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Stochastic bottleneck capacity, merging traffic and morning commute

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ABSTRACT

This paper investigates the impact of stochastic capacity at the downstream bottleneck after a merge and the impact of merging behavior on the morning commuters' departure-time patterns. The classic bottleneck theory is extended to include a uniformly distributed capacity and the commuters' equilibrium departure patterns are derived for two different merging rules. The results show that uncertainty in the bottleneck capacity increases the commuters' mean trip cost and lengthens the peak period, and that the system total cost is lower under give-way merging than under a fixed-rate merging. Capacity paradoxes with dynamic user responses are found under both merging rules.

Key words**:** morning commute; merging behavior; bottleneck; capacity paradox

1. Introduction

The economic analysis of morning commute in congested traffic networks has followed the seminal work of Vickrey (1969) who formulated the morning commute problem to a mono-centric city center as a bottleneck model where commuters choose their departure times to avoid periods of high congestion at the bottleneck. This model represents a common situation during the morning rush hour, where a fixed and very large number of identical (homogeneous) commuters travel from a single origin (e.g. home) to a single destination (e.g. workplace) along a same stretch of road. This road has a single bottleneck with a fixed and commonly known capacity. If the arrival rate at the bottleneck exceeds its capacity, a queue forms. Although all the commuters wish to arrive

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at the common destination at the same time, this is not physically possible because the bottleneck capacity is finite. Consequently, some commuters may choose to depart earlier or later to avoid the cost of waiting in the queue, and pay the penalty cost for doing so. As noted by Arnott et al. (1990, 1998), in determining his/her departure time, each commuter faces a trade-off between journey time and schedule delay (early or late arrival at the destination). Vickrey's model provides a theoretical base to gain qualitative insights into alternative policy measures and to improve our understanding on congestion management possibilities.

Vickrey's model has been extended in various ways (see comprehensive reviews in Arnott et al., 1990, 1998; Lindsey, 2004; de Palma and Fosgerau, 2011). Smith (1984) and Daganzo (1985) proved the existence and uniqueness of the bottleneck equilibrium. The so-called equilibrium refers to a state at which no one can reduce his/her commuting cost through changing the departure time. Arnott et al. (1993a) extended the basic bottleneck model to consider elastic demand. Huang (2000) investigated the pricing and modal split in a system of transit and highway with heterogeneous commuters who differ in their disutility from travel time, schedule delay and transit crowding, whilst Tian et al. (2013) discussed the efficiency of a tradable credit schemes for managing bottleneck congestion and modal split with heterogeneous users. Most of the existing literature, however, is based on deterministic settings, with either a fixed capacity and demand (Arnott et al., 1990; Huang and Lam, 2002; Huang et al., 2007), or a pre-defined elastic demand function (Arnott et al., 1993a; Yang and Huang, 1997). Lindsey (1994) was the first to investigate the optimal departure scheduling when capacity is uncertain. Chen et al. (2002) developed a probability model to represent the variations and their impacts on system performance. Along this line of direction, several other developments on bottleneck models with uncertain capacity have been made (see, for example, Arnott et al., 1999; Fosgerau, 2008, 2010; Siu and Lo, 2009, 2013; Li et al., 2008; Xiao et al., 2013).

On the morning commute problem, most of the existing studies assume that each commuter passes only one bottleneck during the commuting trip. However, along a congested commuter route, it can be often observed that some commuters may pass through two or more bottlenecks during their commute journeys, and en-route they may

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merge with other traffic streams from a different origin. This research follows the pattern of the previous bottleneck analyses but relaxes the above assumption to analyze possible equilibrium queuing patterns in a network with more than one bottlenecks. Kuwahara (1990) developed the equilibrium queuing patterns at a two-tandem bottleneck during morning peak. Arnott et al. (1993b) considered a Y-shaped travel corridor, in a configuration shown in Fig. 2, which consists of two origins, one destination and three links. Two groups of commuters use the corridor, one entering each arm and passing through the corresponding upstream bottleneck and the bottleneck downstream which is common to both groups, on their way to work. Different to the usual ramp-mainline merging configuration where there is only metering control for the ramp, in the Y-shaped network, both upstream links can be controlled. Arnott et al obtained the analytical equilibrium solutions and discussed the capacity paradox arising from users' departure time choice in this Y-shaped corridor.

Lago and Daganzo (2007) adopted a similar Y-shaped highway corridor to study the spillovers of merging traffic. Daniel et al. (2009) conducted a behavioural experiment in a controlled environment with human subjects taking part in their departure time choice in a setting similar to that of Arnott et al. (1993b) and confirmed the theoretical bottleneck paradox by laboratory behaviour. Based on the perspective of the deterministic settings, however, all existing studies assumed a fixed capacity at the downstream bottleneck.

In reality, merging on highway is a major source of conflict and potential causes of flow breakdown, in other word, the capacity downstream of the merge is an exogenous variable in the merge model. Concerns have been raised in recent years about the inadequacy of conventional traffic models in representing the complex interactions at highway merges (Liu and Hyman, 2012). Several models have been proposed to account for capacity fluctuations at merge. For example, Evans et al. (2001) and Kerner (2002) postulated the stochastic approaches. Leclercq et al. (2011) applied the Newell-Daganzo model (Newell, 1982; Daganzo, 1995) to analyze the capacity drops at merges. Wang et al. (2005) and Huang and Sun (2009) employed microsimulation models to investigate merging behaviour. Fig. 1 displays two observed speed-flow relationships from a busy motorway network in England. The data are from two MIDAS (HA, 1994) loop detectors

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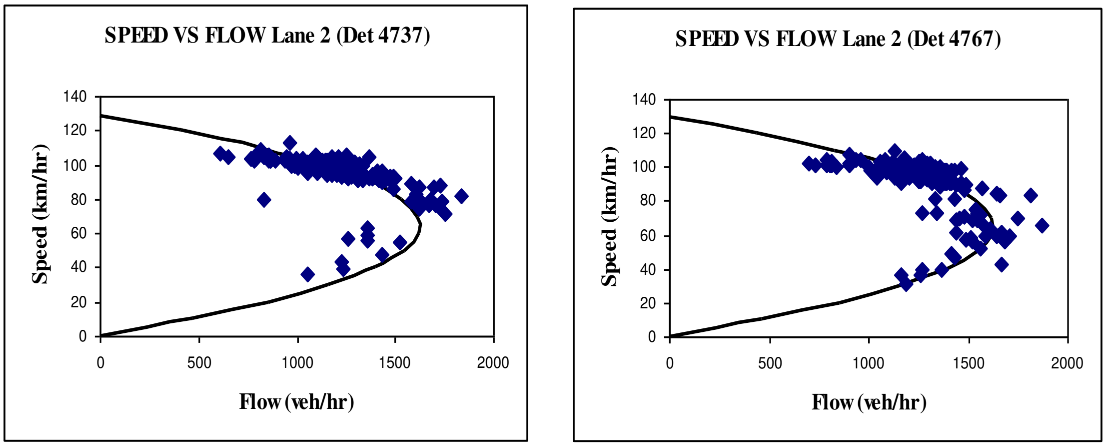
on the M25 motorway in England, and are 5-minute aggregated speed and flow data. Both diagrams show the stochastic maximum flows.



(a) (b)

Fig. 1. Observed speed-flow relationships from the M25 motorway in England.

To highlight the contribution of this paper relative to the literature, Table 1 provides a summary of the existing research on modeling morning commute with bottleneck congestion, categorized in terms of modeling scenarios, characteristics of the models, and selected key references. It is clear that, whilst the integrated problem with consecutive bottlenecks congestion and stochastic capacity is prevalent in reality, it has largely been ignored in the literature. Therefore, the aim of this paper is to understand the departure time choice of commuters travel through two consecutive bottlenecks and how the individual and total travel cost vary with the variability of capacity degradation, and based on which to propose and compare different traffic control strategies under this morning commute problem.

In this paper, we adopt the Vickrey's bottleneck theory to develop a model which consists of two upstream links with fixed capacity and one downstream link with a stochastic bottleneck capacity. We investigate the morning commute problem from two origins to one destination and derive the traffic departure pattern under two merging strategies. Our model setting is similar to that of Arnott et al. (1993b) on a Y-shaped network (shown in Fig. 2), where two groups of commuters travel from home to work,

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one entering each arm and passing through the corresponding upstream bottleneck and a common downstream bottleneck. Arnott et al. (1993b) investigated the capacity paradox and metering of upstream bottleneck for this network with a deterministic capacity at the downstream bottleneck, and they derived the equilibrium traffic patterns without considering for schedule delay late.

Table 1 Morning commute with bottleneck congestion.

Modeling scenarios	Characteristics	Selected key references
First bottleneck model	Equilibrium queuing patterns at a single bottleneck on freeways to a work place during the morning peak period	Vickrey, 1969
Time-varying pricing	The scheme can eliminate the queue delay at the bottleneck	Arnott et al., 1990
Demand elasticity	The trip demand function is treated as price-sensitive in the context of the bottleneck model	Arnott et al., 1993a; Yang and Huang, 1997
Coarse and step tolls	A positive and constant value during a or many certain intervals and zero others	Laih, 1994; Laih, 2004; Lindsey et al., 2012
Heterogeneous commuters	A set of discrete user classes having different value of time (VOT) or a group of users having a continuously distribution VOT	Arnott et al., 1994; Lindsey, 2004; van den Berg and Verhoef, 2011
Stochastic capacity and demand	Bottleneck capacity and demand are uncertain and assumed stochastic and follows a probility distribution	Arnott et al., 1999; Lindsey, 2009; Fosgerau, 2010;
Morning and evening commutes	Integrate morning and evening peaks in a day trip	de Palma and Lindsey, 2002; Zhang et al., 2005
Modal split	A separated transit mode is parallel to a highway with a bottleneck	Tabuchi, 1993; Huang, 2000
Consecutive bottlenecks	Commuters may pass one or two bottlenecks during the commuting trip	Kuwahara, 1990; Arnott et al., 1993b; Lago and Daganzo 2007; Daniel et al., 2009

Similar to Daniels et al. (2009), in this study, we allow for schedule delay early as well as schedule delay late (with higher costs) to the common destination. However, compared

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with previous work, this paper has one significant advance: we allow for the downstream bottleneck capacity to be stochastic. More specifically, our focus is on day-to-day fluctuation of the downstream capacity, assuming that the capacity within a day is constant. The fluctuation leads to variability in queue length behind the downstream bottleneck and to variability of travel time and trip cost, which in turn influences the commuters' departure time choice behavior. The analytical solutions are derived and its properties under two representative merging rules, namely give-way merging and fixed-rate merging, are investigated.

Our objective is to formulate the departure time choice with stochastic capacity under these two different merging strategies, and to investigate any capacity paradox with dynamic user response that may occur. The classical user equilibrium principle is used here to characterize the departure time choice behavior (Hendriksonn and Kocur, 1981). It is noted that on the principle of choice behavior, there has been discussions to use other measures such as reliability-based measures (e.g. Abkowitz, 1981; Siu and Lo, 2013). It is expected that new exploration to morning commute problem with complex configuration can help us design more effective policies in managing traffic congestion. In this spirit, some policy implications will be proposed for merging traffic under stochastic capacity.

The paper is organized as follows. In Section 2, the merge model with one stochastic bottleneck capacity is proposed. In Section 3, the equilibrium departure patterns under stochastic capacity for two different merging strategies are derived. Numerical results are presented in Section 4. Finally, Section 5 concludes the paper.

2. A General Bottleneck Model with Stochastic Capacity at Downstream of a Merge

2.1. Model setting with deterministic capacity

The merge studied in this paper consists of two upstream links and a downstream link. Fig. 2 illustrates the merge configuration which contains two possible upstream bottlenecks (the dotted lines) with service capacities s_1 and s_2 respectively, and a single downstream bottleneck with service capacity s_d. Two groups of commuters, denoted as N_1 and N_2 respectively, travel along this Y-shaped corridor to the central business

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district (CBD), departing from origins at a rate of $d_1(t)$ and $d_2(t)$ at time t respectively. Each commuter passes through two possible bottlenecks, one upstream and another downstream. The downstream bottleneck is common to both groups. For simplicity, we set the free flow travel time of all commuters to be zero, which means the travel time formulated in this paper consists only of queuing times behind the bottlenecks. If the arrival rate of commuters at a bottleneck exceeds its service capacity, a queue forms. The physical length of a queue is not considered. This point-queue assumption implies that vehicle queuing at the downstream bottleneck will not spill back to the merging point.

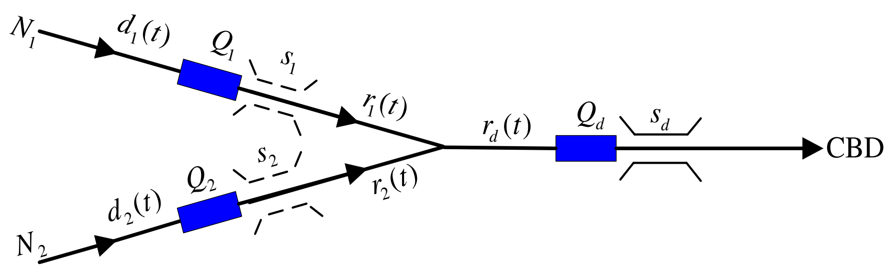

Fig. 2. The merge configuration.

By definition, the cumulative arrivals at a bottleneck at time t, $R(t)$, can be formulated as follows:

$$R(t) = \int_{t^0}^{t} r(x)\mathrm{d}x , \tag{1}$$

where $r(x)$ is the arrival rate at time instant x, and t^0 the earliest time with positive departure rate.

