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Department of Automatic Control and Systems Engineering

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OPTIMAL CONTROL OF THE ARGON OXYGEN  
DECARBURIZING STEELMAKING PROCESS

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Research Report No. 70

June 1978

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Abstract

The argon oxygen decarburizing process is an increasingly important method of stainless steel manufacture. In the present paper, a nonlinear dynamic model is presented which represents approximately the refining process. Finite dimensional optimisation techniques are used to calculate an optimal switched control, and infinite dimensional gradient methods are applied to discover an optimal continuous strategy. Control is achieved by varying the ratio of argon and oxygen in the supplied gas. Results with a realistic cost function indicate that substantial improvements in operating practice may be achieved. The feasibility of implementing the suggested control schemes is discussed.

List of principal symbols

A	=	activity of species indicated by subscript
%C	=	average carbon content of melt
%Cr	=	average chromium content of melt
Cr	=	weight of chromium in melt
f	=	state transition function
Fe	=	weight of iron in melt
g(t)	=	gradient
H	=	Hamiltonian
K	=	equilibrium constant
M	=	atomic weight of element indicated by subscript
$P_{CO}$	=	partial pressure of CO
T	=	temperature of melt
tf	=	total blowing time
u	=	control ratio
$u_1, u_2$	=	rates of oxygen and argon injection
$u_{01}, u_{12}, u_{23}$	=	first, second and third stage controls
V	=	total gas flow rate
$V_{Ar}, V_{O2}$	=	argon and oxygen flow rates
$V_{CO}$	=	amount of CO generated per unit time
W	=	weight of melt
x	=	state vector
$x_1, x_2$	=	carbon and chromium content of melt
$\alpha$	=	empirical rate constant
$\alpha_i$	=	single variable optimisation step length
$\beta_i$	=	parameter in conjugate gradient method
$\gamma_c$	=	activity coefficient of carbon

$\lambda$  = costate vector  
 $\psi$  = integral costs  
 $\psi_1, \psi_2, \psi_3, \psi_4$  = chromium, oxygen, argon and operating costs  
 $\langle a, b \rangle$  = inner product of functions a and b

chemical symbols

Ar = argon  
Cr = chromium  
 $\text{Cr}_2\text{O}_3$  = chromic oxide  
Fe = iron  
FeO = iron oxide  
O = oxygen

1. Introduction

Until recently the established technique for the production of stainless steel has been the melting of scrap in an electric arc furnace, followed by oxygen lancing. The relatively high initial concentration of carbon in the scrap is decreased by oxidation to obtain the desired metallurgical specification of the steel. An unfortunate consequence of the required oxidation is that the useful chromium content of the scrap metal is also decreased. To meet the chromium specification of the end product it is necessary to add expensive low-carbon chromium at the end of the refining period; it being uneconomical to recover a significant amount of chromium from the slag. In order to favour the carbon oxidation it is also possible to maintain the furnace at a high temperature, but this will lead to excessive refractory wear.

The difficulties encountered in stainless steel production using electric arc furnaces, have been largely overcome by the recent introduction of a process termed argon oxygen decarburizing (AOD)<sup>1,2,3</sup>. After melting scrap in a conventional arc furnace the metal is transferred into a separate AOD vessel, where oxidation is achieved by blowing oxygen mixed with argon from openings, or tuyeres, at the base of the vessel. Whereas chromium oxidation produces a solid slag the carbon oxidation produces predominantly carbon monoxide gas which is diluted by argon. The dilution has the effect of encouraging the necessary carbon reaction and hence restricting the undesirable chromium oxidation. The gas blowing through the vessel has the added benefit of producing a thorough stirring action in the melt. The advantages of AOD steelmaking have led to the increasing adoption of the process by world producers<sup>3</sup>.

The major control problem occurring in the AOD process is the selection of an 'optimal' blowing strategy. The ratio of argon to oxygen in the input gas may be varied as the refining proceeds in an attempt to minimise production costs. Although Fruehan<sup>4</sup> and Asai<sup>5</sup> have investigated dynamic models of the process, the control problem has generally been approached by ad hoc methods and systems of recommended operating practice. It is well known that as the refining progresses it is desirable to increase the argon content of the gas<sup>6</sup>. It is therefore common practice to refine over three stages, increasing the A/O ratio between stages. In the present paper, optimal three-stage control strategies are developed using finite-dimensional optimisation methods. If it is possible to continuously change the gas ratio with time then an improved control may be obtained, and in the present work the necessary conditions of the nonlinear optimal control are solved using infinite-dimensional gradient methods. The physical conditions of the AOD process make it virtually impossible to measure the state of the system and consequently metallurgical corrections are made at the end of the process after a sample has been analysed. For the same reason feedback control cannot be implemented, without waste gas analysis and accompanying noise filtering. Open-loop control is therefore proposed, in which a control strategy is specified from known initial conditions and which can be implemented by a small computer or by a manual look up table.

## 2. Mathematical model of the AOD process

A reasonably simple nonlinear dynamic model of the AOD process has been developed by Fruehan<sup>4</sup> based on the following assumptions. The injected oxygen primarily oxidizes chromium in the region of the tuyeres to form chromic oxide ( $\text{Cr}_2\text{O}_3$ ). As the chromic oxide particles move upwards with the argon bubbles, they react with the carbon. The oxidation of carbon by  $\text{Cr}_2\text{O}_3$  is assumed to be the rate controlling reaction and is limited by the liquid-phase mass transfer of carbon to the bubble surface. The validity of the model, which is derived in Section 8, has been confirmed by comparisons with plant data<sup>4</sup>. Expressing the main equations of the process model in state space form, we have

$$\dot{x}_1 = -\alpha \left[ x_1 + \frac{\beta_1 \dot{x}_1}{\left(\frac{u_2}{22.4}\right) - \beta_2 \dot{x}_1} \right] \quad (1)$$

where  $\alpha, \beta_1, \beta_2$  are constants defined in Section 8, and

- $x_1$  = carbon content of the melt (%C)
- $\dot{x}_1$  = rate of carbon oxidation (%C/min.)
- $x_2$  = chromium content of the melt (%Cr)
- $\dot{x}_2$  = rate of chromium oxidation (%Cr/min.)
- $u_1$  = rate of oxygen injection ( $\text{m}^3/\text{min.}$ )
- $u_2$  = rate of argon injection ( $\text{m}^3/\text{min.}$ )

Rearranging eqn. 1 gives

$$\beta_2 (\dot{x}_1)^2 + \left[ \alpha \beta_2 x_1 - \alpha \beta_1 - \left(\frac{u_2}{22.4}\right) \right] \dot{x}_1 - \alpha \left(\frac{u_2}{22.4}\right) x_1 = 0 \quad (2)$$

Equation 2 is a first-order nonlinear differential equation and is in the form of a quadratic expression in  $\dot{x}_1$ . Therefore  $\dot{x}_1$  can be written

$$\dot{x}_1 = \frac{-\left[\alpha\beta_2 x_1 - \alpha\beta_1 - \frac{u_2}{22.4}\right] \pm \sqrt{\left[\alpha\beta_2 x_1 - \alpha\beta_1 - \frac{u_2}{22.4}\right]^2 + 4\beta_2 \alpha \frac{u_2}{22.4}}}{2\beta_2} \quad (3)$$

Since  $\beta_2, \alpha, u_2$  can never be negative, only one of the above roots predicts a decreasing carbon level and is therefore physically meaningful. Consequently, only the negative sign need be considered.

