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Superflow of resonantly driven polaritons against a defect

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In the linear response approximation, coherently driven microcavity polaritons in the pump-only configuration are expected to satisfy the Landau criterion for superfluidity at either strong enough pump powers or small flow velocities. Here, we solve non-perturbatively the time dependent Gross-Pitaevskii equation describing the resonantly-driven polariton system. We show that, even in the limit of asymptotically large densities, where in linear response approximation the system satisfies the Landau criterion, the fluid always experiences a residual drag force when flowing through the defect. We illustrate the result in terms of the polariton lifetime being finite, finding that the equilibrium limit of zero drag can only be recovered in the case of perfect microcavities. In general, both the drag force exerted by the defect on the fluid, as well as the height of Cherenkov radiation, and the percentage of particles scattered by the defect, show a smooth crossover rather than a sharp threshold-like behaviour typical of superfluids which obey the Landau criterion.

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I. INTRODUCTION

In the past two decades, microcavity polaritons[1] have attracted considerable interest because of the possibility of strongly coupling light and matter, leading to an easy manipulation and detection of the non-linear properties of matter via light — see, e.g., Ref.[2] and references therein. Recently, in light of their out-of-equilibrium nature, there has been a growing interest in studying polariton superfluid properties[3]. In fact, polaritons have very short lifetimes (around ~ 30 ps even in the best available samples[4]), therefore any polariton fluid is the result of a steady state balance between pumping and decay. The effects of pump and decay in polariton systems are the subject of several recent theoretical and experimental work.

For condensates in local thermal equilibrium, such as superfluid ^{4}He and the ultracold atom Bose-Einstein condensates, the concept of superfluidity is strongly linked to several paradigmatical properties, such as the Landau criterion, quantised circulation of velocity, and metastable persistent flow. In particular, the Landau criterion connects the frictionless motion of a defect at velocities smaller than a critical one, with the shape of the spectrum of elementary excitations[5]. For weakly interacting Bose gases and a microscopic weak defect, such that a perturbative linear response theory can be applied, the phonon-like dispersion of the Bogoliubov spectrum demands that the critical velocity for dissipationless flow coincides with the velocity of sound[6]. However, for macroscopic defects, the critical velocity for the onset of drag is smaller than the speed of sound and most likely related to vortex nucleation[7].

In contrast, the relevance of the Landau criterion for out-of-equilibrium condensates is questionable, not least because the spectrum of excitations is now complex rather than real. For polariton fluids, one has to singularly assess the system properties in the three different pumping schemes available: i) non-resonant pumping; ii) parametric drive in the optical-parametric-oscillator (OPO) regime; iii) coherent drive in the pump-only configuration.

As far as the spectrum of quasi-particle excitations and the Landau criterion are concerned, the cases of non-resonantly pumped polaritons[8] and parametrically driven polaritons in the OPO regime[9] are similar: in both cases, there is a $U(1)$ phase symmetry which is spontaneously broken above a pump power threshold. This leads to the appearance of a gapless (Goldstone) mode in the spectrum of excitations — gapless means that both real and imaginary part of the excitation energy go to zero at zero momentum. However, in both cases, the effects of pump and decay are such that the real part of the Goldstone mode energy is zero also in a finite interval at small momenta, i.e. the spectrum becomes diffusive[10–12]. This means that a strict application of the Landau criterion would lead to a zero critical velocity, where quasi-particles can be excited at any value of the fluid speed. Therefore, if one would define superfluid properties through a strict application of the Landau criterion, neither non-resonantly pumped polaritons nor parametrically driven polaritons in the OPO regime behave as superfluids. Nevertheless, in both cases there have been evidences for superfluid behaviour. For non-resonantly pumped polaritons, it has been recently shown[13] that, even though strictly speaking there cannot be superfluid behaviour, there are regimes close to equilibrium, where the drag force exerted on a small moving defect shows a sharp threshold at velocities close to the speed of sound. Otherwise, for shorter polariton lifetimes, the threshold-like behaviour of the drag force is replaced by a smooth crossover. Moreover, metastability of supercurrents in non-resonantly pumped microcavities has been theoretically demonstrated[14]. For polaritons in the OPO regime, superfluidity has instead been tested through frictionless flow of polariton bullets[15], through metastability of quantum vortices and persistence of currents[16,17].

The case of coherently driven polaritons in the pump-only configuration strongly differs from the two schemes previously described: Here, there is no phase freedom any longer, because the polariton phase locks to the one of the driving pump. As a consequence, the quasi-particle excitation spectrum is always gapped — i.e., even when the real part of the spectrum energy goes to zero at the pump momentum, the corresponding imaginary part is non zero. For coherently driven polaritons in the pump-only configuration, by making use of a linearised Bogoliubov-like theory, it has been predicted that the Landau criterion can instead be satisfied at either strong enough pump powers or small flow velocities[18,19], regimes where polaritons display a dramatic reduction in the intensity of resonant Rayleigh scattering. For values of the parameters where instead the spectrum allows the excitation of quasi-particles, Cherenkov-like waves were predicted[19], and recently observed in the density profile[20]. Consequently, experiments in this configuration have been analysed in terms of the same theoretical description which is valid for equilibrium superfluids.

