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A dynamo driven by zonal jets at the upper surface: Applications to giant planets

Céline Guervilly^{a,b}, Philippe Cardin^a, Nathanaël Schaeffer^a

^a ISTERre, Université de Grenoble 1/CNRS, F-38041, Grenoble, France

^b Department of Applied Mathematics and Statistics, Baskin School of Engineering,
University of California, Santa Cruz, CA 95064, USA

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Abstract

We present a dynamo mechanism arising from the presence of barotropically unstable zonal jet currents in a rotating spherical shell. The shear instability of the zonal flow develops in the form of a global Rossby mode, whose azimuthal wavenumber depends on the width of the zonal jets. We obtain self-sustained magnetic fields at magnetic Reynolds numbers greater than 10^3 . We show that the propagation of the Rossby waves is crucial for dynamo action. The amplitude of the axisymmetric poloidal magnetic field depends on the wavenumber of the Rossby mode, and hence on the width of the zonal jets. We discuss the plausibility of this dynamo mechanism for generating the magnetic field of the giant planets. Our results suggest a possible link between the topology of the magnetic field and the profile of the zonal winds observed at the surface of the giant planets. For narrow Jupiter-like jets, the poloidal magnetic field is dominated by an axial dipole whereas for wide Neptune-like jets, the axisymmetric poloidal field is weak.

1 Introduction

The zonal (*i.e.* axisymmetric and azimuthally directed) jet streams visible at the surface of the giant planets are a persistent feature of the fluid dynamics of these planets (figure 1). The gas giants (Jupiter and Saturn) display a strong eastward equatorial jet, extending to latitudes $\pm 20^\circ$ with a peak velocity exceeding 100 m/s on Jupiter (Porco et al., 2003), and to latitudes $\pm 30^\circ$ with a peak velocity exceeding 400 m/s on Saturn (Sanchez-Lavega

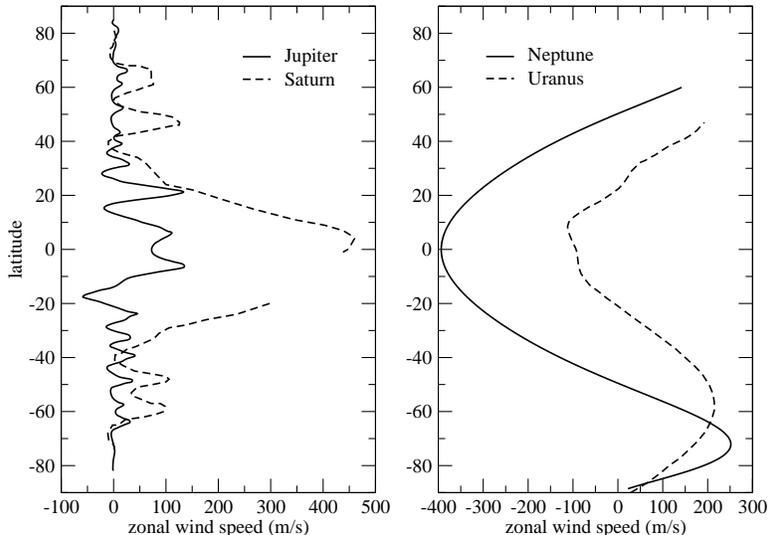


Figure 1: Zonal velocity measured at the surface in the planet’s mean rotating frame for each of the four giants by tracking cloud features in the outer weather layer. Profiles adapted from [Porco et al. \(2003\)](#), [Sanchez-Lavega et al. \(2000\)](#), [Sromovsky et al. \(2001\)](#) and [Sromovsky and Fry \(2005\)](#).

[et al., 2000](#)). At higher latitudes, alternating prograde (eastward) and retrograde (westward) jets of smaller amplitude are observed extending all the way to the poles. These profiles are fairly symmetric with respect to the equator. On the ice giants (Uranus and Neptune) the picture is rather different. A very intense retrograde equatorial current is present with maximum velocity of 100 m/s on Uranus ([Sromovsky and Fry, 2005](#)) and 400 m/s on Neptune ([Sromovsky et al., 2001](#)). At higher latitudes, a single prograde jet of large amplitude is present in each hemisphere. Several decades of observations show that these zonal flows remain approximately steady ([Porco et al., 2003](#)).

The origin of these zonal flows and the associated question of the depth to which they extend into the planets’ interiors have been areas of active research in rotating fluid dynamics for several decades (*e.g.* [Jones and Kuzanyan, 2009](#), and references therein; see also the review by [Vasavada and Showman, 2005](#)). In particular, several models have been proposed to explain the zonal wind pattern of Jupiter, and can be categorized into two main classes: weather layer models and deep convective layer models. The former assume that the zonal flows are produced in a shallow stably stratified region

near cloud level. These models are able to reproduce the high latitude structures with alternating eastward and westward jets and a strong equatorial current (*e.g.* Williams, 1978; Cho and Polvani, 1996). These models tend to produce a retrograde equatorial jet (Yano et al., 2003), so they provide a plausible explanation for the retrograde equatorial flow of the ice giants but not for the prograde flow observed on gas giants. A parametrized forcing such as a strong equatorially-localized baroclinicity is required to force a shallow system to produce a prograde equatorial jet (Williams, 2003). The second class of models is deep convection models which simulate most or all of the whole 10^4 km-thick molecular hydrogen layer (Busse, 1976; Christensen, 2001, 2002; Manneville and Olson, 1996). The presence of deep convection is inferred from the observation that the atmospheres of the major planets emit more energy by long-wave radiation than they absorb from the Sun. Consequently their atmospheres must receive additional heat supplied by the interior of the planet. Recent numerical models using either a Boussinesq approximation (Heimpel et al., 2005) or an anelastic approximation (Jones and Kuzanyan, 2009) and low Ekman numbers (*i.e.* strong rotational effect compared with viscous dissipation) display alternating zonal jets at high latitudes. A strong eastward equatorial jet is a robust feature of these models where the Coriolis force dominates buoyancy, in good agreement with the gas giant observations. Interestingly, deep convection models suggest that the zonal velocity generated by non-linear interactions of convective motions (*i.e.* the motions directly forced by buoyancy) is roughly geostrophic, that is, invariant along the direction of the rotation axis. This feature is also present in strongly compressible models provided that the Ekman number is small enough, despite the increase of density with depth yielding ageostrophic convective motions (Jones and Kuzanyan, 2009; Kaspi et al., 2009). When the convection is more vigorous such that the buoyancy force overcomes the Coriolis force, 3D turbulence homogenizes angular momentum; a retrograde jet forms in the equatorial region and a single strong prograde jet forms in the polar region, in good agreement with the ice giant observations (Aurnou et al., 2007).

Another feature of the giant planets is their strong magnetic fields (figure 2). The observed magnetic fields for gas and ice giants differ drastically (see for instance the recent review by Russell and Dougherty, 2010). Jupiter and Saturn have a main axial dipole component (corresponding to $l = 1$, $m = 0$ in figure 2), a feature shared with the Earth for instance (Yu et al., 2010; Burton et al., 2009). Neptune and Uranus, on the other hand, have strong non-axial multipolar components (corresponding to $l = 2, 3$ in figure 2) compared with the axial dipole component (Connerney et al., 1991;

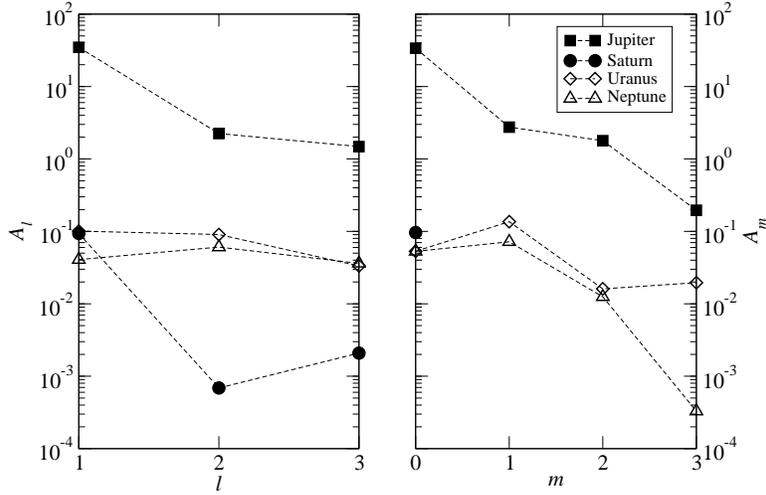


Figure 2: Spectra of the magnetic field squared amplitude at the planetary radius for degrees l and order m up to 3 obtained from inversion models of the magnetic measurements. The squared amplitude for a given degree l is $A_l = \sum_{m=0}^l (l+1) [(g_l^m)^2 + (h_l^m)^2]$ using a Schmidt normalisation for the spherical harmonics. The squared amplitude for a given mode m is $A_m = \sum_{l=m}^{l_{max}} (l+1) [(g_l^m)^2 + (h_l^m)^2]$. g_l^m and h_l^m are the Gauss coefficients in gauss. After [Yu et al. \(2010\)](#) (Model Galileo 15), [Burton et al. \(2009\)](#) (Cassini measurements), [Connerney et al. \(1991\)](#) (model O₈) and [Herbert \(2009\)](#) (AH₅ model from magnetic observations and auroral data).

Herbert, 2009). The magnetic field is generated in the deep, electrically conducting regions of the planets' interiors: a metallic hydrogen layer for Jupiter and Saturn (Nellis et al., 1999; Guillot, 2005, and references therein) and an electrolyte layer composed of water, methane and ammonia (Hubbard et al., 1991; Nellis et al., 1997) or superionic water (Redmer et al., 2011) for Uranus and Neptune.

Numerical models of convective dynamos in rapidly rotating spherical shells typically produce axial dipolar dominated magnetic fields for moderate Rayleigh numbers and moderate Ekman numbers (*e.g.* Olson et al., 1999; Aubert and Wicht, 2004; Christensen and Wicht, 2007). To explain the unusual large scale non-dipolar magnetic fields of Uranus and Neptune, models using peculiar parameter regimes or different convective region geometries have been proposed. The latter models show that a numerical dynamo operating in a thin shell surrounding a stably-stratified fluid interior produces magnetic field morphologies similar to those of Uranus and Neptune (Hubbard et al., 1995; Holme and Bloxham, 1996; Stanley and Bloxham, 2006). Gómez-Pérez and Heimpel (2007) obtain weakly dipolar and strongly tilted dynamo magnetic fields when high magnetic diffusivities are used (or equivalently small electrical conductivity). Their results show that these peculiar fields are stable in the presence of strong zonal circulation and when the flow has a dominant effect over the magnetic fields. This feature is also emphasized by Aubert and Wicht (2004) who find stable equatorial dipole solutions with a weak magnetic field strength and low Elsasser number (measure of the relative importance of the Lorentz and Coriolis forces) for moderately low Ekman numbers. They argue that the magnetic field geometry of the equatorial dipole solution is incompatible with the columnar convective motions and thus this morphology is stable only when Lorentz forces are weak.

Although scaling laws derived from numerical simulations of dynamos driven by basal heating convection predict dipolar magnetic field in planetary parameter regimes (Olson and Christensen, 2006), recent numerical simulations using more realistic parameter values (lower Ekman numbers) have not produced large scale magnetic fields so far, and require larger magnetic Reynolds numbers (measure of magnetic induction versus magnetic diffusion) (Kageyama et al., 2008). Moreover, convection in the interior of Jupiter is often thought to be driven by secular cooling (Stevenson, 2003). Numerical dynamos driven by secular cooling typically produce weak dipole or multipolar magnetic field for larger forcing (Kutzner and Christensen, 2000; Olson and Christensen, 2006) depending on boundary conditions (Hori et al., 2010). Therefore the question of the generation of large scale magnetic field by turbulent convective motions in the planetary parameter regime re-

mains open.