As shown in Section 8 there is a limit to the rate of decarburization represented by eqn. 3 and this may be expressed by

$$\dot{x}_1 = -\frac{2}{\beta_2} \left( \frac{u_1}{22.4} \right) \quad (4)$$

The actual rate is determined by the minimum of eqns. 3 and 4.

The rate of oxidation of chromium is given by

$$\dot{x}_2 = \frac{4M_{cr} f}{3W10^{-2}} \left( \left( \frac{u_1}{22.4} \right) - \frac{\beta_2}{2} \dot{x}_1 \right) \quad (5)$$

where  $M_{cr}$ ,  $f$ ,  $W$ ,  $\beta_2$  are defined in Section 8.

Although the gas ratio is varied it is normal to maintain a constant total blowing rate and therefore the two control inputs may be replaced by one equivalent input. Thus

$$u = \frac{u_2}{u_1 + u_2} \quad (6)$$

The model of the AOD process therefore consists of two nonlinear state equations in two state variables. The nonlinearity is intensified by the necessity of the square root in eqn. 3 and by

the switching condition between eqns. 3 and 4.

In order to optimise the process described by the model equations it is necessary to define a suitable cost function. In the case of the AOD process this is relatively simple, since the costs of the components of the model are accurately quantifiable. A suitable objective function is represented by

$$J = \psi_1(x_{2i} - x_{2f}) + \psi_2 \int_0^{tf} u_1 dt + \psi_3 \int_0^{tf} u_2 dt + \psi_4 \int_0^{tf} dt \quad (7)$$

where  $tf$  = total blowing time

$x_{2i}$  = initial chromium content

$x_{2f}$  = final chromium content

$\psi_1$  = cost of replacing lost chromium

$\psi_2$  = cost of oxygen

$\psi_3$  = cost of argon

$\psi_4$  = overhead running costs (which may be neglected provided that the operating time for the AOD process is less than that of charging and melting in the associated arc furnace)

### 3. Optimal switched control

In order to achieve acceptable control without continuous intervention it is possible to use a three-stage switched control strategy. The control trajectory  $u(t)$  is defined by the piece-wise constant segments,

$$u(t) = \begin{cases} u_{01} & ; & 0 \leq t \leq t_1 \\ u_{12} & ; & t_1 \leq t \leq t_2 \\ u_{23} & ; & t_2 \leq t \leq t_3 \end{cases}$$

where  $t = 0$  corresponds to the initial time and  $t = t_3$  corresponds to the final time  $t_f$ .

The optimisation problem may now be expressed as

$$\text{Min}_{u_{01}, u_{12}, u_{23}} J_1(u_{01}, u_{12}, u_{23})$$

where the function  $J_1$  is obtained by digital simulation of the model, given the control input  $u(t)$ , and assigning the resulting cost  $J$  to  $J_1$ . Any convenient finite-dimensional optimisation method may be used to compute optimal values of  $u_{01}, u_{12}, u_{23}$  from initial guesses, and in the present work the simplex method of Nelder and Mead<sup>8</sup> and a quasi-Newton method of Gill and Murray<sup>9</sup> have been used. An alternative to finding optimal levels  $u_{01}, u_{12}, u_{23}$  is to hold these constant and to optimise the switching times  $t_1$  and  $t_2$ . Programs have been written in FORTRAN for an ICL 1906S computer to implement both schemes. Typical data and results are shown in Table 1 and Figs. 1 and 2. The optima obtained were found to be fairly sensitive and initial guesses within ten percent of the true solution could give twenty percent increases in cost. The most important cost factor which the optimisation was able to reduce was that of the chromium loss.

Table 1: Data for simulation

Parameter	Value
Melt weight $W$	76000 kg.
Melt temperature $T$	1948 $^{\circ}\text{K}$
Gas flow rate $V$	51 $\text{m}^3/\text{min.}$
Initial carbon $x_{1i}$	0.66%
Initial chromium $x_{2i}$	10%
Chromium cost $\psi_1$	2100 units/%Cr
Oxygen cost $\psi_2$	0.05 units/ $\text{m}^3$
Argon cost $\psi_3$	0.4 units/ $\text{m}^3$
Operating cost $\psi_4$	0.0 units/ $\text{m}^3$