Let $T_g(t)$ denotes the travel time of group g commuters who leave home at time t. It follows that:

$$T_g(t) = \frac{Q_g(t)}{s_g} + \frac{Q_d\left(t + Q_g(t)/s_g\right)}{s_d}, \ g = 1,2 , \tag{2}$$

where $Q_g(\cdot)$ is the number of vehicles waiting in the queue behind the upstream bottleneck g, and $Q_d(\cdot)$ the number of vehicles waiting in the queue behind the

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downstream bottleneck d . The queuing lengths for the three bottlenecks can be computed as follows:

$$Q_d(t) = \max\left\{R_d(t) - s_d\left(t - t^0\right), 0\right\}, \quad (3)$$

$$Q_g(t) = \max\left\{R_g(t) - s_g\left(t - t^0\right), 0\right\}, \quad g = 1,2. \quad (4)$$

The commuters' total trip costs consists of costs that are associated with travel time and schedule delay early or late of arriving at destination. A linear trip cost function, for each group of commuters leaving home at time t, can be described as

$$C_g(t) = \alpha T_g(t) + \beta \max\left\{0, t^* - t - T_g(t)\right\} + \gamma \max\left\{t + T_g(t) - t^*, 0\right\}, \quad (5)$$

where α is the value of travel time, β the value of schedule delay early (SDE) and γ the value of schedule delay late (SDL). The relationship $\beta < \alpha < \gamma$ holds according to the estimates of Small (1982). In equilibrium, all commuters who leave the same origin and have the same desired arrival time t^*, should experience the same and minimal trip cost regardless of their departure times.

2.2. Expected trip cost with stochastic capacity

The deterministic models focus on cost equilibrium through adjusting departure time. When merging interactions exist, however, the capacity degradation of downstream bottleneck may occur. In this section, we analyze the commuters' departure time choice following two different merging strategies and stochastic downstream capacity. Throughout the paper, the following assumptions are used.

(A1) Commuters are homogeneous with the same value of time and the same values of schedule delays.

(A2) The capacity of the downstream bottleneck is constant within a day but fluctuates from day to day. The variability of capacity is completely exogenous and independent upon the commuters' departure time choice behavior. This means that our model accounts only for incidents before the peak starts, but not for incidents during the peak (Fosgerau, 2010; Peer et al., 2010).

(A3) In reality, the capacity is a non-negative stochastic variable changing within a range. Following Lo and Tung (2003), Lo et al. (2006) and Li et al. (2008), we adopt the

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same assumption that the stochastic capacity s follows a uniform distribution within interval $[\theta s_d, s_d]$, where s_d is the design capacity of the downstream bottleneck and $\theta(\leq 1)$ is a positive parameter representing the lowest rate of available capacity.

(A4) Commuters are aware of the capacity degeneration probability and their departure time choice follows the user equilibrium (UE) principle in terms of the mean trip cost.

(A5) The stochastic capacity at downstream bottleneck is independent of merging rules or travel demand from the two upstream links. Furthermore, the queuing behind the downstream bottleneck will not extend to the merge point (i.e. no physical queue is considered in this paper).

Incidents such as bad weather conditions, merging accidents or temporary road maintenance might decrease road capacity. At microscopic level, variations in driver behavior, such as in their reaction times to incidents and adherence to speed limits, in the performance of vehicles, in weather and lighting conditions on driving, etc, contribute to the *unpredictability* or the *unreliability* of travel time (e.g. Hollander and Liu, 2008; Li, 2006; Noland and Polak, 2002). Here we represent such supply variability through a stochastic bottleneck capacity. Different from the Vickrey model, we assume the bottleneck capacity is stochastic but the commuters' departure time choice is deterministic. Both commuters' travel time and their schedule delays are stochastic due to capacity fluctuations. Commuters experience the bottleneck day by day so that they can learn the incident probability and adjust their departure times to minimize their expected travel costs.

Under the stochastic condition, definitions (1)-(4) are still valid, and (5) can be directly used to calculate the trip costs of commuters leaving origins at each time instant. However, the trip cost is not deterministic but stochastic instead. The mean trip cost with respect to departure time t can be formulated as follows:

$$E\left[C_g(t)\right] = E\left[\alpha \cdot T_g(t) + \beta \cdot \overline{SDE}_g(t) + \gamma \cdot \overline{SDL}_g(t)\right], \tag{6}$$

where $\overline{SDE}_g(t)$ and $\overline{SDL}_g(t)$ are respectively the schedule delay early and late of group g's commuters who leave home at time t. We have

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$$\overline{SDE}_g(t) = \max\left\{0, t^* - t - T_g(t)\right\} \text{ and } \overline{SDL}_g(t) = \max\left\{t + T_g(t) - t^*, 0\right\}. \tag{7}$$

The user equilibrium is reached if and only if the mean trip cost is a constant for all departure times of each group. It follows

$$\mathrm{d}E\left[C_g(t)\right]/\mathrm{d}t = 0, \tag{8}$$

$$E\left[C_g(t)\right] = C_g^*,\ g = 1,2, \tag{9}$$

where C_g^* denotes the equilibrium mean trip cost of group g 's commuters.

In the next section, we consider two different merging rules: give-way (or priority) merge and fixed-rate merge. We give their definitions and present the UE solutions of their departure patterns under stochastic bottleneck capacity.

3. Equilibrium solutions in merge model with stochastic downstream capacity

The equilibrium solutions must be consistent with node dynamics, i.e. the merging interactions. In this section, we derive the equilibrium under a give-way merging rule and a fixed-rate merging rule, and analyze the solution properties for each scenario.

As shown in Fig. 2, $d_1(t)$ and $d_2(t)$ denote the departure rates of groups 1 and 2 from origins, respectively at time t. Let $r_1(t)$ and $r_2(t)$ be their respective exit flow from links 1 and 2 to the downstream link. Together they form the arrival rate to the downstream bottleneck d. We use $r_d(t)$ to denote the aggregate arrival rate to the downstream merge bottleneck, therefore $r_d(t) = r_1(t) + r_2(t)$.

We define a *give-way (or priority) merge* as such that: (*i*) link 2 traffic is under ramp metering control with a maximum merge rate equals to its capacity s_2, i.e. $r_2(t) \le s_2$; and (*ii*) link 1 traffic is not controlled and it merges to the downstream link with no restriction. This is a situation whereby link 1 traffic has priority over link 2 traffic at the merge.

We define a *fixed-rate merge* as one in which both upstream links are metered, and their maximum merge rates equal to their respective capacity s_1 and s_2, i.e. $r_1(t) \le s_1$ and $r_2(t) \le s_2$.

When an upstream link is controlled with a fixed merge rate, we can consider it as a bottleneck and model it using the classic bottleneck model with a fixed capacity.

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Therefore, depending on the above merging rules, the Fig. 2 network may have one (for the give-way merge) or two (fixed-rate merge) upstream bottlenecks. In addition, for the downstream bottleneck after the merge, we assume a stochastic capacity s, where s follows a uniform distribution as defined in A3 in Section 2.2 above.

3.1. Give-way merging

In this section, we study *give-way* merging. According to the above definition, the network reduces to a corridor with one upstream bottleneck with capacity s_2 and one downstream bottleneck with a stochastic capacity s. Since group 1 commuters face no upstream bottleneck congestion, then $Q_1(t)=0$ holds. The mean travel times of groups 1 and 2 can be respectively formulated as follows:

$$E\left[T_1(t)\right]=E\left[\frac{Q_d(t)}{s}\right], \tag{10}$$

$$E\left[T_2(t)\right]=\frac{Q_2(t)}{s_2}+E\left[\frac{Q_d\left(t+Q_2(t)/s_2\right)}{s}\right]. \tag{11}$$

Let τ_g be the peak period of group g, $\tau_g=[t_g^0,\ t_g^e]$, $g=1,2$, where t_g^0 and t_g^e are the start and end times of the peak period, respectively. Clearly, $C_2 \geq C_1$, since the capacity of link 1 is large enough to never influence the passing through of group 1 commuters. group 1 commuters can always arrange to reach bottleneck d at the same time as commuters from group 2, thereby incurring the same queue time at bottleneck d, and the same schedule delay, but no queue time at link 1. Therefore, under give-way merging rule, the relationship of mean trip costs between two groups can either be $C_2 > C_1$ or $C_1 = C_2$. (These cases are referred to as Case 3 in Kuwahara, 1990 and Case B in Arnott et al., 1993b, respectively). We derive the equilibrium conditions for both cases below.

Case A: $C_2 > C_1$

In this case, the earliest and latest times of leaving home among all commuters should be determined by group 2, i.e. the departure time window of the whole system should be $[t_2^0, t_2^e]$ which is also the arrival time window at the downstream bottleneck. The equilibrium solutions depend on whether $s_2 \geq s_d$ or $s_2 < s_d$. We begin with the case

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$s_2 \geq s_d$, for which the downstream bottleneck is congested from the start and the promised capacity expansion paradox occurs.

Case A1: $C_2 > C_1$, $s_2 \geq s_d$

Similar to the analysis made for a stochastic single bottleneck model (Xiao et al., 2013), there are four situations possible to occur in a merge network: (I) No schedule delay late; (II) Schedule delay either early or late possible; (III) Queuing and schedule delay late; (IV) No schedule delay early but probably queuing. These four situations occur consequently in four time intervals and are separated by three watershed times t_β, t_γ and t_s. We derive the departure rates from origins and arrival rates at downstream bottleneck in each of the four situations. Detailed derivation of the arrival rate at downstream bottleneck can be found in Appendix 1. For simplicity, we set $t^* = 0$.

Situation I. No schedule delay late in $[t_2^0, t_\beta]$

In this situation, no commuters experience schedule delay subject to all possible values of the capacity of the downstream bottleneck.

Before group 1 start to travel from origin, i.e. $t_2^0 \leq t < t_1^0$. The expected trip cost can be formulated as follows,

$$E\left[C_2(t)\right] = \alpha \int_{\theta s_d}^{s_d} T_2(t) f(s) \mathrm{d}s + \beta \int_{\theta s_d}^{s_d} -\left(t + T_2(t)\right) f(s) \mathrm{d}s \,, \tag{12}$$

where $f(s)$ is the probability density function of the stochastic capacity. Since $s_2 \geq s_d$ and there is no departure from group 1, then we can get $Q_d(t) = \left(s_2 - s\right)\left(t - t_2^0\right)$. Substituting (2) into (12), the above equation can be rewritten as:

$$E\left[C_2\right] = \alpha \int_{\theta s_d}^{s_d} \frac{Q_2(t) + \left(s_2 - s\right)\left(t - t_2^0\right)}{s} f(s) \mathrm{d}s - \beta \int_{\theta s_d}^{s_d} \frac{Q_2(t) + s_2\left(t - t_2^0\right) + s t_2^0}{s} f(s) \mathrm{d}s \,. \tag{13}$$

Differentiating (13) with respect to t and noting $\mathrm{d}Q_2(t)/\mathrm{d}t = d_2(t) - s_2$, from equilibrium condition (8), we get, for $t \in [t_2^0, t_1^0)$

$$d_2(t) = \frac{\alpha}{\alpha - \beta} \tilde{\theta} s_d \,, \tag{14}$$

where $\tilde{\theta} = \left(1 - \theta\right) / \ln \theta^{-1}$, $0 < \tilde{\theta} < 1$.

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Hence, the departure rates from the two upstream origins and the aggregate arrival rate to the downstream bottleneck are as follows:

$$d_1(t) = 0\,, \tag{15}$$

$$d_2(t) = \frac{\alpha}{\alpha - \beta}\tilde{\theta} s_d\,, \tag{16}$$

$$r_d(t) = s_2\,. \tag{17}$$

When group 1 begins to travel, i.e. $t_1^0 \le t \le t_\beta$, the departure rates of upstream commuters and the aggregate arrival rate of the both group at the downstream bottleneck can be formulated as follows:

$$d_1(t) = \frac{\alpha}{\alpha - \beta}\tilde{\theta} s_d - s_2\,, \tag{18}$$

$$d_2(t) = s_2\,, \tag{19}$$

$$r_d(t) = \frac{\alpha}{\alpha - \beta}\tilde{\theta} s_d\,. \tag{20}$$

The boundary condition for this situation is $\overline{SDE}_1(t_\beta) = 0$ when $s = \theta s_d$. We then have $R_d(t_\beta) = -t_2^0 \theta s_d$.

Situation II. Schedule delay either early or late possible in $(t_\beta, t_\gamma]$

In this situation, both SDE and SDL may occur. If the capacity of the downstream bottleneck is large enough, only schedule delay early will occur. Otherwise, schedule delay late occurs. The watershed capacity can be derived from $T_g(t) + t = 0$, which results $s = -R_d(t)/t_2^0$.

Before group 1 end to travel, i.e. $t_\beta < t \le t_1^e$, the departure rates of upstream commuters from home and the aggregate arrival rate of the both group at the downstream bottleneck can be formulated as follows:

$$d_1(t) = \frac{\alpha}{A + B\left(\ln R_d(t) + 1\right)} - s_2\,, \tag{21}$$

$$d_2(t) = s_2\,, \tag{22}$$

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$$r_d(t) = \frac{\alpha}{A + B\left(\ln R_d(t) + 1\right)}, \tag{23}$$

where $A = \frac{(\alpha+\gamma)\ln\theta^{-1} - (\beta+\gamma)\left(\ln(-t_2^0 s_d) + 1\right)}{(1-\theta)s_d}$, and $B = \frac{\beta+\gamma}{(1-\theta)s_d}$.

After group 1 ends to travel, i.e. $d_1(t) = 0$, $t_1^e < t \le t_\gamma$, the departure rate of group 2 from home and the arrival rate at the downstream bottleneck are as follows:

$$d_2(t) = r_d(t) = \frac{\alpha}{A + B\left(\ln R_d(t) + 1\right)}. \tag{24}$$

The boundary condition for this situation is $\overline{SDE}_1(t_\gamma) = \overline{SDL}_1(t_\gamma) = 0$ when $s = s_d$. We then have $R_d(t_\gamma) = -t_2^0 s_d$.

