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Water-Sediment Flow Modeling for Field Case Studies in Southwest China

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Abstract

The paper presents a highly robust numerical model to simulate water-sediment mixture flows in practical field studies. The model is composed of an integrated algorithm combining the finite element characteristic splitting method and finite volume Godunov scheme. The former maintains the generality and stability of the numerical algorithm while the latter ensures the conservation and accuracy of the model. The proposed model is first tested by three benchmark flow problems including flood flow in a pool, dam break over a mobile bed and morphological process of a dam removal. Then the model is applied to two practical field case studies to demonstrate its potential engineering values. The first case study is related to the damage of the Polo Hydropower Plant by a sediment flooding event. The second one is the investigation of a well-known 2013 dam break flooding that happened in the Tangjiashan Mountain. It is shown that the simulated water and sediment flows are in good agreement with the documented laboratory and field data and the numerical model is capable of providing useful information on the flow predictions thus making further engineering measures to mitigate these disasters.

Keywords: Water-sediment flow modeling; Southwest China; Flood; Sediment; Field study; Polo Hydropower Plant; Tangjiashan Mountain flood

1 **1. Introduction**

2

In China there are around 50,000 rivers with a coverage area being 100 km² and 1,500 rivers 3 with a coverage area being 1000 km². Severe flood and sediment movement are typical 4 5 features of these rivers due to soil erosion in the catchment area, which caused serious 6 engineering concerns. Especially, the Southwest China region is characterized by 7 mountainous topography and narrow valley, and thus quite a few large hydropower plants 8 have been built to exploit the natural water resource. These dam constructions brought great 9 benefit to the local economy, but meanwhile they also adversely influenced the balance 10 between flow and sediment coexistences. In this sense, timely and accurate predictions of the 11 flood and sediment flows could provide important information in the engineering field. 12 However, the rivers in Southwest China most demonstrate large variation in the water levels 13 and sediment transport capacities. In addition, the mobile bed is composed of different size of 14 sediment materials, which made the process of bed evolution much more complex. Therefore, 15 the development of an efficient and accurate numerical model for these mountainous rivers 16 involves much more challenging tasks than those encountered in the plain rivers.

17

18 The accuracy of any water-sediment mixture flow models should rely on the underlying 19 sediment mechanics. For example, Wang and Song (1995) summarized the sediment research status in Europe and America around the 20th century and Wang (1999) reviewed the river 20 21 sedimentation issues in China. In recent years, great progresses have been made in the study 22 of various sediment mechanics such as the transport capacity, bed form and resistance. For 23 example, 1-D numerical models have been efficiently used for the simulation of flow and 24 sediment processes across large spatial and temporal domains, including the CRS-1 model 25 developed by Liu (2004) at Sichuan University and the HEC model developed by USACE 26 (2003). In order to obtain more detailed flow information along the cross-sectional area, a 27 variety of 2-D numerical models have been developed (Demirbilek and Nwogu, 2007), 28 including the SMS model by Brigham University, RIVCOM by the Delft and MKE21 by the 29 DHI. As these 2-D water-sediment flow models are very CPU efficient, and meanwhile can 30 achieve sufficient accuracy in the flow simulations, they have been widely used in the 31 engineering practice. In comparison, more complicated 3-D models are often used in some 32 refined areas for the purpose of theoretical study and model verification (Liu et al., 2012). 33 Most of the 2-D numerical models are based on the hydrodynamic equations together with 34 different sediment transport modes, in which the 2-D Shallow Water Equations (SWEs) 35 model could be the most widely adopted for large-scale river simulations. On the other hand, 36 the commonly used numerical solution schemes are the Finite Difference method (FDM), 37 Finite Element method (FEM) and Finite Volume method (FVM), respectively, to discretize 38 the flow and sediment equations. Each numerical scheme has some kinds of advantage in 39 certain flow problems but may not perform satisfactorily in a different situation. As a result, 40 the combined or mixed solution schemes have been developed to deal with more complicated

41 issues, such as the FEM-based FVM, FVM-based FEM and hybrid FDM/FVM methods

- 42 (Guillou and Nguyen, 1999; Du, 2000; Casulli and Zanolli, 2002).
- 43

44 In order to investigate the practical sediment-laden flood flows in the field of Southwest 45 China, we should have a competent numerical model that is not only stable and efficient, but 46 also able to deal with complicated boundaries arising from the sediment transport and alluvial 47 deformation. The FEM-based characteristic splitting method and FVM-based Godunov 48 scheme have proved to be robust modeling techniques in numerous flow applications and 49 thus we will develop a hybrid numerical algorithm by combining the above two. The former 50 guarantees the generality and stability of the solution scheme while the latter ensures the 51 conservation and accuracy of the model. The hybrid model will be used to simulate two field 52 cases related to sediment flooding in the Southwest China and the computational findings 53 could provide useful information to mitigate the natural disasters in engineering practice.

54

55

2. Water-Sediment Mixture Flow Model

56 57

58 In the field of water-sediment mixture flow modeling, great achievements have been made in 59 the fundamental equations to describe the physical process and the numerical schemes to 60 solve these equations. Any robust numerical algorithm should possess not only the property 61 of compatibility, stability and convergence, but they should also demonstrate the feature of 62 conservation, non-dissipation/dispersion and computational efficiency/accuracy. In this 63 section, first we will introduce the governing equations in 2-D domain for the water-sediment 64 mixture flow and then the numerical solution technique using the FEM characteristic splitting 65 method and FVM Godunov scheme is developed. The hybrid numerical algorithm inherited 66 the advantages of both FEM and FVM and improved the numerical treatment of advection 67 term.

68

69 2.1 Governing equations for water-sediment mixture flow

70

,

The following 2-D shallow water equations (SWEs), sediment transport and bed deformation equations are used for the water-sediment mixture flows as

73
$$\frac{\partial \eta}{\partial t} + \nabla \cdot (h\mathbf{u}) = 0 \tag{1}$$

74
$$\frac{\mathrm{d}(\mathbf{h}\mathbf{u})}{\mathrm{d}t} = \mathbf{A}_{\mathrm{H}}\mathbf{h}\nabla^{2}\mathbf{u} - \mathbf{g}\mathbf{h}\nabla\eta - \frac{\mathbf{g}\mathbf{n}^{2}|\mathbf{u}|\mathbf{u}}{\mathbf{h}^{1/3}}$$
(2)

75
$$\frac{d(hS)}{dt} = \alpha \omega (S^* - S) + A_H h \nabla^2 S$$
(3)

76
$$(1 - \xi) \frac{\partial z_{\rm b}}{\partial t} + \nabla \cdot \mathbf{q}_{\rm b} = \frac{\alpha \omega (S - S^*)}{\rho_{\rm s}}$$
(4)

in which η = water surface; t = time; h = flow depth; $\mathbf{u} = (\mathbf{u}, \mathbf{v})$ are the horizontal 2-D flow velocities; n = bed roughness; g = gravitational acceleration; S = sediment suspended load concentration; ω = sediment settling velocity; S^{*} = maximum suspended load carrying capacity; α = erosion-deposition coefficient (0.25 is used for deposition, 1.0 for erosion and 0.5 for transition); z_b = movable bed layer thickness; ξ = bed porosity; ρ_s = density of sediment grain; and $\mathbf{q}_b = (\mathbf{q}_{bx}, \mathbf{q}_{by})$ are the horizontal 2-D sediment bed load transport in x and y directions.

84

Here it should be noted that the above water flow equations (1) - (2) are not influenced by any sediment transport parameters in equation (3) - (4), and thus they are represented in the uncoupled form, which is in contrast to those used by Cao et al. (2011). This is based on the rational that the suspended load is not the dominant sediment transport mode in the present field case studies and the flow structure is not significantly modified by the existence of the sediment mixture.

91

92 The horizontal eddy viscosity coefficient A_{H} in equations (2) and (3) is represented as

93
$$A_{\rm H} = C_{\rm s} A \left[\left(\frac{\partial u}{\partial x} \right)^2 + 0.5 \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \right)^2 + \left(\frac{\partial v}{\partial y} \right)^2 \right]^{0.5}$$
(5)

94 in which C_s = Horcon coefficient, taken between 0.1 – 0.2 in this paper; and A = node 95 influence area.

