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Abstract

The NARMAX (Nonlinear AutoRegressive Moving Average model with eXogenous inputs) approach is used to analyse simultaneous measurements of the geomagnetic D_{st} Index and the merging rate of the IMF and the geomagnetic field VB_s. A nonlinear discrete relation which describes the dynamics of the D_{st} index driven by VB_s is derived directly from experimental measurements. This relation can be used to forecast the evolution of the D_{st} index. Higher order spectral analysis of the identified model provides information about nonlinear coupling between the various spectral components.

1 Introduction

Modelling of the geomagnetic activity modelling has received considerable attention in recent years in space physics, especially since it has been realised how important this can be for space weather predictions. One approach considered in the literature for modelling the geomagnetic activity is based on basic physical principles. However it has been realised that geomagnetic activity is a highly complex collection of events, and even the internal relationships are far from being completely understood [Fairfield et al 1992]. A second approach consists of finding a physical analogue model, such as the dripping faucet model [Baker et al, 1990], directly driven model [Goertz et al, 1993], Faraday loop model [Klimas et al, 1996] for auroral electrojet indices or the Burton model [Burton et al, 1975] for the evolution of the D_{st} index.

More recently new data driven approaches have been considered. Neural networks were employed by Wu and Lundsted [1997] to model and forecast the evolution of the D_{st} index. In an alternative approach the nonlinear relationship between the D_{st} index and VB_S was modelled as a collection of local linear models by Klimas and Vassiliadis [Klimas et al, 1997, 1998], [Vassiliadis et al, 1999].

The approach investigated in the present study is based on nonlinear system identification concepts. The NARMAX (Nonlinear AutoRegressive Moving Average model with eXogenous inputs) methodology is used to analyse geomagnetic data, to identify a nonlinear NARMAX model which is a discrete-time difference equation. The identified nonlinear NARMAX model, statistically and dynamically validated, can be used to forecast the D_{st} index. This model can also be readily mapped into the frequency domain to reveal the energy transfer mechanisms that characterise the magnetosphere dynamics.

2 The NARMAX methodology - brief presentation

The NARMAX (Nonlinear AutoRegressive Moving Average model with eXogenous inputs) methodology used in system modelling and identification is a well known procedure in nonlinear system theory. The NARMAX approach which has been developed over the past twenty years, has a wide area of applications, from the analysis of nonlinear differential equation with strong nonlinearities, to real system identification and analysis. The NARMAX representation, proposed by Leontaritis and Billings [1985a,b], is given as

$$y(k) = F[y(k-1), \ldots, y(k-n_y), u(k-1), \ldots, u(k-n_u), \xi(k-1), \ldots, \xi(k-n_\xi)] + \xi(k) \quad (1)$$

where $F[\cdot]$ denotes a nonlinear function, u and y are the discrete-time input and output signals with corresponding maximum lagged values of n_u and n_y. The quantity $\xi(k)$ accounts for possible noise and uncertainties, n_ξ represents the maximum noise lag. The nonlinear function F can be a polynomial, rational function, radial basis functions, wavelet decomposition or any other function subject to some mild constraints. Structure detection, parameter estimation and model validation methods are now well developed for all these model types [Chen and Billings, 1989; Billings and Voon, 1983; Billings and Zhu, 1994].

The structure and parameters in the NARMAX model can be identified using an orthogonal least-squares algorithm, which searches through all the potential model terms and selects the final model terms according to the contribution that they make to the variance of the system output. This allows the user to build the simplest possible model using the most significant model terms. Model validation techniques are then applied to confirm that the model is adequate. The validation is based on both statistical and dynamical criteria. The statistical validation involves high order correlations which ensure that the residuals are unpredictable from all past values of the input and output. The dynamical validation is based on model predictions. A common qualitative validation is based on one step ahead predictions and long term predictions. The one step ahead predictions are obtained from previous measured values of D_{st} and VB_s using the identified NARMAX model (1). A much better test is to compute the long term predictions, often called the model predicted output, where the estimated D_{st} is computed using only input values, without using any measured output values, except for a few initial values to start the recursion.