Situation III. Queuing and schedule delay late in $(t_\gamma, t_s]$

Similar to Situation I, in this situation all commuters experience schedule delay late even though at the maximum value of the downstream bottleneck capacity. The departure rate of group 2 and the arrival rate at the downstream bottleneck in this interval are

$$d_2(t) = r_d(t) = \frac{\alpha}{\alpha+\gamma}\tilde{\theta}s_d. \tag{25}$$

The boundary condition for this situation is $R_d(t_s) = s_d\left(t_s - t_2^0\right)$, i.e. the queuing length behind downstream bottleneck at time t_s equals to zero when $s = s_d$.

Situation IV. No schedule delay early but probably queuing in $(t_s, t_2^e]$

Similar to Situation II, there is a watershed capacity of the bottleneck such that the queuing length falls to zero. The departure rate of group 2 and the arrival rate at downstream bottleneck in this interval are

$$d_2(t) = r_d(t) = \frac{(\alpha+\gamma)R_d(t)\big/\left(t - t_2^0\right) - (\alpha\theta+\gamma)s_d}{(\alpha+\gamma)\left(\ln R_d(t) - \ln\theta s_d\left(t - t_2^0\right)\right)}. \tag{26}$$

The boundary condition for this situation is $r_d(t) = 0$. Equivalently, we have $R_d(t_2^e) = s_d\xi\left(t_2^e - t_2^0\right)$, where $\xi = (\alpha\theta+\gamma)\big/(\alpha+\gamma)$.

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Determination of the watershed time instants

Since the arrival rate $r_d(t)=0$ for $t>t_2^e$, the cumulative arrivals at the downstream bottleneck at time t_2^e equals to the traffic demand, i.e. $R_d(t_2^e)=N_1+N_2$. Therefore, we have $t_2^e=t_2^0+\left(N_1+N_2\right)/\hat{s}_d$, $\hat{s}_d=\xi s_d$. Moreover, the equilibrium condition of the stochastic bottleneck model implies that $E\left[C_2(t_2^0)\right]=E\left[C_2(t_2^e)\right]=-t_2^0\beta$. Thus, we have

$$t_2^0=\frac{N_1+N_2}{s_d}\cdot\frac{1}{\left(k_2-1\right)\xi},\ t_2^e=\frac{N_1+N_2}{s_d}\cdot\frac{k_2}{\left(k_2-1\right)\xi}, \tag{27}$$

where

$$k_2=\frac{\left(\alpha+\gamma\right)\omega-\beta}{\left(\alpha+\gamma\right)\omega+\gamma},\ \omega=\frac{\xi\left(\ln\xi-\ln\theta\right)-\xi+\theta}{\left(1-\theta\right)}. \tag{28}$$

Using the boundary conditions of Situations I, II, and III, we can obtain the watershed times as follows:

$$t_\beta=k_\beta\cdot t_2^0,\ t_\gamma=k_\gamma\cdot t_2^0,\ t_s=k_s\cdot t_2^0, \tag{29}$$

where $k_\beta=1-\dfrac{\alpha-\beta}{\alpha}\tilde{\theta}\theta$, $k_\gamma=\dfrac{\alpha+\beta+\gamma}{\alpha}-\dfrac{\alpha+\gamma}{\tilde{\theta}\alpha}$ and $k_s=1+\dfrac{\gamma+\beta}{\alpha-\tilde{\theta}\left(\alpha+\gamma\right)}$.

Since the departure rate of group 1 equals to zero at time instant t_1^e, i.e. $d_1(t_1^e)=0$. Submitting $d_1(t_1^e)=0$ into Eq. (21), we have the cumulative departure flow at time t_1^e,

$$R_d(t_1^e)=\exp\left(\frac{\alpha/s_2-\left(A+B\right)}{B}\right). \tag{30}$$

Substituting the conservation condition $R_d(t_1^e)=N_1+s_2\left(t_1^e-t_2^0\right)$ into Eq. (30), we obtain the following result,

$$t_1^e=\frac{N_1^e-N_1}{s_2}+\frac{N_1+N_2}{s_d}\cdot\frac{1}{\left(k_2-1\right)\xi}, \tag{31}$$

where $N_1^e=R(t_1^e)=\exp\left(\dfrac{\alpha/s_2-\left(A+B\right)}{B}\right)$.

Since the equilibrium condition requires $E\left[C_1(t_1^0)\right]=E\left[C_1(t_1^e)\right]$, we then have

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$$t_1^0 = \frac{(\beta+\gamma)N_1^e}{(1-\theta)\varphi s_d} + \frac{\alpha N_1}{\varphi s_2} + \frac{\varphi - \theta(\beta+\gamma)/(1-\theta)}{(k_2-1)\xi} \cdot \frac{N_1+N_2}{\varphi s_d}. \tag{32}$$

where $\varphi = \dfrac{(\alpha-\beta)s_2}{\tilde{\theta} s_d} - \alpha$.

In equilibrium, all group 2' commuters should have the same trip cost,

$$C_2^* = E\left[C_2(t_2^0)\right] = -\beta t_2^0, \tag{33}$$

which by Eq. (27) reduces to

$$C_2^* = \frac{N_1+N_2}{s_d} \cdot \frac{-\beta}{(k_2-1)\xi}. \tag{34}$$

The above reflects such a fact that all N_1+N_2 commuters pass through the downstream bottleneck between t_2^0 and t_2^e. Similarly, we have

$$C_1^* = \left((\alpha-\beta)\left(s_2/(\tilde{\theta} s_d)\right) - \alpha\right)\left(t_1^0 - t_2^0\right) - \beta t_2^0. \tag{35}$$

Comparing Eq. (33) and Eq. (35) leads to

$$C_2^* - C_1^* = \left(\alpha - (\alpha-\beta)\left(s_2/(\tilde{\theta} s_d)\right)\right)\left(t_1^0 - t_2^0\right). \tag{36}$$

The total travel cost of the system is

$$TC = N_1 C_1^* + N_2 C_2^*. \tag{37}$$

The departure rate of group 1 should be always nonnegative, i.e. $d_1(t) \geq 0$, $t \in [t_1^0, t_1^e]$. For this, from Eq. (18) and Eq. (21), the following two conditions must be satisfied,

$$\frac{s_2}{s_d} \leq \frac{\alpha}{\alpha-\beta}\tilde{\theta}, \tag{38}$$

$$\frac{s_d}{s_2} \leq \frac{(\beta+\gamma)N_1^e}{\alpha(1-\theta)N_1} - \frac{\theta\left((\alpha+\gamma)\omega+\gamma\right)}{\xi(1-\theta)\alpha} \cdot \frac{N_1+N_2}{N_1}. \tag{39}$$

We present below interesting properties of the equilibrium solution of the proposed Y-shaped network model with stochastic capacity in merging area.

Proposition 1. The first commuter of group 2 leaves home earlier than the first one of group 1, i.e. $t_2^0 < t_1^0$.

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Proof: If not, the first commuter of group 1 departs earlier than that of group 2, i.e. $t_1^0 < t_2^0$. The first commuter of group 1 will not endure waiting time on link 1 and downstream link, then the equilibrium trip cost can be calculated as $C_1 = \beta\left(t^* - t_1^0\right)$. Because the first commuter of group 2 departs from home later than that of group 1 and $s_2 \geq s$, then when the first commuter from group 2 begin to depart, a queue must exist at downstream link, therefore the equilibrium trip cost of group 2 has $C_2 \geq \beta\left(t^* - t_2^0\right)$. Since the case condition satisfies $C_1 < C_2$, then C_1 must be smaller than the minimal value of C_2, i.e. $C_1 = \beta\left(t^* - t_1^0\right) < \beta\left(t^* - t_2^0\right)$; it implies that $t_1^0 > t_2^0$ which violates the previous assumption. Therefore, the assumption is invalid and the Proposition turns out to be true.

Hence, group 1 commuters should leave home later than this time. This proposition is same as that (L1) in Arontt et al. (1993b). □

Proposition 2. The last commuter of group 2 leaves home later than the last commuter of group 1, i.e. $t_1^e < t_2^e$.

Proof: If not, consider a commuter of group 1 who leaves home after time t_2^e. Since s_1 is sufficiently large, he/she arrives at downstream bottleneck immediately and encounters schedule delay late. Differentiating Eq. (6) and using Eq. (10), the equilibrium condition (9) implies

$$r_d(t) = \frac{\alpha}{\alpha + \gamma}\tilde{\theta} s_d < s_2. \tag{40}$$

Eq. (40) means that there is no queue at the upstream bottleneck. The commuter of group 1 would leave home at the same time as someone of group 2 and then arrive at work at the same time. Consequently, they have the same travel cost and the condition $C_1 < C_2$ is then violated. This completes the proof. □

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From Propositions 1 and 2, we know that commuters of group 2 start earlier, and end later than group 1. Hence, the arrival period of group 2 at downstream bottleneck is longer than that of group 1.

Proposition 3. At equilibrium state, for the case scenario $C_1 < C_2$ and $s_2 \geq s_d$, the earliest and the latest departure time of group 2 is independent of s_2, i.e. $\partial t_2^0 / \partial s_2 = 0$, $\partial t_2^e / \partial s_2 = 0$. And, enhancing the upstream capacity s_2 will result in group 1 commuters to depart earlier, i.e. $\partial t_1^0 / \partial s_2 < 0$, $\partial t_1^e / \partial s_2 < 0$.

Proof: Eq. (27) clearly shows that t_2^0 and t_2^e are independent of s_2, so $\partial t_2^0 / \partial s_2 = 0$ and $\partial t_2^e / \partial s_2 = 0$. Eqs. (31) and (32) give the departure times of the group 1's first and last commuters, respectively. We then have

$$\frac{\partial t_1^e}{\partial s_2} = \frac{\partial N_1^e / \partial s_2 - \left(N_1^e - N_1\right)}{\left(s_2\right)^2} \text{ and } \frac{\partial t_1^0}{\partial s_2} = \frac{\beta + \gamma}{(1-\theta)\varphi s_d} \cdot \frac{\partial N_1^e}{\partial s_2} - \frac{\alpha N_1}{\varphi\left(s_2\right)^2}.$$

According to the definition, $N_1^e = \exp\left(\frac{\alpha / s_2 - (A+B)}{B}\right)$ in Eq. (28), we get $\partial N_1^e / \partial s_2 = -\alpha N_1^e \Big/ \left(B\left(s_2\right)^2\right)$ and $N_1^e > N_1$. It is clear that $\partial t_1^0 / \partial s_2 < 0$, $\partial t_1^e / \partial s_2 < 0$. This completes the proof. □

Proposition 4. With a fixed number of commuters, enlarging the value of the parameter θ will result in a decrease in the length of peak period.

Proof: From the definition $\xi = (\alpha\theta + \gamma)/(\alpha + \gamma)$, we have $\mathrm{d}\xi / \mathrm{d}\theta = \alpha / (\alpha + \gamma) > 0$. This implies that ξ is a monotonic increasing function of θ. From Eq. (27), we can obtain the length of peak period as follows:

$$t_2^e - t_2^0 = \frac{N_1 + N_2}{s_d} \cdot \frac{k_2}{\left(k_2 - 1\right)\xi} - \frac{N_1 + N_2}{s_d} \cdot \frac{1}{\left(k_2 - 1\right)\xi} = \frac{N_1 + N_2}{\xi s_d}. \tag{41}$$

Since N_1, N_2, s_d are constant and ξ is a monotonic increasing function of θ, then $t_2^e - t_2^0$ is monotonically decreasing with respect to θ. This completes the proof. □

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Proposition 5. Under the priority merging rule, when the value of the parameter θ approaches to one, the stochastic bottleneck model follows the deterministic model.

Proof: According to the L'Hospital's rule, we have $\lim_{\theta\to 1}(1-\theta)/\ln\theta^{-1}=1$, then

$$\lim_{\theta\to 1}\xi=1,\ \lim_{\theta\to 1}\omega=0,\ \lim_{\theta\to 1}k_\beta=\lim_{\theta\to 1}k_\gamma=\frac{\beta}{\alpha},\ \lim_{\theta\to 1}k_s=\lim_{\theta\to 1}k_2=-\frac{\beta}{\gamma}, \tag{42}$$

and

$$\lim_{\theta\to 1}r_d(t)=\begin{cases} s_2, & \text{if } t_2^0\le t<t_1^0 \\ \alpha s_d/(\alpha-\beta), & \text{if } t_1^0\le t\le t_\beta \\ \alpha s_d/(\alpha+\gamma), & \text{if } t_\gamma<t\le t_s \end{cases} \tag{43}$$

$$\lim_{\theta\to 1}d_2(t)=\begin{cases} \alpha s_d/(\alpha-\beta), & \text{if } t_2^0\le t<t_1^0 \\ s_2, & \text{if } t_1^0\le t\le t_1^e \\ r_d(t), & \text{if } t_1^e<t\le t_2^e \end{cases} \tag{44}$$

$$\lim_{\theta\to 1}d_1(t)=r_d(t)-s_2,\ \text{ if } t_1^0\le t\le t_1^e. \tag{45}$$

Substituting Eqs. (42)-(45) into Eqs. (29)-(32), the watershed times become

$$t_2^0=-\frac{\gamma}{\beta+\gamma}\cdot\frac{N_1+N_2}{s_d},\ t_\beta=t_\gamma=-\frac{\beta\gamma}{\beta+\gamma}\cdot\frac{N_1+N_2}{\alpha s_d},\ t_s=t_2^e=\frac{\beta}{\beta+\gamma}\cdot\frac{N_1+N_2}{s_d}, \tag{46}$$

$$t_1^e=t_2^0+\frac{N_2-s_d t_2^e}{s_2},\ t_1^0=t_1^e-\frac{N_1}{d_1}. \tag{47}$$

The above results are consistent with that reported in Daniel et al. (2009) for a deterministic bottleneck model. □

Both Propositions 3 and 4 show that the departure time period of group 2 is related to the downstream bottleneck capacity s_d, but independent to the upstream bottleneck capacity s_2. This is because $s_2\ge s_d$, so that the real constraint for group 2's commuters is caused by the downstream bottleneck.