96

97 **2.2** Evaluations of relevant sediment parameters

98

99 The choice of bed load transport equation is a difficult issue, as different formulations can 100 predict the results within an error of several orders. Besides, the sediment bed load transport 101 is also highly sensitive to the flow velocity, as the relationship between the two is not a linear 102 function. Thus unrealistic bed deformations can be predicted if the formula is not correctly 103 used. In this study, we will adopt the widely used Meyer-Peter formula as follows:

104
$$\mathbf{q}_{\rm b} = 8\sqrt{(s-1)gd_{\rm i}^3} \frac{\mathbf{u}}{|\mathbf{u}|} \max(\tau_* - \tau_{\rm c}, 0)^{1.5}$$
(6)

- 105 in which s = relative density of the sediment grain to water; d_i = sediment grain size of one 106 group; $\tau_* = \frac{n^2 |\mathbf{u}|^{1.5}}{(s-1)d_i h^{1/3}}$ is the flow shear stress; and $\tau_c = 0.047$ is the threshold shear stress
- 107 of grain initiation. As the Meyer-Peter formula covered a relatively wide range of sediment

108 grain sizes, it should be suitable for the field studies in Southwest China, in which most of the

- 109 river beds are covered by the highly graded sediment materials.
- 110
- 111 The threshold grain size separating the bed load and suspended load materials can be 112 determined by the so-called suspension index, which is represented by (Qian and Wan, 1983)
- 113

$$Rz = \frac{\omega}{\kappa u_*}$$
(7)

114 in which $\kappa = 0.41$ is the von Karmon constant; and $u_* =$ frictional velocity. From the case

115 studies on many field rivers in the Southwest China, the general guidelines stipulate that Rz >116 4.166 falls into the bed load and Rz < 4.166 belongs to the suspended load (Liu, et al. 1991).

117

118 The sediment settling velocity is calculated by the following formula (Zhang, 1961)

119
$$\omega = \sqrt{\left(13.95\frac{\nu}{d}\right)^2 + 1.09\frac{\gamma_s - \gamma}{\gamma} \text{ gd} - 13.95\frac{\nu}{d}}$$
(8)

120 where v = kinematic viscosity of water; γ_s and $\gamma =$ gravity density of sediment grain and 121 water, respectively.

122

123 The general form of sediment carrying capacity of the flow can be represented by

$$\mathbf{S}^* = \mathbf{k} \left(\frac{\mathbf{U}^3}{\mathbf{g} \mathbf{h} \boldsymbol{\omega}} \right)^{\mathrm{m}} \tag{9}$$

in which k and m = empirical sediment coefficient, whose values depend on the particular
river location. In practical sediment simulations, to address the non-uniformity of suspended
load transport and the bed materials adjustment arising from the alluvial deformation,
adequate modifications of equation (9) are provided by and Zhang (1998).

- 129
- 130

1313Numerical Solution Schemes

132

Here the proposed numerical solution method couples the FEM characteristic splitting with 133 134 FVM Godunov schemes to improve the computational efficiency and accuracy. To improve 135 the solution stability of advection-dominated flows, Zienkiewicz and Codina (1995) 136 developed the split characteristic based FEM scheme. This algorithm has the advantage of simplicity and stability and is particularly suitable for the simulation of mountainous river 137 138 flows over relatively steep slope and complicated alluvial topographies. By following 139 Zienkiewicz and Codina (1995), the split schemes in both the spatial and temporal domains 140 are considered in this work.

141

143 On the other hand, the FVM based Godunov scheme is widely used to solve the SWEs type 144 equations of nonlinear hyperbolic feature due to its well-balanced conservation property (Pu et al., 2012). Thus it can compensate for the drawback of FEM that cannot achieve the exact 145 146 property conservation in all of the computational elements. Numerical schemes based on the 147 Godunov are also quite efficient to treat the flows with large energy gradient as well as shock 148 waves. By combining with the approximate Riemann solver, the computational efficiency is 149 greatly improved when solving the momentum flux across the cells and higher order schemes 150 can be easily implemented as a result.

151

152 In the coupled FEM-FVM computations, the FVM Godnuov scheme is used to correct the 153 deficiency cells except those on the boundaries after several time steps of FEM computation. 154 So the conservation property of FEM scheme can be maintained by the coupled FVM scheme. 155 Besides, the FEM uses a first-order accurate element, while the coupled model adopts the 156 TVD algorithm which is second-order accurate. As the FEM and FVM methods use different storage systems to save the flow variables, frequent data interpolation and communication 157 158 should be implemented to transfer the flow information between the two. A brief review of 159 the coupled FEM-FVM solution procedure is provided below.

160

161 The solution of water flow is first carried out based on the FEM scheme. Let $\mathbf{R} = \begin{bmatrix} hu \\ hv \end{bmatrix}$,

162 $\mathbf{r} = \begin{bmatrix} u \\ v \end{bmatrix}, \ \mathbf{q}_{b} = \begin{bmatrix} q_{bx} \\ q_{by} \end{bmatrix}$ and $\mathbf{i} = \begin{bmatrix} i_{x} \\ i_{y} \end{bmatrix}$, in which i_{x} and i_{y} are the slopes of the terrain, then the

163 solution procedure of matrix \mathbf{R}^{n+1} is

164 (1) First step: Solving the variation of velocity $\Delta \mathbf{R}^*$

165
$$\Delta \mathbf{R}^* = \mathbf{R}^* - \mathbf{R}^n = -\mathbf{M}^{-1} \Delta t [(\mathbf{C}\mathbf{R} + \mathbf{K}_m \mathbf{r} - \mathbf{f}) - \Delta t (\mathbf{K}_s \mathbf{R} + \mathbf{f}_s)]^n$$
(10)

166 (2) Second step: Solving the variation of depth Δh

167
$$\Delta \mathbf{h} = \mathbf{M}^{-1} \Delta t \Big[\mathbf{H} \Big(\mathbf{R}^{n} + \theta_{1} \Delta \mathbf{R}^{*} \Big) - \Delta t \theta_{1} \mathbf{G} \mathbf{p}^{n} - \mathbf{f}_{h} \Big]$$
(11)

168 (3) Third step: Using Δh and correcting $\Delta \mathbf{R}^*$ to get the value at next time step t^{n+1}

169

$$\mathbf{h}^{n+1} = \mathbf{h}^n + \Delta \mathbf{h} \tag{12}$$

170
$$\mathbf{R}^{n+1} = \mathbf{R}^{n} + \Delta \mathbf{R}^{*} - \mathbf{M}^{-1} \Delta t \mathbf{H}^{\mathrm{T}} \mathbf{p}^{\mathrm{n}}$$
(13)

The detailed definitions of relevant variables in equations (10) - (13) can be found in Chen et
al. (2011) so they are not repeated here. The computational time step should satisfy the
following Courant criterion

174 $\Delta t \le C_{FL} \frac{l_{em}}{c + |\mathbf{u}|}$ (14)

175 in which l_{em} = characteristic element size; $c = \sqrt{gh}$ is the gravity wave celerity in shallow 176 water; and C_{FL} = Courant stability coefficient, taken as 0.3 ~ 0.7 in the present paper. 177 (4) Fourth step: Then the FVM Godunov scheme is applied to improve the FEM performance 178 and the flow velocity field is updated by

179
$$\mathbf{u}_{i}^{p} = \mathbf{u}_{i}^{n} - \frac{\Delta t}{A} \sum_{j=1}^{3} \left(\mathbf{F} \cdot \mathbf{n}_{x} + \mathbf{G} \cdot \mathbf{n}_{y} \right) \mathbf{l}_{j}$$
(15)

$$\mathbf{u}_{i}^{n+1} = \mathbf{u}_{i}^{p} + \Delta t \mathbf{S}(\mathbf{u}_{i}^{p})$$
(16)

181 in which p represents the prediction step; \mathbf{F} and \mathbf{G} = numerical flux in x and y directions, respectively; 1 = side length of the FV; and S = source term. The numerical flux can be 182 calculated by the general HLLC schemes as 183

0 < 0

184

$$\mathbf{F}_{i+\frac{1}{2}}^{\text{hllc}} = \begin{cases} \mathbf{F}_{L} & 0 \leq \mathbf{S}_{L} \\ \mathbf{F}_{*L} & \mathbf{S}_{L} \leq 0 \leq \mathbf{S}_{*}, \mathbf{F}_{*L} = \mathbf{F}_{L} + \mathbf{S}_{L} (\mathbf{u}_{*L} - \mathbf{u}_{L}) \\ \mathbf{F}_{*R} & \mathbf{S}_{*} \leq 0 \leq \mathbf{S}_{R}, \mathbf{F}_{*R} = \mathbf{F}_{R} + \mathbf{S}_{R} (\mathbf{u}_{*R} - \mathbf{u}_{R}) \\ \mathbf{F}_{R} & 0 \geq \mathbf{S}_{R} \end{cases}$$
(17)

in which $S_{_L}=u_{_L}-q_{_L}\sqrt{gh_{_L}}$, $S_*=u_*$ and $S_{_R}=u_{_R}+q_{_R}\sqrt{gh_{_R}}$. Besides, the accuracy of 185 numerical scheme is further improved by applying the WAF TVD algorithm on the FV 186 187 surface for flux computations.