Once a NARMAX model has been identified, it can be readily translated into the frequency domain [Peyton-Jones and Billings, 1989] to determine all the nonlinear frequency response functions. The nonlinear frequency response functions are generalisations of the linear transfer function, which is defined as the Fourier transform of the impulse response function $h_1(\tau)$ in the convolution integral

$$y(t) = \int_0^\infty h_1(\tau) u(t-\tau) d\tau$$

The first order frequency response function $H_1(\omega)$ gives the measure in which a linear system can modify the amplitude and phase of an input signal.

The nonlinear frequency response functions, also called generalised frequency response functions, are Fourier transforms of the nonlinear impulse response functions

$h_n(\tau_1, \ldots, \tau_n)$ in the nonlinear convolution integral, known as the Volterra series

$$y(t) = \sum_{i=0}^{\infty} \int_0^{\infty} h_i(\tau_1, \ldots, \tau_i) u(t-\tau_1) \ldots u(t-\tau_i) d\tau_1 \ldots d\tau_i \qquad (2)$$

The generalised frequency response functions $H_n(\omega_1, \ldots, \omega_n)$ affect not only the magnitude and phase of an input signal, but also give a measure of the nonlinear coupling between spectral components of the input and of the energy transfer mechanism to new spectral components in the output.

The NARMAX approach will be used in the present work to identify features of the nonlinear dynamics of the terrestrial magnetosphere. In the next section a discrete-time NARMAX model is first identified for the D_{st} and VB_s data sets and translated into the frequency domain, where the generalised frequency response functions reveal in a new way energy transfer phenomena, together with a storage and release energy mechanism.

3 Analysis of the D_{st} dynamics

The input-output data analysed in this paper consist of 4344 hours of data with 5 minutes resolution for D_{st} and the solar wind VB_s taken from January to June 1979[1]. The D_{st} data has not been pressure corrected, and the VB_s data has been propagated ballistically to the magnetopause.

The set of 4000 points of the data set with 40 minutes resolution was chosen as the estimation data set. Applying the NARMAX procedure to the data, using the Orthogonal Least Squares estimator and a polynomial expansion of $F[\cdot]$ in (1), the model process terms were identified as

$$\begin{aligned} D_{st}(k) = \; & +1.0772D_{st}(k-1) - 0.3940D_{st}(k-2) + 0.2711D_{st}(k-3) - 2.5654VB_s(k-2) \\ & +0.8287VB_s(k-4) + 0.4948VB_s(k-6) - 0.1600VB_s(k-1)VB_s(k-9) \\ & +0.3553VB_s(k-10) + 0.3293VB_s(k-2)VB_s(k-8) \\ & +0.2131VB_s(k-1)VB_s(k-8) - 0.1533VB_s(k-9)VB_s(k-9) \\ & +0.0943VB_s(k-1)VB_s(k-7) + 0.1644VB_s(k-2)VB_s(k-10) \\ & -0.4843VB_s(k-2)VB_s(k-7) - 0.1686VB_s(k-3)VB_s(k-4) \end{aligned} \qquad (3)$$

The model (3) consists of 15 process terms, and these contributed 98.95% to the total variance in the output. In this model k represents the discrete-time step, 40 minutes in this case. The model satisfied both the statistical and dynamical validation tests. Note also that a noise model was also estimated but is not given here. The one hour ahead predictions together with ten hours ahead and long term predictions are given in Figure 1. The predictions obtained are qualitatively similar with those obtained by Klimas et al [1997,1998] and Vassiliadis et al [1999], however the model can be improved if solar pressure is taken into account. Furthermore, the NARMAX approach enables not only good forecasting but more importantly, physical interpretation of the nonlinear mechanism that characterise D_{st} in the frequency domain.

