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Speculative bubbles in Bitcoin markets? An empirical investigation into the fundamental value of Bitcoin

Eng-Tuck Cheah, John Fry

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- The current market value of the bitcoin market is currently estimated to be around $3.3 billion
- Bitcoins prices contain a considerable speculative component
- Bitcoin markets are susceptible to bubbles.
- The fundamental value of bitcoins is zero.

Speculative bubbles in Bitcoin markets? An empirical investigation into the fundamental value of Bitcoin

Eng-Tuck Cheah* John Fry†

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Abstract

Amid its rapidly increasing usage and immense public interest the subject of Bitcoin has raised profound economic and societal issues. In this paper we undertake economic and econometric modelling of Bitcoin prices. As with many asset classes we show that Bitcoin exhibits speculative bubbles. Further, we find empirical evidence that the fundamental price of Bitcoin is zero.

Keywords: Bitcoin; Cryptocurrencies; Bubbles; Econophysics
JEL Classification: C1 E4 G1

1 Introduction

Originally introduced in 2008 by a group of programmers, under the pseudonym Satoshi Nakamoto, Bitcoin is a cryptocurrency or virtual money derived from mathematical cryptography and conceived as an alternative to government-backed currencies. It was originally envisaged that its construction and digital "mining" processes would mean that Bitcoin prices would be relatively stable. However, the fact that Bitcoin prices have recently fallen about 60 percent from their peak value, allied to similar boom-bust patterns in other digital currencies, raises two key questions of interest. Firstly, does this dramatic boom-bust phase constitute a bubble? Secondly, and more importantly, does Bitcoin have a positive fundamental value? In this paper we provide empirical evidence to address the existence of bubbles in Bitcoin markets and to determine the fundamental value of Bitcoin. Using methods that originate in physics, but are increasingly becoming part of mainstream finance (see e.g. Bree and Joseph, 2013; Lin et al., 2014), we find that Bitcoin prices contain a substantial speculative bubble component (Dowd, 2014). More importantly, our results show that the fundamental value of Bitcoin is zero.

Bitcoin is the most famous cryptocurrency with an estimated market capitalization of $3.3 billion (`coinmarketcap.com` accessed on Feb 16th 2015) but there are other alternative digital currencies – known as *altcoins*. These altcoins are also gaining in popularity. Rival altcoins to Bitcoin, such as Ripple and Litecoin, also have significant market capitalisations of $441m and $66m respectively. It appears that competition between ever-expanding varieties of altcoins may drive down bitcoin's market share. However, Bitcoin currently accounts for around 83%

*Sheffield University Management School, Conduit Road, Sheffield S10 1FL, UK. E-mail: J.Cheah@sheffield.ac.uk
†Sheffield University Management School, Conduit Road, Sheffield S10 1FL, UK. E-mail: John.Fry@sheffield.ac.uk

of the total estimated cryptocurrency capitalisation of $3.9 billion. Amid huge public and media interest individuals and organisations have increasingly begun to accept Bitcoin and other cryptocurrencies as valid forms of payment. However, some of this media coverage has been negative – reflecting significant downsides. Bitcoin's vast proliferation and lack of regulation (Grinberg, 2012; Plasaras, 2013) may mean its usage encourages black market activities to flourish and raises cybersecurity issues with both individuals and organisations coming under attack. This includes threats to users from malicious software and incidents like the hacking of the Tokyo-based Bitcoin exchange Mt Gox.