188

189 (5) Last step: Finally, the FEM solution procedure is applied again to solve the sediment 190 transport equation and bed deformation equation as below (more detailed derivations can be 191 referred to Chen et al. (2011))

192
$$\mathbf{S}^{n+1} = \mathbf{S}^n - \mathbf{M}^{-1} \Delta t (\mathbf{C}\mathbf{S} + \mathbf{K}_m \mathbf{S} - \mathbf{f})^n + \mathbf{M}^{-1} (\Delta t)^2 (\mathbf{K}_s \mathbf{S} + \mathbf{f}_s)^n$$
(18)

$$z_{b}^{n+1} = z_{b}^{n} - \frac{1}{1 - \xi} (\mathbf{H} \mathbf{q}_{b} + \mathbf{f}_{c})$$
(19)

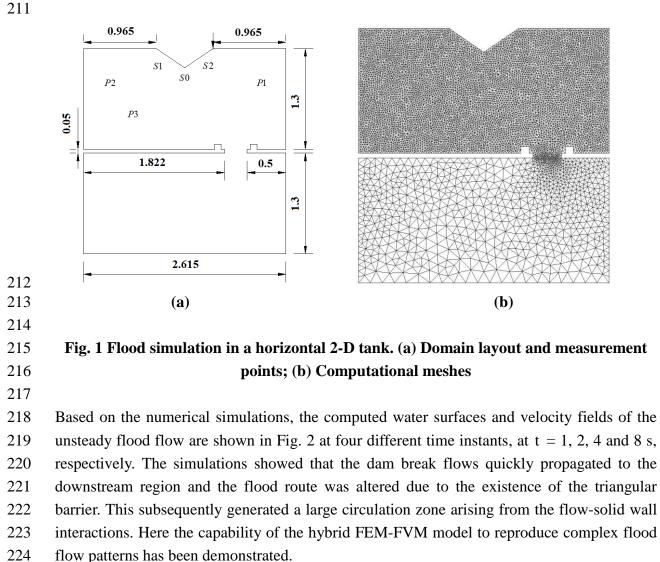
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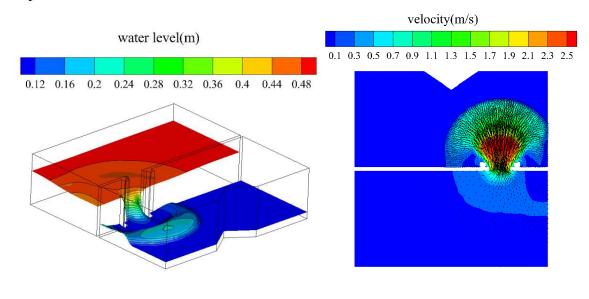
4. **Model Verifications** 196

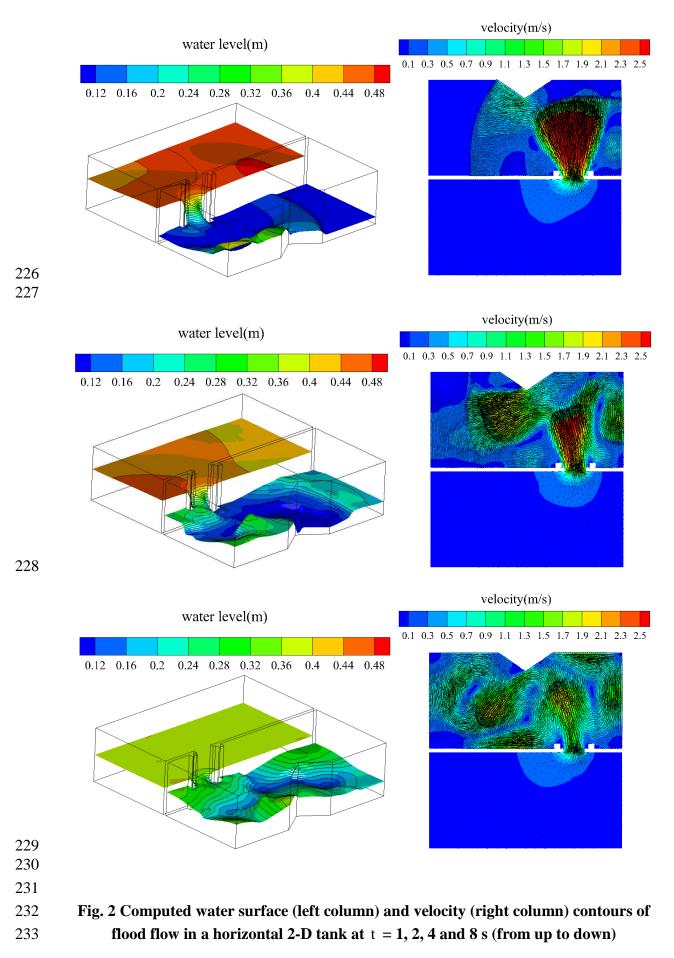
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198 4.1 Flood flow in a horizontal 2-D water tank 199

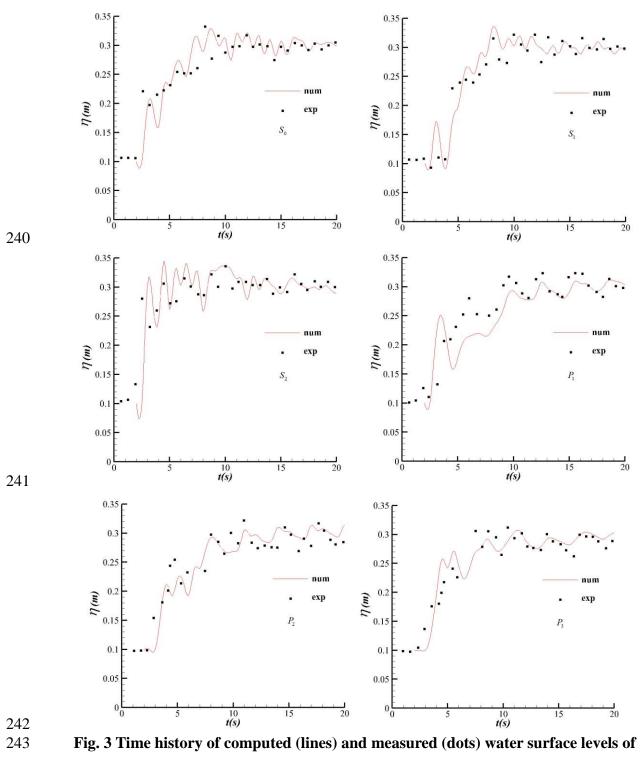
200 This model test is based on the benchmark unsteady flow experiment carried out in the 201 Spanish CITEEC laboratory by J. Puertas (Brufau et al., 2004). The computational domain is 202 composed of two rectangular water tanks, which are connected by a sluice gate and the rest 203 parts are solid walls. The upstream tank contains reservoir water and the downstream tank is 204 the flooded area, which also has a triangular barrier placed on one side of the wall to generate 205 complex geometry and flow conditions. The bed roughness of the whole computational 206 domain is n = 0.018. The initial water depth in the upstream tank is 0.5 m, as compared with 207 0.1 m in the downstream area. The lift of the sluice gate is assumed to be instantaneous. Here 208 we will use the numerical simulations to reproduce the unsteady flooding process. The 209 general layout of the computational domain with the location of six measurement points and 210 the computational meshes are shown in Fig. 1 (a) and (b), respectively.







To quantitatively validate the accuracy of numerical modeling, the computed time-dependent water surface levels at six measurement points (as shown in Fig. 1) are shown in Fig. 3, and compared with the experimental data of Brufau et al. (2004). The comparisons indicated that the general agreement is quite satisfactory and the numerical model is able to predict the unsteady flood flow propagation and interaction with the triangular obstacle. Meanwhile, the location and amplitude of the shock waves are also well captured.