In this work, we show that, despite the fact coherently driven polaritons in the pump-only configuration do satisfy the Landau criterion at large enough densities or small flow velocities, because of the polariton lifetime being finite, the fluid always experiences a residual drag force even in the limit of asymptotically large densities. We show that, only in the limit of perfect microcavities (i.e. infinitely long polariton lifetimes), the residual drag force at large enough densities goes to zero, recovering therefore the equilibrium limit. Otherwise, for finite polariton lifetimes close to the current experimental values, we find that, similarly to the case of incoherently driven polaritons, both the drag force exerted on the polariton fluid by a defect, as well as the height of Cherenkov radiation, and the percentage of particles scattered by the defect show a smooth crossover rather than a sharp threshold-like behaviour which is typical of superfluids which obey the Landau criterion.

The paper is organised as follows: in Sec. II we introduce the model describing polaritons coherently driven in the pump-only configuration in presence of a defect potential. In Sec. III we describe the methods we use for our analysis, in particular the numerical algorithm used to evaluate quantities such as the drag force, the height of Cherenkov radiation, and the percentage of particles scattered by the defect which characterise the crossover from a superfluid-like to a supersonic-like behaviour. In addition, in Sec. III A, we shortly introduce the linearised Bogoliubov-like theory of Refs.[18,19], which will be used later in Sec. IV in order to compare the results obtained with the non-perturbative method with the results obtained in the linear response approximation. Results are discussed in Sec. IV, while conclusions (together with a discussion of the experimental relevance of our findings) are drawn in Sec. V.

II. MODEL

We describe the dynamics of the resonantly-driven polariton system[21] via a Gross-Pitaevskii equation for coupled cavity and exciton fields $\psi_{C,X}(\mathbf{r},t)$, generalised to include the effects of the resonant pumping and decay ($\hbar = 1$):

$$i\partial_t \begin{pmatrix} \psi_X \\ \psi_C \end{pmatrix} = \begin{pmatrix} 0 \\ F_p \end{pmatrix} + \left[\hat{H}_0(-i\nabla) + \begin{pmatrix} -i\kappa_X + g_X|\psi_X|^2 & 0 \\ 0 & -i\kappa_C + V_d(\mathbf{r}) \end{pmatrix} \right] \begin{pmatrix} \psi_X \\ \psi_C \end{pmatrix} . \quad (1)$$

The single-particle polariton Hamiltonian $\hat{H}_0$ can be diagonalised in momentum space,

$$\hat{H}_0(\mathbf{k}) = \begin{pmatrix} \omega_X(\mathbf{k}) & \Omega_R/2 \\ \Omega_R/2 & \omega_C(\mathbf{k}) \end{pmatrix} , \quad (2)$$

by rotating into the lower (LP) and upper polariton (UP) basis,

$$\begin{pmatrix} \psi_X \\ \psi_C \end{pmatrix} = \begin{pmatrix} \cos\theta_{\mathbf{k}} & -\sin\theta_{\mathbf{k}} \\ \sin\theta_{\mathbf{k}} & \cos\theta_{\mathbf{k}} \end{pmatrix} \begin{pmatrix} \psi_{LP} \\ \psi_{UP} \end{pmatrix} ,$$

where $\tan 2\theta_{\mathbf{k}} = \Omega_R/[\omega_X(\mathbf{k}) - \omega_C(\mathbf{k})]$, giving the bare lower and upper polariton dispersions:

$$\omega_{LP,UP}(\mathbf{k}) = \frac{\omega_X + \omega_C}{2} \mp \frac{\sqrt{[\omega_C - \omega_X]^2 + \Omega_R^2}}{2} . \quad (3)$$

At zero energy detuning between excitons and photons, $\omega_C(\mathbf{k}) = \omega_X(\mathbf{k})$, lower and upper polaritons are an equally balanced mixture of exciton and photon, i.e. $\sin^2\theta_{\mathbf{k}} = \cos^2\theta_{\mathbf{k}} = 1/2$. In contrast, at large values of the energy detuning between photons and excitons, $\omega_C(\mathbf{k}) - \omega_X(\mathbf{k}) \gg \Omega_R$, i.e. $\theta_{\mathbf{k}} \to 0$, the lower and upper polariton respectively coincide with the exciton and the photon.

In the following, we will neglect the exciton dispersion, $\omega_X(\mathbf{k}) = \omega_X(0)$, and assume a quadratic dispersion for photons, $\omega_C(\mathbf{k}) = \omega_C(0) + \frac{k^2}{2m_C}$, where the photon mass is $m_C = 2 \times 10^{-5} m_0$ and m_0 is the bare electron mass. Through the paper, we will consider the case of zero detuning at normal incidence, $\omega_X(0) = \omega_C(0)$, and fix the Rabi frequency to $\Omega_R = 4.4$ meV. The parameters κ_X and κ_C are respectively the excitonic and photonic decay rates. We will fix these parameters in order to give a polariton lifetime, $\tau_{LP} = \hbar/\kappa_{LP}$,

$$\kappa_{LP}(\mathbf{k}) = \kappa_X \cos^2\theta_{\mathbf{k}} + \kappa_C \sin^2\theta_{\mathbf{k}} , \qquad (4)$$

close to the experimental values. In addition, we will consider the limit of perfect cavities $\kappa_{LP} \to 0$ in order to recover the equilibrium limit.