The dichotomies observed in the magnetic fields and in the zonal wind profiles of the giant planets are rather striking. Up to now no study has tried to relate them directly, probably because the former is a feature of the deep interior whereas the latter is a characteristic of the surface. However, if some mechanism is able to transport angular momentum from the surface down to the deep, fully conducting region then the zonal motions may influence the generation of the magnetic field. In the non-magnetic deep convection models (Heimpel et al., 2005; Jones and Kuzanyan, 2009), zonal motions extend geostrophically throughout the electrically insulating molecular hydrogen layer down to the bottom of the model. On the other hand, due to the possible rapid increase of electrical conductivity with depth in the outer region, Liu et al. (2008) argued that the ohmic dissipation produced by geostrophic zonal motions shearing dipolar magnetic field lines would exceed the luminosity measured at the surface of Jupiter if the vertical extent of this geostrophic zonal motions exceeds 4% of the planet radius. However, the argument of Liu et al. (2008) is purely kinematic, that is the action of the magnetic forces on the flow and the feedback on the magnetic field are ignored. In a self-consistent magnetohydrodynamic model, the zonal flow would adjust toward a non-geostrophic state due to the action of magnetic forces if the electrical conductivity of the fluid is significant (Glatzmaier (2008), see also the non-linear numerical simulations of convectively-driven dynamos of Aubert (2005)). In this case, angular momentum may be transported along the magnetic field lines leading to a dynamical state close to the Ferraro state. This state minimizes the ohmic dissipation produced by the shearing of the poloidal magnetic field by the zonal flow as the poloidal magnetic field lines are aligned with angular velocity contours. Both scenarios, either geostrophic zonal balance or Ferraro state, imply the existence of multiple zonal jets of significant amplitude at the top of the fully conducting region beneath. The plausibility of each scenario depends on the radial profile of electrical conductivity, which is currently not well constrained within the giant planets (Nellis et al., 1999).

The idea of the work presented in this paper is that these zonal jets may exert, by viscous or electromagnetic coupling, an external forcing at the top of the deeper conducting envelope. From previous studies (Schaeffer and Cardin, 2006; Guervilly and Cardin, 2010) we know that the viscous coupling between a differentially rotating boundary and a low-viscosity electrically conducting fluid can generate a self-sustained magnetic field in different geometries. Zonal motions can be subject to barotropic shear instabilities which have a lengthscale independent of the viscosity, unlike convective in-

stabilities. These instabilities are able to generate large scale magnetic fields, and so they are an interesting source of dynamo action under planetary interior conditions. In order to test the plausibility of a dynamo driven by this source in isolation, we use an incompressible 3D numerical dynamo model with a zonal velocity profile imposed at the top of a spherical shell containing a conducting fluid. We use a dynamical approach, that is non-linear interactions between the flow and the magnetic field are taken into account; therefore the fluid flow is free to adopt a three-dimensional structure as long as it satisfies the imposed viscous boundary conditions.

The dynamics of the deep conducting region is usually assumed to be slower than the dynamics of the outer molecular hydrogen region due to magnetic braking, even if uncertainties remain in the electrical conductivity. The model presented in this paper assumes an idealized one-way coupling between the outer and deep regions. A more realistic model would need to account for the back reaction of the deep layer onto the outer layer; a study of the consistent dynamical interaction of the two layers is beyond the scope of this paper. For studies of more realistic coupling, see promising recent numerical models of self-consistent convectively-driven dynamos in spherical shells including radially variable electrical conductivity of [Heimpel and Gómez Pérez \(2011\)](#) and [Stanley and Glatzmaier \(2010\)](#). In these models, slow convective motions in the interior dynamo region coexist with strong zonal flow near the outer surface. Differential rotation in the interior is only partially inhibited by the strong magnetic field.

In order to assess the role of the zonal wind profile on the topology of the sustained magnetic field, we use both Jupiter-like and Neptune-like zonal wind profiles. In the giant planets, as in rocky planets, it is usually assumed that the dynamo mechanism is driven by convective motions. The giant planets display a strong surface heat flux (with the exception of Uranus) meaning that heat transfer is efficient in the interior of the planet and thus mostly due to convection ([Guillot and Gautier, 2007](#), and references therein). Here we want to assess the efficiency of zonal velocity forcing alone, so we do not model convective motions.

The first goal of this work is to quantify what amplitude of the zonal wind *inside* the conducting layer is needed to trigger the dynamo instability, so we do not model the exact or realistic coupling between the molecular hydrogen upper layer and the deep, electrically conducting region. Our second goal is to test to what extent the pattern of the zonal flow imposed at the top of the conducting layer influences the topology of the self-sustained magnetic field.

We first describe the model and the numerical method used (section 2).

Then we present numerical results from simulations in the non-magnetic case (section 3) followed by results from dynamo simulations (section 4). The application of our results to planetary conditions is discussed in section 5.

2 Model

We model the deep conducting layer of the giant planets as a thick spherical shell. At the top of the conducting layer we impose an axisymmetric azimuthal velocity to represent the zonal flow generated in the overlying envelope. The shell rotates around the z -axis at the imposed rotation rate Ω . The aspect ratio is $\gamma = r_i/r_o$ where r_i is the inner sphere radius, corresponding to a rocky core, and r_o the outer sphere radius, corresponding to the top of the fully conducting region. The fluid is assumed incompressible with constant density ρ and constant temperature, that is, no convective motions are computed. The assumption of incompressibility is made for simplicity, although the pressure scale height at the depths of the conducting layer is roughly 8000km (Guillot et al., 2004), that is, about 1/5 of the thickness of the layer. The effects of compressibility may well play a role in the dynamics of the conducting regions (see for instance Evonuk and Glatzmaier, 2004).

For simplicity we model the angular momentum coupling with the external zonal flow as a rigid boundary condition for the velocity at the outer boundary, rather than as a shear stress condition. The flow is driven through a boundary forcing rather than a volume forcing to avoid directly imposing bidimensionality to the velocity field. As we are interested in the bulk magnetohydrodynamical process, the exact nature of the coupling (electromagnetic or viscous, shear stress or rigid) with the upper molecular hydrogen layer is not crucial for our study. We discuss the implication of the choice of the rigid boundary condition in section 3. The radial profile of electrical conductivity is not well constrained in the gas giants. In particular the existence of a first order or continuous transition between the molecular and metallic hydrogen phase is still an open question, although high-pressure experiments are in favor of a continuous transition (Nellis et al., 1999). We choose to model the outer boundary as electrically insulating to simplify the coupling between the layers. The conductivity is assumed constant throughout the whole modeled conducting layer. As we do not model the molecular hydrogen layer, we assume zonal geostrophic balance within this envelope for simplicity. The amplitude of the zonal motions at the outer boundary of our model is therefore the same as the surface winds. This idealized representation of the dynamics of the molecular hydrogen layer would be altered

if the magnetic forces upset the zonal geostrophic balance. Depending on the magnitude and radial profile of the electrical conductivity, the amplitude of the zonal motions might be reduced, and the zonal flow contours would tend to align with the magnetic field lines, although we do not expect the characteristics of the zonal jets (narrow or wide, relative amplitude of the peaks) to be altered very much.

We use two different synthetic azimuthal velocity profiles for the boundary forcing imposed at the top: a multiple jet profile for the gas giants with a profile based on Jupiter’s surface zonal winds (hereafter profile J) and a 3-band profile based on Neptune’s surface zonal winds (profile N).

For Jupiter, we use the profile given in [Wicht et al. \(2002\)](#)

$$\mathbf{U} = U(s)\mathbf{e}_\phi = U_0 \frac{s}{r_0 \cos(n_0\pi)} \cos\left(n_0\pi \frac{s-r_0}{r_s-r_0}\right) \mathbf{e}_\phi, \quad (1)$$

where $s = r \sin \theta$, r_s is the surface radius of the planet and $U_0 = U(r_0, \theta = \pi/2)$. n_0 controls the numbers of jets. The profile at the radius r_s best matches the observed profile at the surface for $n_0 = 4$ (figure 3). The profile $U(r_o, \theta)$ is used to drive the flow at the top of our simulated metallic hydrogen layer (figure 3). The ratio $\gamma_s = r_s/r_o$ determines the U profile at r_o . We choose $\gamma_s = r_s/r_o = 1/0.8 = 1.25$ following [Guillot et al. \(1994\)](#).

For the Neptune-like profile, we use the zonal velocity profile measured at the surface of Neptune, approximated by a polynomial of order 10 in latitude. We project this surface velocity profile geostrophically down to r_o using $\gamma_s = 1/0.85 = 1.18$ ([Hubbard et al., 1991](#)) (figure 3).

The existence of a rocky core at the centre of the giant planets is uncertain and depends on the poorly constrained composition of the planet. Estimates for the core mass are $0 - 14m_\oplus$ for Jupiter (total mass $318m_\oplus$), $6 - 17m_\oplus$ for Saturn (total mass $95m_\oplus$) and $0 - 4m_\oplus$ for Uranus and Neptune (total mass $15m_\oplus$ and $17m_\oplus$ respectively) where m_\oplus denotes the mass of the Earth ([Guillot, 2005](#)). If present, the rocky cores are therefore believed to be small. Following the interior model of Jupiter proposed by [Guillot et al. \(1994\)](#) we use an aspect ratio $r_i/r_o = 0.2$ for all the simulations performed. The inner core is assumed to be electrically conducting, with the same conductivity as the fluid in the conducting layer. We did not carry out simulations with an insulating core as the effect of the conductivity of the inner core on the dynamo mechanism is believed to be small ([Wicht, 2002](#)). The velocity boundary condition is no-slip at the inner boundary.

The velocity \mathbf{u} is scaled by U_0 , the absolute value of the azimuthal velocity imposed at the equator of the outer sphere. The lengthscale is the radius of the outer sphere r_o . The magnetic field \mathbf{B} is scaled by $\sqrt{\rho\mu_0 r_o \Omega U_0}$

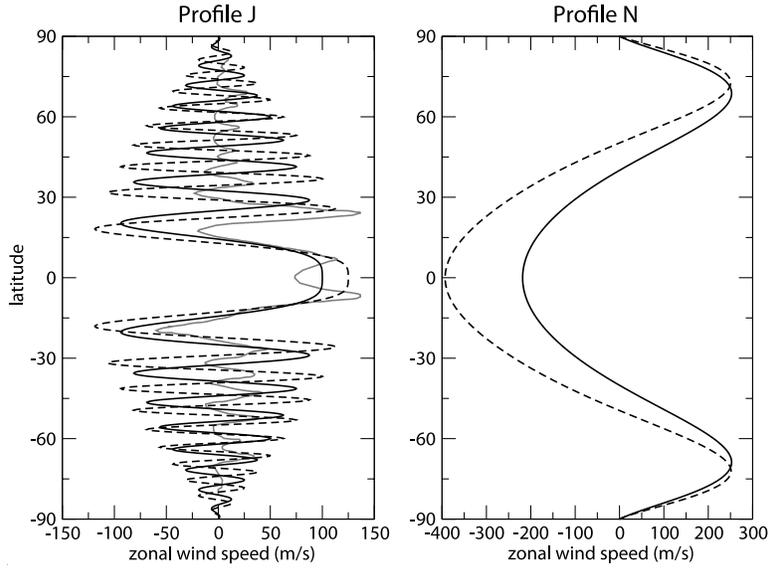


Figure 3: Zonal velocity profile imposed at the surface of model J (left) and model N (right) (solid lines). Both profiles are obtained by assuming that the zonal velocities are geostrophic for $r_s > r > r_o$ and using the profile represented by a dashed line at the surface of the planet ($r = r_s$): model J, profile (1) with $n_0 = 4$, $\gamma_s = r_s/r_o = 1.25$ and $U_0 = 100$; model N: polynomial fit of order 10 in latitude of the zonal wind profile measured at the surface of Neptune (figure 1) with $\gamma_s = 1/0.85 = 1.18$. For comparison the zonal wind profile measured at the surface of Jupiter is plotted in gray.

where ρ is the fluid density and μ_0 is the vacuum magnetic permeability. We numerically solve the momentum equation for an incompressible fluid,

$$Re \frac{\partial \mathbf{u}}{\partial t} + Re (\mathbf{u} \cdot \nabla) \mathbf{u} + \frac{2}{E} \mathbf{e}_z \times \mathbf{u} = -\nabla p + \nabla^2 \mathbf{u} + \frac{1}{E} (\nabla \times \mathbf{B}) \times \mathbf{B}, \quad (2)$$

the continuity equation,

$$\nabla \cdot \mathbf{u} = 0, \quad (3)$$

and the magnetic induction equation,

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B}) + \frac{1}{RePm} \nabla^2 \mathbf{B}, \quad (4)$$

$$\nabla \cdot \mathbf{B} = 0, \quad (5)$$

where p is the dimensionless pressure, which includes the centrifugal potential.