The academic literature on digital currencies, such as Bitcoin, has only recently begun to emerge (see e.g. Grinberg 2012; Plasaras, 2013; Maurer et al., 2013) and is dwarfed by a multitude of popular articles and unpublished working papers. There is currently much debate about the appropriate interpretation of Bitcoin (see Section 2). Precious metals like gold and paper notes do not generate a cash flow but retain their value because they can be exchanged for goods and services. Bitcoin's own digital mining processes are intended to replicate the production costs associated with precious metals. In contrast, Bitcoin's convenience, convertibility and low transaction costs share elements of currencies (Frisby, 2014). Much of the academic literature concentrates upon legal aspects and a more comprehensive analysis related to the fundamental value of these cryptocurrencies is needed. Any currency needs to fulfil a number of functions such as unit of account (Dwyer, 2014), means of payments and store of value. Fundamentally, Bitcoin should represent value. Furthermore, in line with other fiat monies, Bitcoin should command a certain level of confidence among its users. However, recent fluctuations in Bitcoin prices are not suggestive of a constant fundamental value. In fact, Bitcoin prices appear to contain "a substantial speculative component" (Dowd, 2014) that warrants further investigation. Such volatility potentially undermines the role Bitcoin plays as a unit of account. The speculative component could potentially signify bubbles (Dale et al., 2005). Therefore, we address the issue of testing for the existence of speculative bubbles in Bitcoin prices. More importantly, we empirically estimate Bitcoin's fundamental value. Our results may thus have important implications for speculative investors, policymakers and law enforcement agencies alike (Dowd, 2014).

The remainder of the paper is organised as follows. Section 2 discusses economic approaches to the evaluation of Bitcoin. Section 3 introduces a bubble model derived from the theory of complex systems in physics. Section 4 discusses empirical results. Section 5 concludes.

2 Economic considerations

The status of Bitcoin as an alternative currency or another kind of speculative asset is still unclear and subject to on-going debate (European Central Bank, 2012). In order to serve as money Bitcoin should fulfil the roles of unit of account (Dwyer, 2014), means of payments and store of value. In this regard as the speculative aspects of Bitcoin dominates its other functions Bitcoin prices appear to be at the mercy of nothing more that market sentiments (Dwyer, 2014; Shiller, 2005; Weber, 2014). This has resulted in unpredictable volatility. Such volatility would require that merchants accepting Bitcoin incorporate a spread over the price in the original currency as Bitcoin prices vary. Hence, this undermines the role Bitcoin plays as a unit of account. However, others have argued that Bitcoin cannot be considered as a form of money. For example, some economists have argued that, in order to have value, Bitcoin must be backed by a government with tax and spending powers (Van Alystne, 2014). According to the claim theory of money Bitcoin constitutes a social relation representing a claim on the issuer. In addition, Bitcoin can be considered as a legal tender with regard to enforceable debt and tax obligations on citizens (Bell, 2001; Dequech, 2013: Ingham, 2004). In the United States the IRS treats Bitcoin as property and consequently, taxable. Other authorities, such as the German Finance Ministry, categorically endorse Bitcoin as a unit of account for tax and trading purposes (Van Alstyne, 2014).

3 A speculative-bubble perspective

Let P_t denote the price of an asset at time t and let $X_t = \log\ P_t$. The set up of the model is as follows. Following Johansen et al. (2000) our starting point is the equation

$$P(t) = P_1(t)(1-\kappa)^{j(t)}, \tag{1}$$

where $P_1(t)$ satisfies

$$dP_1(t) = \left[\mu(t) + \sigma^2(t)/2\right] P_1(t)dt + \sigma(t)P_1(t)dW_t, \tag{2}$$

where W_t is a Wiener process and $j(t)$ is a jump process satisfying

$$j(t) = \begin{cases} 0 & \text{before the crash} \\ 1 & \text{after the crash.} \end{cases} \tag{3}$$

When a crash occurs κ% is automatically wiped off the value of the asset. Prior to a crash $P(t) = P_1(t)$ and it follows from Itô's formula (Øksendal, 2003; Chapter 4.1) that $X_t = \log(P(t))$ satisfies

$$dX_t = \mu(t)dt + \sigma(t)dW_t - vdj(t), \tag{4}$$

where $v = -\ln[(1-\kappa)] > 0$. Equation (4) shows us how the bubble will impact upon observed prices. Suppose that a crash has not occurred by time t. In this case we have that

$$E[j(t+\Delta) - j(t)] = \Delta h(t) + o(\Delta), \tag{5}$$
$$\text{Var}[j(t+\Delta) - j(t)] = \Delta h(t) + o(\Delta), \tag{6}$$

where $h(t)$ is the hazard rate.