In Eq. (1), the exciton-exciton interaction strength g_X can be set to one by rescaling both fields $\psi_{X,C}$ and pump strength F_p by $\sqrt{\Omega_R/(2g_X)}$. The cavity field is driven by a continuous-wave pump,

$$F_p(\mathbf{r},t) = \mathcal{F}_{f,\sigma}(r) e^{i(\mathbf{k}_p\cdot\mathbf{r}-\omega_p t)} , \qquad (5)$$

with a smoothen top-hat spatial profile with intensity f and full width at half maximum (FWHM) $\sigma = 130\ \mu$m. The pumping laser frequency ω_p has been chosen 0.44 meV blue-detuned above the bare lower polariton dispersion at the pump momentum $\omega_{LP}(\mathbf{k}_p)$. The polariton flow current is determined by the pump momentum $\mathbf{k}_p$, which can be experimentally tuned by changing the pumping laser angle of incidence with respect to the growth direction $\varphi_{\mathbf{k}_p}$:

$$c\mathbf{k}_p = \omega_{LP}(\mathbf{k}_p)\sin(\varphi_{\mathbf{k}_p}) .$$

Finally, in Eq. (1) the potential $V_d(\mathbf{r})$ describes the defect acting on the photonic field, over which the polariton fluid scatters. Specifically, we consider:

$$V_d(\mathbf{r}) = V_d\theta(r - r_d) , \qquad (6)$$

with $r_d = 7\ \mu$m and $V_d = 110$ meV. Defects can be present naturally in the sample's mirror[20]. Alternatively, defects can been artificially engineered by either growing mesas in one of the mirrors[22] or by an additional laser[23].

III. METHODS

We numerically solve Eq. (1) on a 2D grid (256×256) in a 150 μm×150 μm box using a 5$^{\text{th}}$-order adaptive-step Runge-Kutta algorithm, and evaluate both exciton and photon wave-functions $\psi_{X,C}(\mathbf{r},t)$ in the steady-state regime.

We characterise the crossover from a superfluid-like to a supersonic-like regime by evaluating three different quantities. Firstly, we consider the normalised drag force[6] exherted on the flowing polaritons by the defect (6):

$$\mathbf{F}_d = \frac{1}{\int d\mathbf{r}|\psi_C(\mathbf{r})|^2}\int d\mathbf{r}|\psi_C(\mathbf{r})|^2\nabla V_d(\mathbf{r}) . \qquad (7)$$

When shining the laser pump in the x-direction, $\mathbf{k}_p = (k_p, 0)$, the density profile will be symmetric under the transformation $y \mapsto -y$, $\psi_{X,C}(x,y) = \psi_{X,C}(x,-y)$, implying that for defect potentials symmetric under $y \mapsto -y$ only the x-component of the drag force can be non-zero.

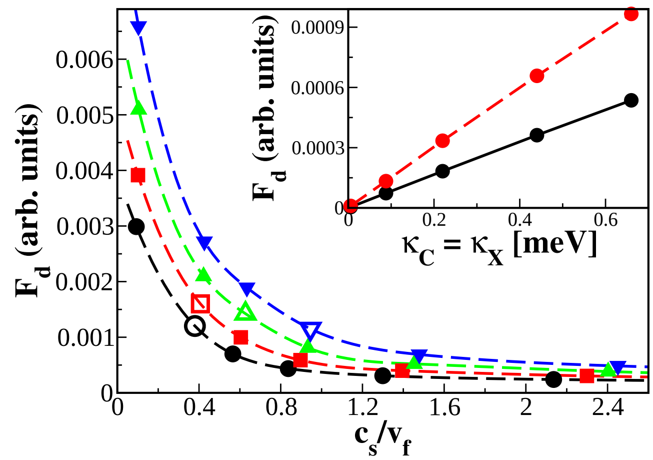

FIG. 1: (Color online) Drag force in the x-direction, F_{d_x} (7), exerted on the polariton fluid by the defect (6) as a function of the velocity ratio c_s/v_f (see Eqs. (12) and (14)), where $n_{LP} = \int d\mathbf{r}|\psi_{LP}(\mathbf{r})|^2/\Omega$ is the average polariton density in the area Ω, and for different values of the pump momentum: $k_p = 0.7\ \mu\text{m}^{-1}$ (black circles), $k_p = 0.8\ \mu\text{m}^{-1}$ (red squares), $k_p = 0.9\ \mu\text{m}^{-1}$ (green up triangles), and $k_p = 1\ \mu\text{m}^{-1}$ (blue down triangles). The exciton and photon decay rates have been fixed to $\kappa_X = \kappa_C = 0.22$ meV, while the pumping laser frequency ω_p has been chosen (as in all the Figures) 0.44 meV blue-detuned above the bare LP dispersion $\omega_{LP}(\mathbf{k}_p)$. The empty symbols indicate the value of c_s/v_f above which in the linear approximation the Landau condition of Eq. (13) cannot be satisfied and therefore where in principle one expects the drag force going to zero. Inset: Residual drag force at asymptotically large polariton densities n_{LP} (i.e., $c_s/v_f \to \infty$) as a function of the exciton and photon decay rates for $k_p = 0.7\ \mu\text{m}^{-1}$ (black solid line) and $k_p = 1\ \mu\text{m}^{-1}$ (red dashed line).