The Reynolds number $Re = r_o U_0 / \nu$ parametrizes the mechanical forcing exerted on the system by controlling the amplitude of the zonal velocity. The magnetic Prandtl number $Pm = \nu / \eta$ measures the ratio of viscous to magnetic diffusivities. The magnetic Reynolds number Rm is defined as $Rm = RePm$. The Ekman number $E = \nu / (\Omega r_o^2)$ measures the importance of the viscous term over the Coriolis force. The Rossby number $Ro = ReE = U_0 / (\Omega r_o)$ is the ratio of inertial force to Coriolis force. Note that in our definition the Rossby number refers to the amplitude of the prescribed zonal jets at the surface, and not to the local flow velocity.

The results presented in this paper were obtained with the PARODY code, a fully three-dimensional and non-linear code. The code was derived from [Dormy \(1997\)](#) by J. Aubert, P. Cardin, E. Dormy in the dynamo benchmark ([Christensen et al., 2001](#)), and parallelised and optimised by J. Aubert and E. Dormy. The velocity and magnetic fields are decomposed into poloidal and toroidal scalars and expanded in spherical harmonic functions in the angular coordinates with l representing the latitudinal degree and m the azimuthal order. A finite difference scheme is used on an irregular radial grid (finer near the boundaries to resolve the boundary layers). A Crank-Nicolson scheme is implemented for the time integration of the diffusion terms and an Adams-Bashforth procedure is used for the other terms.

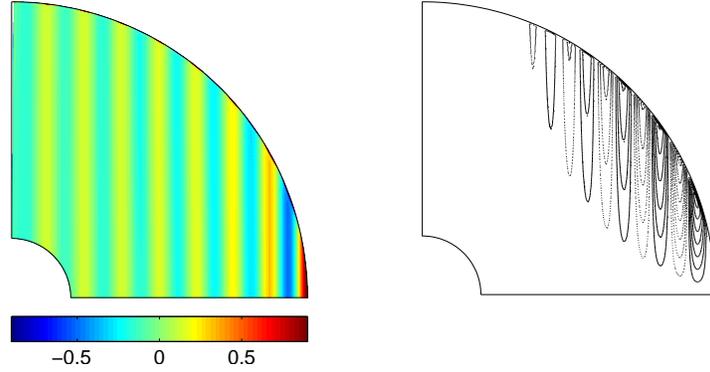
3 Dynamics without the magnetic field

For a rapidly rotating system in which the Coriolis force exactly balances the pressure force, the Proudman-Taylor constraint states that the flow is z -invariant and follows geostrophic contours. For an incompressible fluid in a bounded container, these geostrophic contours correspond to surfaces of equal height. In a sphere the only geostrophic motions are azimuthal and axisymmetric. In the giant planets' conducting envelopes, the Ekman number is about 10^{-16} and the Rossby number is much smaller than 1 (Guillot et al., 2004). In the absence of a magnetic field, we expect the Proudman-Taylor constraint to hold for large scale motions. As we want to reach the dynamical regime in which the flow is strongly geostrophic, the use of small Ekman and Rossby numbers is required. We carried out simulations for $10^{-5} > E > 10^{-6}$ for model J and $10^{-5} > E > 5 \times 10^{-6}$ for model N. The Rossby numbers are always smaller than 0.1. For the profile J, in cases of low Ekman numbers ($E \leq 2 \times 10^{-6}$), we imposed longitudinal symmetry by calculating only the harmonics of a chosen order m_s . The required resolution for $E = 10^{-6}$ is 500 points on the radial grid and $l = 580$ spherical harmonics degrees.

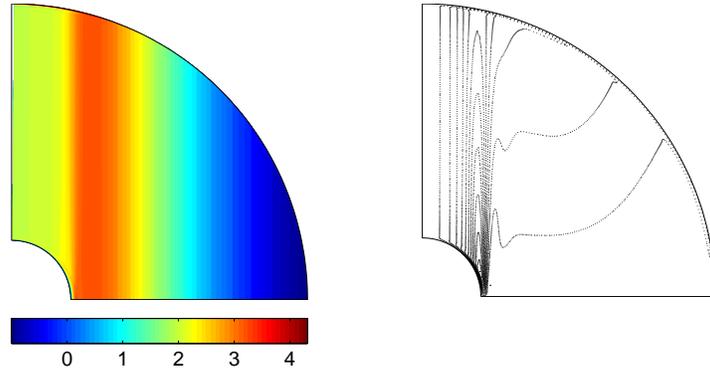
3.1 Axisymmetric flow

When the imposed boundary forcing is small enough, *i.e.* when the Rossby number Ro is less than a critical value Ro_c , the flow is axisymmetric and predominantly azimuthal (figure 4). The zonal jets imposed at the outer boundary extend into the volume along lines parallel to the axis of rotation.

The use of no-slip boundary conditions yields a differential rotation between the boundary and the bulk of the fluid. This differential rotation is accommodated across viscous Ekman boundary layers, which scale as $(E/\cos\theta)^{1/2}$, where θ is the colatitude. By Ekman pumping, viscous forces within the Ekman layers drive axial motions of order $E^{1/2}$ within the bulk of the fluid (figure 4). These meridional circulations advect angular momentum from the boundary layer into the bulk of the fluid and cause the jets to propagate faster than by pure viscous diffusion. At low latitudes, the Ekman layer is thicker so the Ekman pumping is stronger, yielding to a more efficient driving of the zonal motions in the bulk by the outer boundary layer. For model J (figure 5(a)), the zonal velocity in the bulk relative to that imposed at the outer boundary is noticeably weaker for the inner jets than for the outer jets. When E decreases this effect is less marked, and in the $E \rightarrow 0$ limit we expect the basic zonal velocity to be perfectly geostrophic



(a) Model J



(b) Model N

Figure 4: Angular velocity $u_\phi/(r \sin \theta)$ (left) and streamlines of the meridional circulation (isocontours of $\psi = r \sin \theta \frac{\partial u_p}{\partial \theta}$ with u_p the velocity poloidal scalar) (right) of the axisymmetric flow in the northern meridional plane. For the meridional circulation, anti-clockwise (clockwise) flows are shown in solid (dotted) lines. The parameter for the simulations are $E = 5 \times 10^{-6}$ and $Ro = 0.015$ for model J (a) and $E = 10^{-5}$ and $Ro = 0.02$ for model N (b).

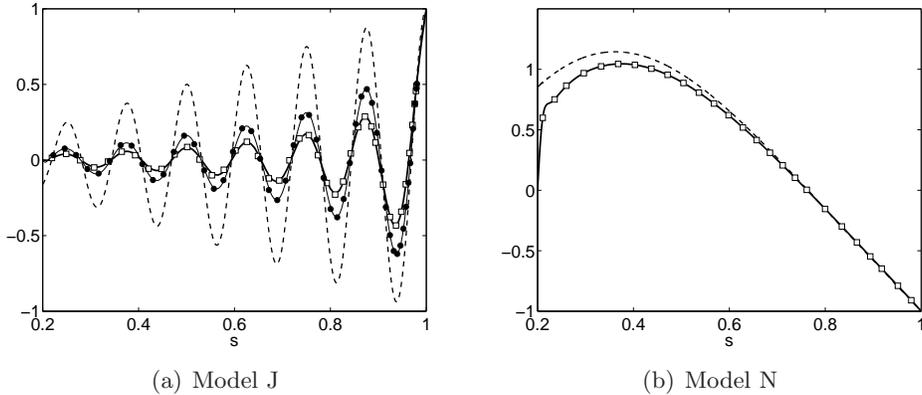


Figure 5: Zonal velocity in the equatorial plane for subcritical numerical simulations (solid lines) compared to the zonal velocity at radius $r = 0.98r_o$ (symbols) and imposed velocity at the top (dashed line) both projected in the equatorial plane for (a) model J ($E = 5 \times 10^{-6}$ (bold solid line and open squares) and $E = 10^{-6}$ (thin solid line and black circles)) and (b) model N ($E = 10^{-5}$ (bold solid line and open squares)).

in the whole volume. The comparison between the zonal velocity just below the Ekman layer and in the equatorial plane (figure 5) shows that the zonal velocity is geostrophic in the bulk of the fluid (outside of the boundary layers). For model N (figure 5(b)), the zonal jets are wider, so the zonal flow already displays a strong geostrophic structure at $E = 10^{-5}$. Note that the azimuthal velocity has to match the no-slip boundary condition at the inner core, and so an internal Stewartson layer forms on the axial cylinder tangent to the inner core (Stewartson, 1966).

3.2 Non-axisymmetric motions

3.2.1 Model J

Rossby wave at the onset When the boundary forcing (measured by Ro) becomes greater than a critical value Ro_c , the axisymmetric basic flow becomes unstable to a non-axisymmetric shear instability. The saturated instability takes the form of an azimuthal necklace of cyclonic and anticyclonic vortices aligned with the axis of rotation, is nearly z -independent and drifts eastward (figure 6). Close to the threshold, the radial extension of the pattern is large and occupies almost half of the gap. The pattern drifts with the same speed over its whole radial extension, even though the advection

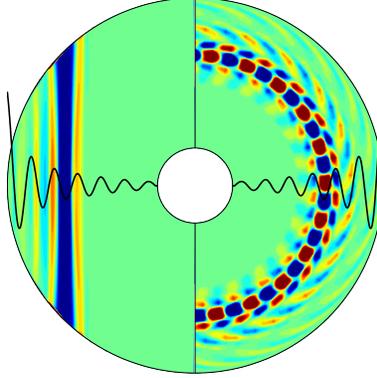


Figure 6: Non-zonal axial vorticity in the equatorial plane (right) and in a meridional slice (left) for model J at $E = 4 \times 10^{-6}$ and $Ro = 1.01Ro_c$ (blue: negative and red: positive). The black curve represents the zonal velocity in the equatorial plane.

by the zonal flow velocity varies with s , implying that it is a single wave.

Wicht et al. (2002) studied the linear stability of the imposed zonal flow (1) in a spherical shell modeling the insulating molecular hydrogen layer of Jupiter (aspect ratio 0.8). For $E = 10^{-4}$ they found nearly bidimensional instabilities that they described as drifting columns aligned with the rotation axis and similar to convective solutions. Although they do not identify these instabilities as waves, their characteristics are very similar to the ones obtained with our non-linear model.

The nearly z -invariant structure and the prograde drift are two characteristics of Rossby waves propagating in a spherical container. The dispersion relation for the Rossby wave given by a local linear analysis is (*e.g.* Finlay, 2008)

$$\omega_{rw}(s) = -2\Omega\beta \frac{m/s}{k_s^2 + (m/s)^2}, \quad (6)$$

where $\beta = h^{-1}(dh/ds) = -s/(r_o^2 - s^2)$ is related to the slope of the upper boundary of the spherical container of height h . k_s and m/s are the radial and azimuthal wavenumbers respectively. The theoretical Rossby wave frequency ω_{rw} can be calculated at a given radius assuming $k_s \approx m/s$ and using the wavenumber m obtained from the numerical simulation. For different E , the frequency ω of the propagating wave observed in our numerical simulations always falls in the range $\omega_{rw}(s_1) < \omega < \omega_{rw}(s_2)$ where $s_1 = 0.56$

($s_2 = 0.87$) is the smallest (resp. largest) radius where a significant vorticity associated with the presence of the wave can be seen in the numerical calculations. This strongly indicates that the shear instability occurs as a Rossby wave.