The generalised frequency response functions can be further obtained based on the method derived by Peyton-Jones and Billings [1989]. The linear frequency response

[1] the data was provided by Prof.A. Klimas and Prof.Y.Kamide

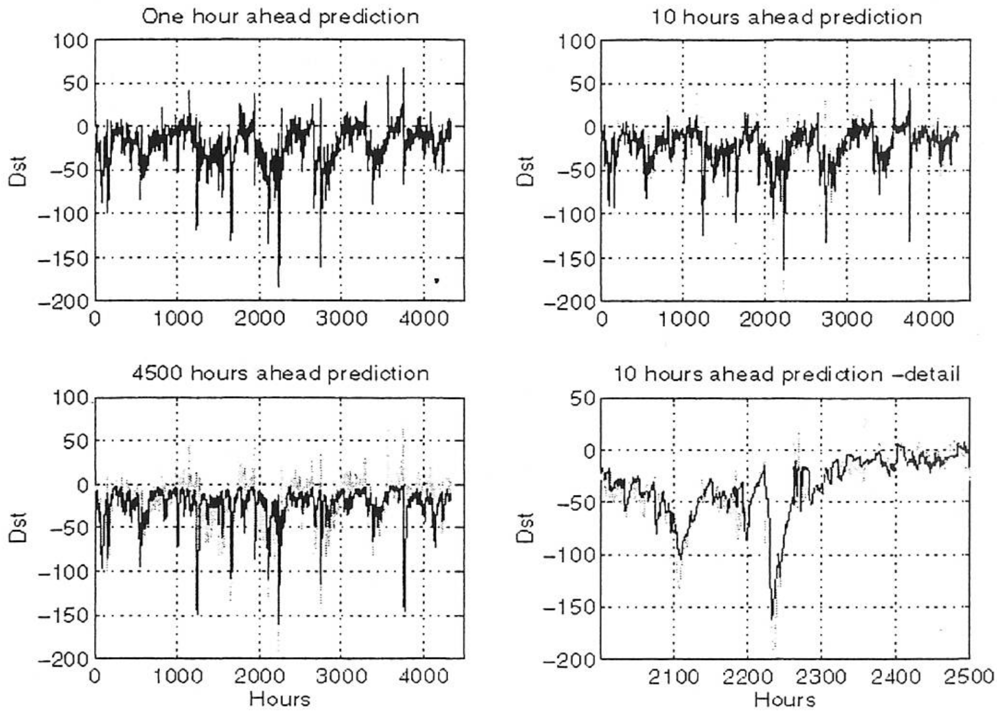
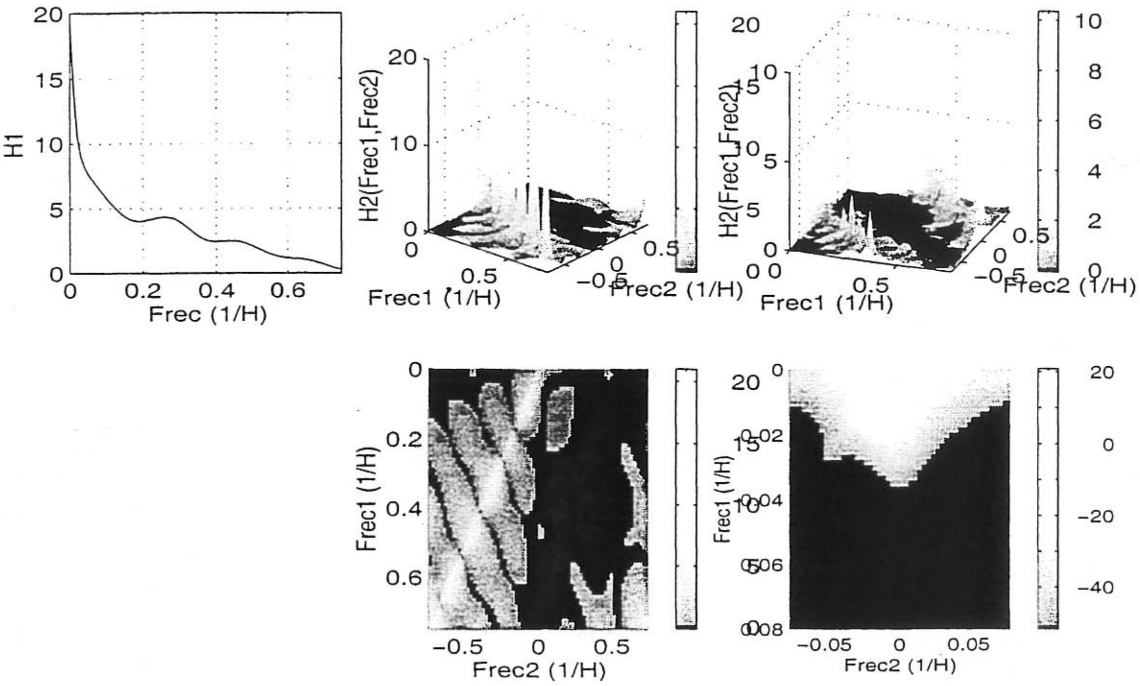


