Article:

Mounce, S.R., Shepherd, W., Sailor, G. et al. (2014) Predicting combined sewer overflows chamber depth using artificial neural networks with rainfall radar data. Water Science and Technology, 69 (6). 1326 - 1333. ISSN: 0273-1223

https://doi.org/10.2166/wst.2014.024

Predicting CSO chamber depth using Artificial Neural Networks with rainfall radar data

S. R. Mounce*, W. Shepherd*, G. Sailor*, J. Shucksmith* and A. J. Saul *
* Pennine Water Group, Department of Civil and Structural Engineering, University of Sheffield, Sheffield, S1 3JD, UK.
(E-mail: s.r.mounce@sheffield.ac.uk)

Abstract Combined sewer overflows (CSOs) represent a common feature in combined urban drainage systems and are used to discharge excess water to the environment during heavy storms. To better understand the performance of CSOs, the UK water industry has installed a large number of monitoring systems that provide data for these assets. This paper presents research into the prediction of the hydraulic performance of CSOs using Artificial Neural Networks (ANN) as an alternative to hydraulic models. Previous work has explored using an ANN model for the prediction of chamber depth using time series for depth and rain gauge data. Rainfall intensity data that can be provided by rainfall radar devices can be used to improve on this approach. Results are presented using real data from a CSO for a catchment in the North of England, UK. An ANN model trained with the pseudo-inverse rule was shown to be capable of providing prediction of CSO depth with less than 5% error for predictions more than one hour ahead for unseen data. Such predictive approaches are important to the future management of combined sewer systems.

Keywords combined sewer overflows**;** artificial neural networks; cross correlation; rainfall radar; depth monitoring; prediction; catchment.

INTRODUCTION

Urban sewerage infrastructure represents highly complex distributed systems; understanding, managing and predicting the performance of these systems is extremely challenging despite being of paramount importance for society. Combined sewer overflow (CSO) structures are common assets within the UK's combined urban drainage systems. Their main purpose is to protect downstream sewers and waste water treatment plants (WWTP) from hydraulic overloads and flooding during extreme rainfall events. Spills can have a significant impact on the quality of receiving waters and cause regulatory failures. Hence, managing CSO spill pollution has become a significant concern for water companies and the efficient management of CSO assets, with catchment wide integration, is likely to be a key enhancement to the operation of systems into the future. Improved spill prediction approaches would considerably enhance this process.

Within the UK, hydraulic monitoring of sewer systems is generally limited to short term flow surveys of around 12 weeks duration. Here, rainfall data is measured by rain gauges for a number of rainfall events and measurements of the sewer flow depth and flow rate are used to calibrate sewer hydraulic models. Measurements in dry weather flow are also made. Decreasing communications costs are allowing longer term monitoring (described by Shepherd et al., 2010) with the possibility of more in depth analysis of large datasets and online data analysis systems. The application of rainfall radar data is one such area which has received attention in recent years and allows the fine grained detection of spatial and temporal rainfall patterns thus potentially improving hydraulic modelling capabilities. Currently, 85% of the UK has a resolution of 2km or better (Met Office, 2009) and most large urban catchments fall within this area. Radar data for the whole of the UK is processed by the Met Office in order to convert the reflectivity measurement into rainfall intensity and to correct potential errors such as attenuation by intervening rainfall and ground clutter. Rainfall radar data is generally supplied at a time resolution of 5 minutes at near real time. Previous studies have suggested methodologies for the application of radar data to urban drainage systems (Einfalt et al., 2004) and investigated the use of rainfall radar data in sewer hydraulic models (Kramer et al., 2005).

Schellart et al. (2012) showed that radar data provides useful measurements of rainfall which can be applied to sewer hydraulic models with similar confidence to rain gauge data. In their research, rainfall radar data at spatial resolutions of 1 km was obtained from the Met Office produced by a network of C-band radars which cover the UK. The work compared predicted flows from InfoWorks with both rainfall inputs from rain gauge and radar methods together with actual measured flow in the sewer. The analysis was carried out by using a verified InfoWorks CS sewer hydraulic model. Similar studies using hydraulic models have explored applying cluster analysis to investigate correlations between rainfall patterns and CSO behaviours (Yu et al., 2013).

