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A MODEL FOR INTEGRATING HOME-WORK TOUR SCHEDULING WITH TIME-VARYING NETWORK CONGESTION AND MARGINAL UTILITY PROFILES FOR HOME AND WORK ACTIVITIES

Muhammad Adnan
Corresponding Author,
PhD Student, Institute for Transport Studies, University of Leeds,
Leeds, England, LS2 9JT
Phone: +44 (0) 113 34 31792
E-mail: **m.adnan04@leeds.ac.uk**
Fax: +44 (0)113 343 5334

David Watling
Institute for Transport Studies, University of Leeds,
Leeds, England, LS2 9JT
Phone: +44 (0) 113 34 36612
E-mail: **D.P.Watling@its.leeds.ac.uk**
Fax: +44 (0)113 343 5334

Tony Fowkes
Institute for Transport Studies, University of Leeds,
Leeds, England, LS2 9JT
Phone: +44 (0) 113 34 35340
E-mail: **A.S.Fowkes@its.leeds.ac.uk**
Fax: +44 (0)113 343 5334

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ABSTRACT:
The existing literature in the activity-based modelling have emphasised the fact that individuals schedule their activities keeping whole-day activity pattern in their mind. Several attempts have been made to integrate this with the network congestion; however, for explicit explanation of travel behaviour of individuals, further improvements are inevitable. In this paper, a combined model is proposed that deals with the scheduling of the home-work tour with time-varied network congestion in a fixed point problem framework. Marginal utility profiles that represent individual time-of-day preferences and satiation effect of the activities are incorporated for the measurement of utility of activity engagement along with the disutility of travel. It has been noticed that consideration of only time-of-day dependent marginal utility profiles of activities in the utility function does not appropriately integrate the activities and travel within the tour. The proof of this has been shown analytically and numerically. This finding contradicts with the earlier researches that have been done to integrate morning-evening commutes together with the network congestion. Additionally, two numerical experiments are conducted and the results are presented in the paper. In the first experiment, an arbitrary dynamic tolling strategy is assumed and then a detailed analysis is performed to show the variation in the balance of trade-offs involved in the process. The second experiment is conducted to assess the sensitivity of the combined model through incorporation of different dynamic traffic loading models. Some meaningful observations are drawn from these experiments and are discussed with the identification of avenues for future research.

INTRODUCTION:

Modelling congestion in conjunction with trip scheduling has been an active area of research over the last four decades. The work presented by Vickrey [1] has remained seminal in this regard. Further extensions and refinement of his work in many dimensions have resulted in a well-known scheduling theory for the morning commute(MC). Attempts have also been made to integrate the MC with the evening commute(EC) through the same framework, with an argument that scheduling of the MC may well depend on the travel cost of the return to home trip, the duration of the work activity and variation of the utility of the work activity with its start and end times [2,3]. Several empirical studies have also recognised the fact that due to growing concerns about congestion on the road network and policies that are aimed at reducing it (e.g. road pricing), people tend to change their activity schedules. Such changes may involve change in departure times, work activity duration, changes in modes and route choice etc[4,5]. Recently, Lam and Yin [6] and Lam and Huang [7] proposed a discrete choice framework in discrete times to model activity, destination and route choice together. They adopted a variational inequality-based formulation in order to assign traffic dynamically and brought out mutual consistency between activity choices and travel times. However, the modelling framework does not model activity duration, which is considered as a vital dimension for linking the MC and EC together [2]. Abdelghany and Mahmassani [8] formulated and analyzed a stochastic dynamic user equilibrium (SDUE) problem in which drivers simultaneously seek to determine their departure time, route choice and sequence of their intermediate activities at the origin to minimize their disutility for travel. Their modelling framework is limited to the MC only with three intermediate stops i.e. it does not deal with the complete activity pattern of an individual for a whole day and also in their model duration at intermediate stops of the MC is treated as exogenously.

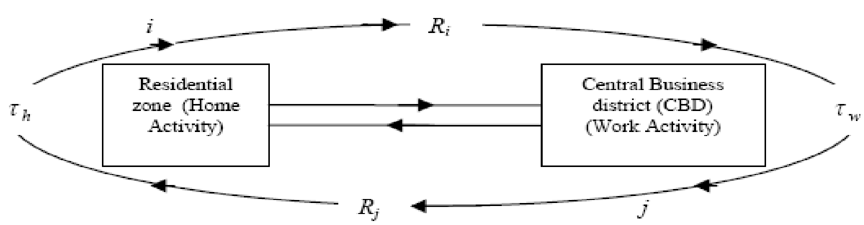
To overcome the deficiencies of earlier works, Zhang et al [2] investigated variation in the departure time within-a-day for the home-work tour as a trade-off between travel cost and the utility of participation in the work activity. The home-work tour is linked with work duration and the model follows a hierarchical nested logit structure utilising a utility framework proposed by Ettema et al [3]. In addition to this, they established an equilibrium condition between the schedule choice pattern and network congestion by solving a fixed-point problem. Their modelling framework utilised bottleneck model for estimation of travel time on a single link between home and work locations, which was then fed into the utility function. A similar sort of work has been presented by Kim et al [9] with the only difference that instead of using the Bottleneck model at supply side, they have utilised DYNASMART (dynamic traffic assignment package) to assign traffic and obtain time-dependent travel times which are feedback to demand model to achieve demand-supply equilibrium. Polak and Heydecker [10] also presented a similar kind of work i.e. combined modelling of home-work tour with dynamic network congestion, with a difference that utility function of their model do not incorporate a random error term i.e. they model home-work tour in manner that it provides deterministic user equilibrium. In all recent work of activity scheduling modelling with network congestion i.e. [2,7,9,10], it has been noticed that the utility specification of their model includes a component that measures the utility of activity engagement. This has been calculated through predetermined time-of-day dependent marginal utility (MU) function/profile for a particular activity. However, in a recent paper of Ettema et al [11], a model is proposed which only deals with the demand side. It is reported that duration based MU function (which represents activity satiation effects) and incorporation of scheduling constraint may also make significant contribution in combination with clock-time based MU function for proper measurement of utility of activity engagement.