Moreover, for step-like defects such as (6), only the values of the field $|\psi_C(\mathbf{r})|^2$ at a distance $r = r_d$ contribute to the integral (7):

$$F_{d_x} = \frac{2V_d r_d}{\int d\mathbf{r}|\psi_C(\mathbf{r})|^2}\int_0^\pi d\phi\cos\phi|\psi_C(r_d,\phi)|^2 . \qquad (8)$$

Therefore the drag force measures the degree of asymmetry of the photon (or alternatively the exciton) density profiles going from a certain angle $\phi < \pi/2$ ahead (with respect to the fluid flow direction) of the defect, to an angle $\pi - \phi > \pi/2$ behind the defect. The asymmetry is caused by the scattering of the fluid passing through the defect. Plots of the drag force are drawn in Fig. 1.

Second, we characterise the superfluid-like behaviour by the suppression of density modulations around the defect known as *Cherenkov waves* — see, e.g., Refs.[24,25] and references therein for atomic Bose-Einstein condensates and Refs.[18,19] for coherently driven polaritons in the pump-only configuration. Cherenkov radiation is generated in the supersonic regime, when the fluid is passing a defect at a velocity higher than the phase velocity of the fluid elementary excitations. As mentioned later, a simple analysis of Cherenkov radiation can be carried on by making use of perturbative Bogoliubov-like analysis: Here, in agreement with the Landau criterion, in the supersonic regime, the kinetic energy of the fluid can be dissipated radiating Bogoliubov modes, giving rise to perturbations which propagate radially from the defect, with a characteristic unperturbed region inside a Cherenkov cone which is related to the singularity of the Bogoliubov mode dispersion evaluated in the reference frame of the moving fluid[24]. In our numerical non-perturbative analysis, we determine the value of the highest crest of the waves, $h_{\max}$, above the mean value of the fluid, h_{mean}, giving $h_{\check{\text{C}}\text{er}} = h_{\max}/h_{\text{mean}}$, and analyse how $h_{\check{\text{C}}\text{er}}$ changes by tuning the fluid velocities and densities.

Finally, a third way of describing the crossover from a superfluid-like to a supersonic-like regime is by evaluating the percentage of particles scattered by the defect with respect to the total number of particles in the system[26], quantity which we label as S_{out}. Both $h_{\check{\text{C}}\text{er}}$ and S_{out} are plotted in Fig. 5.

A. Linear response to a weak external potential

In order to connect our numerical results obtained by solving non-perturbatively the dynamics of the Gross-Pitaevskii equation (1) with the Landau criterion, and the spectrum of quasi-particle excitations, similarly to what is done for conservative weakly interacting Bose gases[5], we follow the same perturbative Bogoliubov-like analysis first introduced for resonantly pumped polaritons in Refs.[18,19] and expand both exciton and photon fields above their mean-field spatially homogeneous and stationary states $e^{-i(\omega_p t - \mathbf{k}_p\cdot\mathbf{r})}\psi^{(0)}_{X,C}$:

$$\psi_{X,C}(\mathbf{r},t) = e^{-i\omega_p t}\left[e^{i\mathbf{k}_p\cdot\mathbf{r}}\psi^{(0)}_{X,C} + \delta\psi_{X,C}(\mathbf{r},t)\right] , \quad (9)$$

where we are assuming the pump to have a homogeneous profile $\mathcal{F}_{f,\sigma}(r) = f$. While the mean-field equations

$$\left[\omega_X(0) - \omega_p - i\kappa_X + g_X|\psi^{(0)}_X|^2\right]\psi^{(0)}_X + \frac{\Omega_R}{2}\psi^{(0)}_C = 0$$
$$\left[\omega_C(\mathbf{k}_p) - \omega_p - i\kappa_C\right]\psi^{(0)}_C + \frac{\Omega_R}{2}\psi^{(0)}_X = -f$$

allow to determine the intensity of exciton and photon fields in absence of the external potential, fluctuations above mean-field need to be introduced in order to evaluate the stability of such a solutions as well as the linear response to a weak perturbing external potential.

The spectrum of the quasi-particle excitations can be found by introducing particle-like $u_{X,C}$ and hole-like $v_{X,C}$ excitations in both the exciton and photon fluctuation fields:

$$\delta\psi_{X,C}(\mathbf{r},t) = \sum_{\mathbf{k}}\left(e^{-i\omega t}e^{i\mathbf{k}\cdot\mathbf{r}}u_{X,C;\mathbf{k}} + e^{i\omega t}e^{-i(\mathbf{k}-2\mathbf{k}_p)\cdot\mathbf{r}}v_{X,C;\mathbf{k}}\right) ,$$

and by solving the eigenvalue equation:

$$\left[(\omega+\omega_p)\mathbb{I} - \mathbb{L}\right]\begin{pmatrix}u_{X;\mathbf{k}} & u_{C;\mathbf{k}} & v_{X;\mathbf{k}} & v_{C;\mathbf{k}}\end{pmatrix}^T = 0 , \quad (10)$$

where $\mathbb{L}$ is a 4×4 matrix given by:

$$\mathbb{L} = \begin{pmatrix} \omega_X(0) + 2g_X|\psi^{(0)}_X|^2 - i\kappa_X & \Omega_R/2 & g_X{\psi^{(0)}_X}^2 & 0 \\ \Omega_2/2 & \omega_C(\mathbf{k}) - i\kappa_C & 0 & 0 \\ -g_X{\psi^{(0)}_X}^{2*} & 0 & 2\omega_p - \omega_X(0) - 2g_X|\psi^{(0)}_X|^2 - i\kappa_X & -\Omega_R/2 \\ 0 & 0 & -\Omega_R/2 & 2\omega_p - \omega_C(2\mathbf{k}_p - \mathbf{k}) - i\kappa_C \end{pmatrix} .$$