Figure 1: (a) Discrete-time model (3) predictions (black) compared with measurements (gray)

function $H_1(\omega)$ shows in Figure 2(a) a high magnitude for low frequencies, meaning that low frequency components of the VB_s will be amplified while the higher components will be reduced. The nonlinear model (3) also has a second order frequency response function $H_2(\omega_1, \omega_2)$, shown in Figure 2(b) as a plan view plot and also as a three dimensional image. As in the linear case, $H_2(\omega_1, \omega_2)$ has a high amplitude along the low frequencies line $f_1 + f_2 = 0$, meaning that only frequency components in the input satisfying this condition will be amplified by second order nonlinearities, the rest of the frequency components being reduced. The phenomena of transferring high frequency components down to a frequency close to the zero frequency component in $f_1 + f_2 = 0$, can be interpreted as energy storage.

A complementary phenomena was detected for a different NARMAX model, identified on the original data set oversampled at one hour resolution. The model is presented in detail in Boaghe et al [1999] and only the third order transfer $H_3(\omega_1, \omega_2, \omega_3)$ described here. This is shown in Figure 2(c) as a hyper-plane section on the line $f_1 = f_3$, the function shows the same type of low frequency amplification on the line $f_1 + f_2 + f_3 = 0$. Again this can be interpreted as a storage of energy, but $H_3(\omega_1, \omega_2, \omega_3)$ also shows an amplification for components which satisfy $f_1 \in [0; 0.005]$ and $f_2 \in [-0.08; 0.08]$. In this case a low frequency component in the interval $f_1 \in [0; 0.005]$ can be transfered to higher frequency values through nonlinear coupling with $f_2 \in [-0.08; 0.08]$ and this phenomena can be interpreted as energy release.

Another advantage of the NARMAX methodology and frequency domain approach



(a) (b) (c)

Figure 2: (a) H_1 response function; (b) H_2 response function (3-D plot and plan view); (c) H_3 response function from Boaghe et al [1999] (3-D plot and plan view detail and logarithmic scale)

is the possibility of continuous-time model extraction. This is very important because the underlying phenomena are continuous and it allows the identified results to be interpreted and related with respect to existing physical models. Based on a method derived by Li and Billings [1998] the following continuous-time nonlinear model was derived from the identified NARMAX model in equation (3)

$$5.38\ddot{D}_{st} + 13.38\dot{D}_{st} + D_{st} = -12.88VB_s - 9.36VB_s^2 + 0.77VB_s^3 \qquad (4)$$

The continuous-time model predicted output agrees very well with the discrete-time model predicted output. Further improvement is expected when solar pressure will be considered.

4 Conclusions

The identification of a mathematical model relating the geomagnetic index D_{st} to the geomagnetic field VB_s has been performed using methods from nonlinear systems identification. A concise representation of the system was identified using the NARMAX approach and was used for D_{st} index forecasting. The dominant nonlinear characteristics of the geomagnetic index were studied using the Generalised Frequency Response Functions computed directly by mapping the identified NARMAX model into the frequency domain.

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