Assumption 1 (Intrinsic Rate of Return) *The intrinsic rate of return is assumed constant and equal to μ:*

$$E[X_{t+\Delta} - X_t|X_t] = \mu\Delta + o(\Delta). \tag{7}$$

First-order condition. From Assumption 1 equations (4-5) and (7) give

$$\mu(t) - vh(t) = \mu; \quad \mu(t) = \mu + vh(t). \tag{8}$$

Equation (8) shows the rate of return must increase in order to compensate a representative investor for the risk of a crash. However, it can be shown that bubbles also impact upon the volatility.

Assumption 2 (Intrinsic Level of Risk) *The intrinsic level of risk is assumed constant and equal to σ^2:*

$$Var[X_{t+\Delta} - X_t|X_t] = \sigma^2\Delta + o(\Delta). \tag{9}$$

Second-order condition. For a bubble to develop a rapid growth in prices alone is not enough. The perceived price risk must also diminish. Similarly, from Assumption 2 equations (4), (6) and (9) give

$$\sigma^2(t) + v^2 h(t) = \sigma^2; \quad \sigma^2(t) = \sigma^2 - v^2 h(t). \tag{10}$$

The model thus states that it is the interplay between risk and return that fundamentally governs the behaviour of financial markets. Assumptions 1-2 show that bubbles can be identified via anomalous behaviour in the drift and volatility in equation (4). During a bubble a representative investor is compensated for the crash risk by an increased rate of return with $\mu(t) > \mu$ the long-term rate of return. This is accompanied by a decrease in the volatility function $\sigma^2(t)$ – a result which though counter-intuitive actually represents market over-confidence (Fry, 2012; 2014a-b). The result thus shows that the historical record alone will not be sufficient to quantify the true level of risk in the market. Inter alia this idea coincides with the notion of seemingly unpredictable and invisible Black Swan events recently popularised by Taleb (2010).

Equations (8) and (10) above mean that we can test for the existence of a speculative bubble by testing the one-sided hypothesis

$$H_0 : v = 0, \quad H_1 : v > 0. \tag{11}$$

On a related theme to equation (11) Fry (2014b) produces the following estimate of fundamental price when $v = 0$:

$$P_F(t) := E(P(t)) = P(0)e^{\tilde{\mu}t}, \tag{12}$$

where $\tilde{\mu} = \mu + \sigma^2/2$. This equation thus makes use of an important stylised empirical fact: namely that financial and economic time series often exhibit approximately exponential behaviour over long time horizons (see e.g Cambell et al., 1997). Further, this approach allows us to account for the fact that prices may undergo substantial periods of growth even in the absence of a bubble. During a bubble ($v > 0$)

$$X_t \sim N(X_0 + \mu t + vH(t), \sigma^2 t - v^2 H(t)), \tag{13}$$

where

$$H(t) := \int_0^t h(u)\ du. \tag{14}$$

Hence, from equation (14)

$$P_B(t) := E(P(t)) = P(0)e^{\tilde{\mu}t + \left(v - \frac{v^2}{2}\right)H(t)}. \tag{15}$$

Equation (15) leads to the following estimate of the bubble component defined as the "average distance" between fundamental and bubble prices:

$$\begin{aligned} \text{Bubble Component} &= 1 - \frac{1}{T}\int_0^T \frac{P_F(t)}{P_B(t)} dt \\ &= 1 - \frac{1}{T}\int_0^T \left(1 + \frac{t^\beta}{\alpha^\beta}\right)^{-\left(v - \frac{v^2}{2}\right)} dt. \end{aligned} \tag{16}$$

Given plug-in estimates of α, β and v the integral in (16) can be calculated numerically.