The velocity of the zonal flow U enters the dispersion relation of the Rossby wave through a Doppler shift

$$\omega(s) = \omega_{rw}(s) + U(s) \frac{m}{s}. \quad (7)$$

As reported earlier, $\omega(s)$ is constant in our numerical calculations so $\omega_{rw}(s)$ must adapt in the s -direction for the wave to be coherent. In a prograde jet $U > 0$, ω_{rw} must decrease, which requires a local increase in k_s in equation (6) and so a local decrease in the radial lengthscale, which can be observed in figure 6. For small enough Ekman number (in practice $E < 5 \times 10^{-6}$), the critical wavenumber m_c of the Rossby mode is independent of E . The radial lengthscale is determined by the width of the jet and the vortices are roughly circular in the equatorial plane (figure 6) suggesting that m_c is controlled by the width of the jets.

In a local approximation that neglects the curvature terms, a criterion of instability of barotropic shear flows has been derived by [Ingersoll and Pollard \(1982\)](#) for an anelastic model in a full rotating sphere and by [Kuo \(1949\)](#) for thin stably stratified “weather” layers. Using an inviscid Boussinesq model and for barotropic instability of a zonal flow U in a sphere, this necessary condition implies a change of sign of a quantity Δ at some radius:

$$\Delta = 2\beta - Ro \frac{d\zeta}{ds}, \quad (8)$$

where ζ is the vorticity of the zonal flow,

$$\zeta = \frac{dU}{ds} + \frac{U}{s}. \quad (9)$$

Note that the curvature terms have been taken into account here. In a sphere, β is negative. Consequently, the zonal velocity profile is more prone to instability where the gradient of zonal vorticity is maximum and negative. Then for a profile U of sinusoidal form, the first shear instability occurs at the maximum of the prograde jets, and thus, perhaps surprisingly, at a null value of the zonal velocity shear dU/ds . Note that our numerical simulations show instabilities with a large radial extent and with maximum amplitude located in a retrograde zonal jet (see figure 6), even though the local instability criterion predicts an onset in a prograde jet. This observation emphasizes

that the local criterion does not predict the location of global saturated modes.

The theoretical critical Rossby number obtained from applying the criterion (8) to the profile (1) imposed at the top of model is $Ro_c^{th} = 0.0011$. The threshold of the first instability of the axisymmetric flow, denoted Ro_c^{nlin} , obtained with the numerical simulations are shown in figure 7. Despite the decrease of Ro_c^{nlin} with the Ekman number, Ro_c^{nlin} is still about four times larger than Ro_c^{th} for $E = 10^{-6}$ because the amplitude of the zonal flow within the bulk is reduced by viscous boundary layers in the numerical simulations. Due to computational limitations, it is not possible for us to carry out simulations at smaller E with a fully non-linear code and prove the existence of an asymptotic regime for the inviscid instability threshold. For this purpose we used a dedicated linear code described in A. The linear code calculates linear perturbation solutions to the momentum equation using the geostrophic profile U as the basic flow in the bulk of the fluid. The computational time is greatly reduced by the linear approach but is restricted to an analysis of the instability threshold. The growing solutions obtained with the linear code exhibit very similar features to the Rossby waves in the non-linear simulations (frequency, bidimensional structure, radial extent, location of the maximum amplitude in a retrograde jet). In figure 7 the threshold Ro_c^{lin} obtained with the linear code approaches asymptotically the value given by the local theory. For the same Ekman number, Ro_c^{lin} is smaller than Ro_c^{nlin} since the geostrophic zonal flow U is used in the linear code, that is the jets in the bulk have greater amplitude than in the non-linear code. From our linear computations we conclude that the theoretical criterion (8) is relevant to explain the onset of instability obtained numerically. More details about the onset of the hydrodynamic instability can be found in Guervilly (2010).

The characteristic time of the Rossby wave is $\tau_{rw} = 1/\omega$. At the instability threshold, the numerical simulations give $\tau_{rw} \approx 18\Omega^{-1}$ for $E < 5 \times 10^{-6}$. The timescale of the zonal jets is $\tau_{zj} = r_o/U_0 = \Omega^{-1}/Ro$. For $Ro = 0.01$, we have $\tau_{zj} > \tau_{rw}$: the Rossby wave propagation is faster than the advection of the fluid by the zonal flow. The turnover time of a fluid particle trapped in a Rossby wave is $\tau_{to} = l/V_s$ where l is the typical radial displacement of the particle and V_s the typical cylindrical radial velocity of the particle. At $Ro = 1.01Ro_c$, V_s is typically $10^{-2}U_0$. In a rough approximation we use $l = \delta$, where δ is the width of the jets, $\delta \approx 0.1r_o$ for the profile J. Then we obtain $\tau_{to} \approx 0.1r_o/(10^{-2}U_0) \approx 10Ro^{-1}\Omega^{-1} \approx 10^3\Omega^{-1}$: the turnover time of the particle is much longer than the timescale of the wave. Consequently the particle oscillates rapidly as the wave propagates and is slowly advected by the zonal flow. In practice the radial displacement l is typically smaller

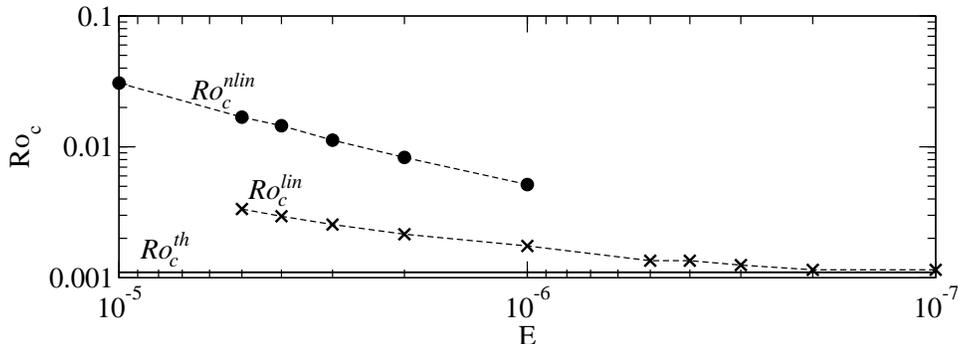


Figure 7: Critical Rossby number obtained from fully non-linear numerical simulations for model J (Ro_c^{nl} , circles) compared to the theoretical Rossby number obtained with the local instability criterion (8) using the geostrophic profile (1) $U(s, \theta = \pi/2)$ (Ro_c^{th} , black line). The critical Rossby number obtained from the linear numerical calculation is also shown (Ro_c^{lin} , crosses).

than δ and so the turnover time is slightly overestimated here.

Supercritical regime When the Rossby number is increased in the supercritical regime, other prograde jets will eventually become unstable. A second Rossby wave appears in the weakly supercritical regime, at $Ro = 1.06Ro_c$ for $E = 5 \times 10^{-6}$, with a maximum velocity located in the retrograde zonal jet at larger radius than the first wave maxima (*i.e.* the wave appearing for $Ro = Ro_c$) (figure 8(a)). To fill the larger circumference at larger radius the instability has a slightly larger wave number, $m = 22$ instead of 21, while the radial width of the jet is comparable. The second wave propagates faster, in agreement with the Rossby wave dispersion relation (6). Barotropic instabilities tend to broaden and weaken narrow jets by redistributing potential vorticity (see for instance Pedlosky, 1979). The smoothing of the jets saturates the amplitude of the Rossby waves. For this slightly supercritical regime the zonal flow profile is only weakly modified. Upon further increasing the forcing ($Ro = 2.94Ro_c$), several Rossby waves of different wavenumbers superpose and interact (figure 8(b)). The structure of the waves and the jets is still mainly bidimensional except in the viscous boundary layers. The typical cylindrical radial velocity is $V_s \approx 0.1U_0$ and the Rossby number is 0.05 so the turnover time is about $20\Omega^{-1}$ assuming that the radial displacement $l = \delta$, about the same order of magnitude as the timescale of the zonal jets.

In figure 9(a) the time-averaged zonal flow in the equatorial plane is

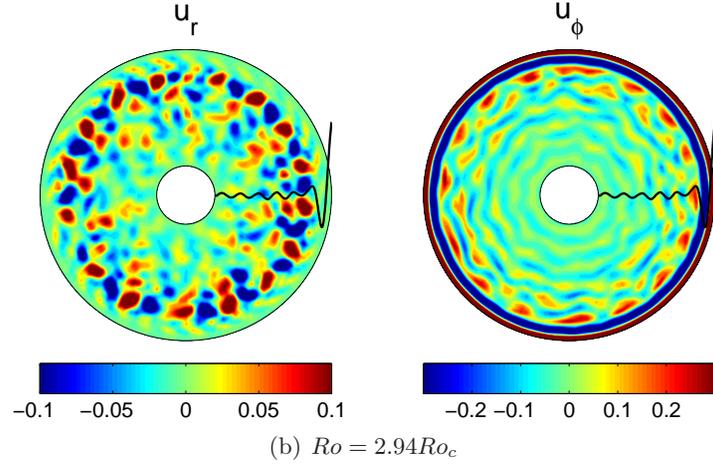
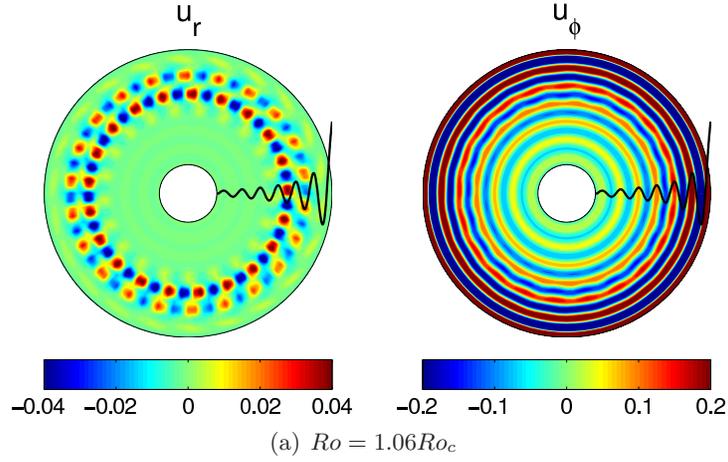


Figure 8: Snapshots of the radial (left) and azimuthal (right) velocity components in the equatorial plane for $E = 5 \times 10^{-6}$ and $Ro > Ro_c$ for model J. The velocities are scaled by U_0 . For u_ϕ the colorscale has been truncated ($u_\phi(r_o, \theta = \pi/2, \phi) = 1$). The black curve represents the zonal velocity in the equatorial plane.

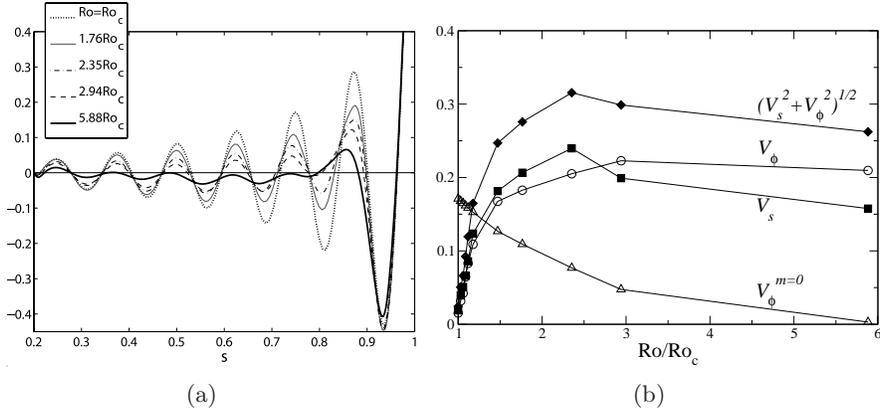


Figure 9: (a) Time-averaged zonal velocity in the equatorial plane for $E = 5 \times 10^{-6}$ and different forcings. (b) Amplitude of the non-axisymmetric radial velocity V_s (squares), non-axisymmetric azimuthal velocity V_ϕ (circles), non-axisymmetric velocity $(V_s^2 + V_\phi^2)^{1/2}$ (diamonds) and zonal velocity at the radius $s = 0.75$ (triangles). All velocities were measured in the equatorial plane in the units of U_0 . The amplitude of the non-axisymmetric velocity corresponds to the maximum in a snapshot, whereas the zonal velocity amplitude has been averaged in time.

plotted for different Ro up to $Ro = 5.88Ro_c$. As the forcing is increased, the Rossby waves gradually reduce the jet strength and broaden the jet width. For $Ro = 2.94Ro_c$, the retrograde jet at $s = 0.81$ has been mostly destroyed leading to the widening of the zonal jet width. We note that the zonal flow becomes mostly westward for the strongest forcings. The amplitude of the zonal flow located at $s > 0.9$ is hardly affected because the threshold to destabilise the outermost jets is high due to the large slope (related to β in equation (8)). For $Ro < 2.35Ro_c$, the amplitude of the non-axisymmetric velocity, relative to U_0 , increases with the forcing (figure 9(b)). After reaching a maximum, at $Ro = 2.35Ro_c$, the amplitude of the non-axisymmetric flow decreases relative to U_0 . The “efficiency” of the forcing to drive the non-zonal velocity is reduced as the Rossby waves smooth the gradient of vorticity and so affect their excitation mechanism.