This paper presents an approach to using an Artificial Neural Network (ANN) for CSO performance prediction. Records of rainfall (rainfall radar) and the depth of flow in the CSO chamber are used as training data to establish the relationship between parameters. This relationship is used to predict chamber depth corresponding to subsequent verification rainfall data, without the use of a hydraulic model. Results are presented from field measurements recorded as part of a catchment case study and appropriate metrics employed to evaluate the technique. The methodology was shown to be successful and it is envisaged that a real-time version of the system could be specifically applicable to manage CSOs which are at a high risk of causing pollution and flooding failures.

BACKGROUND

Water utilities routinely gather large amounts of asset performance data. This provides abundant challenges and opportunities for the application of Machine Learning for monitoring, modelling and forecasting; examples include techniques for time series analysis of urban drainage data (Branisavljevic´ et al., 2010), using fuzzy logic for sewer pumping station control (Ostojin et al., 2011) and for monitoring industrial wastewater treatment using adaptive multivariable approaches based on self-organizing maps (Liukkonen et al., 2013). Recent work has explored utilising rainfall radar data, hydraulic models and machine learning approaches for predicting urban flooding in real-time (e.g. Duncan et al., 2013).

Such strategies will provide the opportunity to improve the performance of existing systems, to reduce costs, meet consents and reduce flooding and pollution incidents. Data-driven modelling has the advantage of not requiring a detailed understanding of the physical, chemical and/or biological processes that affect a system before model inputs can be mapped to outputs. ANNs have become an increasingly popular data driven approach for water industry applications and are a modelling approach based on how biological neural systems are believed to work. Examples include for rainfall-runoff modelling (Solomatine et al., 2003) and river flow forecasting (Dawson et al., 1999). Evora and Coulibaly (2009) presented a review of recent advances in ANN modelling of remote sensing applications in hydrology. Li et al. (2010) reviewed the applicability of ANNs to urban hydraulics and hydrology whilst Kurth et al. (2008) demonstrated that a three hidden-layer Multilayer Perceptron (MLP) ANN trained with backpropagation was capable of learning the underlying relationship between local rainfall occurrence and CSO response. Similarly, Guo and Saul (2011) used an adaptive linear ANN (ADALINE) to model linearly the relationship between the CSO chamber hydraulic condition (water level) and rainfall (from an in catchment rain gauge). The ANN was used to predict, at times of dry weather and in response to rainfall, the hydraulic performance of a CSO in terms of flow depth. Using rainfall and depth as lagged inputs, the chamber water level for 3 time steps ahead (15 minutes) was successfully predicted for a number of assets.

CASE STUDY

Catchment and data sources

Some UK water utility companies currently monitor many of their CSO assets with telemetered ultrasonic water level sensors. The data from one such CSO has been used in this study. Situated in the north of England, the CSO serves as the terminal flow control to a treatment works at the bottom of a steep combined urban drainage catchment. The catchment serves a population of ~11,000 people in several small towns spread over ~ 20km² of a substantively rural area. A schematic of the catchment is shown in Figure 1. The chamber, installed in around 2004, is a recent design of single high side weir (9m long, 2m wide with weir height 1m), incorporating rotary screens along the 5.5 m weir length. Flows from events with a return period greater the 5 years are designed to overtop the screens to preserve hydraulic capacity in the network. Time series water level data within the CSO was recorded using an ultrasonic depth monitor producing an instantaneous reading every 15 minutes, with the depth recorded as a percentage and a figure of 100% generally calibrated to spill level (analysis of the data suggests the spill level at this particular site is over 160% but this calibration discrepancy is unimportant to the key findings of this paper). Rainfall intensity data (mm/hr) from 20 (numbered) rainfall radar pixels (1 km^2 resolution) which covered the sewered area, was collected continuously for a period of six months from 13/06/2012 to 31/12/2012. The rainfall intensity data was supplied at a time resolution of 5 minutes, but was aggregated to 15 minutes to match the CSO level data. Figure 1 also shows the river / canal overlay (blue), urban blocks (grey) and tree areas (green).