Eq. (10) admits four complex eigenvalues for each $\mathbf{k}$ which we indicate as $\omega + \omega_p = \text{LP}^{\pm}(\mathbf{k}), \text{UP}^{\pm}(\mathbf{k})$. Note that the spectrum obtained this way is already evaluated in the polariton flow moving frame. This becomes evident in the limit where the pump momentum $\mathbf{k}_p$ is small enough so that the LP dispersion can be approximated as parabolic, and the UP dispersion can be neglected. In fact, here, one can show[19] that the spectrum of excitation reduces to

$$\text{LP}^{\pm}(\mathbf{k}) - \omega_p \simeq \mathbf{v}_f\cdot(\mathbf{k}-\mathbf{k}_p) - i\kappa_{LP} \pm\sqrt{(\varepsilon_{\mathbf{k}} - \Delta)\left(\varepsilon_{\mathbf{k}} - \Delta + 2g_{LP}|\psi^{(0)}_{LP}|^2\right)} , \quad (11)$$

where

$$\mathbf{v}_f = \frac{\mathbf{k}_p}{m_{LP}} \quad (12)$$

is the polariton fluid velocity at the pump momentum, $\varepsilon_{\mathbf{k}} = (\mathbf{k}-\mathbf{k}_p)^2/(2m_{LP})$, $m_{LP} = m_C/\sin^2\theta_{\mathbf{k}_p}$ is the LP mass, $\Delta = \omega_p - \omega_{LP}(\mathbf{k}_p) - g_{LP}|\psi^{(0)}_{LP}|^2$ is the pump detuning renormalised by the interaction, and $g_{LP} = g_X\cos^4\theta_{\mathbf{k}_p}$. For more details, we refer the reader to the original calculation of Ref.[19]. What we would like to stress here is how to generalise the Landau criterion to the complex spectrum of elementary excitations obtained from Eq. (10). In particular, in the linear approximation,

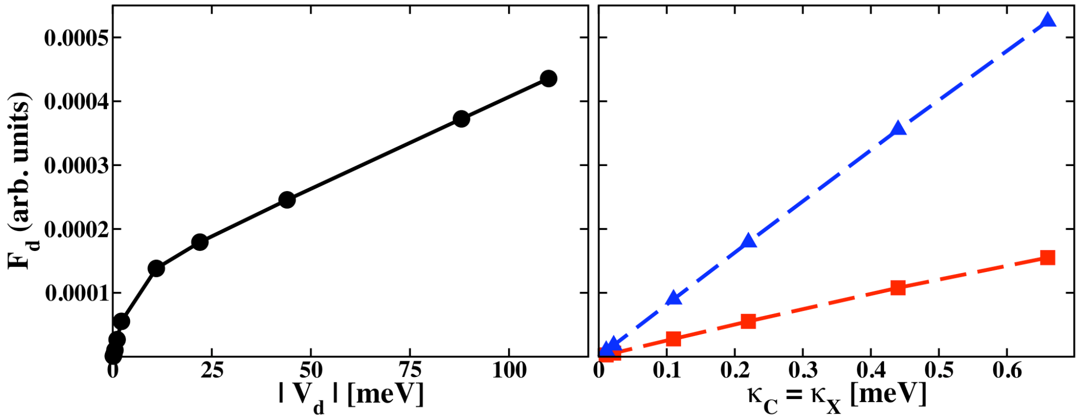

FIG. 2: (Color online) Residual drag force (in arbitrary units) at asymptotically large polariton densities n_{LP} for $k_p = 0.7\ \mu\text{m}^{-1}$. Left panel gives the dependence on the barrier height (in meV) for $\kappa_X = \kappa_C = 0.22$ meV. Right panel shows the dependence on the decay rates κ_X and κ_C (in meV) for two different barriers: $V_d = -22$ meV (blue dashed line) and $V_d = -2.2$ meV (red dashed line).

we expect that the perturbation introduced by the defect is able to excite Bogoliubov-like quasi-particle states with momentum $\mathbf{k}$, when the condition

$$\Re[\text{LP}^+(\mathbf{k})] - \omega_p < 0 \tag{13}$$

is satisfied. Note that, in the limit where the approximation (11) is valid and at resonance, $\Delta = 0$, this recovers the Landau criterion in its original formulation for a conservative system, i.e. close to $\mathbf{k}_p$, $\Re[\text{LP}^+(\mathbf{k})] - \omega_p \simeq (c_s \pm v_f)|\mathbf{k} - \mathbf{k}_p|$ and the critical fluid velocity coincides with the sound velocity given by the usual[5] expression:

$$c_s \equiv \sqrt{\frac{g_{LP} n_{LP}}{m_{LP}}}\ , \tag{14}$$

where $n_{LP} = |\psi_{LP}^{(0)}|^2$ is the mean-field polariton density. In order to be able to draw an analogy with the equilibrium limit, we will later present our results in terms of the ratio c_s/v_f.