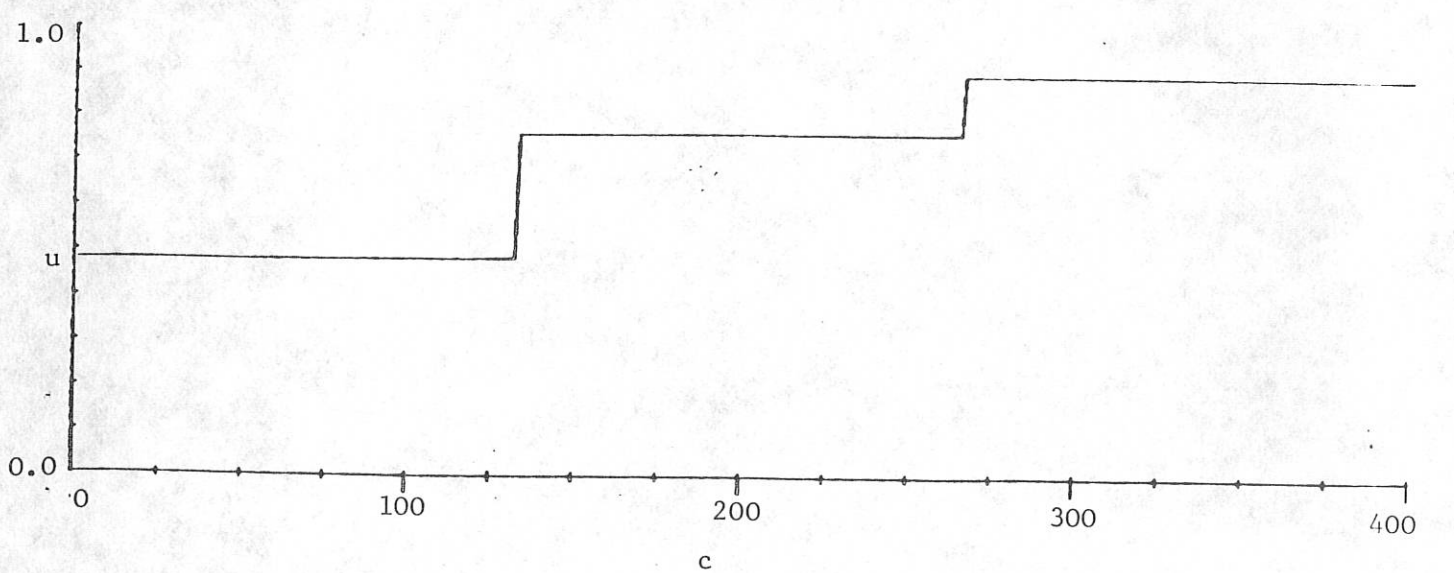
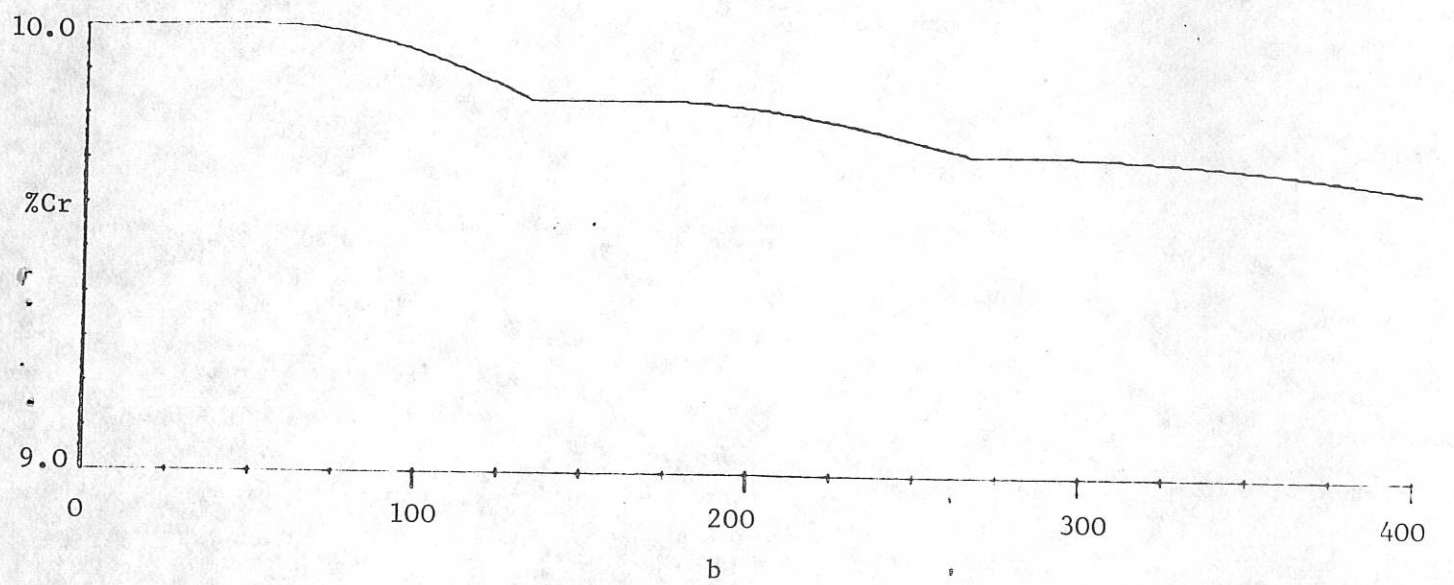
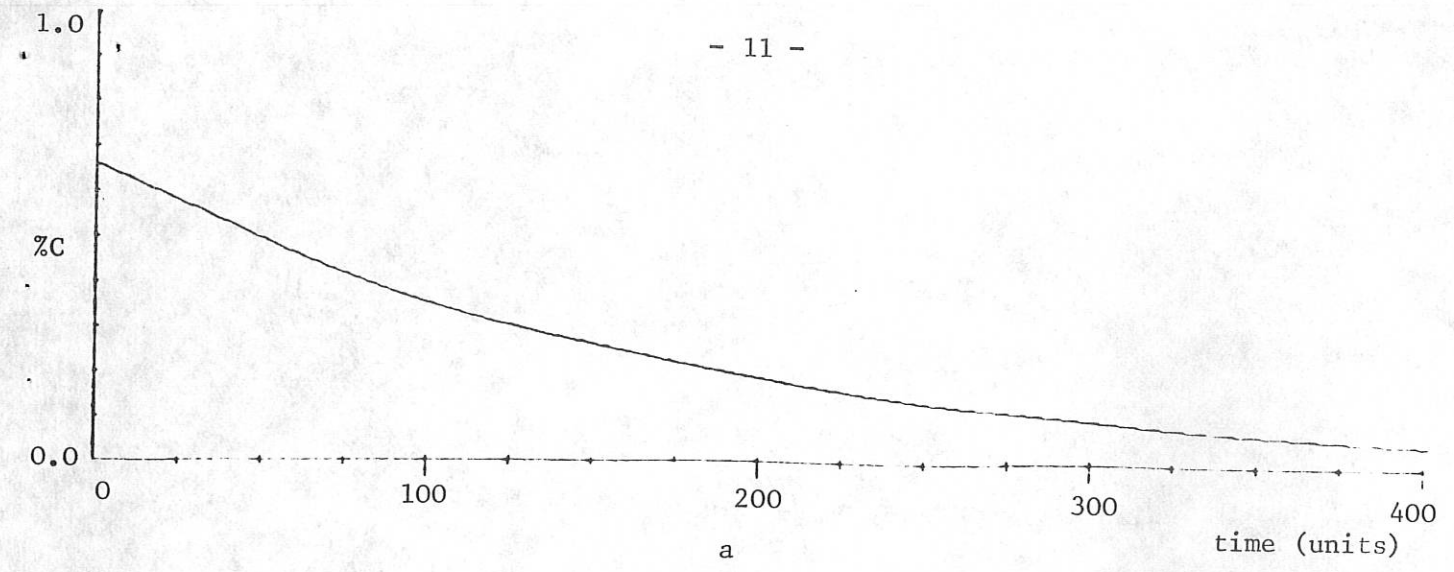