In this paper, a combined home-work tour scheduling model is presented, which brings the system in stochastic dynamic user equilibrium (SDUE), under the above mentioned framework and a detailed investigation is carried out for time-of-day dependent MU functions. It has been proved analytically and numerically that clock-time based MU function for home and work activities do not able to integrate MC and EC together and thus not serve the purpose of the combined modelling. This finding supports the recent Ettema et al [11] work, however, contradicts with the earlier attempts that were made to integrate home-work tour scheduling with network congestion. Additionally, the model presented here can be considered as a generalised model as various forms of discrete choice models can be incorporated at the demand side and in a similar manner variety of supply models (dynamic network loading models) that fulfils the desirable properties for dynamic traffic assignment [12] (e.g. flow propagation, conservation, First-in-First-out, causality, etc.) can also be assimilated. The paper is structured as follows; second section contains a generic illustration of the combined model. Third section describes details of the utility framework of the model with proposed model refinements. Fourth section presents results obtained from the numerical experiments with some meaningful observations and finally conclusion is made with some discussion on future research.

HOME-WORK TOUR COMBINED MODEL

Generic Mathematical Illustration:

We are considering the home-work tour; that contains an origin-destination pair representing home and work locations, connected with a single link between them, and car as only mode of travel. In the context of the above simplified version of home-work tour, the scheduling dimensions involved here are the choice of departure times for work and from work Also there is no question of modelling activity sequence as only two activities are involved. Durations of work and home activities are implicit in this structure and are derived from departure times along with the travel time to get to work and home. Figure 1 further explains this framework in detail. Activity scheduling for this tour can be defined by a pair of discrete departure times from home and work activity denoted by *i* and *j* respectively.



i and *j* are departure times (i.e. clock-times) from home and work location respectively
R_i and R_j are travel times on the link at their respective departure times for the morning and evening commute respectively
τ_w and τ_h are duration of work and home activity and are given by
$\tau_w = j-(i-R_i)$, $\tau_h = 1440-(\tau_w + R_i + R_j)$
Time unit is taken in minutes and a full day is considered that comprises 1440minutes

FIGURE 1 Home-Work Tour Time Cycle

$$\text{Scheduling for home-work tour} = (\text{departure time from home, departure time from CBD}) = (i, j) \in \mathbb{Z}^2$$

where, i = 1, 2, . . . , X; $j = \left(X + R_X + \tau'_w\right)$, . . . 1440; and they belongs to $\mathbb{Z}$ representing the set of integer numbers, we are considering here the duration of each departure time period as one minute. X represents the maximum possible departure period for the MC keeping in mind the minimum feasible duration of work activity τ'_w. Suppose that $\mathbb{K}$ is the set of all possible combinations of (i, j) pairs i.e

$$\mathbb{K} = \left\{(i, j) : \; i \in Z, \;\; j \in Z, \;\; 1 \le i \le X, \; \left(X + R_X + \tau'_w\right) \le j \le 1440\right\}$$

The overall utility of activity scheduling for this tour, according to Ettema et al [3] utility maximisation framework, (which is also adopted in various other researches i.e.[2,9,10]) can be expressed as

$$\max \; V_{ij} = \max\left(V^A + V^T\right) \tag{1}$$

where, V^T is the total utility derived from the travel and V^A is the total utility derived from participation in activities. V^T *and* V^A are themselves the sum of utilities of m number of travels between the activities and n number of activities respectively and are given by

$$V^T = \sum_{m=1}^{m} V^{T_m} \tag{2}$$

$$V^A = \sum_{n=1}^{n} V^{A_n} \tag{3}$$

In the above specification, the utility of a travel made at time t is characterized by its travel time and travel cost. The utility derived from activity participation is dependent on time spend on the activity location. The above three equations can be termed as a generalized utility framework that can accommodate all types of individual daily activity patterns. For home-work tour, which is the prime concerned here, the total utility can be given as

$$V_{ij} = V^h + V^w + V^{T_{h-w}} + V^{T_{w-h}} \tag{4}$$

where, V^h represents utility gained by spending time at home, V^w represents benefits obtained by spending time at work, $V^{T_{h-w}}$ and $V^{T_{w-h}}$ are the utilities of travel from home to work and work to home respectively which are considered negative here. In this way individuals need to trade-off between the overall travel cost of the two travels and benefits gained in participation in home and work activities when taking decision of scheduling for the tour. We will discuss the forms of home and work utility in detail in the next section; however it can be noted that keeping these terms constant for a while the utility of home-work tour scheduling is dependent on travel times R_i and R_j which are actually included in terms $V^{T_{h-w}}$ and $V^{T_{w-h}}$. Therefore, it can be written as

$$V_{ij} = f\left(R_i, R_j\right) \tag{5}$$

To make the above utility framework operational, it is required to estimate probabilities of choosing scheduling of activities in a tour by using random utility model. For example, for the Multinomial logit (MNL) model,

$$P_{ij} = g\left(V_{ij}\right) \qquad \forall (i, j) \in \mathbb{K} \tag{6}$$

and for other discrete choice models i.e. nested logit (NL)

$$P_{ij} = g\left(V_{ij}; \underline{\omega}\right) \qquad \forall (i, j) \in \mathbb{K} \tag{7}$$

where, P_{ij} = Probability of choosing alternative (i , j) in activity scheduling choice set of this tour and $\underline{\omega}$ = Vector of additional parameters in the model form. Suppose that Q is the total number of individuals in the residential zone that are assumed to conduct this type of tour and are fixed in number. The choice rate of individuals who will depart from home and work at time i and j respectively is given by

$$q_{ij} = Q\,P_{ij} \tag{8}$$

The number of trips at departure time *i* from home to work q_i can be determined by summing over all the combined choices q_{ij} over the departure time j and the same strategy can be applied to determine the number of trips at departure time j from work to home q_j , Mathematically,

$$\begin{aligned} q_i &= \sum_j q_{ij} \\ q_j &= \sum_i q_{ij} \end{aligned} \tag{9}$$