The back reaction on the forcing velocity in the upper molecular hydrogen layer is not taken into account in our model although it might significantly affect the zonal profile in the upper layer in the case of strong forcing.

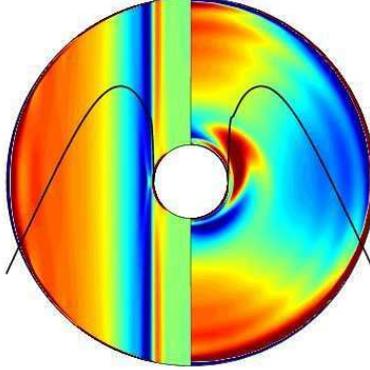


Figure 10: Non-zonal axial vorticity in the equatorial plane (right) and in a meridional slice (left) for model N at $E = 5 \times 10^{-6}$ and $Ro = 1.01Ro_c$ (blue: negative and red: positive). The black curve represents the zonal velocity in the equatorial plane.

3.2.2 Model N

The shear instability takes the form of an $m = 2$ oscillation in the azimuthal direction (figure 10). It is a single wave propagating eastward with the same frequency over the shell, and is nearly z -invariant. The maxima of the non-zonal vorticity are located on each side of the prograde jet. The characteristics of this wave are similar to the Rossby wave obtained with model J. The frequency of this wave is in agreement with the frequency of a theoretical Rossby wave of wavenumber $m = 2$ propagating at a radius $s = 0.53$ (assuming that $k_s \approx m/s$ in the dispersion relation (6)). For $E = 10^{-5}$ and $E = 5 \times 10^{-6}$, the critical Rossby numbers obtained with the non-linear numerical simulations are respectively $Ro_c^{nlin} = 0.0335$ and $Ro_c^{nlin} = 0.0325$. Using the instability criterion (8) with the profile imposed at the surface we obtain a critical Rossby number of 0.026 in good agreement with the non-linear numerical results when the Ekman number decreases.

4 Magnetic field generation

The non-axisymmetric motions are of prime importance for the dynamo mechanism because a purely toroidal flow cannot generate a self-sustained magnetic field. We note that some axisymmetric poloidal flow is present when $Ro < Ro_c$ as a weak meridional circulation is created by the Ekman pumping. However these axisymmetric motions are weak at small Ekman

numbers so we do not expect to find dynamos when the zonal flow is stable, that is when $Ro < Ro_c$, in the asymptotic inviscid regime. Indeed we did not find dynamos when $Ro < Ro_c$ (up to $Pm = 10$). The non-axisymmetry associated with the hydrodynamic shear instability is a crucial element for the dynamo process: the stable zonal flow cannot sustain a magnetic field by itself. This is in agreement with the results obtained by [Guervilly and Cardin \(2010\)](#) with dynamos generated by spherical Couette flows (differential rotation between two concentric spheres).

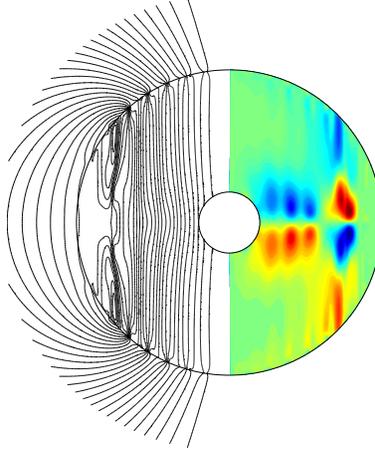
4.1 Characteristics of the magnetic field for model J

We have performed dynamo simulations for $Ro = 1.17 - 1.76Ro_c$ and $E = 5 \times 10^{-6}$. We find that the dynamo threshold occurs at a rather high value of the magnetic Prandtl number, $Pm_c \approx 5$. The critical magnetic Reynolds number (defined via the maximum forcing velocity) required for dynamo action is $Rm_c \approx 20,000$ (see section 5 for an estimate of the critical magnetic Reynolds number defined via the local velocity). For a given forcing, we have performed calculations just above the critical magnetic Prandtl number, Pm_c , and up to $2Pm_c$.

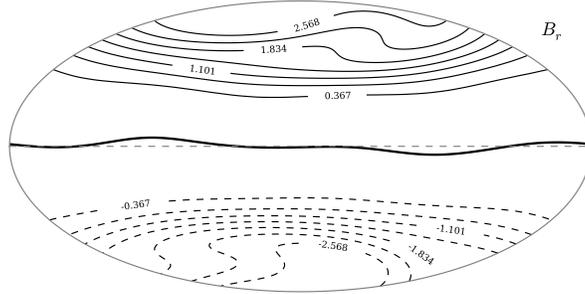
The main features of the self-sustained magnetic field can be observed in figures 11 and 12. The magnetic field displays a dipolar symmetry, *i.e.* antisymmetry with respect to the equatorial plane,

$$(B_r, B_\theta, B_\phi)(r, \pi - \theta, \phi) = (-B_r, B_\theta, -B_\phi)(r, \theta, \phi). \quad (10)$$

The magnetic field is predominantly toroidal and axisymmetric (corresponding to the mode $m = 0$ in figure 12(a)). The toroidal magnetic field does not emerge from the conducting region as the outer region is electrically insulating. The strongest poloidal component is the axial dipole within the conducting region and outside of the outer sphere (corresponding to the harmonic $(l, m) = (1, 0)$ in figure 12). Within the bulk of the flow, the axisymmetric poloidal magnetic field lines are mostly significantly bent where the Rossby wave causes a strong magnetic induction (figure 11(a)). A magnetic field at the scale of the Rossby wave is produced in this region as can be observed on the spectra of magnetic energy (figure 12(a)) with significant peaks at $m = 22$ in the poloidal and toroidal magnetic energies and at $l = 23$ in the poloidal magnetic energy ($l - m$ is odd to preserve the dipolar symmetry). Close to the outer boundary, the axisymmetric poloidal magnetic field lines converge and diverge locally (figure 11(a)). This is due to the induction of axisymmetric magnetic field by the secondary meridional circulation produced by Ekman pumping. This effect is very localized and

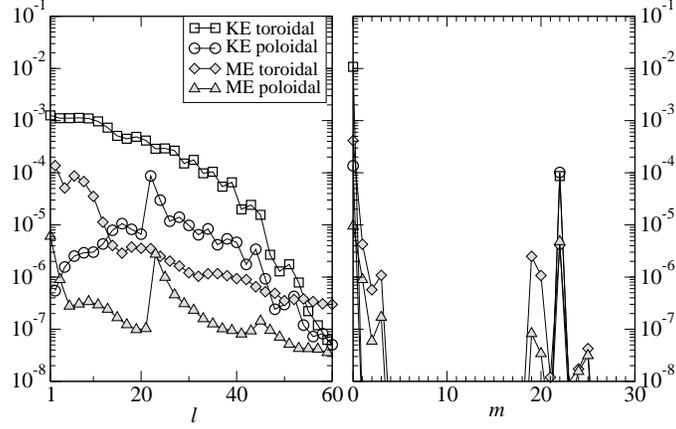


(a) Axisymmetric magnetic field

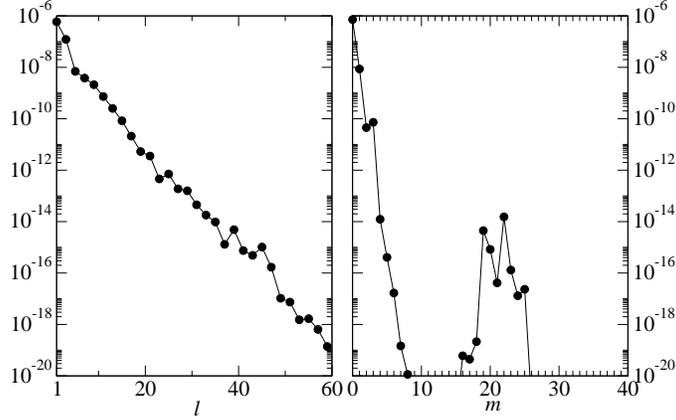


(b) Radial magnetic field at r_s

Figure 11: Magnetic field for model J. (a) Snapshot of the axisymmetric magnetic field in a meridional plane: magnetic poloidal field lines (left) and azimuthal magnetic field (right) (blue: negative and red: positive). (b) Map of the radial magnetic field at the surface of the planet $r_s = 1.25r_o$ in unit of $10^{-3}\sqrt{\rho\mu_0}U_0$ (solid line: positive and dotted line: negative). The poloidal magnetic field at $r = r_s$ is calculated assuming the region between r_o and r_s is electrically insulating. The parameters of this simulation are $E = 5 \times 10^{-6}$, $Ro = 1.17Ro_c$ and $Pm = 5 \approx Pm_c$.



(a) Kinetic and magnetic energies in the fluid



(b) A_l and A_m at r_s

Figure 12: Magnetic energy spectra for model J. (a): Kinetic (KE) and magnetic (ME) energy per unit volume for each spherical harmonics degree l (left) and mode m (right) in the fluid conducting region given in unit of ρU_0^2 . (b): Squared amplitudes of the magnetic field, A_l (left) and A_m (right) as defined in figure 2, at $r_s = 1.25r_o$ given in unit of $\rho\mu_0 U_0^2$. Only the degrees of significant amplitude have been plotted, that is, l even for the kinetic poloidal and magnetic toroidal energies and l odd for the kinetic toroidal and magnetic poloidal energies. These data are taken at a particular instant and have not been time-averaged. Same parameters than figure 11.

generates a magnetic field of small latitudinal scale that decreases rapidly with radius.

The spectrum and map of the radial magnetic field at the surface of our modeled planet (at radius $r_s = 1.25r_o$) (Figs. 11(b) and 12(b)) show that the magnetic field is strongly dominated by the axial dipole. The magnetic field generated at the scale of the Rossby wave ($m = 22$) is still visible in the spectrum of the magnetic field but its amplitude is weak at this radius: about four orders of magnitude smaller than the amplitude of the axisymmetric mode (note that the spectrum in figure 12(b) represents the squared amplitude of the field).

In all the simulations performed, no inversion of polarity of the axial dipole has been observed. The tilt of the dipole is rather weak, at most 2° from the rotation axis. We found a secular variation of the dipole axis of about 1° every 1000 rotation periods or alternatively 0.001 global magnetic diffusion time.

Just above the dynamo threshold ($Pm_c < Pm \leq 2Pm_c$), the magnetic field is weak: the magnetic energy contained within the fluid conducting region is only about 5% of the kinetic energy. The magnetic field does not strongly act back on the flow, except to produce its own saturation. A comparison between the zonal flow in the non-magnetic case and in the presence of the dynamo magnetic field does not reveal significant differences. The magnetic field lines of the poloidal field are almost aligned with the rotation axis and the flow structure (see figure 11(a)) so the flow disruption due to Lorentz forces is weak.