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Theorem 1. At equilibrium state, the total travel cost is a monotonically decreasing function of the downstream capacity s_d, but a monotonically increasing function of the upstream capacity s_2, i.e. $\partial TC/\partial s_d < 0$, $\partial TC/\partial s_2 > 0$.

Proof. See Appendix 2. □

Theorem 1 gives a caution that expanding the capacity of upstream bottleneck with intention to improve the transportation system, may in fact lead to deterioration of the network. Thus, it can be regarded as a dynamic version of some paradox. The implication is that expanding network, if not fully considering the reaction of travelers, may be counter-productive.

Case A2: $C_2 > C_1$, $s_2 < s_d$

In this case, the equilibrium departures from origins and arrival rates at downstream bottleneck in Situations (I)-(IV) remain the same as in Case A1, except the first departure rate of group 2 in time interval $[t_2^0, t_1^0)$. If $s_2 \le \theta s_d < s_d$, there is no queue at downstream bottleneck during time interval $[t_2^0, t_1^0)$. Whilst if $\theta s_d < s_2 < s_d$, depending on capacity s_2, there is possible queue at downstream bottleneck during the same interval. The mean travel time of groups 2 can be formulated as follows:

$$E\left[T_2(t)\right] = \frac{Q_2(t)}{s_2}, \text{ if } s_2 \le \theta s_d < s_d, \tag{48}$$

$$E\left[T_2(t)\right] = \frac{Q_2(t)}{s_2} + \int_{\theta s_d}^{s_2} \frac{Q_d\left(t + Q_2(t)/s_2\right)}{s} f(s)\mathrm{d}s, \text{ if } \theta s_d < s_2 < s_d. \tag{49}$$

Submitting (48) and (49) into (6), and using equilibrium condition (8), the departure rate of group 2 in time interval $[t_2^0, t_1^0)$ can be formulated as follows:

$$d_2(t) = \frac{\alpha}{\alpha - \beta} s_2, \text{ if } s_2 \le \theta s_d < s_d, \tag{50}$$

$$d_2(t) = \frac{\alpha}{\alpha - \beta} \bar{\theta} s_d, \text{ if } \theta s_d < s_2 < s_d, \tag{51}$$

where $\bar{\theta} = \dfrac{1-\theta}{\ln\left(s_2/(\theta s_d)\right) + s_d/s_2 - 1}$.

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As discussed earlier, depending on the degradation of the bottleneck capacity (i.e. the θ value), in the earliest departure time period of group 2 and before group 1 departs, group 2 traffic could queue or not queue at the downstream bottleneck. This makes it impossible to derive analytical solutions for the watershed points for all situations. Furthermore, it is not possible to analyze capacity paradox for Case A2.

Case B: $C_2 = C_1$

Because there is no control to the upstream link 1 traffic, the result that the two groups incurs the same trip costs can only appear if there is no queue to link 2, i.e. if link 2 is also not controlled. In this case, the equilibrium departure rate from origin equals to its arrival rate at downstream bottleneck. The analytical solutions for the equilibrium pattern for this case can be seen in Xiao et al., (2013). Then, the total travel cost can be formulated as follows:

$$TC = \frac{\beta}{(1-k_2)\xi} \cdot \frac{(N_1+N_2)^2}{s_d}. \tag{52}$$

Evidently, $\partial TC/\partial s_d < 0$ and $\partial TC/\partial s_2 = 0$, so there is no paradox. The same conclusion is drawn under a deterministic bottleneck model by Arnott et al. (1993b).

3.2. Fixed-rate merging

In previous subsection, we have investigated the give-way merging rule in a network configuration where the upstream link serving group 1 is not controlled by ramp metering. Our analysis shows that commuters of group 1 leave home at a rate $\tilde{\theta} s_d \alpha/(\alpha-\beta) - s_2$, where $\tilde{\theta} = (1-\theta)/\ln\theta^{-1}$ and $0 < \tilde{\theta} < 1$. To study how vehicles interact at the merge, we now consider a configuration where both upstream links are controlled with the service rate s_1 and s_2, respectively. We assume

$$s_1 < \frac{\alpha}{\alpha-\beta}\tilde{\theta} s_d - s_2, \tag{53}$$

where $s_2 \geq s_d$. In this case, both group 1 and group 2 commuters face upstream bottleneck congestion, the mean travel times for both groups can be formulated as follows:

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$$E\left[T_g(t)\right] = E\left[\frac{Q_g(t)}{s_g} + \frac{Q_d\left(t + Q_g(t)/s_g\right)}{s}\right],\ g = 1,2\,. \tag{54}$$

The congestion degree of bottleneck η_g can be measured as follows:

$$\eta_g = \frac{N_g}{s_g}, \quad g = 1,2\,.$$

where N_g is the demand and s_g denotes the bottleneck capacity of link g.

There are two cases that need to be considered: (*i*) the congestion degree at bottleneck 1 is lighter than bottleneck 2 ($\eta_1 < \eta_2$) such that the trip cost of group 1 is smaller than that of group 2, i.e. $C_2 > C_1$, thus, the first commuter of group 2 leaves home earlier than any group 1 commuter; (*ii*) the congestion degree at bottleneck 1 is heavier than bottleneck 2 ($\eta_1 > \eta_2$) such that $C_2 < C_1$ holds, the first commuter of group 1 leaves home earlier than group 2 commuters. These two cases are symmetric as far as their departure time choices are concerned; hence we only study the first case in the following.

Similar to the give-way merge case in Section 3.1, all commuters face four situations to make their decisions about departure times.

Situation I. No schedule delay late in $[t_2^0, t_\beta]$

If $t \in [t_2^0, t_1^0)$, the departure rate of upstream commuters and the arrival rate to the downstream bottleneck are the same as Eqs. (15)-(17).

If $t \in [t_1^0, t_\beta]$, we can get

$$d_1(t) = \frac{\alpha}{\alpha - \beta} \cdot \frac{s_2}{s_2 + s_1}\tilde{\theta} s_d\,,\ d_2(t) = \frac{\alpha}{\alpha - \beta} \cdot \frac{s_2}{s_2 + s_1}\tilde{\theta} s_d \text{ and } r_d(t) = s_1 + s_2\,.$$

The boundary condition for this situation is $\overline{SDE}_1(t_\beta) = 0$ when $s = \theta s_d$, we then have $R_d(t_\beta) = -t_2^0 \theta s_d$.

Situation II. Schedule delay either early or late possible in $(t_\beta, t_\gamma]$

If $t \in (t_\beta, t_\gamma]$, we can obtain

$$d_1(t) = \frac{s_1\alpha/\left(s_1 + s_2\right)}{A + B\left(\ln R(t) + 1\right)},\ d_2(t) = \frac{s_2\alpha/\left(s_1 + s_2\right)}{A + B\left(\ln R(t) + 1\right)} \text{ and } r_d(t) = s_1 + s_2\,.$$

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The boundary condition for this situation is $\overline{SDE}_1(t_\gamma)=\overline{SDL}_1(t_\gamma)=0$ when $s=s_d$, we then have $R_d(t_\gamma)=-t_2^0 s_d$.

Situation III. Queuing and schedule delay late in $(t_\gamma,t_s]$

If $t\in(t_\gamma,t_1^e]$, we can get

$$d_1(t)=\frac{\alpha}{\alpha+\gamma}\cdot\frac{s_1}{s_1+s_2}\tilde{\theta}s_d,\ d_2(t)=\frac{\alpha}{\alpha+\gamma}\cdot\frac{s_2}{s_1+s_2}\tilde{\theta}s_d \text{ and } r_d(t)=s_1+s_2.$$

The boundary condition in this interval is $R_d(t_1^e)=s_1\left(t_1^e-t_1^0\right)$, i.e. the queuing length in upstream bottleneck 1 at time t_1^e equals to zero.

If $t\in(t_1^e,t_s]$, we can have

$$d_1(t)=0,\ d_2(t)=\frac{\alpha}{\alpha+\gamma}\tilde{\theta}s_d \text{ and } r_d(t)=s_2.$$

The boundary condition in this interval is $R_d(t_s)=s_d\left(t_s-t_2^0\right)$, i.e. the queuing length in downstream bottleneck at time t_s equals to zero when $s=s_d$.

Situation IV. No schedule delay early but probably queuing in $(t_s,t_2^e]$

Similar to Situation II, we can get

$$d_1(t)=0,d_2(t)=r_d(t)=\frac{\left(\alpha+\gamma\right)R_d(t)/\left(t-t_2^0\right)-\left(\alpha\theta+\gamma\right)s_d}{\left(\alpha+\gamma\right)\left(\ln R_d(t)-\ln\theta s_d\left(t-t_2^0\right)\right)}.$$

The boundary condition for this situation is $r_d(t)=0$. Equivalently, we have $R_d(t_2^e)=s_d\xi\left(t_2^e-t_2^0\right)$, where $\xi=\left(\alpha\theta+\gamma\right)/\left(\alpha+\gamma\right)$.

Note that the definitions to the abbreviated parameters, including $A,B,\tilde{\theta}$ and watershed times, are the same as those used in subsection 3.1.

To get the travel cost for each group, we need to derive all watershed times. Using the boundary conditions and equilibrium conditions, we have

$$t_2^0=\frac{N_1+N_2}{s_d}\cdot\frac{1}{\left(k_2-1\right)\xi},\ t_2^e=\frac{N_1+N_2}{s_d}\cdot\frac{k_2}{\left(k_2-1\right)\xi},\ t_\beta=k_\beta\cdot t_2^0,\ t_\gamma=k_\gamma\cdot t_2^0,\ t_s=k_s\cdot t_2^0,$$

where k_2 is given by (28), k_β,k_γ and k_s are the same as that used in (29).

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Noting the conservation condition $s_1\left(t_1^e - t_1^0\right) = N_1$ and the trip cost equilibrium $E\left[C_1(t_1^0)\right] = E\left[C_1(t_1^e)\right]$ for commuters of group 1, we have

$$t_1^0 = \frac{\tilde{\theta}\alpha s_d/s_2 - (\alpha+\gamma)s_1/s_2 - \alpha - \gamma}{\gamma+\beta} \cdot \frac{N_1}{s_1} + \frac{1-\tilde{\theta}s_d/s_2}{(k_2-1)\xi} \cdot \frac{N_1+N_2}{s_d},$$

$$t_1^e = \frac{\tilde{\theta}\alpha s_d/s_2 - (\alpha+\gamma)s_1/s_2 - \alpha + \beta}{\gamma+\beta} \cdot \frac{N_1}{s_1} + \frac{1-\tilde{\theta}s_d/s_2}{(k_2-1)\xi} \cdot \frac{N_1+N_2}{s_d}.$$

Given that all group 2 commuters have the same trip cost as the first commuter, using the equilibrium condition, we then get

$$C_2^* = \frac{N_1+N_2}{s_d} \cdot \frac{-\beta}{(k_2-1)\xi}.$$

For group 1, similarly we have

$$C_1^* = E\left[C_1(t_1^0)\right] = \left((\alpha-\beta)\left(s_2/(\tilde{\theta}s_d)\right) - \alpha\right)\left(t_1^0 - t_2^0\right) - \beta t_2^0.$$

Substituting with t_1^0 and t_2^0, leads to

$$C_1^* = -\varphi\frac{\phi + (\alpha+\gamma)\tilde{\theta}s_1/s_d}{\phi-\varphi} \cdot \frac{N_1}{s_1} - \left(\varphi\frac{s_d}{\tilde{\theta}s_2} + \beta\right)\frac{N_1+N_2}{s_d} \cdot \frac{1}{(k_2-1)\xi},$$

where $\varphi = (\alpha-\beta)s_2/\left(\tilde{\theta}s_d\right) - \alpha$, $\phi = (\alpha+\gamma)s_2/\left(\tilde{\theta}s_d\right) - \alpha$. Finally, the total travel cost is

$$TC = N_1C_1^* + N_2C_2^*.$$

Proposition 6. Under the fixed-rate merging rule, when the value of the parameter θ approaches to one, the stochastic bottleneck model follows the deterministic model.