Fig. 1 Optimal level, three-stage control  
a,b,c Carbon, chromium and control trajectories  
400 time units = 36.2 minutes  
Objective function value = 1318.2 units

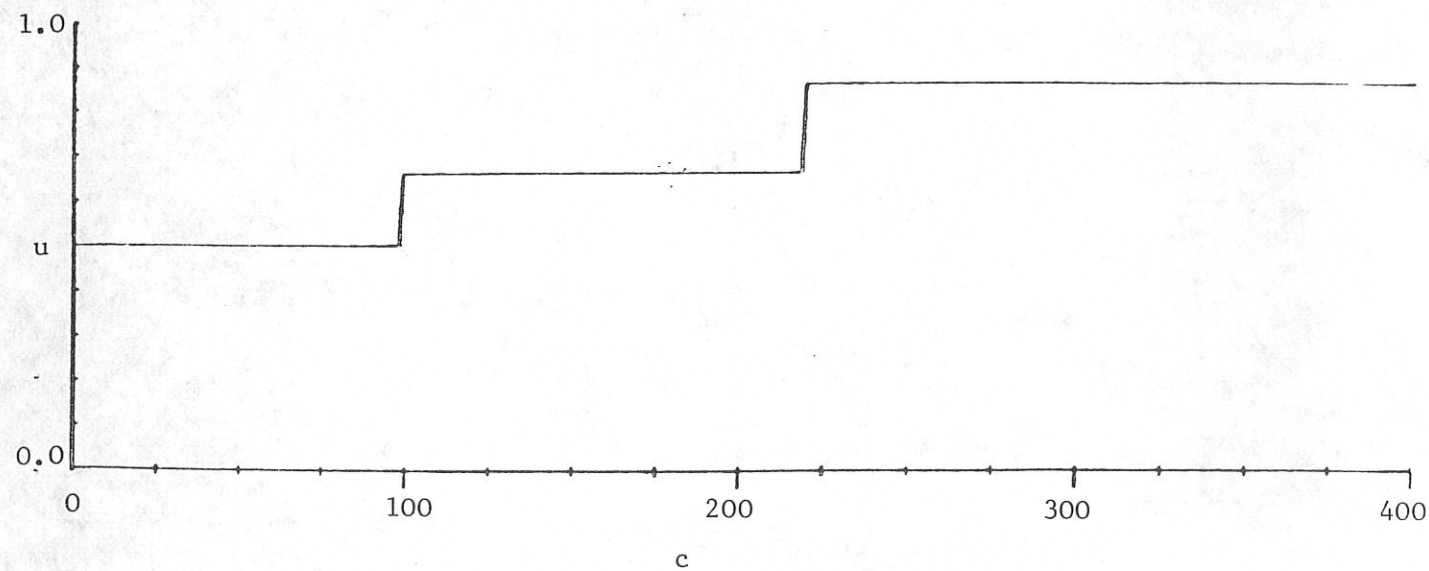
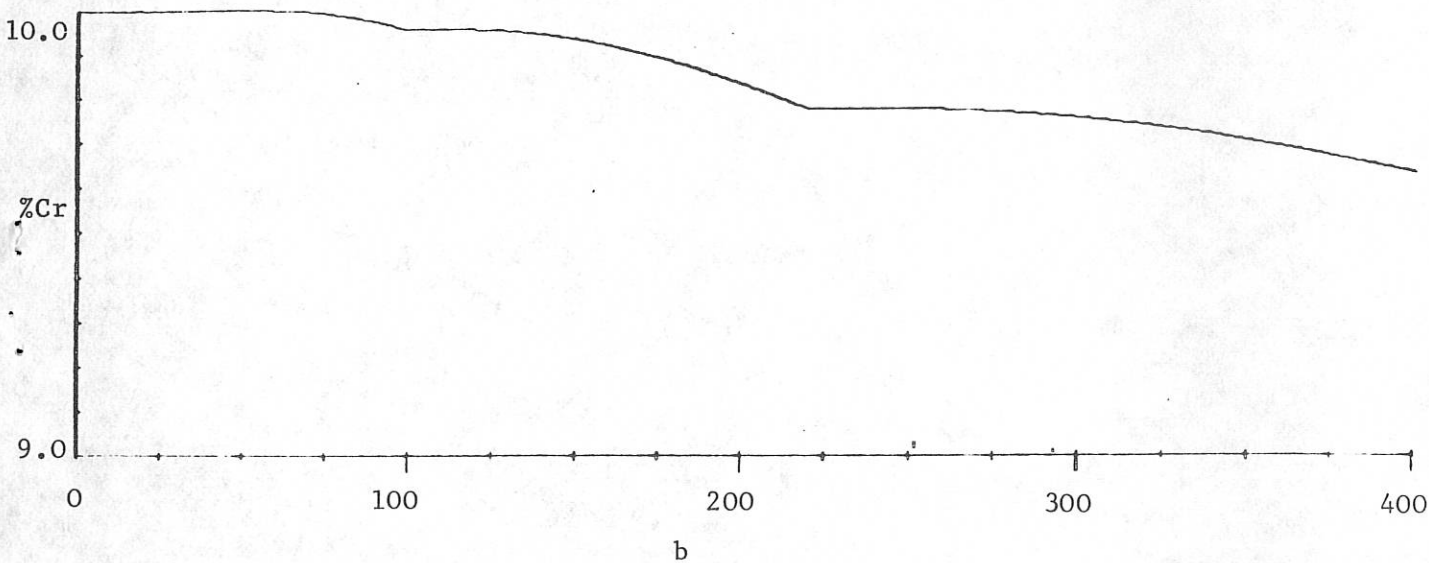
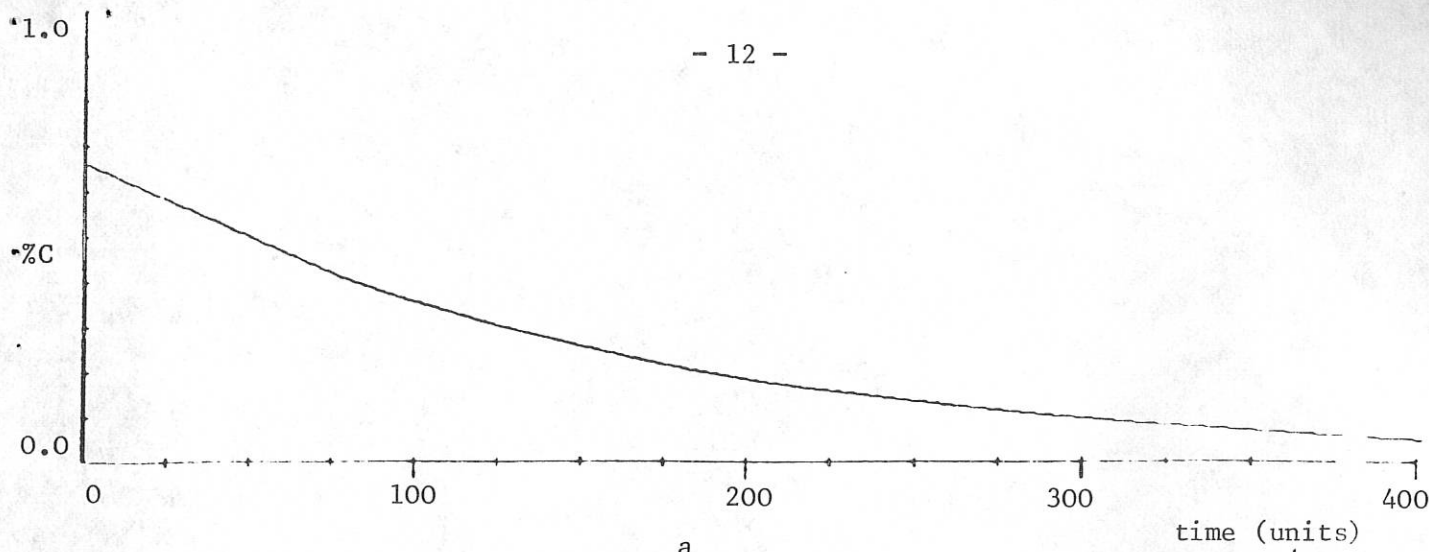