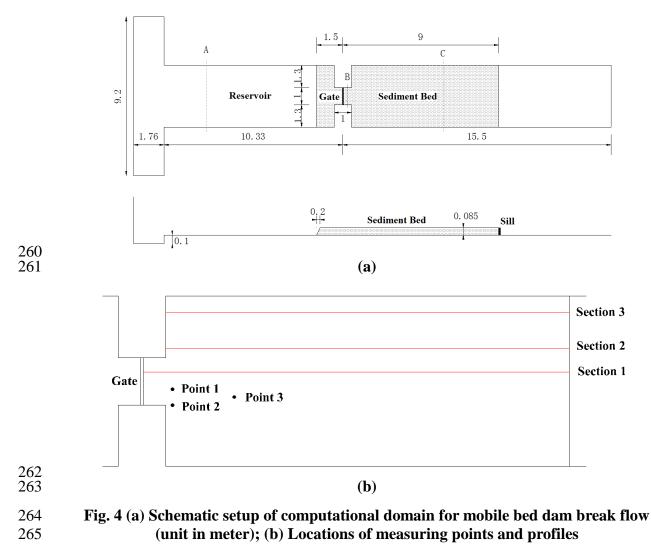
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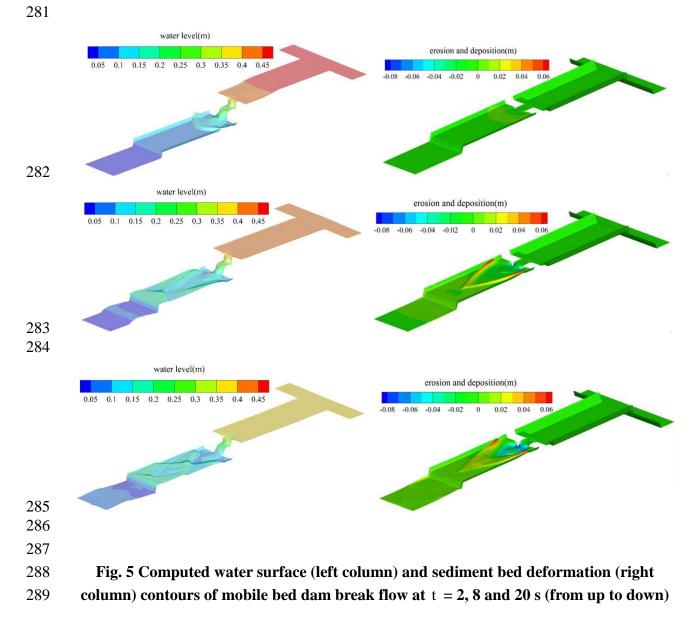
unsteady flood flow at measurement points $S_0, S_1, S_2, P_1, P_2, P_3$

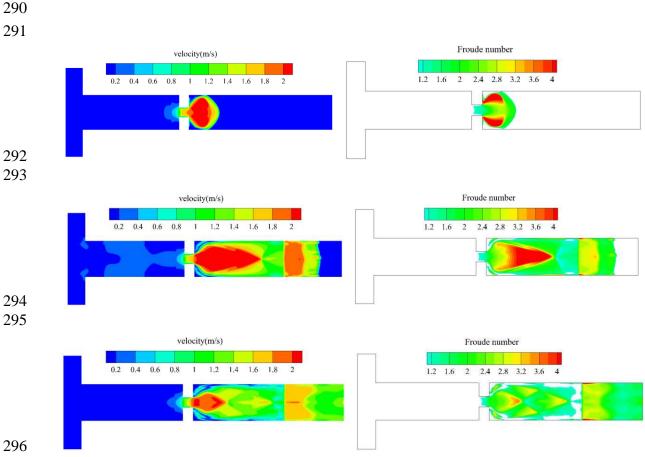
4.2 Dam break flow over a mobile bed channel

In this section, we use the proposed hybrid FEM-FVM model to reproduce a laboratory 247 248 experiment of mobile bed dam break flow carried out in the Hydraulics Unit of the LEMSC, 249 Catholic University of Louvain (Soares-Frazao et al., 2012). The computational domain is 27 250 m long, the width of upstream and downstream channels is 3.6 m and a sluice gate is placed 251 in the middle of the channel connection. The thickness of sediment layer is 0.085 m, which 252 extends 1.5 m in the upstream side and 9.0 m in the downstream side. The mean diameter of 253 sediment material is $d_{50} = 1.61$ mm, relative density $\rho_s / \rho_w = 2.63$, and bed porosity $\xi =$ 254 0.42. The roughness is n = 0.0165 for the sediment bed and n = 0.01 for the fixed bed. The 255 initial water depth in the upstream reservoir is 0.47 m and the downstream channel is dry. The 256 dam break was assumed to occur instantly and the subsequent flood process lasted for 20 s. A schematic setup of the computational domain is shown in Fig. 4(a). To validate the numerical 257 258 simulations, three measurement points and measurement profiles in the longitudinal direction of the channel are taken in the computational domain as shown in Fig. 4(b). 259



267 The computed water surface levels and sediment bed deformations are shown in Fig. 5, and 268 the velocity fields and flow Fr numbers are shown in Fig. 6, respectively, at three selected 269 times at t = 2.0, 8.0 and 20.0 s, respectively. The erosion and deposition results in Fig. 5 270 indicated that as the dam break flow propagated over the mobile bed it caused severe erosion 271 at the original dam site. Later on, with the change of local flow and bed conditions the dam 272 break flow reduced its capacity and deposited the sediment materials in the downstream area, 273 forming something like an alluvial fan. Besides, the water surface contours showed the 274 generation and evolution of strong dam break shock waves propagating in the downstream 275 direction, which led to irregular wavy water surfaces. Generally speaking, the dam break flow 276 could generate significant sediment erosions and depositions in a very short period of time 277 and thus constitute a rapid alluvial process. For example, the sediment bed layer at the dam 278 site was nearly eradicated at time t = 8 s after the dam break, and the maximum deposition 279 height added up to 7 cm around t = 20 s. As a result, a total of around 14 cm topographic 280 difference in the original flat bed was generated within a short time scale.





298Fig. 6 Computed flow velocity (left column) and Fr number (right column) contours of299mobile bed dam break flow at t = 2, 8 and 20 s (from up to down)

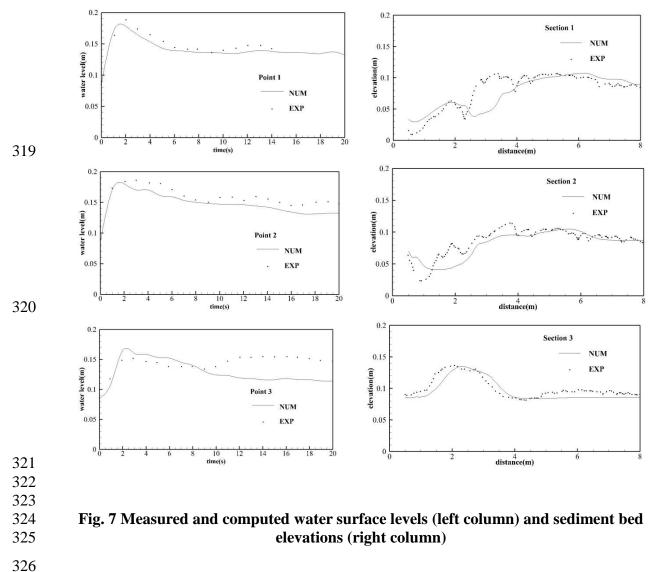
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301 The velocity fields in Fig. 6 show that in the early stage of dam break, the high velocity area concentrates near the dam site, but it spreads out along the centreline of the channel as time 302 303 goes on and the flow intensity also tends to decrease. The mobile sediment layer has an 304 obvious effect on the dam break process and acts as a buffer zone to reduce the flow velocity, which is demonstrated by the fact that the velocity in the fixed bed portion of the channel as 305 306 shown at time t = 8 s is larger than that in the sediment channel in the cross-sectional area. 307 Besides, the Froude number contours have also showed a similar trend. Although the flow regime is supercritical in most of the downstream areas, the flow intensity becomes smaller 308 309 and more uniformly distributed at the later stage of dam break process.

310

The comparisons between the simulated and measured water surfaces as well as sediment bed elevations are shown in Fig. 7. Generally speaking, the agreement between the numerical simulations and experimental measurements is quite satisfactory in the engineering interest by considering the complex water-sediment interactions and efficient simulation time of the model using a hybrid FEM-FVM solution scheme. However, relatively larger errors can be 317 computational errors between the water surface level and bed deformation.





326 327

329

328 4.3 Morphological modelling of dam removal experiment

330 To validate the numerical model in the prediction of morphological changes at the dam site 331 area, the benchmark laboratory experiment of Cantelli et al. (2004) is used. The original work 332 reported on the erosion of a deltaic front induced by the removal of a dam. The installation of 333 a dam on the river induces sedimentation at the upstream end of the reservoir. The removal of 334 a dam causes erosion into the resulting deposit and subsequent disastrous sediment flows. Not 335 many numerical models have been developed to model such a complex process including the 336 erosion, transport and deposition of sediment due to the difficulties in treating the steep 337 streamwise bed slope created by the dam removal that induces high flow velocities and 338 sediment transport rates, and the variations in the width of the channel incising into the 339 deposit.