Proof: According to Proposition 3, we have

$$\lim_{\theta\to1}\xi = 1,\ \lim_{\theta\to1}\omega = 0,\ \lim_{\theta\to1}k_\beta = \lim_{\theta\to1}k_\gamma = -\frac{\beta}{\gamma},\ \lim_{\theta\to1}k_s = \lim_{\theta\to1}k_2 = \frac{\beta}{\alpha}. \tag{55}$$

Then, under fixed-rate merging rule, we get

$$\lim_{\theta\to1} r_d(t) = \begin{cases} s_2, & \text{if } t_2^0 \le t < t_1^0 \\ s_1+s_2, & \text{if } t_1^0 \le t \le t_1^e \\ s_2, & \text{if } t_1^e < t \le t_2^e \end{cases} \tag{56}$$

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$$\lim_{\theta\to 1} d_2(t)=\begin{cases}\alpha s_d/(\alpha-\beta), & \text{if } t_2^0\le t<t_1^0\\ \alpha s_d s_2/((\alpha-\beta)(s_1+s_2)), & \text{if } t_1^0\le t\le t_\beta\\ \alpha s_d s_2/((\alpha+\gamma)(s_1+s_2)), & \text{if } t_\gamma<t\le t_1^e\\ \alpha s_d/(\alpha+\gamma), & \text{if } t_1^e<t\le t_2^e\end{cases} \tag{57}$$

$$\lim_{\theta\to 1} d_1(t)=\begin{cases}\alpha s_d s_1/((\alpha-\beta)(s_1+s_2)), & \text{if } t_1^0\le t\le t_\beta\\ \alpha s_d s_1/((\alpha+\gamma)(s_1+s_2)), & \text{if } t_\gamma<t\le t_1^e\end{cases} \tag{58}$$

Furthermore, the watershed times are

$$t_2^0=-\frac{\gamma}{\beta+\gamma}\cdot\frac{N_1+N_2}{s_d},\ t_\beta=t_\gamma=-\frac{\beta\gamma}{\beta+\gamma}\cdot\frac{N_1+N_2}{\alpha s_d},\ t_s=t_2^e=\frac{\beta}{\beta+\gamma}\cdot\frac{N_1+N_2}{s_d}, \tag{59}$$

$$t_1^0=\frac{\alpha s_d/s_2-(\alpha+\gamma)s_1/s_2-\alpha-\gamma}{\gamma+\beta}\cdot\frac{N_1}{s_1}+(1-s_d/s_2)t_2^0, \tag{60}$$

$$t_1^e=\frac{\alpha s_d/s_2-(\alpha+\gamma)s_1/s_2-\alpha+\beta}{\gamma+\beta}\cdot\frac{N_1}{s_1}+(1-s_d/s_2)t_2^0. \tag{61}$$

This completes the proof. □

Theorem 2. Under the fixed-rate merging rule, the total travel cost is a monotonically decreasing function of capacity s_1 and of capacity s_d, but a monotonically increasing function of capacity s_2, i.e. $\partial TC/\partial s_1<0$, $\partial TC/\partial s_2>0$ and $\partial TC/\partial s_d<0$.

Proof. See Appendix 3. □

Theorem 2 shows a capacity paradox in that increasing capacity s_2 leads to an increase in total travel cost. This paradox suggests that metering the capacity of upstream bottleneck 2 is beneficial. Moreover, the total travel cost is a decreasing function of upstream capacity bottleneck 1, indicating that metering the capacity of upstream bottleneck 1 is harmful.

4. Numerical examples

In this section, we present numerical results for the Y-shaped merge network model with stochastic downstream bottleneck under the give-way merging in subsection 4.1 and

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fixed–rate merging rule in subsection 4.2. In subsection 4.3, we compare and discuss the analytical and numerical results in our model. We set the shadow values of travel time, early arrival time, and late arrival time as α =6.4 (\$/hour), β =3.9 (\$/hour), and γ =15.21 (\$/hour), respectively; these shadow values are chosen in accordance with the empirical findings in Small (1982). The other inputs in the model are set as, $N_1 = 0.5$, $N_2 = 1.0$, $s_1 = 0.8$, $s_2 = 0.8$ or 1.0 and $s_d = 0.8$. According to Arnott et al. (1990), the ratio of demand to capacity equals to the length of morning commute rush hour under deterministic bottleneck model. Thus $(N_1 + N_2)/s_d = 1.85$ means that the rush hour at this merge area lasts for 1.85 hours. The value of $\hat{s}_d$ can be computed from $\hat{s}_d = s_d(\alpha\theta+\gamma)/(\alpha+\gamma)$.

4.1. Give-way merging

Fig. 3 shows the cumulative departures and arrival distributions of both groups under the give-way merging rule. In Fig. 3(a), s_2 is set as equal to s_d. The individual mean trip cost for groups 1 and 2 are $\mathrm{E}[C_1] = 5.34$ and $\mathrm{E}[C_2] = 6.23$, respectively. The total trip cost is $TC = 8.91$. In Fig. 3(b), the capacity of the upstream bottleneck s_2 is increased to 1.0. As a result, the first commuter of group 1 leaves home earlier than in Fig. 3(a) which is consistent with Proposition 3 and the corresponding costs are $\mathrm{E}[C_1] = 5.83$, $\mathrm{E}[C_2] = 6.23$ and $TC = 9.14$. Therefore, from Fig. 3(a) to Fig. 3(b), one of upstream bottlenecks is expanded, the individual trip costs do not come down and the total network cost goes up. Thus, enhancing bottleneck capacity results in a degradation of the system efficiency. This provides a specific instance about bottleneck paradox. The trip cost of group 2 remains unchanged, which verifies the analytical result that the trip cost of group 2 is independent of the capacity s_2.

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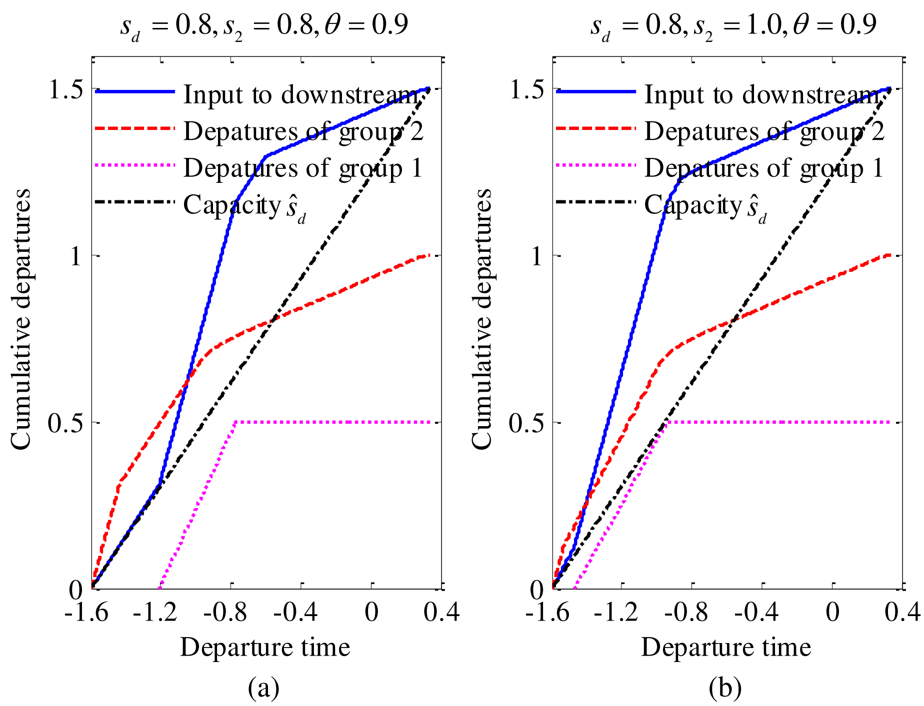

Fig. 3. Cumulative departures under give-way merging rule for two different levels of bottleneck 2 capacity: (a) $s_2 = 0.8$ and (b) $s_2 = 1.0$

Table 2

Influence of θ on mean trip costs and watershed time instants under the give-way merging rule.

θ	$E[C_2]$	$E[C_1]$	t_2^0	t_1^0	t_β	t_γ	t_s	t_1^e	t_2^e	$t_1^e - t_1^0$	$t_2^e - t_2^0$
1.00	5.82	5.11	-1.49	-1.28	-0.80	-0.80	0.38	-0.80	0.38	0.48	1.88
0.95	6.02	5.46	-1.54	-1.37	-0.87	-0.81	0.33	-0.86	0.36	0.51	1.90
0.90	6.23	5.83	-1.60	-1.47	-0.94	-0.68	0.27	-0.93	0.34	0.54	1.93
0.85	6.45	6.23	-1.66	-1.58	-1.03	-0.54	0.20	-1.01	0.31	0.57	1.96
0.80	6.70	6.66	-1.72	-1.70	-1.11	-0.38	0.14	-1.91	0.28	0.61	1.99

It is interesting to investigate the sensitivity of parameter θ on the solution of the stochastic merge model. We set $s_2 = 1.0$ and $s_d = 0.8$, and vary the θ-value from 0.8 to 1.0. The mean trip costs and watershed time instants derived are presented in Table 2. It can be seen that $t_\beta = t_\gamma = t_1^e$ and $t_s = t_2^e$ when $\theta = 1.0$. This is consistent with Proposition 5. It can also be seen that the length of peak period increases as the θ-value decreases. This is consistent with Proposition 4. Since decreasing the θ-value is equivalent to

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increasing the travel time uncertainty, this means that commuters will leave home earlier with smaller θ-value to avoid the potential loss caused by uncertainty risk.

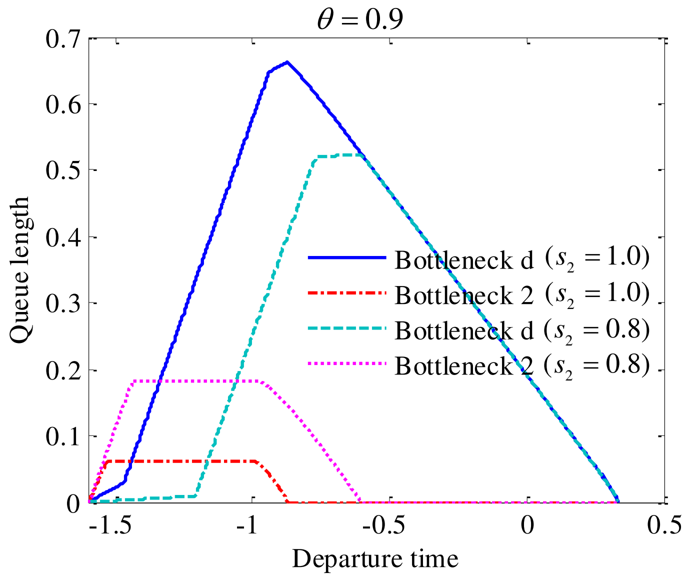

Fig. 4. Queue length by departure time behind bottleneck d and 2 for two levels of s_2 under give-way merging rule.

To illustrate the paradox in a different way, Fig. 4 depicts the mean queue lengths behind bottleneck d according to mean capacity $\hat{s}_d$ and bottleneck 2 according to capacity s_2 during the peak period time. It can be seen that commuters of group 2 experience less queuing congestion at bottleneck 2 when s_2 is increased from 0.8 to 1.0. However, the queuing congestion at bottleneck d becomes more serious. Moreover, it should be noted that the peak period length doesn't change with the s_2 value. This is consistent with Proposition 4.

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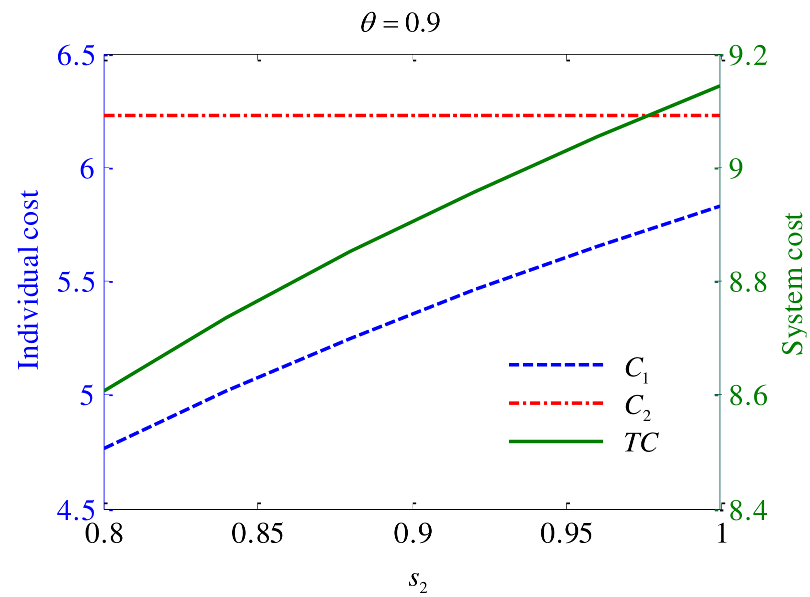

Fig. 5. Variation of the individual group costs C_1 and C_2, and the system cost TC with s_2 under give-way merging rule.

Furthermore, to illustrate the capacity paradox through traffic cost under give-way merging rule, we calculate the traffic equilibrium in the network under varying capacity of the upstream link 2, and the results are depicted in Figure 5. It shows clearly that when s_2 increases, the individual trip cost of group 1 (C_1) increases, whilst the cost of group 2 (C_2) remains unchanged. Overall, increasing capacity s_2 results in an increase of total trip cost (TC). This is consistent with Theorem 1.

Figures 6 and 7 depict the different effects of the stochasticity on traffic performance. Firstly, the time-varying arrival rates to downstream bottleneck against the θ-value are presented in Figure 6. Here, the service rate s_2 and the designed capacity of downstream bottleneck s_d are both set to be 0.8, which implies that the stochastic downstream capacity s can fluctuate in the interval [0.8θ, 0.8] day-to-day and $\theta \leq 1$. One can observe from the figure that the merge model with stochastic downstream bottleneck immediately follows the deterministic model when the θ-value approaches to one.

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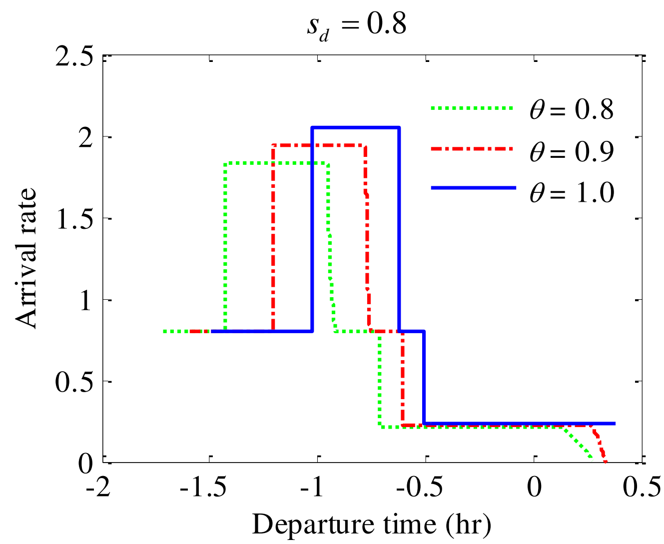

Fig. 6. Influence of parameter θ on the arrival rate at downstream bottleneck under give merging.