Our model also gives us additional insights into the long-term fundamental value of an asset. Consider equation (12). If $\tilde{\mu} < 0$ the fundamental value satisfies

$$\lim_{t \to \infty} P_F(t) = 0. \tag{17}$$

4 Empirical results

The data consists of daily closing prices for the Bitcoin Coindesk Index from July 18th 2010 to July 17th 2014. Figures 1 and 2 show Bitcoin prices from July 18th 2010 to July 17th 2014 and from July 18th 2010 to December 31st respectively. From Figure 1 Bitcoin prices during this period appear relatively stable before peaking dramatically in late 2013. However, as shown in Figure 2 even in the earliest years of this period the price rises observed are considerable. Following similar studies (e.g. Geraskin and Fantazzini, 2013) we investigate whether or not the google trends search index can be used to provide additional insight. Figure 3 plots the google trends search index for the term "Bitcoin" and shows a notable peak in late 2013 reinforcing an important social dimension to bubbles (Kindelberger and Aliber, 2005).

[Insert Figures 1-3 about here]

According to Shiller (2014) speculative bubbles are characterized by a peculiar kind of fad or social epidemic following the principles of social psychology, imperfect news media and information channels. History is replete with examples of people who gambled and lost during economic booms (Reinhart and Rogoff, 2009). Further, evidence suggests that even rational speculation may still be accompanied by episodes of mass hysteria (Zeira, 1997). Given that 70% of existing Bitcoins are held in dormant accounts (Weber, 2014) Bitcoin seems to behave more like an asset than a currency. Bitcoin's main attraction seems to lie in being an object of speculation instead of functioning as money. However, the question of Bitcoin's fundamental value remains important given both the range of related empirical work that has appeared in the literature (see e.g. Campbell et al. 1997 for a review) and wider questions about Bitcoin's long-term sustainability.

Speculative bubbles can be categorized as either rational or irrational (Dale et al., 2005). Possible explanations for the formation of bubbles include self-fulfilling expectations (rational bubble), mispricing of fundamentals (intrinsic rational bubble) and the endowment of irrelevant exogenous variables with asset pricing value (extrinsic rational bubble). Rational bubbles exist when investors anticipate that they can profitably sell an overvalued asset at an even higher price. In contrast, irrational bubbles are formed when investors are driven by psychological factors unrelated to the asset's fundamental value. This can occur when investors resort to simple heuristics driven by market sentiments or via irrationally optimistic expectations, fashions and fads (Dwyer, 2014; Shiller, 2005; Weber, 2014). Under these circumstances, the relationship between fundamental value and price breaks down (Dale et al., 2005).

Table 1 presents summary statistics for the log-returns series and shows that Bitcoin prices exhibit rich and volatile market dynamics. In particular, the high mean return points to a dramatic bubble-like price rise. Finally, we ran a BDS test (Brock et al., 1996) to test for dependence in the log-returns. Results were significant ($p = 0.000$) and are suggestive of some hidden underlying structure. In applications such hidden nonlinearity or nonstationarity is often interpreted to mean that the system (here Bitcoin prices) may be approaching a critical transition. Thus, information from multiple sources suggests that Bitcoin may have been subject to a speculative bubble.

[Insert Table 1 about here]

Following MacDonell (2014) we test for bubbles in the Bitcoin index from January 1st 2013-November 30th 2013 to determine if the crash of December 2013 is preceded by a bubble.

Results are shown in Table 2. We reject the null hypothesis (11) and have significant evidence of a bubble. Further, the parameter $\tilde{\mu}$ is not statistically different from zero suggesting that equation (17) applies. Thus, during the bubble price rises are so dramatic that the estimated long-term fundamental value is not statistically different from zero. This result reinforces the fact that the bubble dominates observed prices and occurs amid popular warnings that Bitcoin is "voodoo". Equation (16) suggests that the bubble accounts for around 48.7% of observed prices.