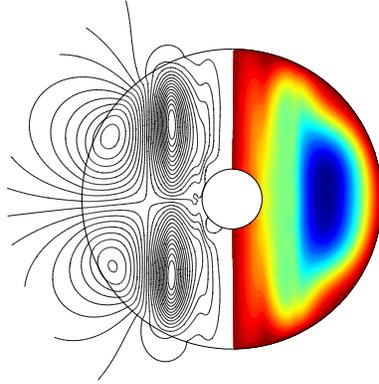
4.2 Characteristics of the magnetic field for model N

We performed simulations at $Ro = 1.05 - 1.5Ro_c$ and $E = 10^{-5}$. We find the dynamo threshold at $Pm_c \approx 1$, that is, the critical magnetic Reynolds number is $Rm_c \approx 4000$.

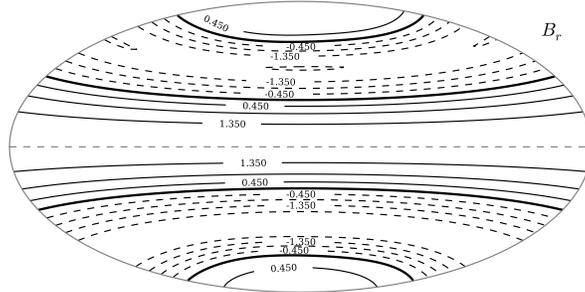
The main features of the self-sustained magnetic field can be observed in figures 13 and 14. The self-sustained magnetic field displays an equatorial symmetry, *i.e.*

$$(B_r, B_\theta, B_\phi)(r, \pi - \theta, \phi) = (B_r, -B_\theta, B_\phi)(r, \theta, \phi). \quad (11)$$

Within the fluid conducting region, the axisymmetric toroidal field is the strongest component whereas the poloidal field is dominated by the $m = 2$ mode, not the axisymmetric $m = 0$ mode. The $m = 2$ mode corresponds to the magnetic field generated at the scale of the Rossby wave (figure 14(a)). The axisymmetric poloidal field is multipolar, mainly composed by the

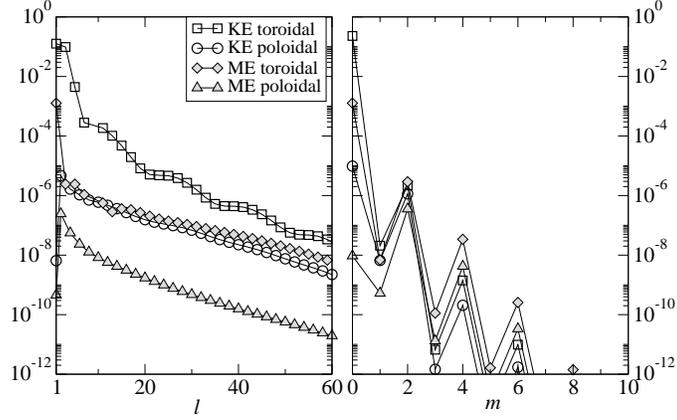


(a) Axisymmetric magnetic field

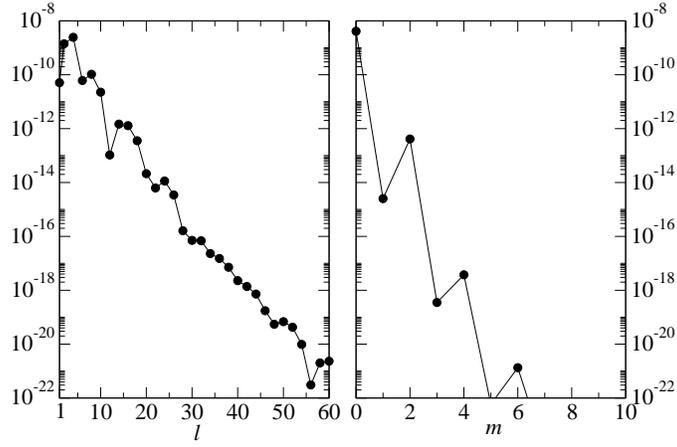


(b) Radial magnetic field at r_s

Figure 13: Magnetic field for model N (same as figure 11). For the axisymmetric azimuthal field, blue corresponds to negative values and red to zero values. The radial magnetic field is plotted at the surface of the planet $r_s = 1.18r_o$ in unit of $10^{-5}\sqrt{\rho\mu_0}U_0$. The parameters of this simulation are $E = 10^{-5}$, $Ro = 1.20Ro_c$ and $Pm = 2 \approx 2Pm_c$.



(a) Kinetic and magnetic energies in the fluid



(b) A_l and A_m at r_s

Figure 14: Magnetic energy spectra for model N (same as figure 12). Only the degrees of significant amplitude have been plotted, that is, l even for the kinetic poloidal and magnetic poloidal energies (plus $l = 1$) and l odd for the kinetic toroidal and magnetic toroidal energies. Same parameters than figure 13.

$(l, m) = (2, 0)$ (axial quadrupole) and $(l, m) = (4, 0)$ modes. At the surface of the planet (figure 14(b)), the magnetic field appears to be mainly axisymmetric (with the $l = 2$ and $l = 4$ harmonics degrees dominant). The amplitude of the $m = 2$ structure is weak, about two orders of magnitude smaller than the $m = 0$ mode but still visible at high latitudes on the map of the radial field at the surface (figure 13(b)).

Due to the equatorial symmetry of the field, the magnetic field lines in the equatorial plane are roughly perpendicular to the cylindrical structure of the flow whereas they are nearly aligned at higher latitudes (figure 13(a)). As a result magnetic braking acting on the flow is stronger in the equatorial region than at high latitude regions. In the simulation performed here ($Pm_c < Pm \leq 2Pm_c$), the magnetic energy is weak compared to the kinetic energy (about 5%) so the feedback of the magnetic field on the flow remains weak. For a stronger magnetic field (at larger magnetic Reynolds numbers), we expect that the flow disruption would become important. As a result the equatorially symmetric solution may become unstable and the magnetic field may switch to an axial dipolar symmetry. This is the result obtained by [Aubert and Wicht \(2004\)](#) in convectively-driven dynamos: they found equatorial dipolar magnetic fields for Rayleigh numbers close to the convection onset; these solutions become unstable as the convective forcing is increased and an axial dipolar configuration is preferred.

In summary, the flows driven by the profiles J and N produce very different poloidal magnetic fields: mainly a strongly axisymmetric dipole for the profile J and a weak multipolar axisymmetric field dominated by the magnetic field induced at the scale of the Rossby waves for the profile N. In both cases the magnetic field within the conducting region is mainly an axisymmetric toroidal field. The different magnetic field morphology is quite surprising given that the flows are quite similar: strong zonal flows and propagating Rossby waves. In the next section we review the dynamo mechanism that has been proposed to operate for similar flows and suggest the key difference between profiles J and N that determines the topology of their self-sustained magnetic fields.

4.3 Dynamo mechanism

Using a quasi-geostrophic flow and a kinematic approach (no Lorentz force in the momentum equation), [Schaeffer and Cardin \(2006, hereafter SC06\)](#) obtain numerical dynamos generated by an unstable axisymmetric shear layer (Stewartson layer): for a strong enough forcing, the Stewartson layer is unstable to non-axisymmetric shear instabilities, which appear in the form of

Rossby waves (of wavenumber about 10 for the Ekman numbers and Rossby numbers they investigated). The self-sustained magnetic field has a strong axisymmetric toroidal component and a mostly axisymmetric poloidal component. SC06 show that the time dependence of the flow is a key ingredient for the dynamo effect: time-stepping the magnetic induction equation using a steady flow taken either from a snapshot or a time-average leads to the decay of the magnetic field. They characterize the dynamo process as an $\alpha\omega$ mechanism. In mean field theory, the α effect parameterizes the generation of an axisymmetric poloidal magnetic field from the correlation of small scale magnetic field and velocity. The α effect usually requires that the flow possess some helicity, the correlation between fluid velocity and vorticity, $H = \mathbf{u} \cdot \boldsymbol{\omega}$ (e.g. Moffatt, 1978). Flows displaying a columnar structure aligned with the axis of rotation, such as Rossby waves or convection columns (Olson et al., 1999), typically possess strong mean helicity. As these columns are essentially bidimensional vortical structures, the helicity is mainly produced by the term $u_z \omega_z$. In nearly z -invariant flow, the axial (z) velocity is mostly due to two terms: the slope effect and the Ekman pumping. The slope effect comes from the combination of mass conservation and impenetrable boundaries: a (cylindrical) radial velocity u_s creates an axial velocity $u_z \sim z\beta u_s$ with $\beta = h^{-1}(dh/ds)$. In the limit of rapid rotation in a spherical container, this contribution is much larger (of order 1) than the Ekman pumping ($u_z \sim E^{1/2}\omega_z$). However, the axial velocity produced by the slope effect is phase shifted by $\pi/2$ with ω_z , and so does not allow the production of mean helicity. On the contrary, axial velocity produced by Ekman pumping is in phase with the axial vorticity and a dynamo mechanism based on the Ekman pumping associated to an azimuthal necklace of axial vortices is plausible (Busse, 1975). In a numerical experiment at small Ekman numbers ($E = \mathcal{O}(10^{-8})$), SC06 artificially remove the Ekman pumping and observe dynamo action with nearly the same threshold showing that the Ekman pumping is unimportant in their dynamo mechanism. The crucial importance of the time dependence of the flow and the negligible contribution of the Ekman pumping lead SC06 to consider the involvement of the Rossby waves in the dynamo process. They conjecture that the propagation of the Rossby waves yields a proper phase shift between the non-axisymmetric magnetic field and velocity field in order to produce the axisymmetric poloidal magnetic field.

Avalos-Zuniga et al. (2009) have calculated the α tensor, describing the generation of a large scale magnetic field by correlation of small scale velocity and magnetic field, with a flow geometry corresponding to Rossby waves. In the absence of Ekman pumping, they show that the diagonal components of

the α tensor, which are the relevant coefficients for the α effect, are non-zero if and only if the flow pattern is drifting relative to the mean flow.

Tilgner (2008) explains that the time dependence of a velocity field can lead to dynamo action even when any particular snapshot of the velocity field cannot because the linear operator associated with the induction equation is non-normal. In particular, he shows that the simple time dependence of a propagating wave is enough for dynamo action. Several numerical studies report the importance of the time dependence of the velocity field, mainly of oscillating nature (Reuter et al., 2009; Gubbins, 2008).

The idea that the propagation of Rossby waves may maintain a dynamo action is very appealing as their presence is ubiquitous in rotating fluid dynamics. A system in which no wave propagation occurs, and which is unable to produce u_z by another mechanism, such as buoyancy, will rely on Ekman pumping to create axial velocity with the proper phase shift. However, in the limit of small Ekman number, the Ekman pumping vanishes and the dynamo threshold should become infinitely high. The dynamo mechanism relying on the propagation of Rossby waves is robust in the limit of small Ekman number as the presence of these waves does not rely on the action of viscosity.