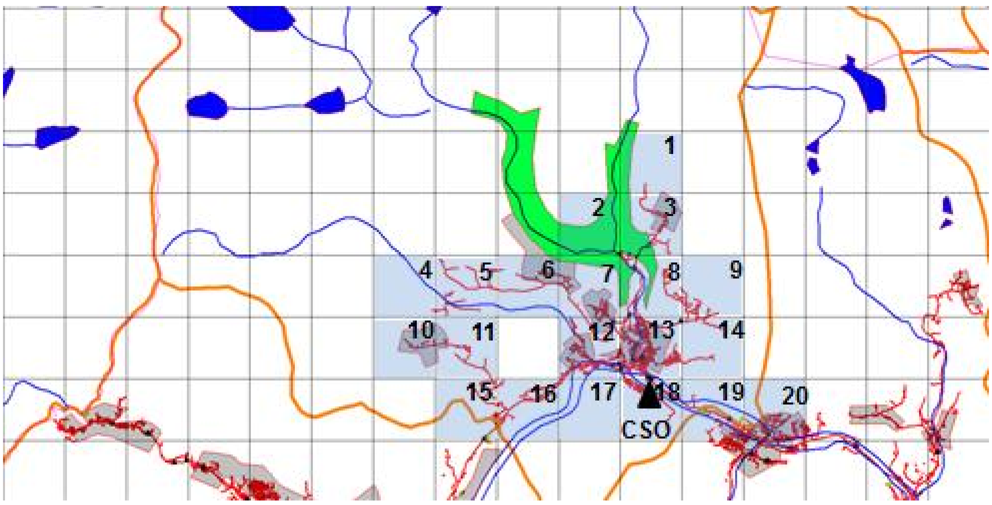

Figure 1: Case study CSO and rainfall radar grid square coverage

Runoff from a storm over an urban catchment initially flows overland, before entering the sewer system to be carried downstream to the CSO. This runoff time and sewer flow time causes a time difference between recorded rainfall and the response at the CSO chamber, this may be exacerbated by the spatial and temporal distribution of the rainfall over the catchment, including speed and direction of travel. Note that there is a slope ratio of ~ 1 in 20 present.

METHODS

Data analysis and determination of input data

Initial data analysis was conducted in order to determine appropriate inputs. Firstly, all data was assembled and pre-processed for any missing data points. The coefficient of correlation was then calculated between rainfall intensity (mm/h) from each of the 20 grid squares and the CSO depth. These positive values varied between 0.175 and 0.241 (mean 0.218) with the top 6 squares being 1, 2, 3, 5, 6 and 7. However, Pearson's r does not provide any information concerning lagged versions of time series data. The underlying relationship between local rainfall and water level in a CSO chamber will occur with a certain lag time. When a rainfall event occurs in the contributing catchment, the CSO reacts with a rising water level in its chamber, whereas under normal conditions during dry weather the water level presents a relatively stable diurnal pattern.

Cross-correlation is a measure of the similarity of two variables (signals) as a function of a time lag between them (Bracewell, 1965). It achieves this by aligning peaks (or troughs) across the two signals at different lags and hence can be used to determine the time delay between two signals. The cross correlation between the CSO depth and rainfall data (and the serial correlation to explore auto correlation for depth) were thus investigated, this method has previously been successfully used for similar studies (Fernando et al., 2006) in order to determine the size of the model input in order to capture the underlying process effectively. Equation 1 gives the cross correlation and equation 2 the serial correlation.

$$y \otimes u = \bar{y}(-t) * u(t) \qquad (1)$$

$$y \otimes y = \bar{y}(-t) * y(t) \qquad (2)$$

where y is the depth, u the rainfall intensity, $*$ a convolution function and $\bar{y}(t)$ is the complex conjugate of $y(t)$

Twenty cross-correlations were applied to the datasets, using the XCORR function in MATLAB® R2012a (The MathWorks Inc., Massachusetts). The maximum of the cross-correlation function indicates the point in time where the signals are best aligned. Figure 2 shows a graph of the correlations for each rainfall radar cell for a range of time lags. The cross correlation maximum varied between 0.29 and 0.38 at either time lag -4 or -5 (corresponding to one hour to one hour and fifteen minutes). The larger maximum correlation squares were 1, 3, 6 and 7. The longer time delay of -5 was observed in the far western grid squares (4, 5 and 10). While a full hydraulic model for this catchment was not available, this figure is close to an estimate of the time of concentration. A greater geographic separation in rainfall radar squares would be expected to show a wider range of peak cross correlation values.