As already mentioned, the supply side of the combined model provides dynamic representation of congestion on the network through estimation of time-dependent travel times. For this purpose, whole-link models have been utilised that requires inflow profiles which are basically the outcome of demand side i.e. equation (9) and (10). Here we are using whole link models in which travel time of the vehicle entering at time *i* is considered as a linear function of number of vehicles exist on the link at time *i*. Therefore, for the morning trip i.e. trip from home to work, travel time R_i is given by

$$R_i = ff + c\left(x_i\right) \tag{10}$$

where, x_i represents the number of vehicles on the link at departure time i , *ff* is the free-flow travel time on the link and *c* represents inverse of the capacity of the link. According, to the linear whole-link model formulation [20], x_i is given by the flow conservation equation (i.e. difference between the cumulative inflows and outflows at time *i* assuming that link is empty at initial time) and according to the flow propagation equation, outflows from the link at time *i* are function of inflows, therefore it can be written as

$$x_i = \phi_i\left(\underline{q}_i\right) \tag{11}$$

where, $\underline{q}_i$ is the vector that represents inflow q_i for the MC, $_i$ is the functional parameter that ensures the compatibility of the above equation. From the above equation it can be shown that travel time experienced by vehicles at time *i* on the link is basically a function of inflows provided from demand side of the model such as

$$R_i = v_i\left(\underline{q}_i\right) \tag{12}$$

And in the similar manner, the travel time for the EC can be written as

$$R_j = v_j\left(\underline{q}_j\right) \tag{13}$$

Suppose that $\underline{R}_i$ and $\underline{R}_j$ are the vectors that represents profiles of travel times for trip to work and home respectively, and $\underline{R}$ is a vector that contains $\underline{R}_i$ and $\underline{R}_j$ as its elements and in the same manner q represents a vector whose elements are and then a fixed point problem can be formulated in a general way i.e.

$$\underline{q} = Q\,\underline{P}\left(\underline{V}\left(\underline{R}\left(\underline{q}\right)\right)\right) \tag{14}$$

where, $\underline{P}$ and $\underline{V}$ are two dimensional vectors containing elements P_{ij} and V_{ij} respectively, the above equation can be also be expressed as

$$\underline{q} = \left(\underline{q}_i, \underline{q}_j\right) = Q\,\underline{P}\left(\underline{V}\left(\underline{R}_i\left(\underline{q}_i\right), \underline{R}_j\left(\underline{q}_j\right)\right)\right) \tag{15}$$

The solution of equation (15) may result in a SUE equilibrium condition which may be defined as "*At SUE no motorist can improve his/her perceived utility of scheduling of the tour by unilaterally changing the schedules. This follows directly from the interpretation that of the choice probability as the probability that the perceived utility of the chosen schedule for the tour is the highest of all the schedules for the tour.*"

The following mathematical expressions represent an optimization problem of the above fixed point problem in order to find out solution for $\underline{x}$ from the standard minimization algorithms.

Let $\underline{x} = \left(\underline{q}_i, \underline{q}_j\right)$

min $S\left(\underline{x}\right) = \sum k^2\left(\underline{x}\right)$

The constraints are ; $\underline{x} = \left(\underline{q}_i, \underline{q}_j\right) \geq 0$ and $\sum \underline{q}_i = Q$, $\sum \underline{q}_j = Q$

where, $k\left(\underline{x}\right) = \left[\underline{x} - G\left(\underline{x}\right)\right]$ and $G\left(\underline{x}\right) = G\left(\underline{q}_i, \underline{q}_j\right) = Q\,\underline{P}\left(\underline{V}\left(\underline{R}_i\left(\underline{q}_i\right), \underline{R}_j\left(\underline{q}_j\right)\right)\right)$

UTILITY FRAMEWORK OF THE MODEL

The essential aspect of the model is lies within the utility specification for the scheduling of the tour. It is very clear from the above mathematical illustration that utility of scheduling home-work tour contains two major component, first one deals with utility of activity engagement and the another one caters for the utility of travel. We will start our discussion by further elaborating equation (4), which is rewritten here.

$$V_{ij} = V^h + V^w + V^{T_{h-w}} + V^{T_{w-h}} \tag{16}$$

The Utility of Activity Engagement

In the literature of activity scheduling, the major emphasis is given to find the proper ways to measure utility of participating in an activity. Polak and Jones [13] introduced the idea of time-of-day dependent MU i.e. there exist a MU (which may vary over time), expressing the utility gained from one additional time unit of activity participation. Many researchers has proposed various functional forms for the MU of an activity, however most common in literature are bell-shaped and piece-wise constant profiles dependent on clock time [2,3,9,14]. These profiles assumed that the MU of an activity is high for a preferred period and that it decreases if one moves further away from this period. Figure 2a shows MU functional forms for work activity with the general notion that work activity has higher MU during working hours i.e. 8 to 5.

The earlier works for scheduling of activities in the home-work tour context [2,3,9,10,15], have considered MU profiles for an activity as a function of clock-time only with a claim that their model integrates both MC and EC with time-varied network congestion. If this is considered as true then the activity engagement related components in equation (16) can be written as

$$V^h = \int_0^i V'^h(t)\,dt + \int_{j+R_j}^{1440} V'^h(t)\,dt \quad \text{and} \quad V^w = \int_{i+R_i}^{j} V'^w(t)\,dt$$

where, $V^{'h}$ and $V^{'w}$ are MU functions for home and work activities respectively, and these function may follow any form either bell-shaped or piece-wise constant profile which are actually dependent on time-of-day. It should be noted here that we have consider here full day, starts from midnight 0000hours and ends at 0000hours again at on the next day and the unit of time is taken in minutes past midnight. The MU functions for home and work activities is integrated over the time duration individual have spend while participating in these activities. So, the overall utility of activity engagement for home-work tour is given by

$$V^h + V^w = \int_0^i V^{'h}(t)dt \;+\; \int_{i+R_i}^{j} V^{'w}(t)dt \;+\; \int_{j+R_j}^{1440} V^{'h}(t)dt \qquad (17)$$

In the next sub-section, we will prove that if the MU of activities is taken only as a function of time or in other words individual time-of-day preference is only considered in measuring utility of activity engagement, then this utility specification doesn't properly integrate two commutes together i.e. there is no difference either two commutes are modelled in combination to each other or modelled separately.