[Insert Table 2 about here]

As a simple crosscheck we applied two related models that have regularly appeared in the literature. Using the model in Johansen et al. (2000) we reject the null hypothesis of exponential growth in favour of the alternative hypothesis of a nonlinear super-exponential bubble ($p = 0.000$). Similarly, using the model in Andersen and Sornette (2004) we reject the null hypothesis of a random walk in favour of the alternative hypothesis of an explosive bubble ($p = 0.000$).

Following similar approaches taken in Geraskin and Fantazzini (2013) and Philips et al. (2011) we test for bubbles over a moving time window. Results in Table 3 give evidence in favour of a bubble irrespective of the time interval chosen. However, results suggest a new, more dramatic, bubble phase begins around January 2013.

[Insert Table 3 about here]

5 Conclusions

Amid increasing levels of interest and popularization (see e.g. Frisby, 2014), Bitcoins and other cryptocurrency markets have been under-explored academically. Firstly, as with other asset classes, Bitcoin prices are prone to speculative bubbles. Secondly, the bubble component contained within Bitcoin prices is substantial (Dowd, 2014). Thirdly, the fundamental value of Bitcoin is zero. These results therefore reflect wider academic and popular concerns about Bitcoin's long-term viability. Results from this study show that cryptocurrency markets share some stylised empirical facts with other markets – namely a vulnerability to speculative bubbles. Future work will undertake further empirical analyses of cryptocurrency markets and the comparison with findings reported for other asset classes appears interesting.

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Figures and Tables

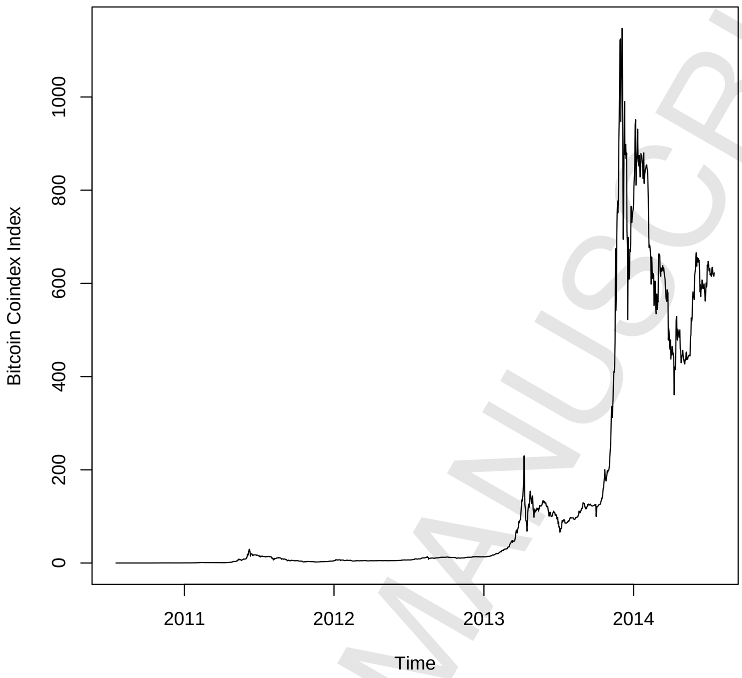

Figure 1: Daily closing prices of the Coindesk Bitcoin Index (US Dollars).

Mean	0.006088821
Median	0.002512208
Standard Deviation	0.07117108
Skewness	-0.3519514
Kurtosis	11.97878
Jarque-Bera	4934.438

Table 1: Summary statistics of the log-returns series

Parameter	Estimate	E.S.E.	t-value	p-value
v	0.546	0.090	6.060	0.000
$\tilde{\mu}$	0.00166	0.00195	0.852	0.394