Fig. 2 Optimal switching time, three-stage control  
a,b,c Carbon, chromium and control trajectories  
Objective function value = 1332.8 units

#### 4. Optimal continuous control

If it is possible to adjust continuously the supply of argon and oxygen to the AOD vessel, then an improved control strategy may be obtained. The computation of such a strategy is a significantly more complex problem than the 2- or 3-dimensional optimisations discussed in the previous section. A simple approach would be to discretise the control in 50 to 100 constant time intervals. This would lead to a very high dimensional optimisation problem which may require excessive computer resources for solution. A more attractive approach is to apply the theory of infinite-dimensional optimisation recently developed by a number of authors<sup>10,11,12,13,14</sup>. The objective function which it is desired to minimise may be restated as

$$J = M[x_{tf}] + \int_0^{tf} \psi(x,u) dt \quad (8)$$

where M represents the end point chromium cost and  $\psi$  represents the remaining costs.

The minimisation must be carried out subject to the dynamic constraints of the process model given by

$$\dot{x} = f(x,u) , \quad x(0) = x_{ti} \quad (9)$$

Defining the Hamiltonian

$$H(x,\lambda,u) \triangleq \lambda^T f(x,u) + \psi(x,u) \quad (10)$$

where

$$\dot{\lambda} = - \frac{\partial H}{\partial x} = - \left( \frac{\partial f}{\partial x} \right)^T \lambda - \frac{\partial \psi}{\partial x} \quad (11)$$

$$\lambda(tf) = \left( \frac{\partial M}{\partial x} \right)_{t=tf} \quad (12)$$

and letting

$$g(t) = \frac{\partial H}{\partial u}(x, \lambda, u) \quad (13)$$

The infinite dimensional analogue of the steepest-descent method may be written as<sup>10</sup>

- (1) Select an arbitrary initial guess  $u_0(t)$ .
- (2) Using  $u_0(t)$  integrate the state equations (9) forward in time, and the adjoint equations (11) and (12) backward in time. Evaluate  $g_1(t)$  for the current approximation using analytical expressions for eqn. 13.

(3) Select a scalar multiple of the negative gradient which produces a minimum cost and add this to the control trajectory. This step requires a one-dimensional optimisation represented by

$$u_{i+1}(t) = u_i(t) - \alpha_i g_i(t) \quad (14)$$

where  $J(u_i(t) - \alpha_i g_i(t)) \leq J(u_i(t) - \alpha g_i(t))$  for all  $\alpha$ .

- (4) If convergence has been obtained stop, otherwise repeat from step (1).

If the algorithm is implemented on a digital computer it is necessary to discretise the control trajectory  $u_i(t)$ , however the infinite-dimensional nature of the algorithm is preserved by applying numerical integration techniques in computing the desired functions.

The steepest-descent method is a first-order method and as such has good initial convergence but does not perform well in the neighbourhood of the minimum. When applied to the AOD control problem, 113 iterations were required for convergence, corresponding to 13 minutes of computing time on a large mainframe computer. In

order to promote rapid convergence the method of conjugate gradients<sup>10</sup> may be applied. This technique has often been used in finite-dimensional problems<sup>15</sup>, and for n-dimensional quadratic objective functions, is known to locate the minimum in n steps. The method modifies the search direction used in step (3) above as follows:

(3) Select a scalar multiple of a search direction  $S_i(t)$  which produces minimum cost and add this to the control trajectory, where

$$S_i(t) = -g_i(t) + \beta_i S_{i-1}(t) \quad (15)$$

$$\beta_i = \frac{\langle g_i(t), g_i(t) \rangle}{\langle g_{i-1}(t), g_{i-1}(t) \rangle} \quad (16)$$

At the first iteration, some of the above quantities are undefined and a steepest descent step is taken.

Convergence of the conjugate-gradient method is often improved in the finite-dimensional case by resetting the algorithm every kth step with a steepest-descent iteration. Luenberger<sup>16</sup> has shown that for quadratic problems this may be related to the spectrum of the quadratic operator. In order to investigate the value of resetting in the infinite-dimensional case a number of experimental computer runs were made. The results are shown in Fig.3, and it can be seen that for the AOD control problem an optimum resetting constant is 10-12 iterations.