Numerical Example of the Proof

The travel component in the utility formulation, which explains the disutility of travel an individual experiences during travel from one point to another, can be measured by considering actual travel times and cost spent on travelling. Therefore, the following can be written

$$V^{T_{h-w}} + V^{T_{w-h}} = \lambda R_i + \lambda R_j \qquad (18)$$

where, λ is the negative parameter, which represents the disutility individual experiences while travelling (in-vehicle disutility), this cannot be confused with travel time parameter that represent the value of time of the individual, which contains some other parameters that are part of the utility of activity engagement here. Therefore the total utility of the scheduling of home-work tour can be given as

$$V_{ij} = \int_0^i V^{'h}(t)dt \;+\; \int_{i+R_i}^{j} V^{'w}(t)dt \;+\; \int_{j+R_j}^{1440} V^{'h}(t)dt \;+\lambda R_i + \lambda R_j \qquad (19)$$

With the above specification of the utility function, the following assumptions are made to practically apply the model. Suppose that there are in total Q = 5000 commuters living in the residential zone shown in figure 1. Let us assume that the feasible departure time is from 0600hours to 1000hours for the MC and for simplicity we are assuming departure times for the EC as well from 1400hours to 1800hours; so the consideration of minimum possible duration for the work activity is implicit in these assumptions. Free-flow travel time on the link is considered as 10 minutes with a capacity equals 1800veh/hr. The in-vehicle travel-time parameter is assumed as λ = -0.08£/min. For home activity, bell shaped MU function is assumed which depends on clock-time. This represents that the utility of stay-at-home is supposed to be higher at early morning and evening than the day time because people prefer to stay at home for regular home activities in normal time such as having a family dinner, watching TV and sleeping. The functional form according to [2,3] is given by

$$V^{'h}(t) = h_0 - \frac{\beta \gamma U_0}{\exp[\beta(t-\alpha)]\,[1+\exp(-\beta(t-\alpha))]^{\gamma+1}}$$

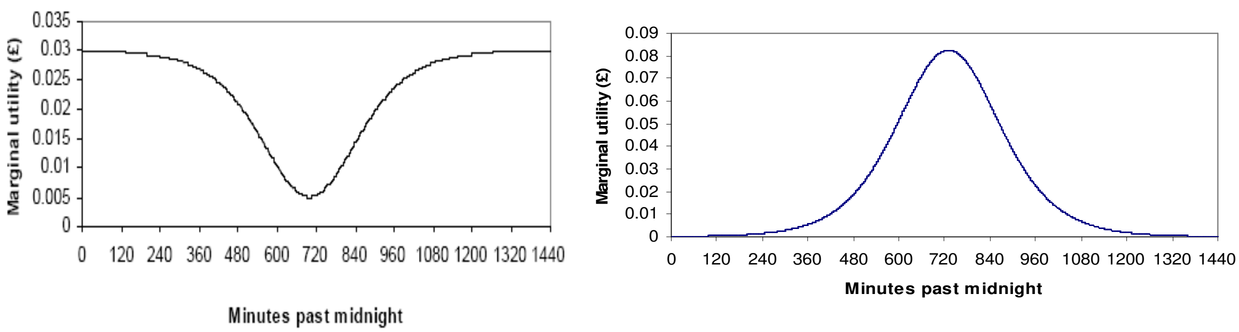
For work activity, the bell-shaped time-of-day dependent MU profile is assumed which provides high utility around mid-day with an argument that workers start to warm up after arrival at office and work most efficiently at about mid-day. In the afternoon, workers efficiency keeps declining until one leaves office. This is given by

$$V'^{w}(t)=\frac{\beta\,\gamma\,U_0}{\exp[\beta\,(t-\alpha)]\,[1+\exp(-\beta(t-\alpha))]^{\gamma+1}}$$

where, h_0, α, β, γ, U_0 are the parameters that controls the shape of the bell-shaped MU profiles, figure 2a and 2b shows the MU distribution with the following parameters values considered for the numerical proof

Home Activity: h_0=0.03, α = 700, β = 0.01, γ =1.0, U_0 =10 £.

Work Activity: α = 720, β = 0.01, γ =1.0, U_0 =30 £.



2(a) : home activity, V'^{h} **2(b) :** Work activity, V'^{w}

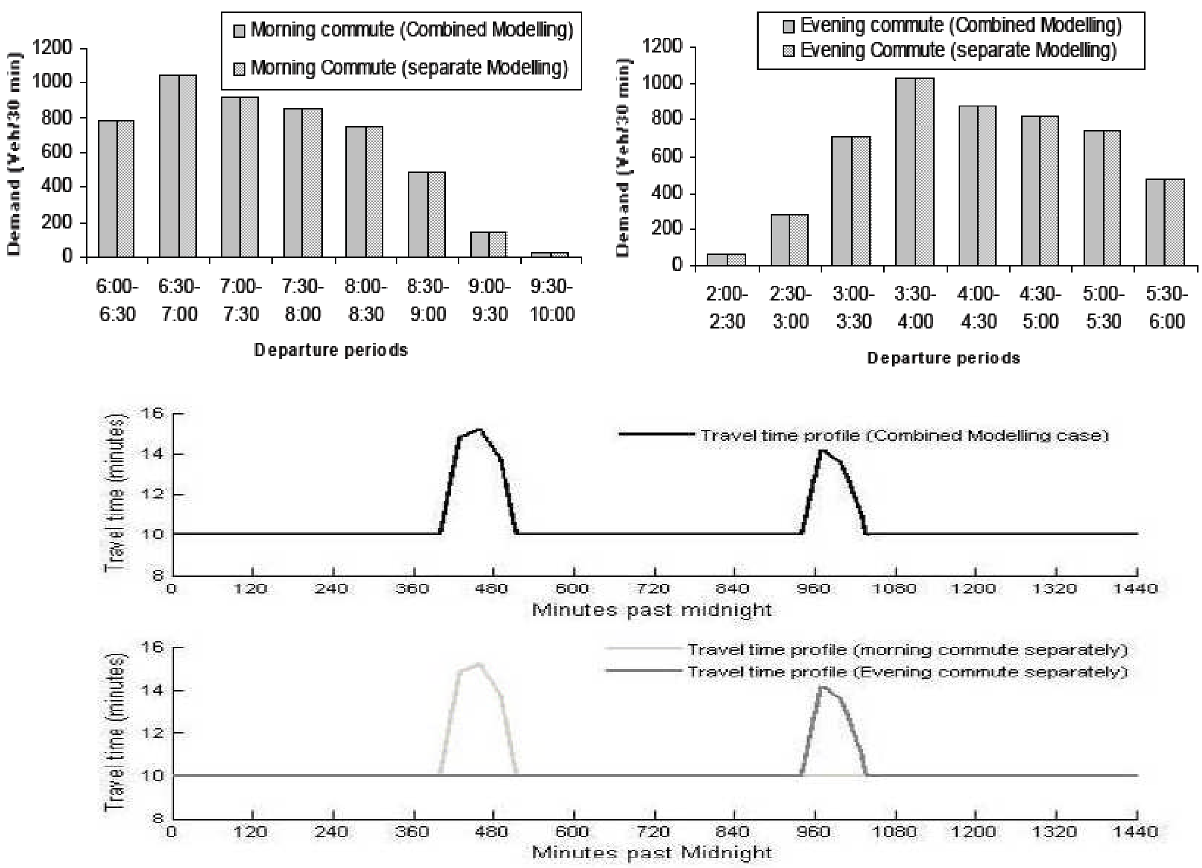


FIGURE 2 Marginal Utility of Activities, Demand and travel time profiles for combined and separate modelling cases