According to Cantelli et al. (2004), the experiments were performed at St. Anthony Falls Laboratory, University of Minnesota. A rectangular glass-walled flume 14 m long, 0.61 m wide and 0.48 m high was modified to model a long reservoir of uniform width. The initial slope of the sediment bed was 1.8%. The dam was located 9 m from the inlet of the flume. To simplify the problem, non-cohesive sediment was used in the experiment. Ten runs using different water flow discharges and dam removal procedures were considered in the experiment and we reproduce Run 6, which is an instantaneous collapse.

348

The model simulation time duration is 5600 s, which is set to be consistent with the physical experiment (Cantelli et al., 2004). The numerical domain is 14 m long and 0.61 m wide. Nonstructured grids were used with 29518 nodes and 57570 grids. The roughness of the channel bed was taken as 0.016. Computational time step was automatically adjusted based on a CFL condition of 0.4. The flow discharge rate is $Q = 0.3 \times 10^{-3}$ m³/s and the sediment grain size is $D_{50} = 0.8$ mm.

355

356 Fig. 8 shows the evolutions of the bed profiles along the channel center during the erosion of 357 the reservoir deposits at different time instants. The erosion upstream of the deposit front and 358 the deposition downstream are clearly visible. The dotted line in the figure represents the 359 initial bed profile. It shows that after the sudden removal of the dam, the flow incises into the 360 reservoir deposit, first rapidly and then more slowly. Towards the end of model simulation 361 around time t = 5600 s, the equilibrium bed slope is approximately equal to the initial slope 362 value. A very good agreement in the morphological profiles has been found between the 363 numerical simulations and measurement data of Cantelli et al. (2004) in Fig. 8. Slightly large 364 discrepancies only appear at t = 5600 s near the far downstream side of the deposit, which is 365 probably due to the numerical parameter uncertainties after the long-time simulation. 366

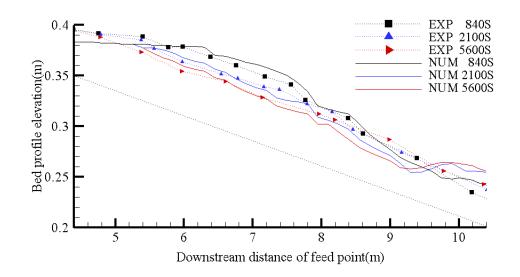
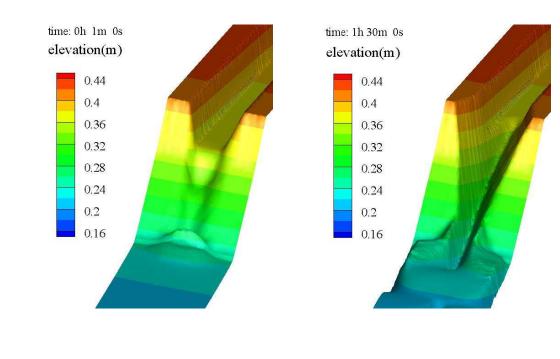


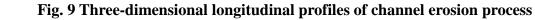


Fig. 8 Computed and measured time-evolution of bed profiles along the channel center (Dam located 9 m downstream of the feed point and flow from left to right, following Cantelli et al., 2004)

Furthermore, the computed 3D longitudinal channel profiles are shown in Fig. 9, for the time instants at t = 1 min and 1.5 hour, respectively, after the erosion started. It clearly demonstrated the spatial and temporal variations of the incision and widening of the channel. The so-called erosional narrowing as found by Cantelli et al. (2004) in their laboratory experiment was also found in the present numerical simulations.

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5. Model Applications in Field Case Study 387

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5.1

389 390

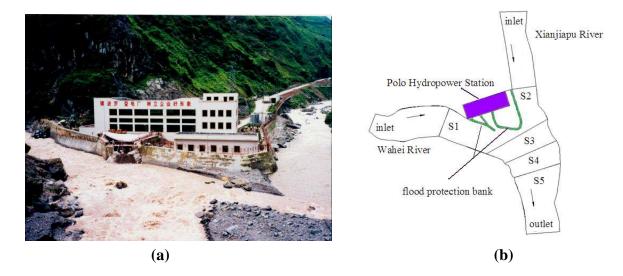
Sediment-laden flood flow at Polo Hydropower Plant

391 The Polo Hydropower Plant is located at the two river confluence in the Yi Autonomous 392 Region of Sichuan Province, which was put into operation in December 1999. During the 393 heavy rainstorms of July 28, 2001, the sediment-laden floods from the two rivers inundated 394 the power plant. The situation was further deteriorated by the large quantities of sediment 395 deposition, which raised the river bed by about $5 \sim 7$ m. The two rivers near the confluence 396 region demonstrated mountainous river features with relatively steep bed slope. The flow 397 regime was quite complicated due to the drastic changes of topographic and boundary 398 conditions. To re-investigate this natural disaster and provide useful information for the 399 engineering practice in future, in this study we carry out both the physical experiment and numerical simulation to reproduce the flood and sediment transport process near the river 400 401 confluence. The physical experiment was carried out at the State Key Laboratory of 402 Hydraulics and Mountain River Engineering, Sichuan University.

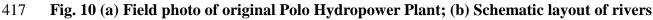
404 The topographic data of the site were obtained from the field survey carried out two years 405 before the 2001 sediment-laden flood disaster. The field photo of the original Polo 406 Hydropower Plant and a schematic layout of the nearby rivers are shown in Fig. 10 (a) and 407 (b), respectively. The detailed river systems are introduced as follows: The upstream flow 408 inlet of one tributary river, which is called the Wahei River, is 250 m away from the 409 confluence junction. The upstream inlet of another tributary river, called the Xianjiapu River, 410 is 200 m away. The downstream flow outlet is 150 m away from the confluence point. For a 411 comparison, the field photos of the hydropower plant after being damaged by the flood and 412 sediment flows in July 2001 are shown in Fig. 11 (a) and (b).

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Fig. 11 Field photo of damaged Polo Hydropower Plant caused by (a) Flood flows; (b) Sediment deposits

425 **5.1.1 Modeling parameters**

427 The detailed modeling conditions are taken as follows. Due to the lack of real time 428 observations of flow and sediment hydrographs during the flood event, here we simply use 429 the peak flow discharges of the July 28 flood as input parameters for the present physical and 430 numerical models. That is to say, we assume the Wahei River has a flow discharge of 1350 m^3/s (P = 2%) and Xianjiapu River has a flow discharge of 712 m^3/s (P = 5%). The sediment 431 432 concentration is assumed to be in saturation at the upstream river inlets and the free outflow 433 boundary conditions are imposed at the downstream flow outlet. The gradation of the 434 sediment materials is shown in Table 1.

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Table 1. Gradation of sediment materials at upstream inlet

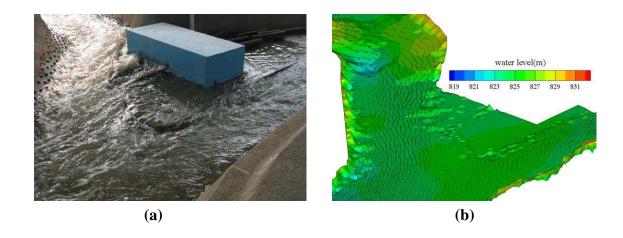
Diameter (mm)	>500	500-200	200-60	60-20	20-5	5-2	2-0.5	0.5-0.25	< 0.25
Percentage (%)	0.0	28.8	10.5	18.7	16.8	17.3	6.1	1.0	0.8

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440 **5.1.2 Flood flow simulation results and analysis**

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In the first phase of the study, we carried out physical experiment and numerical simulations for the water flow only without considering the sediment transport. Fig. 12 (a) and (b) shows the inundation of the hydropower plant due to the confluence of flood water from the two river tributaries. The physical experimental photo in Fig. 12 (a) and the numerical water level in Fig. 12 (b) are qualitatively compared together and it shows that the two converging rivers caused significant flooding damages to the hydropower plant.