The control and state trajectories found using the conjugate gradient method are presented in Fig.4. It is apparent that chromium loss has been eliminated by the optimal strategy and that the value of the cost function is significantly lower than those of the switched controls. Although it has not been proved that

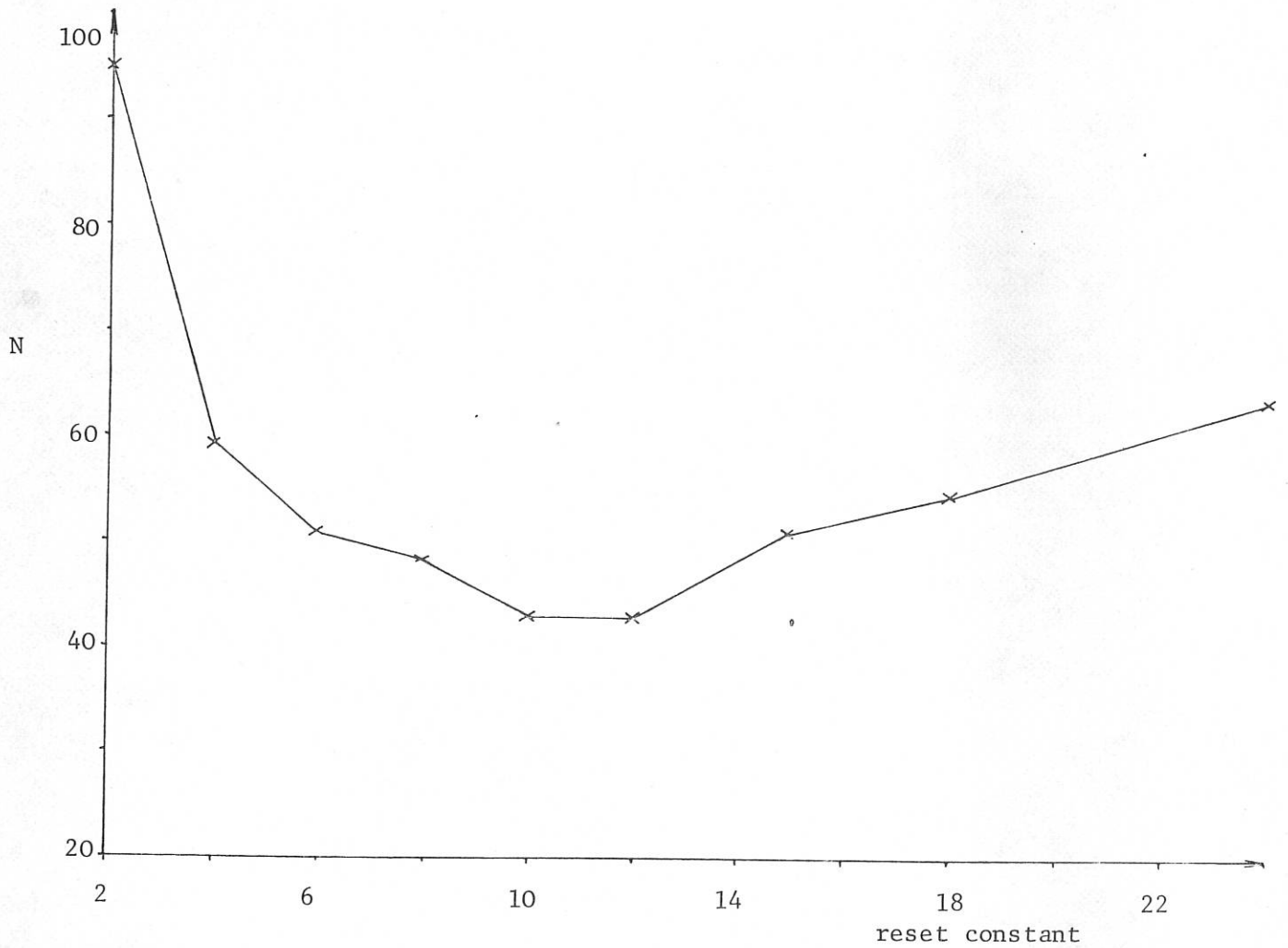
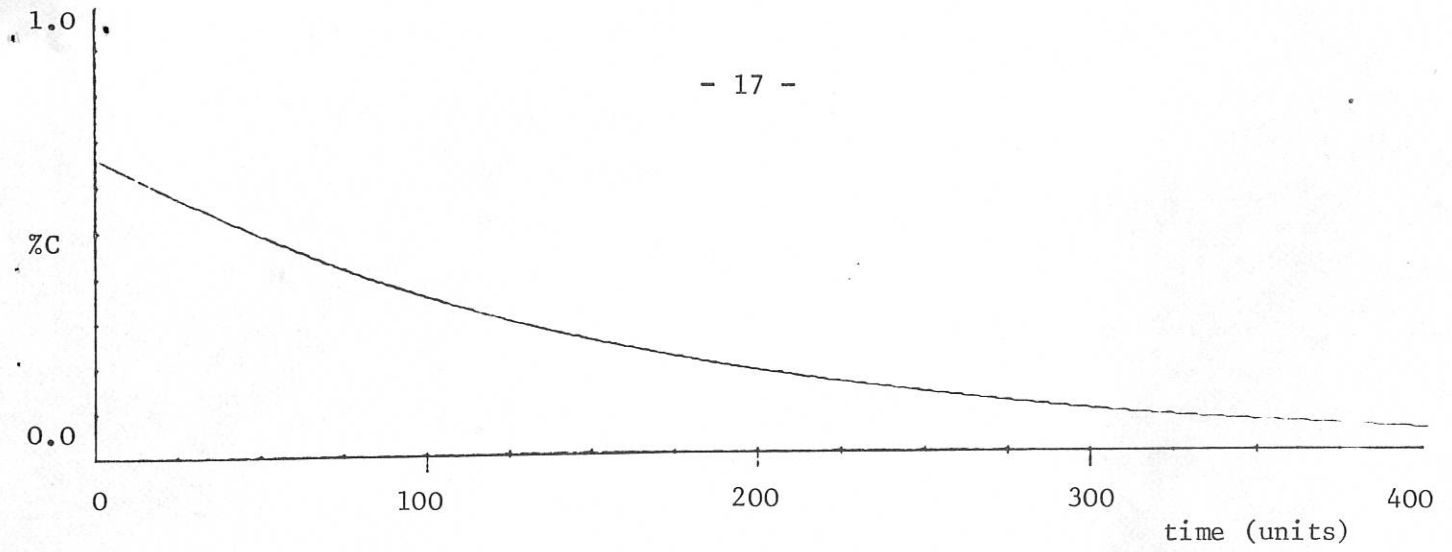
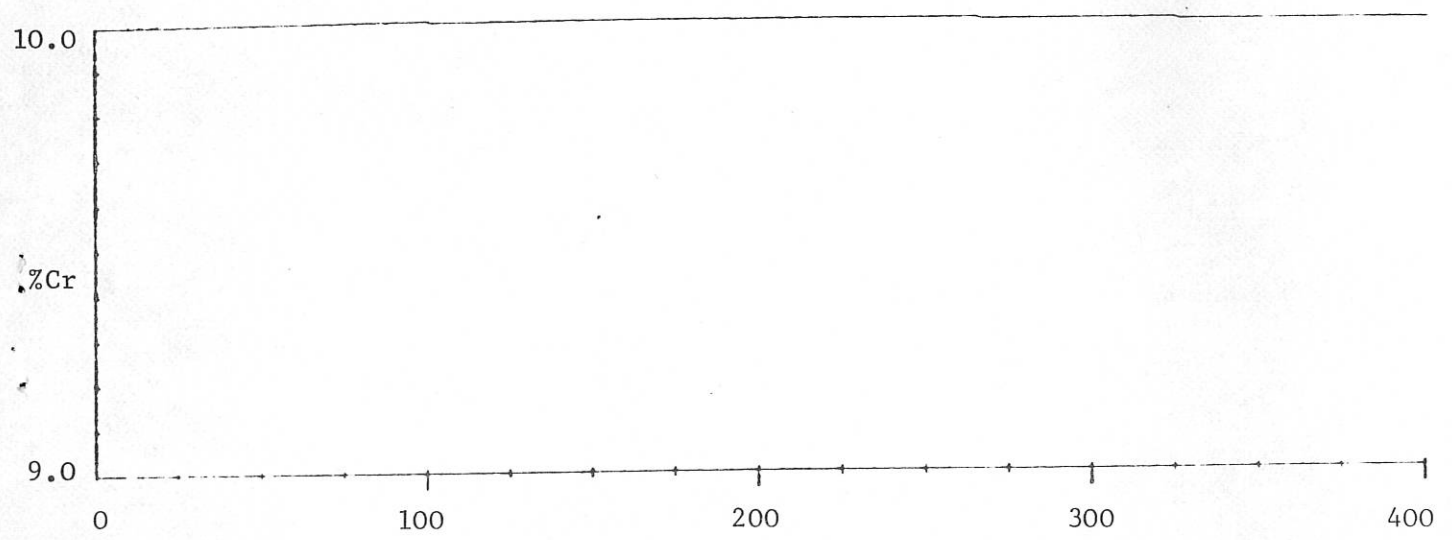