Secondly, with the same parameter setting, the mean trip cost and its individual component costs: mean travel time cost, the mean schedule delay early and late costs (SDE and SDL) for the two groups of commuters are shown in Figure 7. We can observe that the mean trip costs for all commuters in each group are the same and equal to \$4.76 of group 1 and \$6.23 of group 2 respectively, but the commuters endure a trade-off between the cost of travel time and the cost of schedule delay. From Figure 7(a), we can see that SDL curve is non zero at the end of the peak period. This means that commuters from group 1 can arrive early or late under the stochastic capacity assumption. Daniel et al (2009) found that, under deterministic capacity, commuters for group 1 can only arrive early. Furthermore, for group 2 traffic in Figure 7(b), we note that the SDE and SDL curves cross at a point where their costs are non zero and the travel time cost at the crossing point does not reach the mean trip cost. Again, these results under the stochastic capacity are different to those under deterministic capacity as discussed in Daniel et al. (2009).

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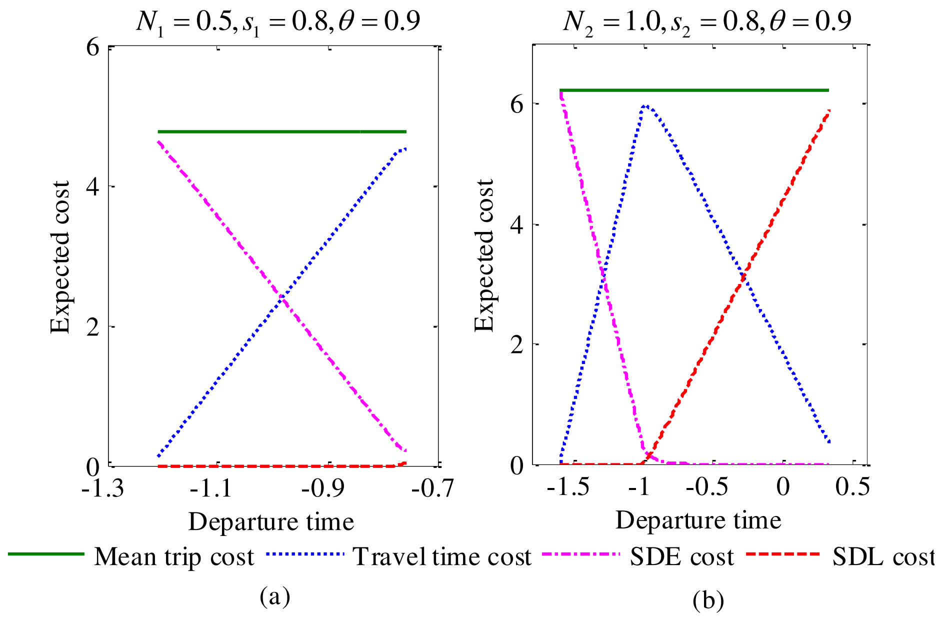

Fig. 7. Mean trip cost and its individual component costs of (a) group 1 and (b) group 2 under give-way merging.

4.2. Fixed-rate merging

With the definition of fixed-rate merging, both of the upstream links can be treated as bottlenecks with capacity $s_1 = 0.8$ and $s_2 = 1.0$, respectively. Taking the other parameters settings as in Section 4.1, Table 3 presents the mean trip costs and the watershed time instants against different θ-values under the fixed-rate merging rule. It can be seen that the length of the peak period for group 2 becomes shorter when θ-value increases, whilst the peak period length of group 1 remain unchanged. The first departure times of both group decrease with decreasing θ-value. This suggests that commuters would leave home earlier when uncertainty increases.

Table 3

Influence of θ on mean trip costs and watershed time instants under the fixed-rate merging rule.

θ	$E[C_2]$	$E[C_1]$	t_2^0	t_1^0	t_β	t_γ	t_s	t_1^e	t_2^e	$t_1^e - t_1^0$	$t_2^e - t_2^0$
1.00	5.82	5.53	-1.49	-1.40	-0.79	-0.79	0.38	-0.79	0.38	0.63	1.88
0.95	6.02	5.72	-1.54	-1.45	-0.85	-0.81	0.33	-0.83	0.36	0.63	1.90
0.90	6.23	5.98	-1.60	-1.52	-0.92	-0.68	0.27	-0.89	0.34	0.63	1.93

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0.85	6.45	6.30	-1.66	-1.60	-1.01	-0.54	0.20	-0.98	0.31	0.63	1.96
0.80	6.70	6.66	-1.72	-1.71	-1.10	-0.38	0.14	-1.08	0.28	0.63	1.99

Fig. 8 shows the cumulative departures from origins and integrated arrival distributions of both groups at downstream bottleneck under fixed-rate merging rule. In Fig. 8(a) with $\theta = 0.8$, it can be seen that the times of first commuters of both groups leave home are very close. Whilst in Fig. 8(b) with $\theta = 1.0$, the first commuter of group 1 leaves home significantly later than the first commuter from group 2. Hence, it is conceivable that under fixed-rate merging rule, when the stochastic downstream capacity converges to the deterministic case, commuters of group 1 would benefit more than group 2 commuters.

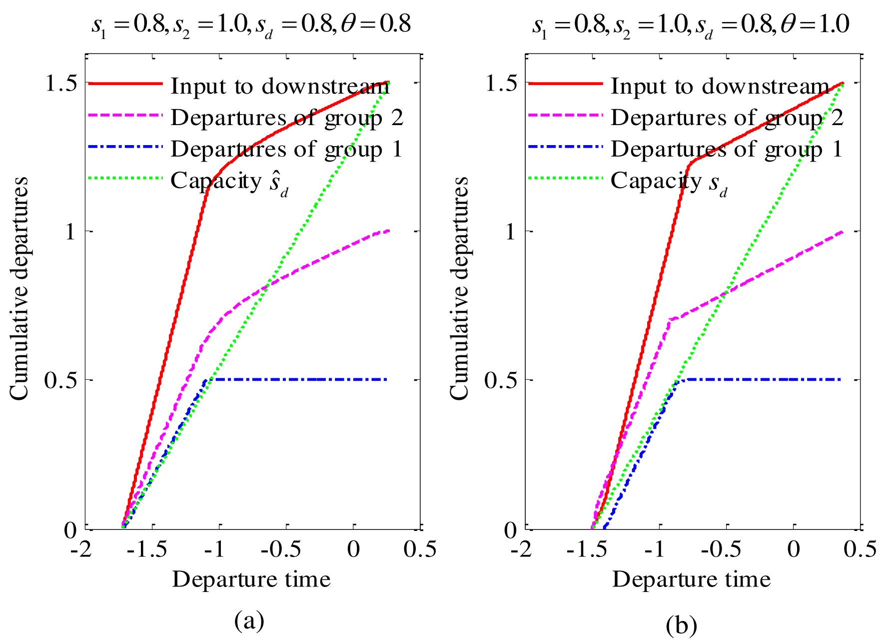

Fig. 8. Cumulative departures under fixed-rate merging rule under (a) a stochastic bottleneck and (b) a deterministic bottleneck.

Fig. 9 depicts the queue lengths at each of the three bottlenecks under fixed-rate merging rule under a stochastic ($\theta = 0.8$) and the deterministic case $\theta = 1.0$. It can be seen that both upstream bottlenecks are controlled and the queue length at downstream bottleneck d according to mean capacity $\hat{s}_d$ is the most, followed by upstream

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bottleneck 2 according to capacity s_2 and then upstream bottleneck 1 according to capacity s_1. We can also see that, the total queuing delay (i.e. the area under the three curves representing queue length) under the stochastic case is bigger than that under the deterministic case; their values are respectively 0.7059 and 0.5865. This implies that the downstream bottleneck becomes much more congested when considering stochastic bottleneck capacity than that of the deterministic case. On the other hand, the queue lengths of upstream bottlenecks are slightly higher in the deterministic case than in the stochastic case.

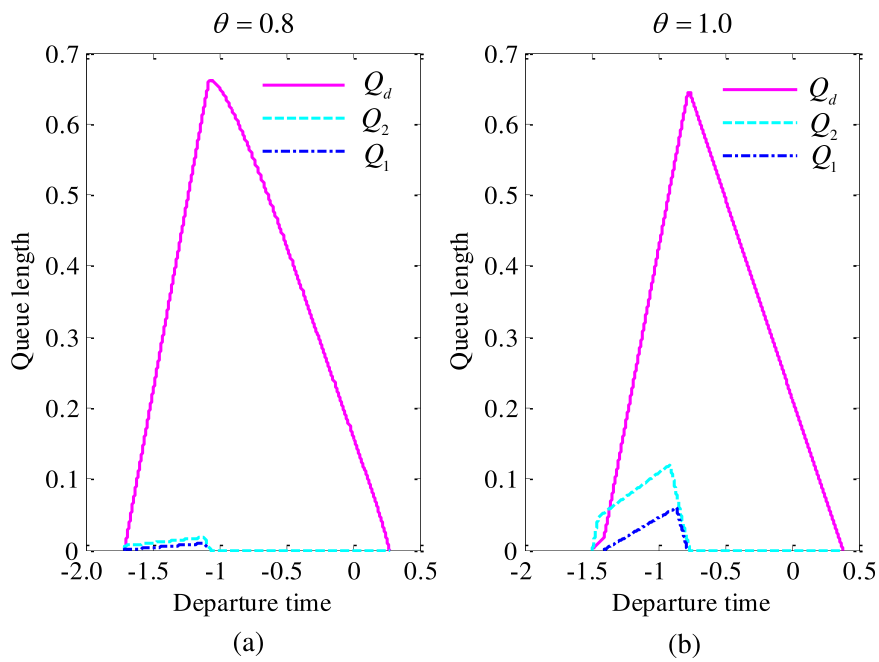

Fig. 9. Queue length under fixed-rate merging rule at (a) a stochastic bottleneck and (b) a deterministic bottleneck.

For comparison with the priority merging case, we also set $s_1 = 0.8$, $s_2 = 0.8$ and $s_d = 0.8$ to investigate the change of time-varying arrival rate at downstream bottleneck with the variability of downstream capacity under fixed-rate merging. The equilibrium arrival rates are shown in Figure 10. Clearly, the arrival rates converge to that of the deterministic merge model when θ approaches to one. This is consistent with Proposition 6.

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Using the same parameter setting as in Figure 7, we consider both links 1 and 2 are controlled by ramp metering. Then commuter from each origin has to traverse two bottlenecks to the destination. Figure 11 shows the mean trip cost and its individual component costs: mean travel time cost, the mean schedule delay early and late costs (SDE and SDL) for the two groups of commuters under fixed-rate merging rule. Compared to the results in Fig. 7, we can observe that the cost patterns for group 2 remain unchanged with respect to give-way merging rule, whilst for group 1, in equilibrium, commuters depart earlier and endure larger trip cost by ramp metering.

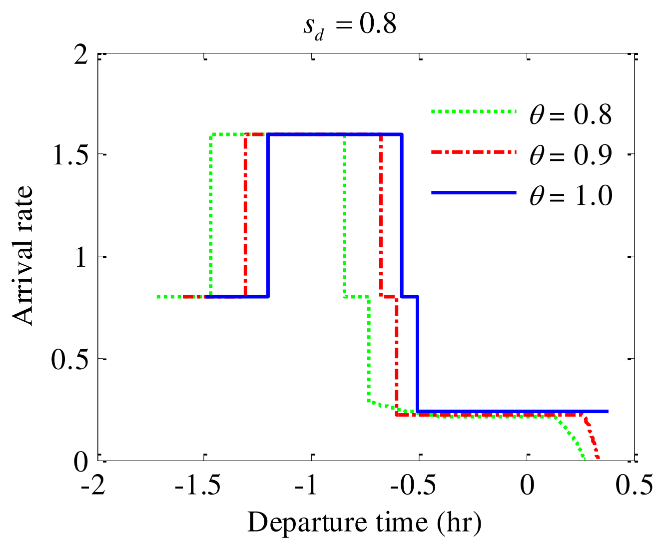

Fig. 10. Influence of parameter θ on the arrival rate at downstream bottleneck under fixed-rate merging.

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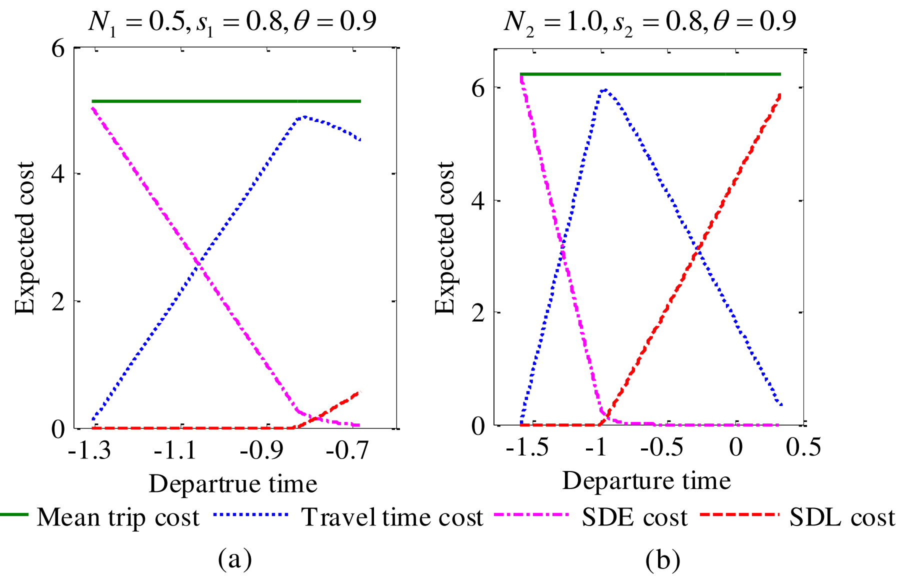

Fig. 11. Mean trip cost and its individual component costs of (a) group 1 and (b) group 2 under fixed-rate merging.