IV. RESULTS

We now turn to the non-perturbative numerical analysis of the Gross-Pitaevskii equation (1) and the analysis of the behaviour of the superfluid properties of the resonantly driven polariton system in the pump-only regime when changing the laser pump strength. In particular, by fixing the pump momentum $\mathbf{k}_p$ and energy ω_p, we evaluate the dependence of the drag force (7), the height of the Cherenkov waves $h_{\check{\text{C}}\text{er}}$ and the percentage of scattered particles S_{out}, on the the steady-state average density of polaritons, $n_{LP} = \int d\mathbf{r} |\psi_{LP}(\mathbf{r})|^2/\Omega$, pumped into the cavity — here, Ω is the circle pumping area we are averaging the density over. Rather than as a function of n_{LP}, we present the results as a function of the ratio between sound and polariton fluid velocities, c_s/v_f, defined in Eqs. (12) and (14).

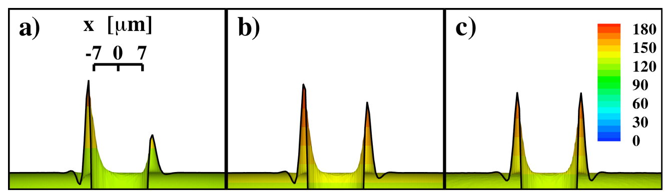

FIG. 3: (Color online) Perturbation of the steady state photon density $|\psi_C(\mathbf{r})|^2$ (side view) and the $y = 0$ cut of the profile, $|\psi_C(x, y = 0)|^2$ (solid black line), close to the defect, for values of the pump power asymptotically large ($c_s/v_f \to \infty$), where the residual drag force of Fig. 1 does not have any appreciable variation. The pump is shined on the cavity at a momentum $k_p = 1\ \mu\text{m}^{-1}$, and the three panels, (a), (b), and (c), correspond to increasing values of the polariton lifetime: $\hbar/\kappa_X = \hbar/\kappa_C = 1$ ps (a), 3 ps (b) and 120 ps (c). In each panel the intensity of the pumping laser has been adjusted in order to give the same polariton density. Note that in this limit, the defect induces an asymmetric perturbation of $|\psi_C(\mathbf{r})|^2$ only ahead and behind the defect, similarly to the left bottom panel of Fig. 4. The asymmetry disappears for perfect cavities.

We plot in Fig. 1 the drag force exerted on the polariton fluid by the defect (6) in the x-direction, F_{dx} (7), as a function of the ratio c_s/v_f (14), for different values of the pump wavevector $\mathbf{k}_p$. We find that the drag force decreases fast as a function of the polariton density, when $c_s/v_f < 1$, while, for finite values of the polariton lifetime, reaches a finite asymptotic value, a *residual drag force*, at large densities, when $c_s/v_f \gg 1$. While for conservative superfluid systems the drag force has a threshold-like behaviour and in particular, for perturbatively weak delta-like defects, is finite only for superfluid velocities above the speed of sound[6], we now observe a smooth crossover as a function of c_s/v_f. In addition, as

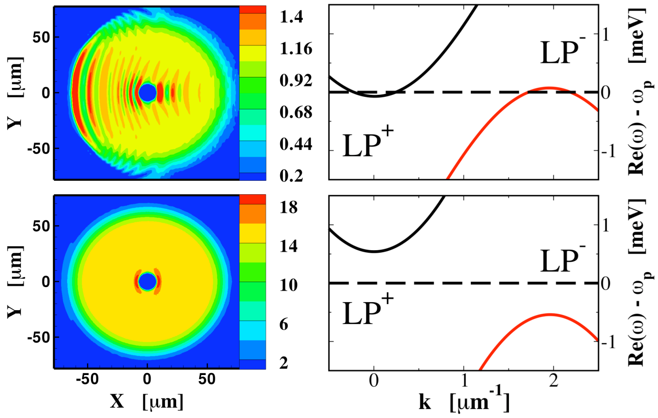

FIG. 4: (Color online) Steady state photon density profile, $|\psi_C(\mathbf{r})|^2$ (left panels) and corresponding quasi-particle excitation spectrum (10) (right panels) evaluated in the homogeneous case for the same system parameters of the left panels. Here, $k_p = 0.9\ \mu\text{m}^{-1}$, $\hbar/\kappa_C = 7$ ps, $\hbar/\kappa_X = 120$ ps, and the values of the pump power correspond to a state where scattering is allowed in the linear approximation at $c_s/v_f = 0.54$ (top panels) and no quasi-particle excitation in allowed in the linear approximation at $c_s/v_f = 1.49$ (bottom panels). Both pump strength values considered here corresponds to photon densities on the upper branch of the bistability curve.