452 Fig. 12 Flood inundation of the hydropower plant due to river confluence. (a) Physical 453 experiment; (b) Numerical simulation

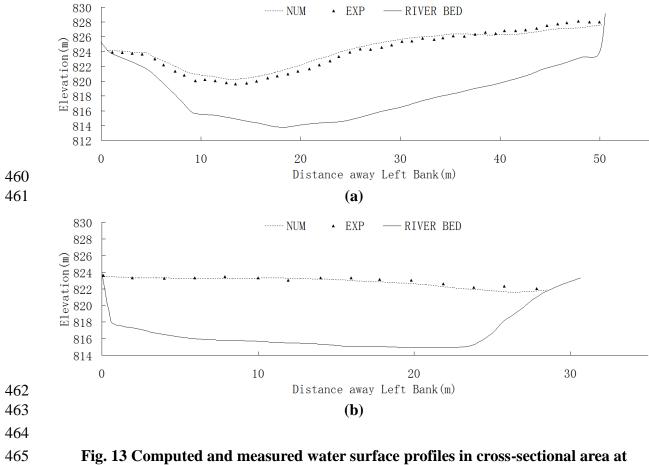
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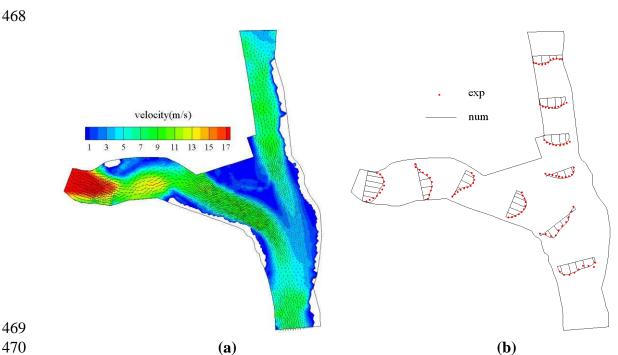
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To further investigate the flood flow features, the water surface profiles at two measurement sections S1 and S2 (as shown in Fig. 10 (b)) are compared between the numerical results and experimental data in Fig. 13 (a) and (b), respectively. Besides, the computed flow velocity field and its good agreement with the experimental measurement are shown in Fig. 14 (a) and (b).



measurement sections (a) S1; (b) S2



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467

472 Fig. 14 (a) Computed velocity field; (b) Comparisons of experimental and numerical
 473 velocities near the river confluence area

- 475 It can be seen from Figs. 13 and 14 that the inundation of the hydropower plant was caused 476 by two different flood scenarios: (1) As the tributary river - Wahei River has relatively steep 477 bed slope, irregular channel cross-section and meandering flow pattern, the direct flow thrust 478 and deflection of this river caused the flood water to overtop over the protection bank. This 479 can be easily understood through Fig. 13 (a) for the cross-sectional water level at Section S1, 480 where there exists a large difference in the water surface levels of about 7.45 m. (2) On the 481 other hand, another tributary river - Xianjiapu River inundated the hydropower plant due to 482 the flood water from the two tributary rivers running against each other. The interaction of the 483 two rivers reduced the local flow velocity and accordingly increased the flow depth, resulting 484 in the water level being higher than the protection bank. This can be partly demonstrated by 485 the cross-sectional water level in Fig. 13 (b) at measuring section S2, as well as the flow 486 velocity field near the confluence area in Fig. 14. Besides, in both Figs. 13 and 14 the 487 numerical results agree quite well with the experimental data, indicating the accuracy of the 488 proposed FEM-FVM model in water flow simulations.
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491 **5.1.3** Sediment simulation results and analysis

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In the second phase of the study, the physical experiment and numerical simulations for the mobile bed condition were carried out, based on the previous fixed bed works by feeding the 495 sediment materials into the river system at the upstream flow inlets. The aim is to investigate the sediment transport and deposition process near the river confluence and evaluate its 496 influence on the inundation of the Polo Hydropower Plant. An illustrative photo of the 497 498 experimental site is shown in Fig. 15, which shows that the sediment deposition near the river 499 confluence also contributed to the damage of the power plant.

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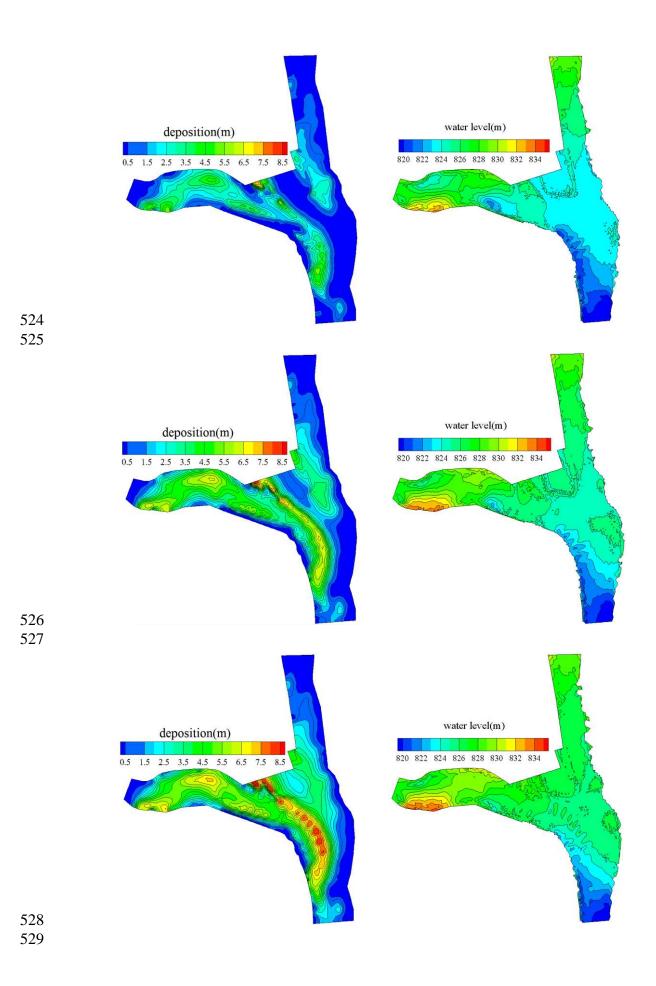
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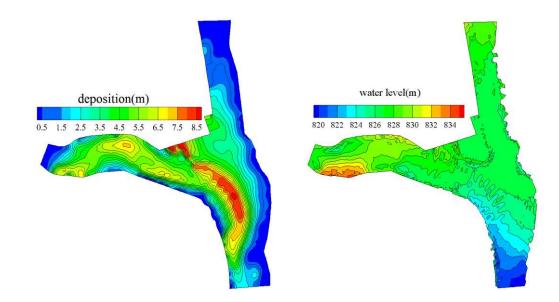
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Fig. 15 Experimental photo of sediment deposition near the hydropower plant

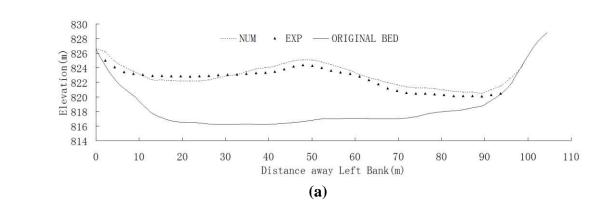
507 Fig. 16 shows the sediment deposition thickness and water level contours at time t = 1, 2, 4508 and 6 hr of the numerical simulations. The left column is the deposition layer thickness and 509 the right one is the water level. Fig. 16 demonstrates the importance of river confluence on 510 the sediment transport and deposition features, which contributed to the raising of river bed and deteriorated the flood disasters. Due to the blocking effect of the Wahei River along the 511 512 Xianjiapu River's flow route, the sediment-laden capacity of the latter was greatly reduced, 513 leading to severe sedimentation of the river bed. On the other hand, due to that the upstream 514 sediment input of the Wahei River was very rich and the existing river training works blocked 515 part of the flow channel, the water surface slope in the confluence area was much smaller as 516 compared with its upstream value. This situation was made more serious by the two river 517 interactions, i.e. one river flowing against another as mentioned before. As a result, some 518 large sediment materials such as gravels were first deposited in the confluence region. 519 However, this deteriorated the situation by further reducing the local water surface slope and 520 caused more medium and even small sediment materials to be settled. Finally, the thickness 521 of the sediment deposits gradually exceeded the height limit of the river protection bank and 522 large quantities of the sediment materials were dumped into the hydropower plant. 523





533Fig. 16 Computed sediment deposition thickness (left column) and water surface level534(right column) contours near river confluence at t =1, 2, 4 and 6 hr (from up to down)

To quantitatively validate the accuracy of the numerical model for long time flow and sediment simulations, Fig. 17 (a) ~ (c) shows the computed and measured sediment bed profiles for the measurement sections S3 - S5 as shown in Fig. 10 (b). In spite of some discrepancies in the data set due to the complexity of the problem, the two results agree quite well in view of the engineering interest and it shows the numerical simulations realistically reproduced the sediment transport and deposition process during the two river confluence. According to the field observation records, large scale of landslides also happened during the storm flood and thus plenty of the debris stones were dumped into the power plant, with some gravel stones having diameter as large as 1.0 m. These have generated catastrophic effect on the damage of the building but they were difficult to model in either the physical experiment or numerical simulations.