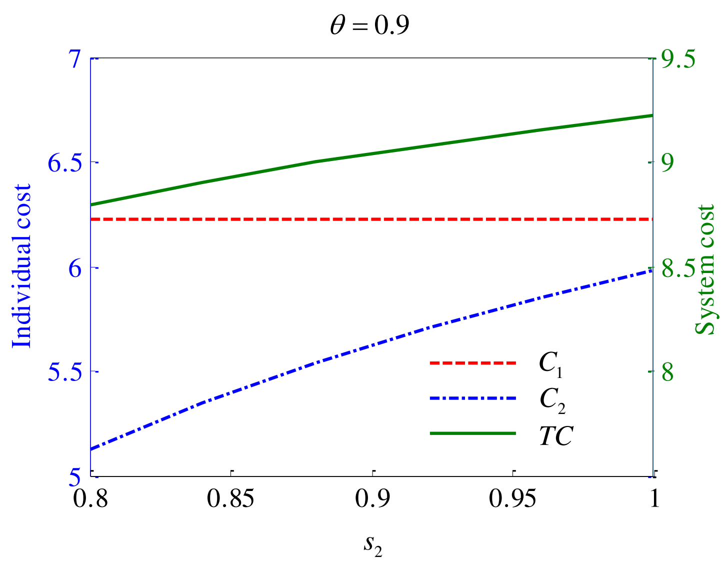

Fig. 12. Variation of the individual cost and the system cost with s_2 under fixed-rate merging rule.

Figure 12 shows the variation of the individual trip cost of both group and the system cost with capacity s_2. It is clear that the mean trip cost of group 1 and the system cost both increase with increasing capacity s_2, whilst the individual trip costs of group 2 remain unchanged. This is consistent with Theorem 2.

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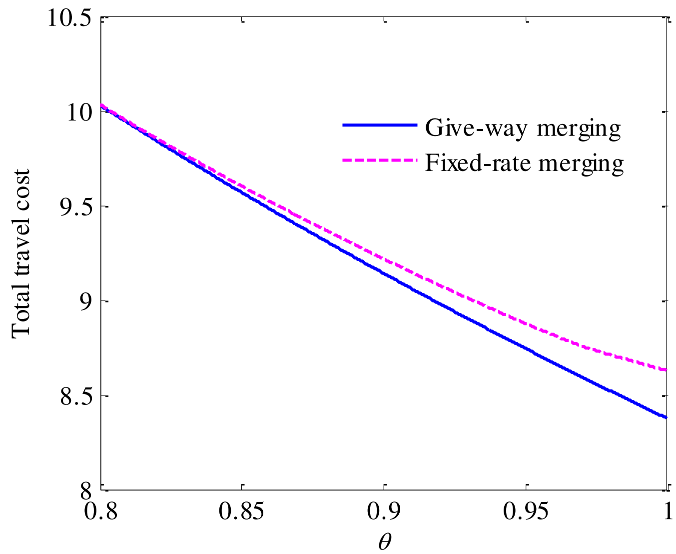

Fig. 13. Variation of the total travel cost with θ -value under both merging rules.

Fig. 13 shows the total travel costs (system cost) generated under both give-way metering and fixed-rate metering rules. Under both metering rules, the total travel cost decreases as the θ -value increases. The cost difference between the two rules increases with θ -value and reaches its maximum at $\theta = 1.0$.

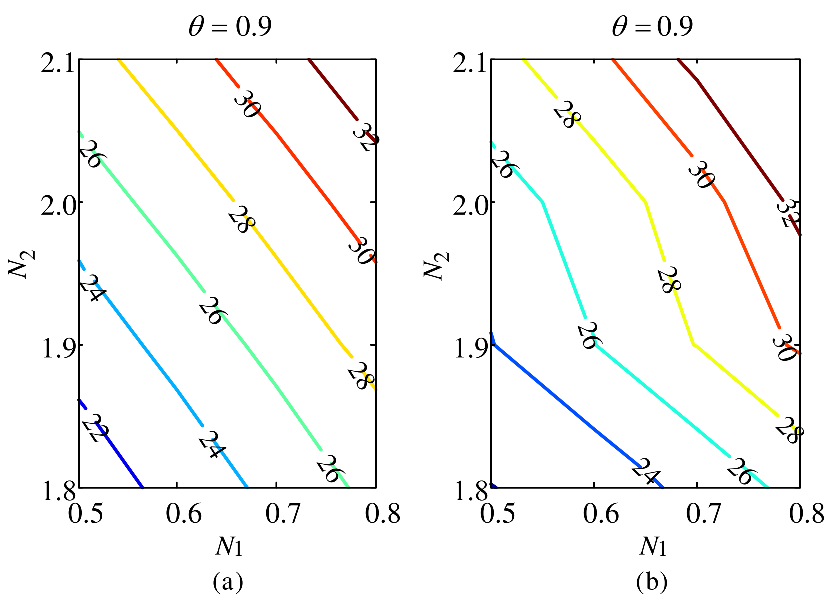

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Fig.14. Variation of the system trip cost against various demands under (a) the give-way merging and (b) the fixed-rate merging.

Figure 14 depicts the system trip costs for both rules when the demands N_1 and N_2 between the intervals [0.5, 0.8] and [1.8, 2.1], respectively. It is clear that the total trip cost increases with respect to the demands N_1 and N_2 growing up under both merging rules.

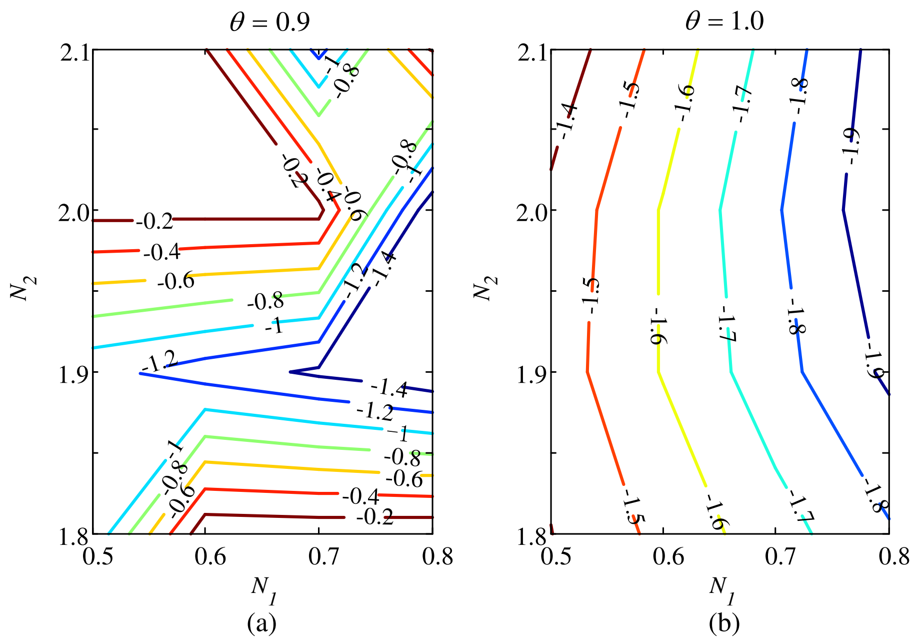

Fig. 15. Variation of the difference in system trip cost between the two merging rules against various demands under (a) a stochastic bottleneck and (b) a deterministic bottleneck.

Finally, we investigate the changes of the system trip costs under both merging rules. We vary the demands N_1 and N_2 between the intervals [0.5, 0.8] and [1.8, 2.1], respectively. We show in Fig. 15 contour plots how the difference in total system trip cost between the two merging rules varies with upstream demands N_1 and N_2. The numbers in Fig. 15 represent the difference between the system trip cost under the give-way rule and that under the fixed-rate rule. We can see that the system trip cost under the give-way merging rule is always smaller than that under the fixed-rate merging rule. When the value of parameter θ is one, the difference becomes larger. This conclusion is

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subject to the conditions that $N_1 < s_1$ and $N_2 > s_2$, which is due to the assumption for the give-way merging but not the fixed rate merging rule.

4.3. Remarks

This paper extends the classic bottleneck model to consider a Y-shaped merge network and day-to-day degradation of downstream bottleneck capacity simultaneously. The purpose of this paper is first to capture the commuters' departure time choice behavior with the minimized expected travel cost. The paper also aims to demonstrate that the capacity increasing paradox also occurs under different merging rules by considering travel time variability, in particular, in terms of stochastic merge capacity.

Two merging rules are considered for this Y-shaped corridor network: *give-way merging* and *fixed-rate merging*. For the *give-way merging*, it states that traffic from link 2 is controlled, whilst for the *fixed-rate merging*, traffic from both upstream links are controlled and they merge at a rate not exceeding their respective bottleneck capacity s_1 and s_2. Equilibrium departure time patterns are derived for both cases.

It is observed that there are four possible arrival-time intervals in this corridor when users always arrive early, they can arrive early or late, always arrive late and incur a queuing delay, or always arrive late and may not incur a queuing delay. Under the *give-way rule*, group 1 can only experience the first two situations. However, when both upstream links are controlled in the *fixed-rate rule*, group 1 would experience the first three situations, not the last one.

Our analytical and numerical results suggest that, when experiencing uncertainty in network supply (represented in terms of bottleneck capacity here), commuters will respond by shifting their temporal travel patterns. They compensate for the uncertainty by departing earlier. The overall peak period is longer and total travel cost is higher with larger stochasticity in bottleneck capacity.

Capacity paradoxes are found under both give-way and fixed-rate rules for upstream link 2, such that the total network trip cost (the system trip cost) rises as the capacity of the upstream capacity increases, but the effectiveness on the two groups are different. Under both rules, an upstream capacity increase results in an increase of individual trip

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cost of group 1, however, the mean trip cost of group 2 remains unchanged. Furthermore, we find that, compared to *give-way merging*, the *fixed-rate merging* can advance the earliest departure time of group 1, whilst the earliest departure time for group 2 is unaffected.

These results have strong policy implications. Firstly, the increased travel costs and lengthened peak period under stochastic conditions both have impact on the evaluation of a network performance, and need to be appropriately accounted for in the formal appraisal of a congested network and of a new transport scheme. Secondly, the capacity paradox identified here suggests that expanding network capacity, if not fully considering the reactions of travelers, could adversely reduce the efficiency of a congested network. This suggests that it may be counter-productive to solve road congestion by increasing road capacity, at least in the short term. Furthermore, the design and construction of highway networks should be carefully determined in terms of layout and control rules, as our results show that depending on the control mechanism (merging rules), metering one upstream bottleneck can be beneficial, whilst metering the other can be harmful.

It is worth noting that, in this paper, a simple uniform capacity distribution is adopted, and a simple travel cost function of departure time choice and simple user equilibrium (UE) condition are assumed. These assumptions are made to facilitate an analytical solution. It may be possible that some of these assumptions could be relaxed; we will investigate this in our future work.

There are a number of possible extensions to the existing study on stochastic capacity. Firstly, it may be possible to extend the analysis on the capacity paradox to a more general network by exploiting the analytical formula of the solution derived in this paper. It would be interesting to see if the capacity paradox exists in a general network. Secondly, it would be interesting to analyze more realistic cases where the assumption of UE condition is relaxed, for example, to consider the variance of trip cost along with travelers' risk preferences in choosing an appropriate departure time (see Siu and Lo, 2013). Finally, the studies on stochastic capacity should be extended to consider the case with physical queues which has shown to cause very complex phenomena (e.g. Lago and Daganzo, 2007); comprehensive studies on this topic would be indispensable for a clear understanding of the properties of dynamic network flows.

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5. Conclusions

In this paper, we extend the Vickrey's bottleneck theory to include a stochastic bottleneck capacity in a Y-shaped merge network. The stochastic capacity is considered to be a result of the merging interactions. Each commuter using the merge network is assumed to have the same travel cost function which consists of time-varying costs due to queuing delay (waiting time in a queue) and schedule delay (the time difference between his/her actual and desired arrival time at the work place). To obtain the equilibrium traffic pattern of this model, we assumed the downstream capacity follows a uniform distribution and the commuters' departure time choice follows UE principle in terms of their mean trip costs.

Considering the possibility that some commuters pass of one or both bottlenecks during the morning peak, we have developed the model under two merging rules, namely a *give-way merging* and a *fixed-rate merging*. We derive the analytical solutions and provide numerical results for both scenarios. The results show that uncertainty in the downstream bottleneck capacity increases the commuters' mean trip cost and lengthens the peak period. Moreover, a capacity paradox is found under both merging rules for bottleneck 2, such that expanding one of the upstream bottlenecks may adversely increase the total network trip cost (the system trip cost). Furthermore, we find that, compared to *give-way merging*, the *fixed-rate merging* can advance the earliest departure time of group 1, whilst the earliest departure time for group 2 remains unchanged.