Table 2: Parameter estimates of the stochastic bubble model

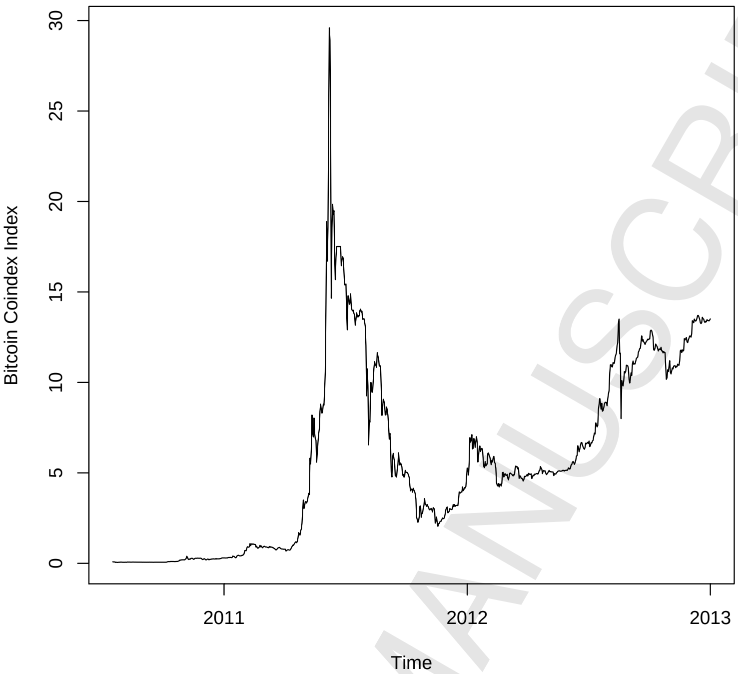

Figure 2: Daily closing prices of the Coindesk Bitcoin Index (US Dollars) July 2010-December 2012.

Time Window	p-value
July 2010-December 2012	0.000
November 2012-January 2013	0.078
December 2012-February 2013	0.001
January 2013-March 2013	0.000
February 2013-April 2013	0.000
March 2013-May 2013	0.000
April 2013-June 2013	0.000
May 2013-July 2013	0.000
June 2013-August 2013	0.000
July 2013-September 2013	0.000
August 2013-October 2013	0.000
September 2013-November 2013	0.000

Table 3: Likelihood ratio tests of the null hypothesis of no speculative bubble over a moving time window

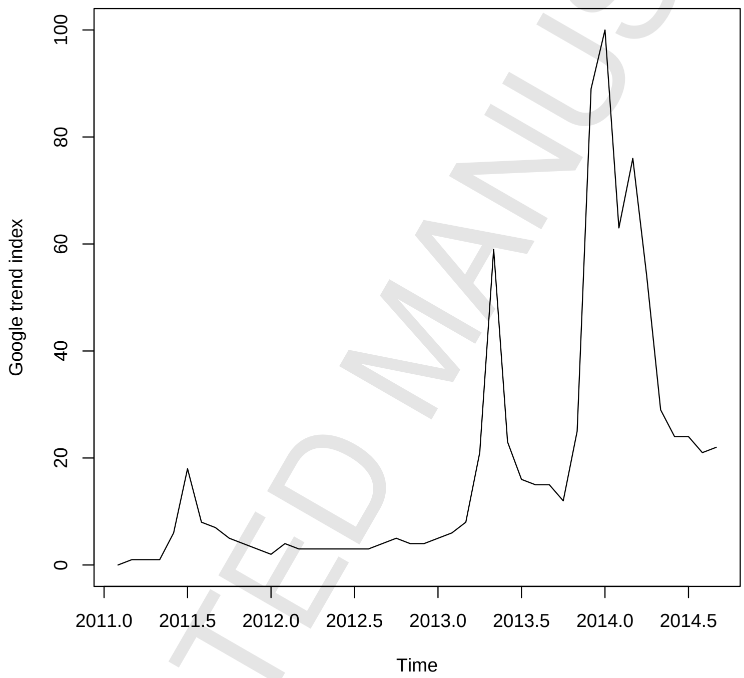

Figure 3: Google trends search index for “Bitcoin”.