


Fig. 13. BLAC operation with flux weakening ( $U_{dc} = 200$  V, 2010 r/min, 0.27 N·m). (a) Measured waveforms. (b) Rotor position and error.

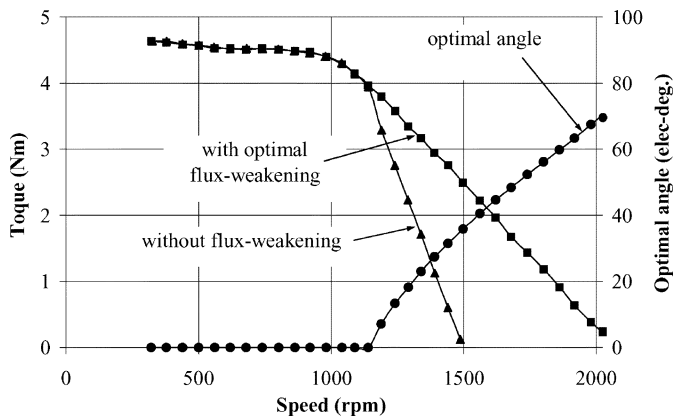


Fig. 14. Torque-speed curves for BLAC operation with and without flux-weakening control, and optimal phase angle between current vector and  $q$  axis.

$-I_{\max}$ . The  $q$ -axis current and the equivalent commutation-advance angle  $\theta_{ad}$  can then be calculated from (9). This enables a seamless transition between the maximum torque per ampere mode (without flux weakening) and maximum power operating modes (with flux weakening) to be achieved according to the motor speed and parameters.

Third harmonic back-EMF sensorless operation has been assessed both with and without flux weakening, representative waveforms being shown in Figs. 12 and 13, together with the estimated and measured rotor position and rotor position error. Again, good agreement is achieved. Fig. 14 compares

TABLE III  
SPECIFICATION AND PARAMETERS OF EXPERIMENTAL MOTOR

|                                                  |                         |
|--------------------------------------------------|-------------------------|
| Rated DC link voltage ( $U_{dc}$ ):              | 200 (V)                 |
| Rated power ( $P_o$ ):                           | 400 (W)                 |
| Rated torque ( $T_e$ ):                          | 4.6 (Nm)                |
| Rated speed ( $n$ ):                             | 830 (rpm)               |
| Number of pole-pairs ( $p$ ):                    | 3                       |
| Number of slots                                  | 18                      |
| Phase resistance ( $R$ ):                        | 3.4 (ohm)               |
| Self-inductance ( $L$ ):                         | 20.7 (mH)               |
| Mutual-inductance ( $M$ ):                       | -2.8 (mH)               |
| Fundamental back-emf ( $k_{Em1}$ ):              | 287.3 (mV/(elec-rad/s)) |
| 3 <sup>rd</sup> Harmonic back-emf ( $k_{Em3}$ ): | 64.5 (mV/(elec-rad/s))  |
| 5 <sup>th</sup> Harmonic back-emf ( $k_{Em5}$ ): | 15.6 (mV/(elec-rad/s))  |
| 7 <sup>th</sup> Harmonic back-emf ( $k_{Em7}$ ): | 2.5 (mV/(elec-rad/s))   |

the torque-speed characteristics which result both with and without optimal flux-weakening control.

## V. CONCLUSION

The application of third harmonic back-EMF-based sensorless rotor position estimation to both BLDC and BLAC machines has been demonstrated. It caters for phase currents which flow continuously, when the zero crossings of the phase back-EMF waveforms are not detectable. However, as with all EMF-based sensorless methods, an open-loop starting procedure still has to be employed [1]–[3].

## APPENDIX

For the specification and parameters of the experimental motor, see Table III.

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