The contribution of this paper to the existing literature (such as Arnott et al., 1993b; Lago and Daganzo, 2007; Daniels et al., 2009), is on the consideration of a stochastic bottleneck capacity in a merging network under two different merging rules. We show empirical observations on capacity fluctuations (Fig.1), and demonstrate through our modeling results the impact of stochastic capacity on trip cost and travel patterns. Understanding such cause-and-effect would help transport managers to better predict the impact of network supply changes on travel patterns and resulting traffic congestion. Furthermore, the study reveals a capacity paradox in that increasing the capacity of one

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of the upstream links would result in an increase in total travel cost. Such modeling result will also have real-world implication, for example, in deciding whether/where a capacity improvement scheme is due and what affect that may have.

For future work, we will further extend the stochastic merge model to investigate various congestion toll schemes and metering policies. In addition, we will consider risk preference, demand uncertainty, multiple transport modes and flexible work schedule in the model development.

Acknowledgments

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Appendix 1. Derivation of arrival rate at downstream bottleneck for give-way merging

Ignoring the travel time along the link from the origin to the merging point, the departure time window $[t_2^0, t_2^e]$ of group 1 is also the arrival time window of commuters at downstream bottleneck. We consider here the equilibrium arrival behaviour at the stochastic downstream bottleneck. The time window $[t_2^0, t_2^e]$ is divided into four intervals according to the schedule delay and queuing delay experienced by commuters. We consider $t^* = 0$ for simplicity.

In $[t_2^0, t_\beta]$, there is no schedule delay late, the expected travel cost of a commuter who arrives at downstream bottleneck at time t, is

$$E\left[C(t)\right] = \alpha \int_{\theta s_d}^{s_d} \left(\frac{R(t)}{s} + t_2^0 - t \right) f(s) \mathrm{d}s + \beta \int_{\theta s_d}^{s_d} -\left(\frac{R(t)}{s} + t_2^0 \right) f(s) \mathrm{d}s,$$

where $f(s)$ is the probability density function of the stochastic capacity, $f(s) = 1/(s_d - \theta s_d)$. At equilibrium, $E\left[C(t)\right]$ must be constant, i.e. $\mathrm{d}E\left[C(t)\right]/\mathrm{d}t = 0$ which directly leads to

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$$r(t)=\frac{\alpha}{\alpha-\beta}\tilde{\theta}s_d,\quad t_2^0\le t\le t_\beta,$$

where $\tilde{\theta}=(1-\theta)/\ln\theta^{-1}$.

In $(t_\beta,t_\gamma]$, both schedule delay early and late may occur, we then have

$$E\left[C(t)\right]=\alpha\int_{\theta s_d}^{s_d}\left(\frac{R(t)}{s}+t_2^0-t\right)f(s)\mathrm{d}s+\beta\int_{-R(t)/t_2^0}^{s_d}-\left(\frac{R(t)}{s}+t_2^0\right)f(s)\mathrm{d}s$$

$$+\gamma\int_{\theta\overline{s}}^{-R(t)/t_2^0}\left(\frac{R(t)}{s}+t_2^0\right)f(s)\mathrm{d}s.$$

Letting $\mathrm{d}E\left[C(t)\right]/\mathrm{d}t=0$ leads to

$$r(t)=\frac{\alpha}{A+B\left(\ln R(t)+1\right)},\quad t_\beta<t\le t_\gamma,$$

where $A=\dfrac{(\alpha+\gamma)\ln\theta^{-1}-(\beta+\gamma)\left(\ln(-t_2^0s_d)+1\right)}{(1-\theta)s_d}$, $B=\dfrac{\beta+\gamma}{(1-\theta)s_d}$.

In $(t_\gamma,t_s]$, there is no schedule early, we have

$$E\left[C(t)\right]=\alpha\int_{\theta s_d}^{s_d}\left(\frac{R(t)}{s}+t_2^0-t\right)f(s)\mathrm{d}s+\gamma\int_{\theta s_d}^{s_d}\left(\frac{R(t)}{s}+t_2^0\right)f(s)\mathrm{d}s.$$

Letting $\mathrm{d}E\left[C(t)\right]/\mathrm{d}t=0$ gives

$$r(t)=\frac{\alpha}{\alpha+\gamma}\cdot\frac{s_d(1-\theta)}{\ln\theta^{-1}}=\frac{\alpha}{\alpha+\gamma}\tilde{\theta}s_d,\quad t_\gamma<t\le t_s.$$

In $(t_s,t_2^e]$, there is no schedule early but queue may exist. We can find a watershed capacity of the bottleneck such that the queuing length equals zero, i.e. $R(t)=s\left(t-t_2^0\right)$, and hence the watershed capacity is $R(t)/\left(t-t_2^0\right)$. We then have

$$E\left[C(t)\right]=\alpha\int_{\theta s_d}^{\frac{R(t)}{t-t_2^0}}\left(\frac{R(t)}{s}+t_2^0-t\right)f(s)\mathrm{d}s+\gamma\int_{\theta s_d}^{\frac{R(t)}{t-t_2^0}}\left(\frac{R(t)}{s}+t_2^0\right)f(s)\mathrm{d}s+\gamma\int_{\frac{R(t)}{t-t_2^0}}^{s_d}tf(s)\mathrm{d}s.$$

Letting $\mathrm{d}E\left[C(t)\right]/\mathrm{d}t=0$ leads to

$$r(t)=\frac{(\alpha+\gamma)R(t)/\left(t-t_2^0\right)-(\alpha\theta+\gamma)s_d}{(\alpha+\gamma)\left(\ln R(t)-\ln\left(\theta s_d\left(t-t_2^0\right)\right)\right)},\quad t_s<t\le t_2^e.$$

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Appendix 2. Proof of Theorem 1

Differentiating Eq. (34) with respect to s_d and s_2, respectively , we have

$$\partial C_2^* / \partial s_d = \frac{N_1 + N_2}{\left(s_d\right)^2} \cdot \frac{\beta}{\left(k_2 - 1\right)\xi},$$

$$\partial C_2^* / \partial s_2 = 0,$$

where $\xi = \left(\alpha\theta + \gamma\right) / \left(\alpha + \gamma\right)$. Therefore, to prove $\partial C_2^* / \partial s_d < 0$, we only need to prove $k_2 - 1 < 0$. Since $\beta < \gamma$, and beta is a positive value, then $-\beta < \gamma$. From Eq. (25), we have

$$k_2 = \frac{(\alpha+\gamma)\phi - \beta}{(\alpha+\gamma)\phi + \gamma} < \frac{(\alpha+\gamma)\phi + \gamma}{(\alpha+\gamma)\phi + \gamma} = 1.$$

Thus, $k_2 - 1 < 0$ holds.

Substituting Eq. (24) and Eq. (32) into Eq. (35) and re-arranging it, we get

$$C_1^* = -\frac{\left(\beta+\gamma\right)}{\left(1-\theta\right)}\left(\frac{N_1^e}{s_d} + \theta t_2^0\right) + \alpha\frac{N_1}{s_2} - \beta t_2^0,$$

where $N_1^e = \exp\left(\frac{\alpha / s_2 - \left(A+B\right)}{B}\right)$. Taking the first-order derivative with respect to s_d and s_2, respectively, leads to

$$\partial C_1^* / \partial s_d = -\frac{\beta+\gamma}{1-\theta}\left(\frac{\partial\left(N_1^e / s_d\right)}{\partial s_d}\right),$$

$$\partial C_1^* / \partial s_2 = -\frac{\beta+\gamma}{s_d\left(1-\theta\right)} \cdot \frac{\partial N_1^e}{\partial s_2} - \alpha\frac{N_1}{\left(s_2\right)^2}.$$

For proving $\partial C_1^* / \partial s_d < 0$, we only need to prove the derivative of N_1^e / s_d be positive. This is true as

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$$\frac{\partial\left(N_1^e/s_d\right)}{\partial s_d}=\frac{2-(1-\theta)\left(\alpha/(\beta+\gamma)\right)\left(s_d/s_2\right)}{\left(s_d\right)^2}N_1^e>\frac{N_1^e}{\left(s_d\right)^2}>0.$$

So, $\partial C_1^*/\partial s_d<0$ holds.

Substituting N_1^e into C_1^*, using the equation $R_d(t_1^e)=N_1+s_2\left(t_1^e-t_2^0\right)>N_1$ and differentiating C_1^* with respect to s_2, we obtain

$$\partial C_1^*/\partial s_2=-\frac{(\beta+\gamma)}{s_d(1-\theta)}\cdot\frac{\partial N_1^e}{\partial s_2}-\alpha\frac{N_1}{\left(s_2\right)^2}=N_1^e\frac{\alpha}{\left(s_2\right)^2}-N_1\frac{\alpha}{\left(s_2\right)^2}>0.$$

Noting $TC=N_1C_1^*+N_2C_2^*$, we then conclude

$$\frac{\partial TC}{\partial s_d}=N_1\frac{\partial C_1^*}{\partial s_d}+N_2\frac{\partial C_2^*}{\partial s_d}<0,$$

$$\frac{\partial TC}{\partial s_2}=N_1\frac{\partial C_1^*}{\partial s_2}+N_2\frac{\partial C_2^*}{\partial s_2}>0.$$

This completes the proof.

Appendix 3. Proof of Theorem 2

From Theorem 1 Eqs. (24) and (29), we have

$$t_1^0-t_2^0=-\frac{\tilde{\theta}s_d}{s_2}t_2^0-\frac{(\alpha+\gamma)\left(s_2/(\tilde{\theta}s_d)\right)-\alpha+(\alpha+\gamma)\left(s_1/(\tilde{\theta}s_d)\right)}{\gamma+\beta}\cdot\frac{N_1}{s_1}\cdot\frac{\tilde{\theta}s_d}{s_2}.$$

By denoting

$$h=-t_2^0-\frac{(\alpha+\gamma)\left(s_2/(\tilde{\theta}s_d)\right)-\alpha+(\alpha+\gamma)\left(s_1/(\tilde{\theta}s_d)\right)}{\gamma+\beta}\cdot\frac{N_1}{s_1},$$

we get $t_1^0-t_2^0=h\tilde{\theta}s_d/s_2$. To be consistent with the premise $C_2^*>C_1^*$, $t_1^0-t_2^0>0$ is required, then $h>0$.

Considering the equilibrium condition $C_1^*=E\left[C_1(t_1^0)\right]$, we have

$$C_1^*=\left((\alpha-\beta)\left(s_2/(\tilde{\theta}s_d)\right)-\alpha\right)\left(t_1^0-t_2^0\right)-\beta t_2^0.$$

Note that $\partial t_2^0/\partial s_1=0$ and $\partial t_2^0/\partial s_2=0$ hold. The first derivate of the travel cost C_1^* with respect to s_1 and s_2, respectively, can be written as follows,

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$$\frac{\partial C_1^*}{\partial s_1} = \left((\alpha - \beta) - \alpha \left(\tilde{\theta} s_d / s_2 \right) \right) \frac{\partial h}{\partial s_1},$$

$$\frac{\partial C_1^*}{\partial s_2} = \alpha \frac{\tilde{\theta} s_d}{(s_2)^2} h + \left((\alpha - \beta) \left(s_2 / (\tilde{\theta} s_d) \right) - \alpha \right) \frac{\tilde{\theta} s_d}{s_2} \frac{\partial h}{\partial s_2}.$$

Since $h > 0$, $(\alpha - \beta)\left(s_2 / (\tilde{\theta} s_d) \right) - \alpha < 0$ and

$$\frac{\partial h}{\partial s_1} = \frac{(\alpha + \gamma)\left(s_2 / (\tilde{\theta} s_d) \right) - \alpha}{\gamma + \beta} \cdot \frac{N_1}{(s_1)^2} > 0,$$

$$\frac{\partial h}{\partial s_2} = -\frac{\alpha + \gamma}{\gamma + \beta} \cdot \frac{1}{\tilde{\theta} s_d} \cdot \frac{N_1}{s_1} < 0,$$

$\partial C_1^* / \partial s_1 < 0$ and $\partial C_1^* / \partial s_2 > 0$ hold.

Let $g = (\alpha - \beta)\left(s_2 / (\tilde{\theta} s_d) \right) - \alpha$, and the first derivate of the travel cost C_1^* with respect to s_d can be calculated as follows,

$$\frac{\partial C_1^*}{\partial s_d} = \frac{\partial C_2^*}{\partial s_d} + \left(t_1^0 - t_2^0 \right) \frac{\partial g}{\partial s_d} + g \frac{\partial}{\partial s_d} \left(t_1^0 - t_2^0 \right),$$

Since $\partial \left(t_1^0 - t_2^0 \right) / \partial s_d = \alpha \tilde{\theta} N_1 / \left((\gamma + \beta) s_1 s_2 \right) > 0$, $\partial g / \partial s_d = (\beta - \alpha) s_2 / \left(\tilde{\theta} {s_d}^2 \right) < 0$, $g < 0$, $t_1^0 - t_2^0 > 0$, and from the Equation (34), we get $\partial C_2^* / \partial s_d < 0$. Hence $\partial C_1^* / \partial s_d < 0$ hold.

Considering $\partial C_2^* / \partial s_1 = 0$, $\partial C_2^* / \partial s_2 = 0$ and , we furthermore conclude

$$\frac{\partial TC}{\partial s_1} = N_1 \frac{\partial C_1^*}{\partial s_1} + N_2 \frac{\partial C_2^*}{\partial s_1} < 0,$$

$$\frac{\partial TC}{\partial s_2} = N_1 \frac{\partial C_1^*}{\partial s_2} + N_2 \frac{\partial C_2^*}{\partial s_2} > 0,$$

$$\frac{\partial TC}{\partial s_d} = N_1 \frac{\partial C_1^*}{\partial s_d} + N_2 \frac{\partial C_2^*}{\partial s_d} < 0.$$

This completes the proof.

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