Due to the close resemblance of the flow (zonal motions and propagating Rossby wave) in our 3D numerical model and the kinematic quasi-geostrophic model of SC06, we now try to establish if the dynamo mechanism evoked in SC06 is at work in our 3D model. To formalize their idea, let us first consider a simple theoretical model. The velocity field is composed by a zonal flow, $\overline{U}(s)\mathbf{e}_\phi$, and the small scale velocity of a Rossby wave \mathbf{u}^m with

$$\mathbf{u}^m(s, \phi, z, t) = (u_s^m(s, z)\mathbf{e}_s + u_\phi^m(s, z)\mathbf{e}_\phi + u_z^m(s, z)\mathbf{e}_z)e^{i(m\phi - \omega t)} \quad (12)$$

where u_s^m , u_ϕ^m and u_z^m are complex and ω is the frequency of the wave. The magnetic field is composed of an axisymmetric magnetic field $\overline{\mathbf{B}}$, and a magnetic field perturbation induced at the scale of the Rossby wave \mathbf{b}^m with

$$\mathbf{b}^m(s, \phi, z, t) = (b_s^m(s, z)\mathbf{e}_s + b_\phi^m(s, z)\mathbf{e}_\phi + b_z^m(s, z)\mathbf{e}_z)e^{i(m\phi - \omega t) + \lambda t} \quad (13)$$

where b_s^m , b_ϕ^m and b_z^m are complex and λ is the growth rate of the magnetic field. The equations for the evolution of the poloidal components of $\overline{\mathbf{B}}$ in

cylindrical coordinates \overline{B}_s and \overline{B}_z are

$$\frac{\partial \overline{B}_s}{\partial t} = -\frac{\partial}{\partial z} (\overline{u_z^m b_s^m - u_s^m b_z^m}) + \eta \left(\nabla^2 \overline{B}_s - \frac{\overline{B}_s}{s^2} \right), \quad (14)$$

$$\frac{\partial \overline{B}_z}{\partial t} = \frac{1}{s} \frac{\partial}{\partial s} s (\overline{u_z^m b_s^m - u_s^m b_z^m}) + \eta \nabla^2 \overline{B}_z, \quad (15)$$

where the overbar denotes an azimuthal average. It is immediately apparent that if u_s^m (u_z^m) is out of phase by $\pi/2$ with b_z^m (resp. b_s^m), then \overline{B}_s and \overline{B}_z will be decaying in time. If we suppose that $\overline{B}_\phi \gg \overline{B}_s, \overline{B}_z$ the equations for b_s^m and b_z^m are

$$(\lambda - ic \frac{m}{s}) b_s^m = \frac{im}{s} u_s^m \overline{B}_\phi + \eta \left(\nabla^2 b_s^m - \frac{2}{s^2} \frac{\partial b_\phi^m}{\partial \phi} - \frac{b_s^m}{s^2} \right), \quad (16)$$

$$(\lambda - ic \frac{m}{s}) b_z^m = \frac{im}{s} u_z^m \overline{B}_\phi + \eta \nabla^2 b_z^m. \quad (17)$$

where $c = (\omega/(m/s) - \overline{U})$ is the phase speed of the wave relative to the mean flow \overline{U} . In the case of marginal stability ($\lambda = 0$), if we neglect the magnetic diffusivity η then we obtain that b_s^m (b_z^m) is in phase with u_s^m (u_z^m resp.). Moreover if the axial velocity is mainly due to the slope effect then $u_z^m = z\beta u_s^m$ and so according to the equations (16)-(17) $b_z^m \approx z\beta b_s^m$. This implies that the first term of the right hand side of equations (14)-(15) is almost zero and thus \overline{B}_s and \overline{B}_z are decaying. Consequently magnetic diffusivity at the scale of b_s^m and b_z^m must play a role in the generation of the axisymmetric poloidal magnetic field by introducing a short phase lag between the velocity and magnetic modes. This phase lag depends on the spatial structures of b_s^m and b_z^m , and hence the terms $\overline{u_s^m b_z^m}$ and $\overline{u_z^m b_s^m}$ do not cancel out. Note that the importance of magnetic diffusivity is well established in the α effect (Roberts, 2007). On the other hand, if the wave is not propagating, $c = 0$, then

$$-\frac{im}{s} u_s^m \overline{B}_\phi = \eta \left(\nabla^2 b_s^m - \frac{2}{s^2} \frac{\partial b_\phi^m}{\partial \phi} - \frac{b_s^m}{s^2} \right), \quad (18)$$

$$-\frac{im}{s} u_z^m \overline{B}_\phi = \eta \nabla^2 b_z^m. \quad (19)$$

In this case the magnetic field perturbations b_s^m and b_z^m are out of phase with u_s^m and u_z^m (as u_s^m and u_z^m are correlated by the slope effect) and so the averaged products $\overline{u_z^m b_s^m}$ and $\overline{u_s^m b_z^m}$ are zero. We can conclude that in order for this simple model to work as a mean-field dynamo (i) the wave must propagate and (ii) the magnetic diffusivity must act on the magnetic

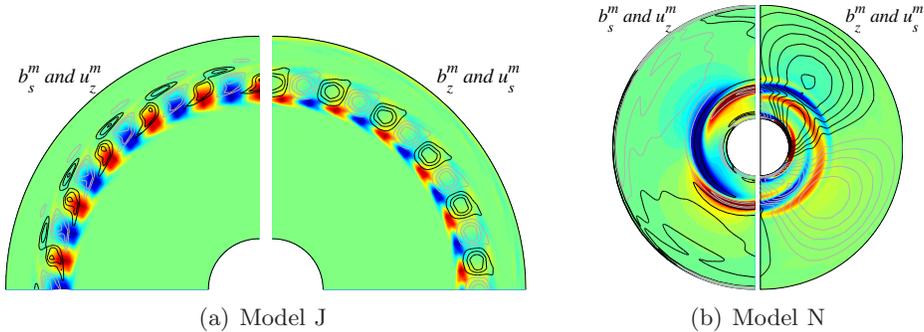


Figure 15: Non-axisymmetric magnetic field (coloured) and non-axisymmetric velocity (black lines: positive, and gray lines: negative) in a plane a few degree of latitude above the equatorial plane (northern hemisphere). Same parameters than figures 11 and 13.

field generated at the scale of the waves. As the Rossby wave propagates, the location of the induction of the magnetic field perturbation is forced to drift with the same rate, but with a phase-shift. The phase-shift between the magnetic field perturbation and the Rossby wave depends on both the phase speed c and the magnetic diffusivity η . The argument above implies that this phase-shift is essential for the dynamo mechanism.

Using any particular snapshot of the velocity field for time stepping the magnetic induction in our numerical simulations with models J or N leads to the decay of the magnetic field. The failure of dynamo in the kinematic numerical experiment with both models is readily explained by our simple theoretical model.

In figure 15, we plot the non-axisymmetric components of the velocity, u_z^m and u_s^m and magnetic field, b_s^m and b_z^m obtained in the numerical simulations for model J and model N in a plane located just above the equatorial plane (b_s^m and b_z^m are zero in the equatorial plane by dipolar symmetry in model J). The correlation of u_z^m with u_s^m confirms that u_z^m is mainly produced by the slope effect for both models. For model J (figure 15(a)), we observe that u_z^m and b_s^m are in phase so $\overline{u_z^m b_s^m}$ has a significant amplitude. However, b_z^m is out of phase with u_s^m , which means that $\overline{u_z^m b_s^m} \gg \overline{u_s^m b_z^m}$. This may be an effect of the magnetic diffusivity as b_s^m and b_z^m have different spatial structures, or due to radial derivatives of $\overline{B_s}$ and $\overline{B_z}$ that we neglect in equation (17). Consequently $\overline{u_z^m b_s^m}$ mainly contributes to the generation of strong $\overline{B_s}$ and $\overline{B_z}$.

For model N (figure 15(b)) strong positive (negative) crescent-shaped

patches of b_s^m and b_z^m are visible in the cyclonic (resp. anticyclonic) vortices, out of phase by $\pi/2$ with u_s^m and u_z^m . Consequently these crescent-shaped structures of b_s^m and b_z^m do not contribute to the terms $\overline{u_z^m b_s^m}$ and $\overline{u_s^m b_z^m}$. The presence of these maxima of b_s^m and b_z^m are not explained by the theoretical model (equations (16) and (17)) likely because of the neglect of the axial and radial derivatives of $\overline{B_s}$ and $\overline{B_z}$, which are important in this region (see figure 13(a)). Round-shaped lobes of b_s^m and b_z^m of weaker amplitude (located in the middle of the gap) are observed in phase with u_s^m and u_z^m . Consequently these round-shaped structures of b_s^m and b_z^m contribute to the terms $\overline{u_z^m b_s^m}$ and $\overline{u_s^m b_z^m}$. Unlike model J (where $\overline{u_z^m b_s^m} \gg \overline{u_s^m b_z^m}$), $\overline{u_z^m b_s^m} \sim \overline{u_s^m b_z^m}$ so only a weak axisymmetric multipolar magnetic field is maintained in this case. At the surface of the planet this axisymmetric field is the dominant component but in comparison with the strongly axisymmetric dipolar field produced in model J, the field is of small amplitude: the amplitude of the axisymmetric radial field is about $10^{-3} \sqrt{\rho \mu_0} U_0$ for model J at $Rm = 1.17 Rm_c$ ($Ro = 1.17 Ro_c$ and $Pm \approx Pm_c$) (figure 12(b)) while it is only $10^{-5} \sqrt{\rho \mu_0} U_0$ for model N at $Rm_c = 2.4 Rm_c$ ($Ro = 1.20 Ro_c$ and $Pm = 2 Pm_c$) (figure 14(b)).

The main difference between the Rossby waves in models J and N is their size. The phase speed of the Rossby wave, $c \approx \Omega \beta / (m/s)^2$, is about 100 times larger for a $m = 2$ wave than a $m = 22$ wave, for a fixed radius s and rotation rate Ω . On the other hand, the magnetic diffusion acts more rapidly on small scale structures. The typical propagation timescale for a Rossby wave of size d is $\tau_{rw} = 1/(\Omega \beta d)$ assuming that the radial and azimuthal lengthscales of the wave are similar. The magnetic diffusion timescale at the scale of the vortex d is $\tau_\eta = d^2/\eta$. The ratio of the two timescales is

$$\frac{\tau_\eta}{\tau_{rw}} = \frac{d^3 \Omega \beta}{\eta}. \quad (20)$$

The dependence to the third power of the size, $d \propto 1/m$, shows that the magnetic diffusion timescale relative to the propagation timescale is about three orders of magnitude smaller for an $m = 22$ mode than an $m = 2$ mode for the same parameter values. For the simulation presented for model N, the ratio τ_η/τ_{rw} is about 10^5 . For model J the ratio τ_η/τ_{rw} is about 500 so the propagation of the Rossby wave is still much more rapid than the magnetic diffusion. For both models, we found that the values of the small scale magnetic field in phase with the velocity is of the same order of magnitude. The velocity field of the vortices is also about the same order of magnitude for the two models. The difference between the two models is that, in model N, the magnetic diffusion acts too slowly on the $m = 2$ magnetic structures

compared to the wave propagation to produce a significant enough phase lag between b_s^m (b_z^m) and u_z^m (u_s^m). Consequently, the term $\overline{u_s^m b_z^m} - \overline{u_z^m b_s^m}$ is weak and leads to little generation of axisymmetric poloidal magnetic field.

The last stage of the dynamo mechanism is the generation of the axisymmetric toroidal field. It can either be produced from the correlation of small scale velocity and magnetic field (as an α effect) or an ω effect, that is the shearing of the axisymmetric poloidal magnetic field by the mean zonal flow \overline{U} . SC06 find that the ω effect from the Stewartson layer is dominant in their numerical model. The zonal shear produced in the Stewartson layer is stronger than the shear we obtained with the profiles J and N, so it is not clear that the ω effect is important in our model *prima facie*. In α^2 dynamos, both toroidal and poloidal components are typically of similar magnitudes (Olson et al., 1999). Here, the strong toroidal magnetic field suggests that the ω effect is more important. To confirm this, we plot in figure 16 the term responsible for the ω effect in the azimuthal component of the magnetic induction equation (Gubbins and Roberts, 1987), $r\overline{B_r}\partial_r(r^{-1}\overline{U}) + r^{-1}\sin\theta\overline{B_\theta}\partial_\theta(\sin\theta^{-1}\overline{U})$. For model J, as we expect, this term is most significant in the region where the poloidal magnetic field lines are bent and misaligned with the zonal flow structure (see figure 11(a)). The correlation of sign and location of the maxima of the ω effect in the bulk of the fluid with the axisymmetric azimuthal field indicates that it is mainly generated by the ω effect. Note that some ω effect is also present close to the outer boundary, where the poloidal magnetic field lines converge and diverge locally due to induction by the Ekman pumping. However no particularly strong axisymmetric azimuthal magnetic field is produced in this region (figure 11(a)) so this small scale field diffuses probably very rapidly. For model N the outer part of the jet ($s > 0.5$) is retrograde and creates a negative ω effect whose sign and location correlate with the axisymmetric azimuthal field, implying that the main dynamo process in the outer region is indeed the ω effect. However, $\overline{B_\phi}$ and the ω effect are anti-correlated in the inner region ($s < 0.5$) so another dynamo process such as a correlation of small scale velocity and magnetic field must be at work there.