In the separate modelling case, to find out the utility for the MC; the MU of home and work activities are integrated over the half-day period, starting from midnight and end at 12 noon. For the EC utility, the remaining day is considered i.e. start at 12:00 noon and end at 12:00 midnight. The utility of activity scheduling for both these trips is given as

$$V_i = \int_0^i V'^h(t)dt + \int_{i+R_i}^{720} V'^w(t)dt + \lambda R_i \quad \text{, for MC} \tag{20a}$$

$$V_j = \int_{720}^j V'^w(t)dt + \int_{j+R_j}^{1440} V'^h(t)dt + \lambda R_j \quad \text{, for EC} \tag{20b}$$

The travel times for the MC and EC are derived from the Point-queue model and for both these trips two fixed point problems are solved independently, unlike combined home-work tour. For this numerical test, eight departure periods each of 30 minutes duration were considered for the morning trip as well as for the evening trip. At the supply side the time interval is considered as 1 min, as we need to feedback the travel times into the demand model, therefore first travel times were averaged for 30 min duration and then fed into the demand model. The results obtained are shown in figure 2 as demand and travel time profiles. The figure shows that there are absolutely no differences in the demand distribution and travel time profiles of separate and combined modelling cases. The possible explanation of this phenomenon lies within the MU profiles used in this experiment, which are the major source of obtaining utilities. This finding is further strengthened by the following analytical proof.

Analytical Proof

We start our analytical proof with equations (19), (20a) and (20b), with the assumption that the MU of home and work activities follows any general form and they are only dependent on time-of-day. We can rewrite these equations as

For overall utility of scheduling of the tour

$$V_{ij} = \int_0^i V'^h(t)dt + \int_{i+R_i}^j V'^w(t)dt + \int_{j+R_j}^{1440} V'^h(t)dt + \lambda R_i + \lambda R_j \tag{21}$$

For MC

$$V_i = \int_0^i V'^h(t)dt + \int_{i+R_i}^T V'^w(t)dt + \lambda R_i \tag{22a}$$

For EC

$$V_j = \int_T^j V'^w(t)dt + \int_{j+R_j}^{1440} V'^h(t)dt + \lambda R_j \tag{22b}$$

where, T is an arbitrary time that follows $((i + R_i) \leq T \leq j)$ and indicates the time-of-day up to which MU profile of work activity is integrated for the MC and the remaining portion of it is integrated in the EC in order to gain utility from the work activity. Now if (21) is compared with (22a) and (22b) then we can clearly write as

$$V_{ij} = V_i + V_j \tag{23}$$

Now we will prove that when the MU of home and work activities are taken as a function of time-of-day then there is no difference in the modelling of MC and EC separately or jointly, provided that equation (23) holds. Mathematically it is equivalent to say that

$$\sum_{j=\left(X+R_X+\tau'_w\right)}^{1440} q_{ij} = q_i \tag{24}$$

where, $q_{(i,j)}$ is the demand predicted for an alternative (i , j) using (21) and q_i is the demand predicted from the separate modelling of the MC for an alternative i using (22a). Equation (24) can be written in the probabilistic terms as

$$\sum_{j=\left(X+R_X+\tau'_w\right)}^{1440} P_{ij} = P_i \tag{25}$$

where, P_{ij} is the calculated probability for an alternative (i, j) and p_i is the calculated probability from the separate modelling of the MC for an alternative i. If it is supposed that MNL model is used to calculate the probabilities shown in (25) then we can write as follows

$$\sum_{j=\left(X+R_X+\tau'_w\right)}^{1440} \frac{\exp\left(V_{ij}\right)}{\sum_{i=1}^{X} \sum_{j=\left(X+R_X+\tau'_w\right)}^{1440} \exp\left(V_{ij}\right)} = \frac{\exp\left(V_i\right)}{\sum_{i=1}^{X} \exp\left(V_i\right)} \tag{26}$$

Now consider the left side of the above equation, we will show that with the use of (23) and some application of algebraic operations, the left side of (26) is equal to its right side. Using (23) we can write down the left side of the equation (26) as

$$\Rightarrow \sum_{j=\left(X+R_X+\tau'_w\right)}^{1440} \frac{\exp\left(V_{ij}\right)}{\sum_{i=1}^{X} \sum_{j=\left(X+R_X+\tau'_w\right)}^{1440} \exp\left(V_{ij}\right)} = \sum_{j=\left(X+R_X+\tau'_w\right)}^{1440} \frac{\exp\left(V_i + V_j\right)}{\sum_{i=1}^{X} \sum_{j=\left(X+R_X+\tau'_w\right)}^{1440} \exp\left(V_i + V_j\right)}$$

By using properties of *exp,* we can write down further as

$$= \sum_{j=\left(X+R_X+\tau'_w\right)}^{1440} \frac{\exp\left(V_i\right) . \exp\left(V_j\right)}{\sum_{i=1}^{X} \sum_{j=\left(X+R_X+\tau'_w\right)}^{1440} \exp\left(V_i\right) . \exp\left(V_j\right)} = \exp\left(V_i\right) \left[\sum_{j=\left(X+R_X+\tau'_w\right)}^{1440} \frac{\exp\left(V_j\right)}{\sum_{i=1}^{X} \sum_{j=\left(X+R_X+\tau'_w\right)}^{1440} \exp\left(V_i\right) . \exp\left(V_j\right)} \right]$$

$$\text{since} \quad \sum_{v}^{V} \sum_{w}^{W} a_v \cdot b_w = \sum_{v}^{V} a_v \sum_{w}^{W} b_w$$

$$= \exp\left(V_i\right) \left[\frac{\sum_{j=(X+R_X+\tau'_w)}^{1440} \exp\left(V_j\right)}{\sum_{i=1}^{X} \exp\left(V_i\right) \sum_{j=\left(X+R_X+\tau'_w\right)}^{1440} \exp\left(V_j\right)} \right] = \frac{\exp\left(V_i\right)}{\sum_{i=1}^{X} \exp\left(V_i\right)} = \text{Right side of the equation 26}$$

The above analytical proof shows that equation (23) plays vital role in detaching the MC and EC in a combined model. Equation (11) also suggest that utility of choosing departure time for MC and EC is independent to each other, this is the consequence of using only time-of-day specific MU for home and work activities. As in these MU functions assumes that one unit of activity engagement at time-of-day t will always yield the same utility, irrespective of the activity start and end times. This suggests that only time-of-day specific MU of activities are not enough to model scheduling of home-work tour, therefore, it is necessary to look for other possible alternatives to measure utility of participation in an activity.