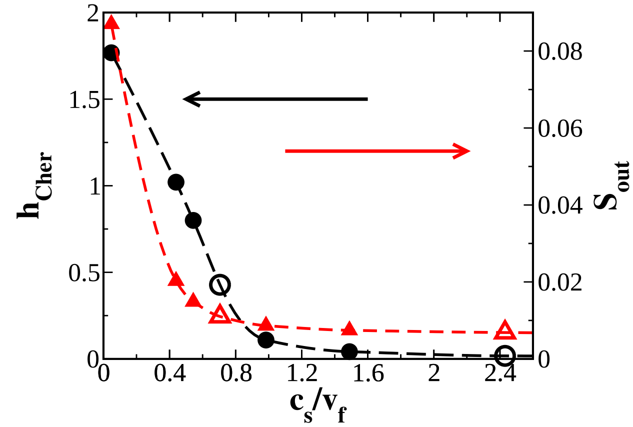

FIG. 5: (Color online) Height of the Cherenkov waves $h_{\check{\mathrm{C}}\mathrm{er}}$ (black circles) and percentage of scattered particles S_{out} (red triangles) as defined in Sec. II, plotted as a function of the normalized density c_s/v_f. The parameters are the same as the ones of Fig. 4: $k_p = 0.9\ \mu\text{m}^{-1}$, $\hbar/\kappa_C = 7$ ps, $\hbar/\kappa_X = 120$ ps. The empty symbols corresponds to the values $c_s/v_f = 0.54$ and $c_s/v_f = 1.59$ chosen to plot respectively the top and bottom panels of Fig. 4.

shown in the inset of Fig. 1, we find that the residual drag force vanishes only in the limit of perfect microcavities, $\kappa_X = \kappa_C \to 0$, i.e., for infinitely long-lived polaritons. Note that this recovers the equilibrium limit, as the condition $\kappa_X = \kappa_C \to 0$ automatically requires a pump strength $f \to 0$, as the laser pump strength needed to reach a given polariton-density in the cavity decreases with the increasing polariton lifetime.

The two main results to be drawn from Fig. 1 are the existence of a cross-over instead of an abrupt transition and the appearance of the residual drag force. Since these results have been obtained with a practically infinite barrier ($V_d = 110$ meV), one can wonder to what extend they remain valid for actual potential barriers which are much lower. We have repeated the analysis for different values of V_d, including negative ones, and always a cross-over occurs for finite decay rates. More interesting is the behavior of the residual drag force: it always quenches linearly with the decay rates as shown in the right panel of Fig. 2 for $k_p = 0.7\ \mu\text{m}^{-1}$ in two cases with $V_d = -22$ meV and $V_d = -2.2$ meV. The dependence of the residual drag force on V_d is shown in the left panel for $k_p = 0.7\ \mu\text{m}^{-1}$ and $\kappa_X = \kappa_C = 0.22$ meV. Calculations have been done with negative values of V_d for numerical reasons. A clear conclusion is drawn from Fig. 2: the residual drag force is non zero as far as both barrier height and decay rates are finite.

The origin of the residual drag force for asymptotically large values of c_s/v_f at finite polariton lifetimes can be understood in terms of the asymmetry of the perturbation generated by the defect in the (e.g., photonic) wave-function, $|\psi_C(\mathbf{r})|^2$ — see Fig. 3. While, as explained later, we observe the disappearance of the Cherenkov waves for large enough polariton densities, even at asymptotically large polariton densities we always observe two small perturbations in the fluid wave-function around the defect, one just in front and one just behind the defect in the x-direction, i.e., in the polariton flow direction. We plot in Fig. 3 cut at $y = 0$ of the wave-function around the defect. It is clear from the definition (8) that, in this limit, the drag force measures the degree of asymmetry of such two perturbations. As the perturbation becomes symmetric around the defect in the limit of large polariton lifetimes, the drag force vanishes. We find that the shorter the polariton lifetime the smoother is the crossover observed in the drag force from supersonic to superfluid behaviour. Instead, for high quality cavities, i.e. long polariton lifetimes, the transition from supersonic to subsonic behaviour is sharper and the residual value of the drag force is smaller.

In Fig. 1, for every value of the pump momentum $\mathbf{k}_p$, we also identify the region of values of c_s/v_f above which in the linear approximation the Landau condition of Eq. (13) cannot be satisfied, and therefore where in the linear approximation one would not expect any drag (such values are plotted as empty symbols). Two examples of the linear excitation spectrum are plotted in Fig. 4 together with the corresponding profiles evaluated beyond the linear approximation — Note that in order to do that the pump strength f in the mean-field equations of Sec. III A needs to be fixed so that to give the same average density of polaritons evaluated by solving numerically Eq. (1), $|\psi^{(0)}_{X,C}|^2 = \int d\mathbf{r} |\psi_{X,C}(\mathbf{r})|^2/\Omega$. We find that,