We have not yet addressed the question of the selection of the axial dipolar symmetry or the axial quadrupolar symmetry. In kinematic dynamo calculations, Gubbins et al. (2000) show that minor changes in the flow can select very different eigenvectors. For a self-consistent system the selection rules are thus very subtle. As discussed in section 4.2, Aubert and Wicht (2004) found that axial quadrupolar symmetry is incompatible with the vertical structures of cyclones and anticyclones in convectively-driven dynamos, and so these solutions are unstable for strong convective

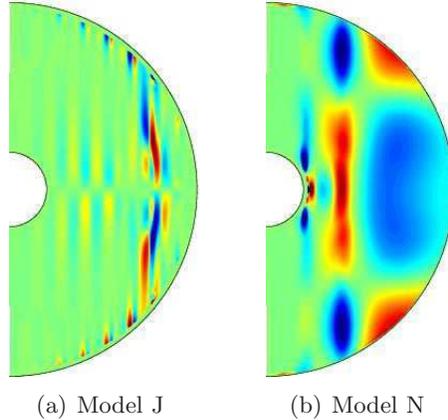


Figure 16: ω effect in the meridional plane in the bulk (outside the Ekman layers) (blue: negative and red: positive). Same parameters than figures 11 and 13.

flows. In our simulations of model N, this conclusion suggests that the axial quadrupolar symmetry would be unstable for larger magnetic Reynolds numbers, and an axial dipolar field would be preferred. The selection of a given symmetry does not modify our argument that the wavenumber of the Rossby mode determines the amplitude of the axisymmetric magnetic field since no particular latitudinal symmetry is assumed.

In this study, it appears that the dynamo mechanism relies on a subtle balance between the Rossby wave propagation and the magnetic diffusion and therefore is closely related to the size of the Rossby waves. The dynamo field produced with this mechanism requires high magnetic Reynolds numbers ($Rm_c \approx 20,000$ for model J and $Rm_c \approx 4000$ for model N). However, in the limit of small Ekman number, this dynamo mechanism is expected to keep a finite value of the critical magnetic Reynolds number (Schaeffer and Cardin, 2006), whereas for dynamos that rely on Ekman pumping the critical magnetic Reynolds number becomes infinitely high.

5 Summary and discussion

We have numerically studied the dynamics of zonal flows driven by differential rotation imposed at the top of a conducting layer and how they sustain a magnetic field.

5.1 Hydrodynamical instability

In our hydrodynamical simulations, we found that the destabilisation of the zonal flow takes the form of a global (large radial extension) Rossby mode, even though the instability threshold is governed by a local criterion. The wavenumber depends on the width of the jets, and is independent of the viscosity and rotation rate provided that the former is sufficiently small. In the supercritical regime, several Rossby waves appear and saturate the amplitude of the zonal flow in the bulk of the fluid. They produce a widening of the jets and a strong damping of their amplitude, even for relatively small supercritical forcing ($Ro = 2.94Ro_c$).

5.2 Constraints on the dynamo mechanism

In the limit of small Ekman number, we find that the Rossby wave appears for $Ro_c \approx 0.001$ for a Jupiter-like zonal wind profile (model J) and $Ro_c \approx 0.02$ for a Neptune-like profile (model N). In our numerical calculations, non-axisymmetric motions are necessary for dynamo action to occur. As the viscosity is large in the numerical simulations compared to the planetary values, the Reynolds number is much smaller in the simulations. To reach a sufficiently high magnetic Reynolds number, the magnetic Prandtl number is of order 1, much larger than the expected planetary values. The critical magnetic Reynolds number Rm_c is about 20,000 for model J and 4000 for model N. To make this dynamo mechanism work, two constraints must be satisfied: (i) $Ro > Ro_c$ and (ii) $Rm > Rm_c$. Equivalently this gives constraints on the amplitude of the zonal motions at the top of the conducting region, $U_0 > Ro_c \Omega r_o$, and on the electrical conductivity within the conducting region, $\sigma > Rm_c / (U_0 r_o \mu_0)$.

The extrapolation of the constraint (i) to the giant planets is straightforward as the hydrodynamical instability threshold is independent of the Ekman number, which is of order $10^{-15} - 10^{-16}$ for Jupiter (Guillot et al., 2004) and Neptune (Stevenson, 1983). For Jupiter ($r_o \approx 56,000$ km and $\Omega = 1.8 \times 10^{-4} \text{s}^{-1}$), the equatorial velocity at the top of the conducting region, U_0 , must be larger than 10 m/s to have $Ro > Ro_c = 0.001$. For Neptune ($r_o \approx 21,000$ km and $\Omega = 1.08 \times 10^{-4} \text{s}^{-1}$), U_0 must be larger than 45 m/s to have $Ro > Ro_c = 0.02$. For both cases, this constraint is quite strong as it only allows for a factor 10 decrease of the amplitude of the zonal wind between the surface of the planet and the top of the deep conducting region, independently of the location of the top of this region.

The extrapolation of the constraint (ii) to the giant planets requires

knowing how the critical Reynolds number scales with the Ekman number. When varying the Ekman number from 10^{-6} down to 10^{-8} , [Schaeffer and Cardin \(2006\)](#) found that Rm_c remains constant (of the order of 10^4 in their simulations, close to the values found in our study). Based on their results, we assume that Rm_c is of the same order of magnitude when the Ekman number is close to the planetary values. For Jupiter, we obtain that the electrical conductivity should be larger than 30S/m to have $Rm > Rm_c = 20,000$ (using $U_0 = 10$ m/s). For Neptune, the electrical conductivity should be larger than 10S/m to have $Rm > Rm_c = 4000$ (using $U_0 = 45$ m/s). This constraint on the conductivity is less restrictive than the constraint on the amplitude of the zonal motions and should be satisfied in the deep conducting layer of Jupiter ([Nellis et al., 1999](#)) and Neptune ([Nellis et al., 1997](#)).

We conclude that the differential rotation imposed by the zonal winds at the top of the conducting regions is a plausible candidate to drive the dynamo mechanism in the giant planets although a strong constraint on the amplitude of the zonal jet applies. Given the assumptions used in our model, such as incompressibility, constant conductivity, unrealistically large viscosity and viscous coupling between electrically insulating and conducting regions, this conclusion remains tentative. However, the robust nature of Rossby waves in the asymptotic limit of small Ekman numbers makes this dynamo mechanism appealing for planetary physical conditions.

5.3 Generation of the axisymmetric field and width of the jets

With a simple theoretical model, we show that the production of the axisymmetric field depends on the propagation of the Rossby waves and on the magnetic diffusion acting at the scale of the vortices. This model is in agreement with our numerical results: the magnetic diffusion rate of the $m = 2$ magnetic structures induced by the Rossby waves in model N is nearly negligible compared to the propagation rate of the wave: as a result a weak axisymmetric poloidal magnetic field is generated; the magnetic diffusion acting on the $m = 22$ magnetic structures is not negligible compared to the propagation rate of the small size ($m = 22$) Rossby wave of model J: a dominant axisymmetric poloidal magnetic field is therefore generated. Consequently, in this model, the width of the zonal jets has an important influence on the generation of the axisymmetric magnetic field by controlling the size of the Rossby waves. Our results suggest that the difference in the magnetic fields and the surface zonal winds may be related

if a (hydrodynamic or magnetohydrodynamic) mechanism can transport angular momentum between the surface and the deep, electrically conducting region.

The critical magnetic Reynolds number of this dynamo mechanism is large. However, in order to compare with other dynamos, a more significant number may be the critical local magnetic Reynolds number associated with magnetic induction by the Rossby wave velocity $Rm_c^l = V_s d / \eta$ where V_s is the typical non-axisymmetric radial velocity and d is the lengthscale of the Rossby mode. For the dynamo obtained in model J ($Rm_c = 20,000$), $V_s \approx 0.1U_0$ and $m = 22$ so we find $Rm_c^l \approx 570$. For the dynamo obtained in model N ($Rm_c = 4000$), $V_s \approx 0.01U_0$ and $m = 2$ so $Rm_c^l \approx 130$. Thus Rm_c^l is roughly 2–10 times larger than the magnetic Reynolds number needed for dynamo action with a convective forcing (Christensen and Aubert, 2006).

5.4 Magnetic field at the planets’ surfaces

In our numerical model, we obtain a peak at small azimuthal scale in the magnetic field spectrum correlated with the width of the hydrodynamically unstable zonal jets. This is a testable prediction as the magnetic measurements of the forthcoming *Juno* mission (arrival at Jupiter in 2016) are expected to be of extraordinary quality due to the absence of a crustal magnetic field on Jupiter.

For model J, we obtain a secular variation of the dipole tilt of about 1° in 1000 rotation periods or equivalently 0.001 global magnetic diffusion time. The dipole is strongly axisymmetric with a tilt that does not exceed 2° . On Jupiter, the dipole axis tilt measured with the *Pioneer* and *Voyager* data compared with the *Galileo* measurements is larger (about 10°) and displays a secular variation of about 0.5° in 20 years (Russell et al., 2001). The strong axisymmetry of the dipolar field of model J is in better agreement with the magnetic field of Saturn with a dipole tilt less than 1° (Russell and Dougherty, 2010).

5.5 Convective motions within the conducting region

In this work we have not taken into account the convective motions within the deep conducting region. Wicht et al. (2002) studied the linear stability of an imposed zonal flow in a spherical shell modeling the molecular hydrogen layer of Jupiter. They found that the critical Rossby number of the shear instability onset is almost independent of the Rayleigh number, which measures the strength of the convection. They concluded that the

shear instability is only weakly modified by the presence of convection. On the other hand, they showed that the convection onset is strongly influenced by the presence of the zonal circulation, with the convection either enhanced or damped depending on the direction of the shear. However, their study is linear, and so the results cannot be extrapolated beyond the weakly non-linear regime of convection. Whether or not our results apply in the presence of convection is a subject for future studies. In the presence of convection (which produces strong zonal motions and Rossby waves), and even for a convectively-driven dynamo (see for instance [Aubert, 2005](#); [Grote and Busse, 2001](#)), the mechanism described here may still impose a similar relationship between the magnetic field morphology and the zonal wind profile.

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A Linear code used to compute the hydrodynamical instability threshold

In order to study the linear stability threshold at very low Ekman numbers, we designed a linear code derived from [Gillet et al. \(2011\)](#). This three-dimensional spherical code uses second order finite differences in radius and pseudo-spectral spherical harmonic expansion. The linear perturbation \mathbf{u} of the imposed background flow \mathbf{U} is time-stepped from a random initial field with the following equation:

$$\left(\frac{\partial}{\partial t} - \nabla^2\right) \mathbf{u} = -\left(\frac{2}{E} \mathbf{e}_z + \nabla \times \mathbf{U}\right) \times \mathbf{u} + \mathbf{U} \times \nabla \times \mathbf{u} - \nabla p, \quad (21)$$

together with the continuity equation $\nabla \cdot \mathbf{u} = 0$, which allows us to eliminate the pressure term by using a poloidal-toroidal decomposition. The left hand side of equation 21 is treated with a semi-implicit Crank-Nicolson scheme, whereas the right hand side is treated as an explicit Adams-Bashforth term. Thanks to the cylindrical symmetry of the base flow \mathbf{U} , all azimuthal modes m of the perturbation \mathbf{u} are independent, and we can compute them separately. The coupling with the background flow and the Coriolis force are handled in physical space, but a very fast implementation of the spherical harmonic transform (SHTns library) makes the code quite efficient.

In order to determine the stability threshold at $E = 10^{-7}$, we used 350 points in the radial direction, and spherical harmonics truncated at $l_{max} = 300$. We use no-slip boundary conditions.

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