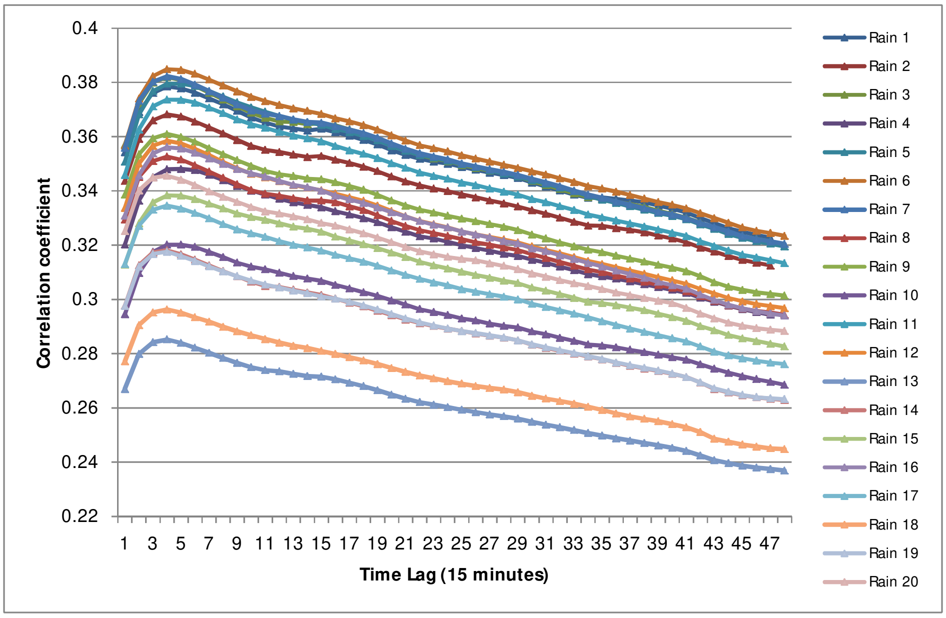

Figure 2: Cross correlations between depth (y) and rainfall intensity (u)

For most radar squares a good choice for a temporal window with a high correlation would be a lag of between 2 and 12. For serial-correlation in depth on the other hand, the correlation values decrease gradually from unity, as would be expected, with increasing lag time.

Model implementation

One of the most straightforward ANN architectures is a single layer feed-forward network with single output. One such structure is the standard perceptron with weights, a bias and a summation function. A number of learning rules can be used to train this network. The Hebb rule is based on the correlations of each input with the output through every prototype. A generalisation of the Hebb rule is the pseudo-inverse rule defined by equation (3).

$$\mathbf{w} = \mathbf{b}\mathbf{A}^{+} \tag{3}$$

Where **w** is the weight matrix, **b** the output vector, and $\mathbf{A}^{+}$ the Moore–Penrose pseudoinverse of the input vector (Penrose, 1955). A common use of the Moore–Penrose pseudoinverse is to compute a least square errors solution to a system of linear equations that lacks a unique solution. Consequently the matrix defined by (3) is the one which minimises the error $\|\mathbf{wA} - \mathbf{b}\|$ on the output space.