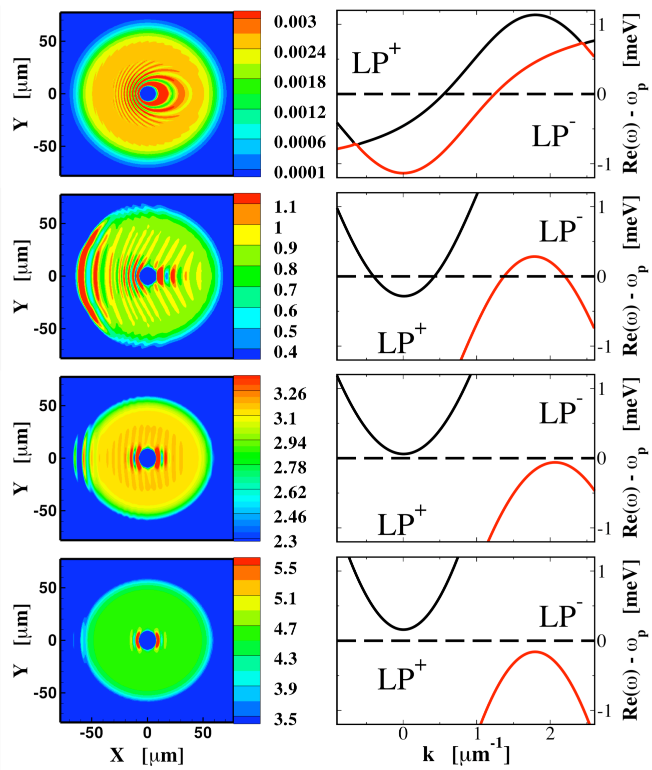

FIG. 6: (Color online) Steady state photon density profile, $|\psi_C(\mathbf{r})|^2$ (left panels) and corresponding quasi-particle excitation spectrum (right panels) for $k_p = 1\ \mu\text{m}^{-1}$, $\hbar/\kappa_C = \hbar/\kappa_X = 7$ ps, and values of the pump power giving $c_s/v_f = 0.05$ (first row), $c_s/v_f = 0.7$ (second row), $c_s/v_f = 1.25$ (third row) and $c_s/v_f = 1.5$ (fourth row). Note in that in the third panel, while in the linear approximation no scattering and therefore no Cherenkov waves are allowed, the full solution of the problem still allows scattering, as Cherenkov waves are observed in the profile of the corresponding left panel.

while eventually the Cherenkov waves disappear at large enough densities, the persistence of asymmetric perturbations around the defect, which we ascribe to finite lifetime effects, contributes to give a finite drag force. This is also apparent in Fig. 5, where we plot the height of the Cherenkov waves as a function of c_s/v_f for the same system parameters as Fig. 4. Here, it is clear that the the Cherenkov waves height is strongly suppressed for $c_s/v_f \simeq 1$, and go to zero for $c_s/v_f \gg 1$.

In addition, in Fig. 5 we plot the percentage of the particles scattered by the defect. Here, like for the drag force, we find a residual value of the percentage of scattered particles at asymptotically high polariton densities. The difference with the drag force is here that, even for perfect cavities when the drag goes to zero, the percentage of scattered particles keeps retaining a residual value (see panel (c) of Fig 3).

In Fig. 6 we plot the photon density profiles $|\psi_C(\mathbf{r})|^2$ (left panels) for increasing pump power, obtained by solving the time dependent Gross-Pitaevskii equation (1), together with the spectrum of linear excitations obtained by solving the eigenvalue problem (10) (right panels). Left panels show Cherenkov waves evolving smoothly from a 'closed' to an 'open' shape till they disappear when the subsonic superfluid regime is reached. In particular, the angle formed between the waves and the propagation direction increases by increasing the polariton density. Note also that, while qualitatively the phenomenology of the Cherenkov waves seems to be well described by the linear approximation theory, the transition from the supersonic to the subsonic superfluid regime is not. In particular, we find a value range of the pump power (like the one shown in the third row of Fig. 6), where no scattering and therefore no Cherenkov waves are allowed in the linear approximation description, while the full numerical solution of the time dependent Gross-Pitaevskii equation (1) still allows scattering of polaritons and the associated Cherenkov waves.

V. CONCLUSIONS AND DISCUSSION

In this paper we have proposed three different ways to analyse the superfluid properties of coherently driven polaritons in the pump-only configuration in presence of a defect potential. We have evaluated the drag force when the fluid passes the defect, the height of Cherenkov waves, and the percentage of particles scattered by the defect. By making a comparison with the linearised Bogoliubov-theory introduced in Refs.[18,19], we have found that, the disappearance of the Cherenkov waves, characterising the transition from a supersonic to a subsonic superfluid behaviour, is not well described by the linear theory. In addition, we have found that non-equilibrium effects which go beyond the linear approximation cause a finite residual drag force even at asymptotically large polariton densities, where the Landau criterion predicts that no elementary excitation can be emitted by the defect. Only in the limit of infinitely long polariton lifetimes, the residual drag force at large enough densities goes to zero, recovering the equilibrium limit. The drag force exerted on the polariton fluid by a defect, as well as the height of Cherenkov radiation, and the percentage of particles scattered by the defect show a smooth crossover rather then a sharp threshold-like behaviour which is typical of superfluids obeying the Landau criterion. We have characterised this crossover as a function of the fluid velocity, the polariton density and the polariton lifetime.

The three observables which we here evaluate theoretically, can in principle be measured in current state-of-the art experiments on semiconductor microcavities. For example, the defect can be carefully engineered by either patterning a metal grating on the microcavity top mirror or growing mesas in one of the mirrors[22]. This allows having a predetermined shape and size of the defect, suitable for a direct comparison with our theoretical analysis. Alternatively, the defect can be switched on and off externally by an additional laser[23]. The second scheme would allow a direct comparison between the fluid motion in presence and absence of the defect. Both schemes are within the current experimental reach, and so with this work we intend to motivate further experimental investigations, which would lead to a better understanding of the novel non-equilibrium superfluid phenomena in microcavities.

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