Model Refinement

In the activity scheduling literature it has been found out that time-of-day dependent MU is criticised by various authors, as it does not incorporate the activity fatigue or satiation effect which is a very likely phenomenon for most of the activities. This implies that the utility derived from one additional time unit of activity participation diminishes with increasing duration. This is also in line with the basic principle of economics. If MU of an activity is taken as a function of duration for scheduling of home to work tour, then it is very obvious that it interlinks the utility of MC and EC as both utilities are dependent on each other. And the combined utility of the tour cannot be detached into two parts as shown above for time-of-day specific MU function, therefore, equation (23) would not hold in this case. Yamamoto et al [16] and Bhat and Misra [17] presented duration based MU profiles for activities that follows a logarithmic function, with an argument that these utility profiles are in line with the economic theory of diminishing MU. According to them, utility of an activity, for example work is given by

$$V^{w}(\tau_{w}) = \eta_{w} \ln(\tau_{w}) \qquad (27)$$

This gives MU function for work activity as

$$V'^{w}(\tau_{w}) = \eta_{w} \frac{1}{\tau_{w}} \qquad (\tau_{w} \geq 1) \qquad (28)$$

where, τ_w denotes the duration of work activity and η_w is the scaling parameter.

Although, the duration based MU of activity is able to addresses the short comings of the time-of-day dependent MU, but it should be noted that relying entirely on duration based MU for modelling scheduling of the home-work tour is not practicable as in that case individuals time-of-day preferences in participating work and home activities are completely ignored. Therefore, both of these ingredients are important to accurately model the scheduling of home-work tour. Recently, Ettema et al [11] argued that time-of-day dependent MU functions proposed in the literature are continuous in their nature. These functions neglect the fact that most of the every day activities are not flexible in terms of time-of-day, e.g. work and school arrangement and opening hours of stores are the constraints that play vital role in determining the schedule of the various fixed in time activities. They mentioned that schedule delay formulation presented by Small [18] is efficient to deal with such discontinuities, as it is assumed that there exist a certain preferred start time of each activity, and deviations from that time result in a negative utility and these are termed as early and late arrival penalty. Moreover, Ettema et al [11] estimated a model to empirically test their approach for home to work tour. It has been shown in the results that some correlation exist between parameters of time-of-day and duration based MU profiles which shows that time-of-day component also implicitly address duration dependency. Furthermore, parameters of schedule delay are found significant only for work activity due to its relatively less flexible nature than other activities. They concluded that there are rather subtle relationship between the components of the utility and trips, which may partly overlap and correlate.

Based on the above, we can conclude that the scheduling of whole-day activity pattern is dependent on the types of activities actually involved in the pattern. And it is due to the nature of these activities because of which different components shows their significance in the total utility measurement e.g. non-flexible nature of work activity causes significance of schedule delays parameters and fatigue-less nature of home activity in comparison to other out-home activities cause irrelevance to duration component. Therefore, modelling schedules of entire activity pattern is specific to the nature and type of activities involved in the whole pattern. Therefore for the Home-Work tour scheduling model, the following is proposed

- For the home activity; MU based on time-of-day would be the most significant. So the overall home participation activity can be given as

$$V^{h} = \int_{0}^{i} V'^{h}(t)\, dt + \int_{j+R_j}^{1440} V'^{h}(t)\, dt$$

- For work activity; duration based MU function and schedule delay constraints specification would be the most significant i.e. the utility for work activity is given by

$$V^{w} = \left(\int_{i+R_i}^{j} V'^{w}(i + R_i + t)\, dt \right) + g(i + R_i - a)$$

where, $V'^{w}(i+R_i+t)$ is the duration dependent MU function, s is the activity start time and t is the current time at which utility is measured for this activity. Another function in the above expression, $g(i+R_i-a)$ represent the scheduling cost posed on an individual in the form of penalty, here a represents the preferred start time of an activity. Therefore, in accordance with the above MU and utility specification for home and work activity, the complete home-work tour utility can be given as a combination of above

$$V_{ij} = \int_{0}^{i} V'^{h}(t)\, dt + \left(\int_{i+R_i}^{j} V'^{w}(i + R_i + t)\, dt \right) + g(i + R_i - a) + \int_{j+R_j}^{1440} V'^{h}(t)\, dt + \lambda R_i + \lambda R_j \quad (29)$$

The following section presents and discusses the results obtained from the numerical experiments conducted with the model proposed in (29).

NUMERICAL EXPERIMENTS

Analysis of the model with Dynamic Tolls

To conduct this experiment, it is assumed that the arbitrary dynamic tolls are induced to reduce congestion on the link at peak periods. This has been done by adding a term (in monetary units) for the MC and EC in expression (29). The same setup is followed as presented in the numerical example proof. Figure 3 represents the dynamic tolls assumed in this experiment. The utility expression shown in (29) is used here with the same assumption of different parameters as mentioned under the numerical proof above with the difference that MU of work activity is considered here as a function of its duration and its functional form is taken as (28) with the parameter $\eta_w = 18$£-min and preferred start time for work activity is taken as 0830am with equal parameter value of early and late arrival penalties, -0.04£/min.