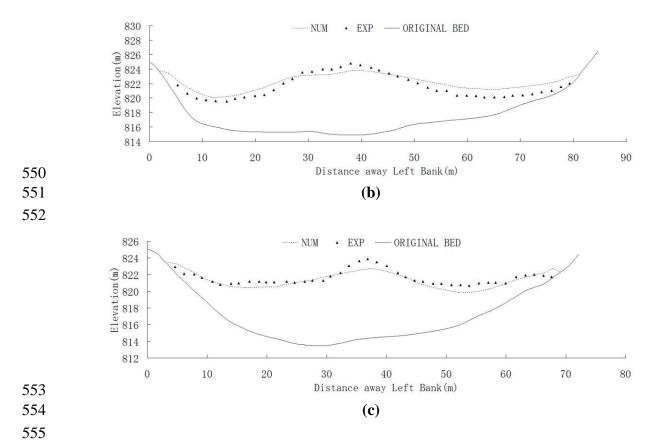


Fig. 17 Computed and measured sediment bed profiles in cross-sectional area at measuring sections (a) S3; (b) S4; (c) S5

The disagreement between the physical experiment and numerical simulations could be attributed to several reasons. Firstly, the simulation of mountainous rivers over relatively steep bed slope requires more stable numerical schemes, and some numerical treatments adopted in this paper could bring uncertainties and affect the simulation accuracy. Secondly, the present river systems have a bed slope around 0.04 and the characteristic sediment diameter is 35 mm, while the Meyer-Peter formula used in the FEM-FVM model only applies to the bed slope being less than 0.02 and sediment diameter being in the range of $0.4 \sim 29$ mm. Lastly, the proposed numerical model uses the averaged horizontal velocities to compute the bed load transport. This approach did not adequately address the influence of secondary flows which are quite important in mountainous rivers.

577 5.2 Tanjiashan Mountain 2013 Flood

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579 **5.2.1 Engineering background**

580

581 Tanjiashan Mountain Lake was formed during the Wenchuan Earthquake that happened in 582 May 12, 2008 and it is located 5 km upstream of the Old Town of Beichuan. The length of 583 the dam is 803 m along the river flow and the maximum cross-sectional width is 611 m. Due 584 to the adequate engineering works carried out after the Wenchuan Earthquake, the water level 585 in the lake reservoir has been maintained at a safety level of 713 m and the water storage capacity is around 86 million m³. On July 9, 2013 the heavy precipitations in the catchment 586 587 area reached as large as 285 mm, which raised the water level of the Tanjiashan Lake by 8.0 m. As a result, the right portion of the dam was partially breached leading to a flood with 588 589 maximum discharge rate of 5000 m^3/s . Subsequently, quite a few downstream areas were 590 inundated with the submergence depth exceeding 7.0 m. The damaged areas include some 591 heritage sites such as the Beichuan Hotel and Cemetery of Victims. Fig. 18 shows the site 592 photo of the flood and sediment inundation areas near Beichuan Hotel.

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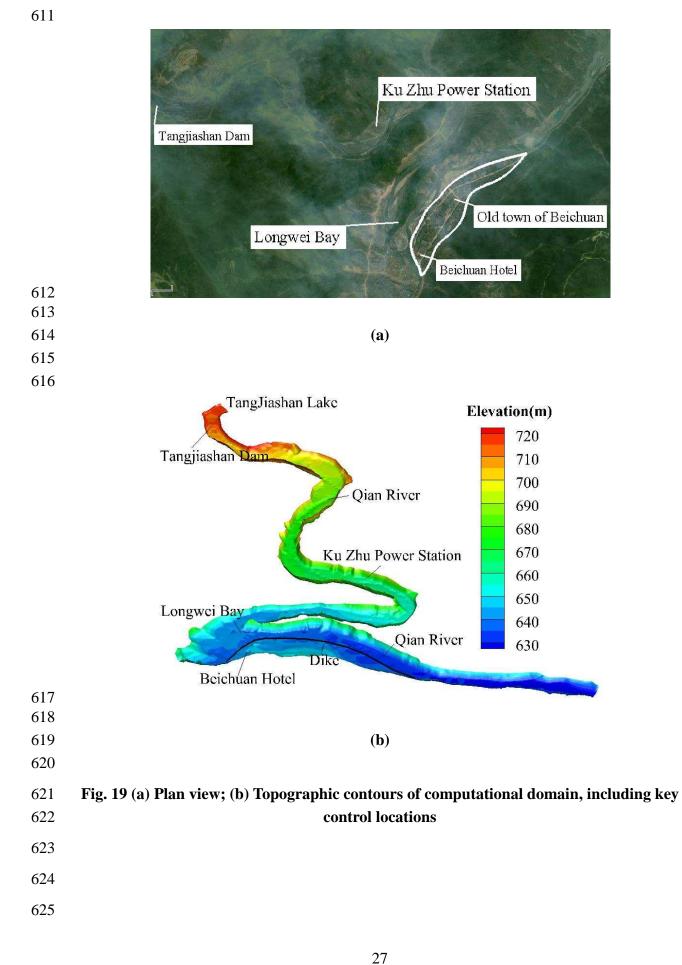
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 596 Fig. 18 Site photo of flood and sediment (circled in red line) inundations near Beichuan
 597 Hotel (http://www.chinanews.com/shipin/2013/07-12/news251654.shtml)
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600 5.2.2 Computational domain and model parameters

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602 The whole computational domain includes the Tanjiashan Lake in the most upstream area and 603 1.85 km downstream of the Old Town of Beichuan, with a total length of 8.6 km. The plan 604 view and topographic contours of the computational area are shown in Fig. 19 (a) and (b). 605 According to the field survey, the river channel upstream of the Beichuan Hotel has a bed 606 roughness of n = 0.042 and the downstream area from this location has n = 0.04. The 607 numerical simulations aim to reproduce the flooding and sediment process for the next 7 days 608 after the dam break flow disaster. For the boundary conditions, the upstream inflow 609 hydrograph and downstream rating curve are shown in Fig. 20 (a) and (b), respectively.



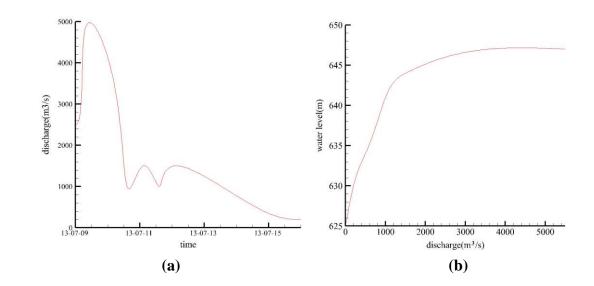


Fig. 20 (a) Upstream hydrograph; (b) Downstream rating curve for model input

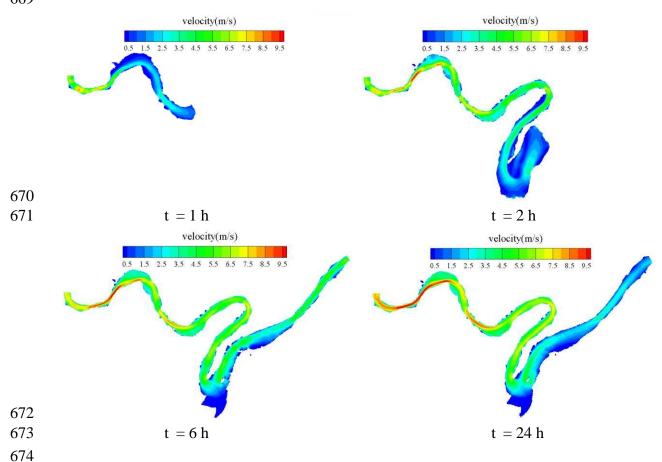
At the beginning of the computation, the river channel is treated as being dry. As described by Chen et al. (2011), in order to obtain a good-shaped wet-dry boundary line, a minimum threshold water depth h_{min} is used. A node is regarded as being dry when its water depth is less than h_{min}, otherwise it is marked as being wet. A grid including any dry node is treated as a dry grid unless it contains all of the wet nodes, in which case it is regarded as being wet. The upstream sediment input is assumed to be from the breached dam materials rather than from the Tanjishan Lake area. The bed materials of the channel are composed of three layers and the sediment size gradations for each layer are shown in Table 2 as below.