Fig. 3 Variation of convergence with reset constant

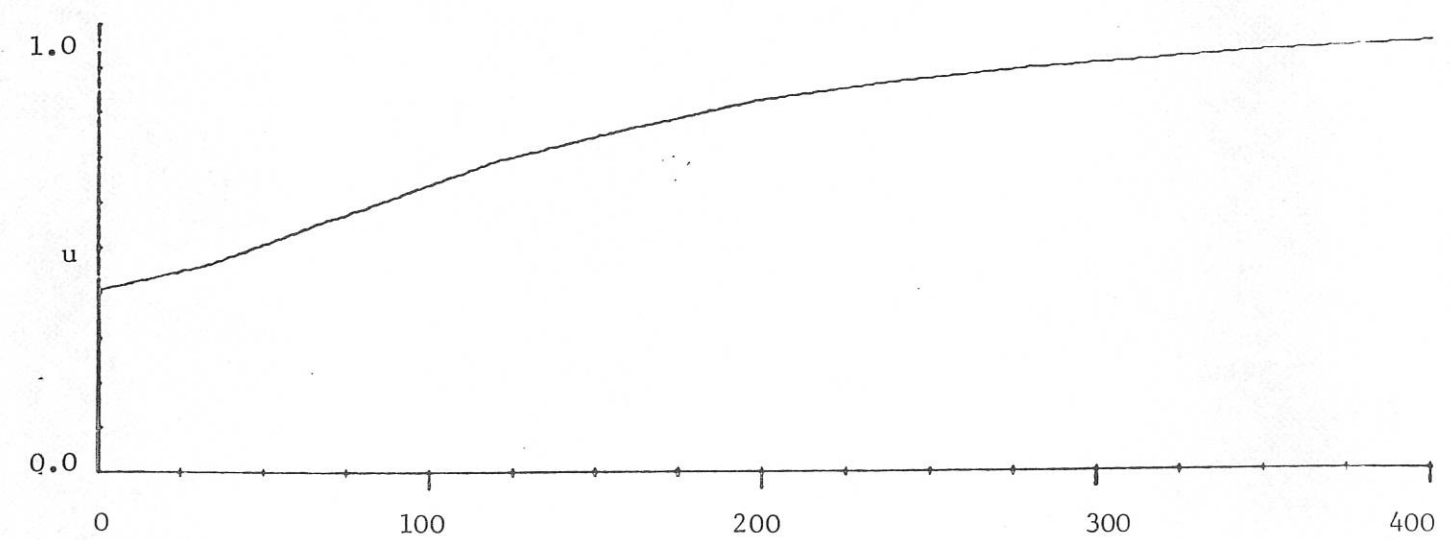
N = no. of iterations required to achieve cost  $\leq$  589.7 units



a



b



c

Fig. 4 Optimal continuous control

a,b,c Carbon, chromium and control trajectories

Objective function value = 577.8 units

the minimum located by the present technique is an absolute minimum, experience with different initial guesses and alternative solution methods has indicated that the minimum is indeed a global one.

In both the switched and continuous control methods it is possible to allow the end time  $t_f$  to vary. In the three-stage control optimisation  $t_f$  may be specified by continuing the integration until the end point carbon is satisfactory, or alternatively fixing the end time and penalizing any deviation in carbon end point with an added quadratic cost. In the infinite-dimensional case a time scaling method proposed by Quintana and Davison may be applied<sup>13</sup>.

##### 5. Discussion of results

Optimal three stage trajectories for both variable control ratios and variable switching times have confirmed the validity of current operating practice in AOD steelmaking. By repeatedly running the optimisation program with different initial conditions it would be possible to build up a table of optimal strategies. Depending on the validity of the model used, such a table may provide some guidance to operators concerned with selecting a gas input strategy for a particular melt. The main advantage of this scheme would be ease of implementation and the establishment of a look-up table is the only requirement. If a process computer was available, either online optimisation or the maintenance of a data base of controls would be feasible. Also the availability of a computer model of the system would enable operators to experiment, and become more familiar with the process.

Objective function values for optimal continuous control have been found to be much lower than those of equivalent switched controls, the improvements being mainly caused by the elimination of chromium loss. However, it is questionable if such an improvement could be realized in practice. The model of the process neglects various factors such as the occurrence of other metals in the melt. The improvements predicted by the model are therefore certainly over optimistic. However it is probable that large cost savings may be achieved by implementing some form of continuous control. The difficulty involved in applying this type of control is greater than that of the switched strategies, in that new equipment would be required. In current practice, the gas supply valves are set manually, and for continuous control, automation would be necessary. However, the cost of the hardware required would not be expected to be high, and the savings achieved could outweigh the initial expenditure. Once again, provided that the control was described by a small number of linear segments, the data processing requirement would be limited to a small online look-up table supported by a large number of off line studies.