It is revealed from figure 3 that when there is no toll the middle departure periods contains higher volume of traffic and when dynamic tolls are introduced in a manner that higher demand departure periods have higher value of tolls, then as a result of this traffic volume has been shifted towards early departure periods in the MC and in the EC it is shifted to the later departure periods to a considerable extent. This suggests that peak is dispersed significantly due to the introduction of tolls and as a result individuals change their departure times in order to balance the trade-off between overall travel cost and benefits gained through participation in work and home activities. Further to that, significant differences are noted in the travel time profile for the MC, i.e. for without toll case the peak travel time is equivalent to around 40 min, and with the induction of toll this has been reduce to significant extent i.e. around 20 min. So the additional travel cost individuals are paying in terms of toll, is basically reducing there travel time to a significant extent, however it is also noted that the average duration of work activity is increased to approximately 15 minutes (i.e. 8.44 hours to 8.70 hours) when tolls are incorporated. It is therefore, useful to examine the change in the components of utility function to better understand the complicated trade-offs involved in the process when dynamic tolls are incorporated. For this purpose, table 1 is shown which represents the calculation of utility components for both with toll and without toll cases.

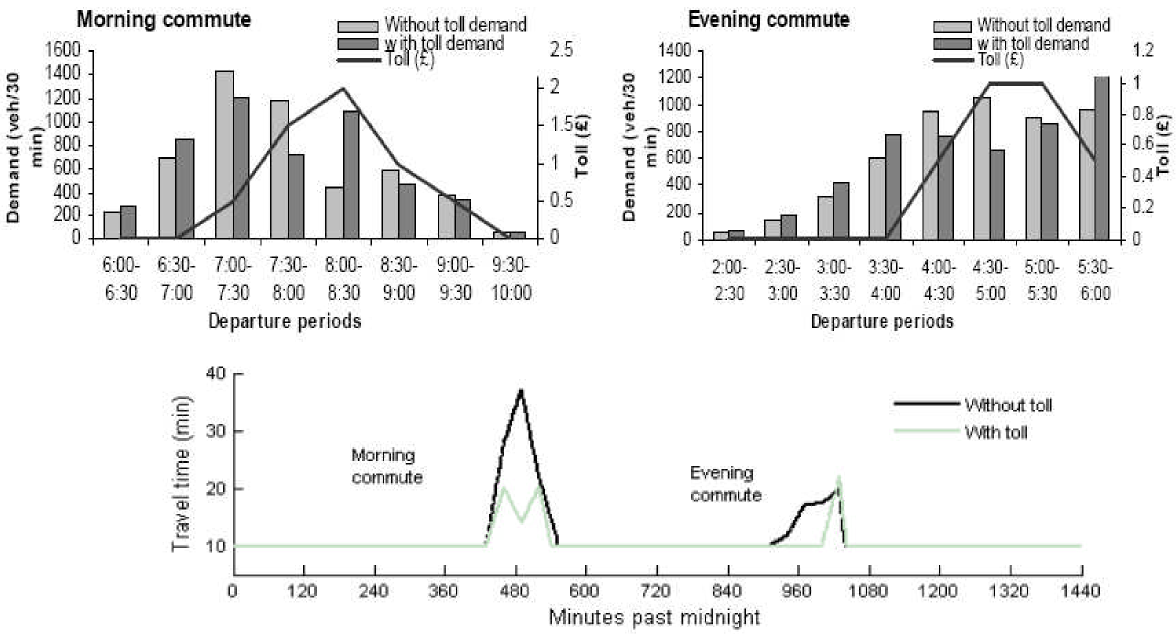


FIGURE 3 Demand and Travel Time Profiles for Without Toll and With Toll Cases

Table 1 revealed that if the individual chooses the departure period 4 (465 minutes past midnight) for the MC and departure period 4 (945 minutes past midnight) for the EC; he has to spend around 11 more minutes at work due to the reduction of travel time of the same extent when tolls are incorporated. So the benefits are two fold, reduction in travel time along with the more utility gains by indulging in work activity for same amount of time. If we consider an individual who has chosen departure period 5 for the MC and departure period 4 for the EC; due to the toll of around 2£, the saving in travel time is around 18 minutes (1.39£) which are utilized to gain more utility of around 0.73£ at work location. On the other hand it is beneficial to the employer, as workers are spending more time at work location. The similar results were reported by [10] when they analyzed the effects of congestion charging on work duration.

This analysis is carried out with the assumption that individual wages are flexible, which is a very rare case in the reality, however, it is possible to analyse the case from the model in which individuals wages are fixed i.e. by assuming a MU function for the work activity that represents the uniform MU for a fixed duration. Another way thorough which the increase in work duration can be explained is through modelling the home-work tour for the entire week, constrained with the fixed number of work hours per week and at the same time individual have a provision to participate in other one or two activities once in a week (i.e. shopping, recreational) for the same additional amount of time individual have spend at the work location due to tolls. At present, the developed model is considering a typical working day of the week; however, it can be extended further on the above mentioned theme to model the entire weekdays with one or two additional activities along with the home and work activities. Under this notion the further work is underway as it will extend the model further with the incorporation of more dimensions i.e. additional activities, sequence of activities and route choice.

TABLE 1 Utility Function Analysis for the Experiment

Morning Departure time	Evening Departure time	Demand (persons)	Home Utility (£)		Work Utility (£)	Early and Late Arrival Penalty (£)		Travel Cost			Toll (£)		Total Utility (£)	Work Duration (min)
			Before work	After work		Early	Late	M (min)	E (min)	TC (£)	M	E		
465	945	164.0	13.25	13.508	110.004	-0.65	---	29.05	10	-3.12	-	-	132.98	450.95
465	1005	247.2	13.25	12.110	112.252	-0.65	---	29.05	12.87	-3.35	-	-	133.60	510.94
495	945	39.22	13.90	13.508	108.588	---	-0.70	33.16	10	-3.45	-	-	131.84	416.83
495	1005	68.9	13.90	12.110	111.009	---	-0.70	33.16	12.87	-3.68	-	-	132.63	476.83
465	945	111.9	13.25	13.508	110.434	-1.10	----	18.13	10	-2.25	1.5	0	132.34	461.86
465	1005	98.38	13.25	12.180	112.633	-1.10	----	18.13	10	-2.25	1.5	1	132.21	521.86
495	945	150.7	13.90	13.508	109.322	---	-0.03	15.81	10	-2.06	2.0	0	132.63	434.18
495	1005	151.0	13.90	12.180	111.652	---	-0.03	15.81	10	-2.06	2.0	1	132.64	494.18

M = Morning Commute
E = Evening Commute
TC= Total Travel cost of the tour, using value of In-vehicle travel-time equals 8 pence/ minute

Model Sensitivity with Different Dynamic Traffic Performance Models

To conduct this experiment no changes have been done at demand side of the model and in the numerical setup mentioned above. The changes made are related to supply side of the combined model. The experiments were performed using the linear travel time model [21], divided-linear travel time model [23] and Point-Queue model [22]. Figure 4 show results of this experiment at equilibrium.