Table 2. Sediment layer thickness and grain size gradation of each layer

Sediment layer	Gradation of sediment grain size in percentage (%)								
thickness (m)	Gradation of sediment grain size in percentage (%)								
Sediment size (mm)	200~60	60~20	20~5	5~2	0.5~0.25	<0.25			
2 (upper layer)	37.2	27.8	16.3	6.6	6.7	3.4			
2 (middle layer)	24.8	24.5	18.3	12.1	15.5	4.8			
5 (lower layer)	48.2	14.6	8.2	10.2	14.4	4.4			

5.2.3 Modeling results and discussions

652 Fig. 21 shows the computed velocity fields after the dam break flood at time t = 1 h, 2 h, 6 h, 653 24 h, 3 d and 7 d, respectively. It shows that the flooding water propagates about 2.9 km downstream of the dam at t = 1 h. The flow velocities decrease from the dam site, which are 654 around 5.0 m/s, to about 1.0 m/s towards the downstream region due to the widening of the 655 channel and the flow energy dissipation from bed friction. At t = 2 h, the flood flow reaches 656 657 the downstream of Longwei Bay with a smaller front velocity of 1.2 m/s as a result of the 658 spacious flow path. It has been observed in the model simulation that the flow arrives at the 659 outlet boundary of the model at time t = 2.7 h. Due to the continuous supply of flooding 660 water from the dam reservoir, the general velocity amplitude increases at t = 6 h, which is 661 demonstrated by the fact that the main flow velocity upstream of the Longwei Bay exceeds 5 m/s ~ 10 m/s, while the corresponding downstream velocity is also as high as $3 \text{ m/s} \sim 5 \text{ m/s}$. 662 After t = 24 h of the dam break, due to the reduction of the upstream flood input, the general 663 664 flow velocities tend to decrease. However, due to the erosion of the upstream main channel, the converged water still maintains sufficient momentum. Between t = 3 d and 7 d, the flow 665 666 velocities start to demonstrate a decreasing trend, due to the continuous reduction in the 667 upstream water supply. Finally, all of the flooding water returns to the main channel and there 668 exists a clear difference in the flow patterns at upstream and downstream of the Longwei Bay. 669



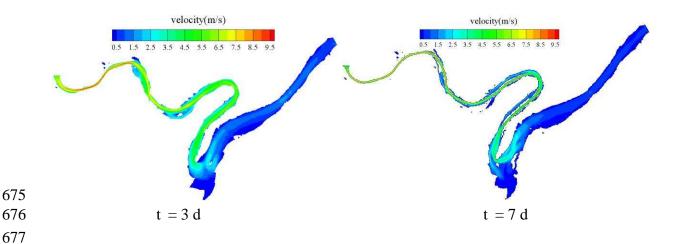
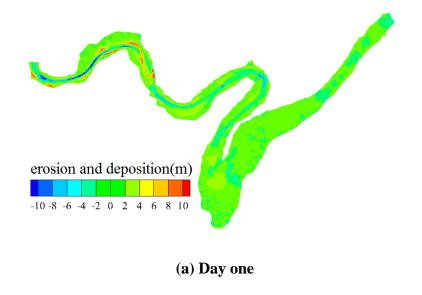
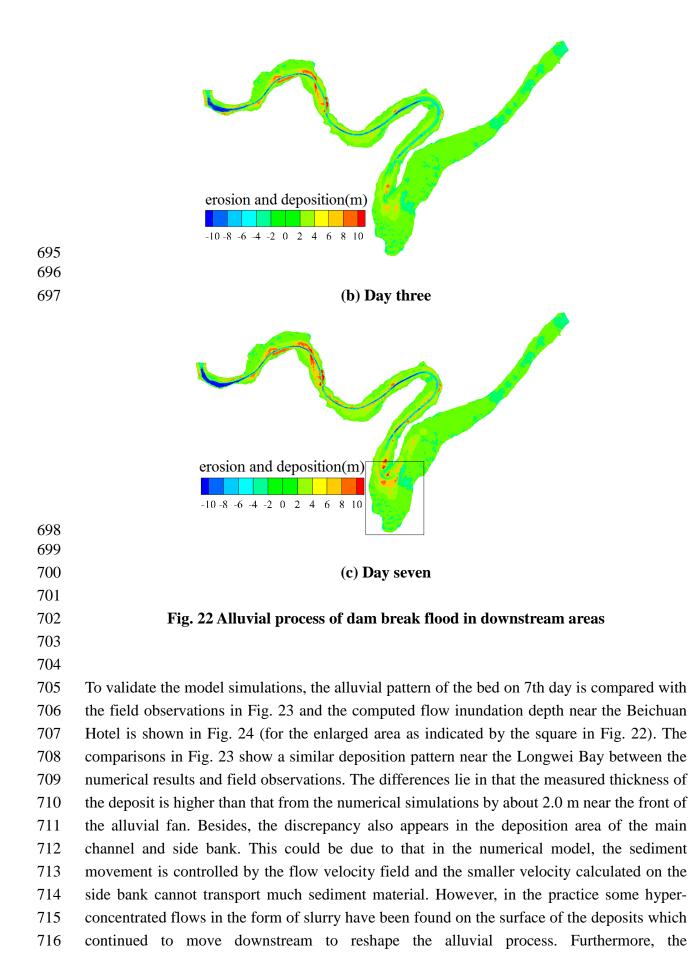


Fig. 21 Computed velocity fields of flood flow at different times

The sediment erosion and deposition of the river bed are shown in Fig. 22 for the next three days after the dam break flood. It is shown that by one day later, most of the alluvial processes happen near the dam area. This is demonstrated by the fact that severe erosions are found immediately downstream of the Tongjiashan Dam and further downstream channels show the features of erosion in the main channel and deposition on the side bank. Then three days later, the erosion of the main channel has progressed to somewhere upstream of the Longwei Bay. Finally seven days after the dam break, a quasi-equilibrium state of the alluvial evolution has achieved, demonstrated by the clear erosion of the main channel which deposited the sediment materials onto the side bank. The deposition thickness near the Longwei Bay reaches as high as $5 \sim 10$ m.





simplifications of initial and boundary conditions in the numerical model could also cause uncertainty in the result predictions. In addition, the inundation depths computed in Fig. 24 indicate that most of the areas near the Beichuan Hotel and Dike have been deposited by the sediments up to more than 8.0 m. This is also quite consistent with the field observations and the media report.

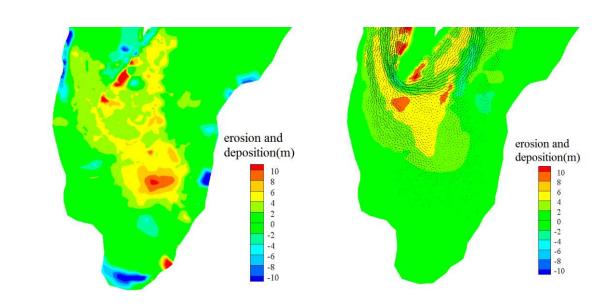
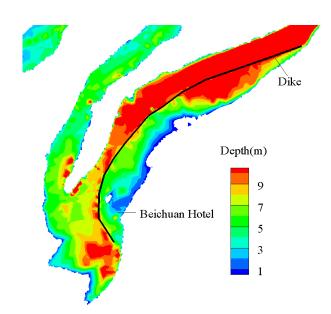


Fig. 23 Alluvial pattern of bed 7 days after the dam-break flood (left: field observation;
 right: numerical result)



733 Fig. 24 Computed flow inundation depths near Beichuan Hotel and Dike area

735 **6.** Conclusions

736

737 The paper combined the FEM characteristic splitting method and FVM Godunov scheme and 738 developed a water-sediment mixture flow model for the field river studies in Southwest 739 China. The FEM numerical algorithm is robust for the treatment of advection term and 740 flexible in fitting the complicated physical boundary, while the FVM solution scheme has 741 satisfactory conservation property and numerical accuracy. Through a series of benchmark 742 unsteady flow tests and two field case applications, the hybrid FEM-FVM model has been 743 found to have good stability and accuracy when the numerical results were compared with the 744 experimental data. The study demonstrates that the present numerical model could provide a 745 simple and useful prediction tool for the water-sediment mixture flow problems in 746 engineering field as an economic substitute for the physical experiment and field observation.

747

In order to more accurately forecast the practical sediment transport and alluvial deformation, more sound sediment erosion and deposition mechanisms should be explored in the future work. As fundamental sediment properties could behave significantly different from one region to another (Wang and Zhang, 2012), further investigations on the sediment initiation, sediment-laden capacity, sediment settling velocity, etc. should be of more priority than the development of higher-order numerical schemes. This is especially true shall the model be applied to the practical field cases in different regional areas.

755

Finally, we would like to mention that the main objective of the present research is to use the numerical model to evaluate the dam break flood in the downstream areas. However, in many engineering situations, the upstream flow structures and sediment erosion and sedimentation properties, such as the dam-failure hydrograph and morphological changes at the dam site, are of particular interest in the engineering community as documented by Wang et al. (2008) and Liu et al. (2012). Also, these could provide a more robust test on the capacity of the numerical models.

763 764

765 Acknowledgements

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