## 6. Conclusions

A nonlinear dynamic model of the AOD process has been presented. The optimal three-stage switched control has been found by the application of standard finite-dimensional optimisation methods, and if dynamic adjustment of gas input is possible a more general continuous control strategy is feasible. The optimal continuous control has been computed using the infinite-dimensional conjugate gradient method and the convergence of the algorithm has been

accelerated by periodically resetting the iteration. Implementation of the proposed schemes is discussed, and it is concluded that cost benefits could be obtained by the application of optimal switched controls, and also of the more sophisticated continuous control action.

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8. Appendix

Reaction model of Fruehan

Studies by Choulet et al<sup>1</sup> have shown that for argon to dilute the carbon monoxide effectively it is necessary to inject gas low down in the vessel, by means of tuyeres. It is assumed<sup>4</sup> that in the neighbourhood of the tuyeres most of the oxygen is consumed by chromium oxidation. Iron is also oxidized in this region, but the FeO is quickly reduced by chromium and only acts as an intermediary. The main reactions in the tuyere zone are therefore given by



As the chromic oxide rises in the melt with the argon bubbles the following reaction occurs:



By introducing an empirical rate parameter  $\alpha$ , which is proportional to the average liquid-phase mass transfer coefficient for carbon transport to the Ar-CO bubbles, the average rate of oxidation of carbon may be represented by

$$\frac{d(\%C)}{dt} = -\alpha [\%C - (\%C)_e] \quad (20)$$

where  $\%C$  = the time dependent average carbon content of the melt

$(\%C)_e$  = the average carbon content at the bubble surface in local equilibrium with the average chromium content and the average CO partial pressure in the gas bubbles.

The empirical rate constant  $\alpha$  represents the average rate constant for the reaction and is determined from operational data. It is temperature independent, assuming that the rate is controlled by mass transfer.

Assuming that the activity of  $Cr_2O_3$  is approximately equal to one, the equilibrium of reaction eqn. 19 is given by

$$K^{1/3} = \frac{(A_{cr})^{2/3} P_{CO}}{A_c} \quad (21)$$

where  $K$  = the equilibrium constant for the reaction

$P_{CO}$  = the partial pressure of CO in the bubbles

$A_{cr}$  = the average activity of chromium in the melt relative to pure chromium

$A_c$  = the average activity of carbon in the melt relative to pure carbon

The equilibrium constant  $K$  is temperature dependent and its value is determined by<sup>4</sup>

$$\log_{10} K = \frac{-40970}{T} + 27.31 \quad (22)$$

where  $T$  = temperature of the metal ( $^{\circ}K$ ).

Assuming that the amount of minor elements such as nickel and manganese in the vessel is negligible, the average activity of chromium  $A_{cr}$  is determined from the following expression

$$A_{cr} \approx \frac{\frac{Cr}{M_{cr}}}{\frac{Cr}{M_{cr}} + \frac{Fe}{M_{fe}}} \quad (23)$$

where Cr = weight of chromium in the melt

Fe = weight of iron in the melt

M = atomic weight of the element indicated by  
the subscript

From eqns. 22 and 23, the local equilibrium carbon concentration at low carbon levels, can be represented by

$$(\%C)_e = \frac{(A_{cr})^{2/3}}{K^{1/3}} \frac{10^2 M_c P_{co}}{M_{fe} \gamma_c} \quad (24)$$

where  $\gamma_c$  = the activity coefficient of carbon relative to graphite

The parameter  $\gamma_c$  is a function of chromium and carbon contents and is represented by<sup>7</sup>

$$\log_{10} \gamma_c = -0.024 (\%Cr) + 0.22 (\%C) - 0.25 \quad (25)$$

The partial pressure of CO in the bubbles is given by

$$P_{CO} = \frac{V_{CO}}{V_{CO} + V_{Ar}} P \quad (26)$$

where P = average total pressure in the vessel

$V_{Ar}$  = argon flow rate

$V_{CO}$  = amount of CO generated per unit time

$V_{CO}$  may be expressed in kmols per unit time, and given by

$$V_{CO} = - \frac{d(\%C)}{dt} \frac{10^{-2}W}{M_c} \quad (27)$$

where  $W$  = the weight of the melt.

Substituting eqns. 24, 26 and 27 into eqn. 20 gives the rate of decarburization of the process as

$$\frac{d(\%C)}{dt} = - \alpha \left( \%C + \frac{\beta_1 \frac{d(\%C)}{dt}}{V_{Ar} - \beta_2 \frac{d(\%C)}{dt}} \right) \quad (28)$$

where  $\beta_1 = \frac{(A_{cr})^2 / 3 W \cdot P}{K \cdot 3 M_{Fe} \gamma_c}$  ,  $\beta_2 = \frac{10^{-2}W}{M_c}$

However, there is a limit to the rate of decarburization that is possible due to the limited supply of oxygen. Therefore at high carbon levels the rate is given by

$$\frac{d(\%C)}{dt} = - \frac{2 M_c V_{O2}}{10^{-2} W} \quad (29)$$

where  $V_{O2}$  = oxygen flow rate

Hence, the rate of decarburization is described by eqn. 28 or 29, whichever predicts the slower rate.

Initially, all the oxygen in the form of chromic oxide is used to oxidize carbon. As the decarburization rate decreases, the amount of chromium lost into the slag increases. This is caused by the accumulation of chromic oxide in the slag which is not reduced by carbon.

The amount of oxygen in kmols required to oxidize,  $\Delta\%C$  to CO is given by

$$V_{O_2} = \frac{10^{-2} W}{2M_c} (\%C_i - \%C) \quad (30)$$

where  $\%C_i$  = the initial carbon content of the melt

$\%C$  = the present carbon content of the melt

Therefore, from eqn. 17, the net amount of chromium  $\Delta\%Cr$ , oxidized to form  $Cr_2O_3$  is

$$\Delta\%Cr = 4M_{cr} f \left[ V_{O_2} t - \frac{10^{-2} W}{2 M_c} (\%C_i - \%C) \right] \quad (31)$$

where  $V_{O_2}$  = rate of oxygen input (kmols/min)

$f$  = a parameter governed by the relative amounts of chromium and iron oxidized during the process.

Using eqn. 31 the rate of chromium oxidation in the process can be represented by

$$\frac{d(\Delta\%Cr)}{dt} = \frac{4 M_{cr} f}{3 W 10^{-2}} \left[ V_{O_2} - \frac{10^{-2} W}{2 M_c} \frac{d(\%C)}{dt} \right] \quad (32)$$

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