It is revealed from figure 4 that travel times obtained from the point-queue model are lower than other two models. Higher travel times are obtained when linear travel time model is used. Divided linear travel time model provides moderate values of travel times. This is due to the inherited properties of these models, as it is already mentioned in the literature that point –queue model underestimate the travel time when there is no congestion on the road because this model always gives travel time equal to free flow travel time of the link unless inflow to the link exceeds its capacity [12]. Linear travel time model estimate the higher value of travel times because the structure of the model is such that it calculates the travel time for the incoming vehicle at a particular time by considering all the existing vehicles on the link at that time, even when there are few vehicles on the link. Therefore, this model overestimates travel time when there is no congestion on the link and this effect propagate further which results in higher values of travel time. This property of the linear travel time model is termed as double counting effect in the DTA literature [12]. Divided linear travel time model presented by Mun [19] is a result of the modification proposed in the linear travel time model. This model addresses the overestimation problem exists in the linear travel time model, as in this model the link is divided into two sections and traffic is supposed to propagate the first section with the free flow speed. When traffic reaches the second section of the link whose free flow travel time is recommended to be equal to the time interval at the supply side, the flow propagate according to the linear travel time model. This results in consideration of congestion effects of the vehicles only in second section which is the limited part of the whole link. All the supply models considered here are linear models i.e. travel time increases linearly with the flow, however in practical terms it is observed that travel time follows a convex curvilinear path. This is still an active area of research that a model would need to be developed which fulfils all the desirable properties of DTA and allows change of travel time in accordance with the empirical observations.

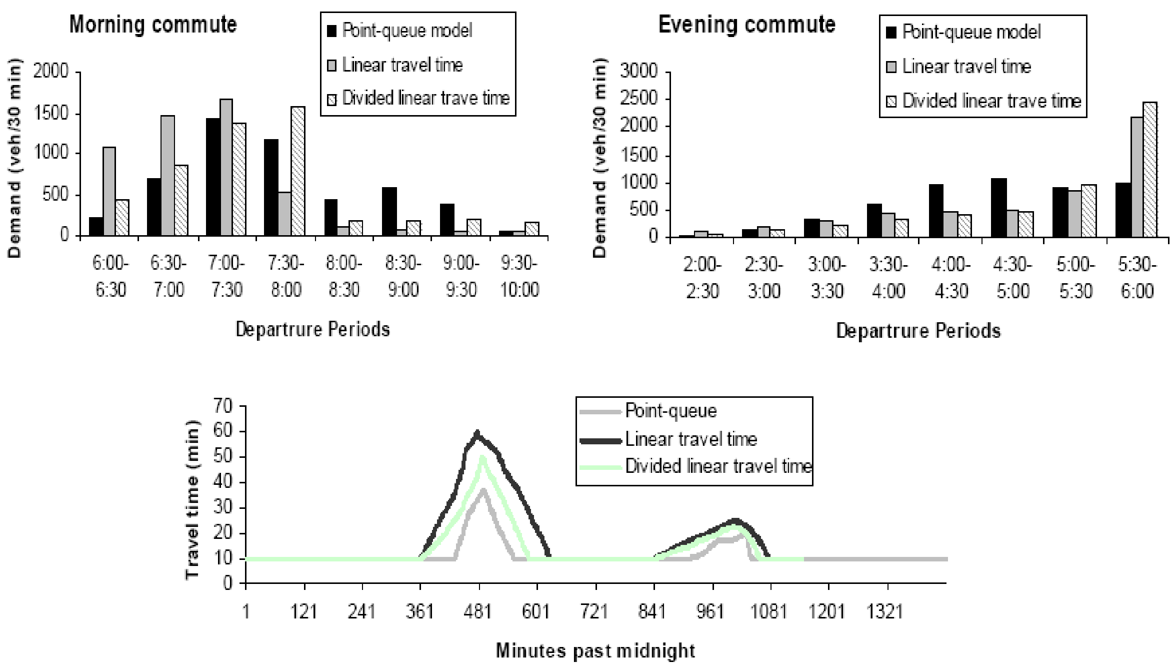


FIGURE 4 Demand and travel time profiles for different supply models

CONCLUSIONS AND FUTURE RESEARCH

This paper reported an integrated model for scheduling of activities an individual supposed to do in a given day with the representation of road network congestion effects. A generic mathematical illustration of the developed model is presented for a simplified version of the home-work tour. It has been found out that when MU of an activity is considered as a function of clock-time only, the model detached the MC and EC i.e. the effect of any change in the MC are not transferred to the EC. The numerical and analytical proof of this has been presented in the paper. This finding supports the utility specification presented by Ettema et al [11]. It has been noted that duration based MU which represents the activity satiation effect is an important ingredient along with the time-of-day representation, in order to properly integrate the two commutes in the home-work tour. The results from the numerical experiments suggested that the model is behaving well and yielding prediction as per expectation, as the model allows the dispersion of peak when dynamic tolls are introduced. The analysis of the utility function presented for the dynamic toll experiment was useful to examine the complex trade-off involved in the process. Introduction of toll seems to have a direct effect on travel time, but it is found out that individuals are utilising that time in participating home and work activities to gain more utility. The increase in work duration seems beneficial not only to individual but for the employer as well. The 2nd numerical test suggested that a care should be taken for the choice of dynamic loading model at the supply side as this would potentially effects the overall results obtained from the model because of their inherited properties.

In the future, research work would be carried out to presents the mathematical and numerical illustration of the developed model for any realistic medium size network with complex activity pattern. The model could be further extended in various ways: (1) incorporation of secondary and tertiary tours within the framework of the proposed model, (2) incorporation of activity location and mode choice, (3) linking the proposed model with an activity-generation model to develop a full activity-based model with supply component, (4) theoretical examination of the convergence pattern and stability of the solution and (5) development of a more integrated generalised package that would be further user friendly and practically applicable for any reasonable sized road network and activity centres within it.

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