An alternative learning rule is the ADALINE rule, also referred to as the delta rule or Widrow-Hoff rule (Widrow, 1962) as defined in equation (4), with *t* time η_t the learning rate:

$$\mathbf{w}(t+\mathbf{1}) = \mathbf{w}(t) + \eta_t(\mathbf{b} - \mathbf{w}(\mathbf{t})\mathbf{A})\mathbf{A}^{\mathrm{T}} \tag{4}$$

The ADALINE rule produces its best solution on the convergent point, which is $\mathbf{w} = \mathbf{b}\mathbf{A}^{+}$ (Mayoraz, 1990). Hence the pseudo inverse rule is utilised here for hydraulic performance prediction. Guo (2011) found that the relationship between CSO hydraulic condition (flow depth) and rainfall (from rain gauge data) was capable of being modelled in this way i.e. with a single layer ANN (no hidden layers). Hence a moving time-window approach can be implemented for the case study CSO whereby lagged time-series signals (rainfall intensity and depth) are provided in parallel over the time-window as inputs to the network. The model development and data pre-processing (such as normalisation) was carried out using MATLAB®. Several ANN models (ANN-N) were consequently developed for forecasting the current to p future value of the depth rate y(t) to y(t+p) thus consisting of n+m input nodes (n antecedent depth data y and m antecedent rainfall data u from grid square X). For example, ANN-1 used rainfall radar data from grid 6, with rainfall input to forecast y(t) being u(t), u(t-1), u(t-2), u(t-3), u(t-4), u(t-5), u(t-6), u(t-7), u(t-8), u(t-9), u(t-10) and depth input values y(t-1), y(t-2), y(t-3), y(t-4), y(t-5), y(t-6), y(t-7), y(t-8). Hence the rainfall intensity parameter u was always one data step ahead of the chamber water depth parameter y.

RESULTS AND DISCUSSION

Several ANN models, as described in the methodology, were constructed and the data for the case study catchment used to assess performance. Predictions for the chamber depth were attempted for p time steps ahead (15 minutes to 1 hour and 15 minutes). A representative training set was constructed containing both dry and wet weather periods (approximately 50% of the overall period). This model was then applied to a subsequent test period. A number of rainfall radar pixels were used and three are included here. Training and test performance is given in Table 1. In this paper the Root Mean Squared Error (RMSE), as defined in equation (5), has primarily been used to evaluate the predictive accuracy of the model.

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n}\sum_{j=1}^{m}(o_{ij} - t_{ij})^2}{n}} \qquad (5)$$

where n is the number of patterns in the validation set, m is the number of components in the output vector, o is the output of a single neuron j, t is the target for the single neuron j, and each input is denoted by vector i. A value was also calculated which scaled the error by the predicted water depth i.e. to give a percentage error per time step.

Table 1. ANN models and metric results

	Radar	Architecture			Train RMSE	Train %	Test RMSE	Test %
	Grid square	*u* delay (m)	*y* delay (n)	p				
ANN-1	6	11	8	1	5.19	2.74	3.97	1.99
ANN-2	6	8	6	1	5.20	2.73	3.97	1.98
ANN-3	6	15	10	1	5.18	2.75	3.97	2.00
ANN-4	6	11	8	2	7.91	4.66	4.54	2.72
ANN-5	6	11	8	3	10.03	6.34	5.42	3.84
ANN-6	6	11	8	4	11.97	8.22	6.11	3.97
ANN-7	6	11	8	5	13.73	10.23	6.58	4.28
ANN-8	5	11	8	1	5.23	2.74	3.94	1.97
ANN-9	5	11	8	5	13.84	10.39	6.35	4.32
ANN-10	18	11	8	1	5.37	2.68	3.98	2.27
ANN-11	18	11	8	5	14.55	11.27	5.71	4.07

It can be seen from Table 1 that the model prediction accuracy is reduced as the prediction range is increased. A range of input lags (identified by cross correlation) provide a good performance, and it is concluded that model accuracy is sufficient to provide a prediction of CSO depth with only 2% error on a one time step ahead prediction (15 minutes) for unseen data, and with less than 5% error in all models for predictions 5 time steps ahead (75 minutes). An example of the application of the ANN-1 model, with a prediction of performance 15 minutes (one time step) into the future is shown in Figure 3. There is excellent agreement between the predicted and observed depth for unseen data. Figure 4 shows the prediction for a period in which spilling occurred following rainfall. A one hour ahead (4 time steps) prediction is plotted in order to assess the potential effect of rainfall at the CSO (so the prediction has been shown four time steps advanced). It can be observed that for the periods of rainfall an increase in chamber depth is anticipated ahead of time, thus illustrating the potential of the model.

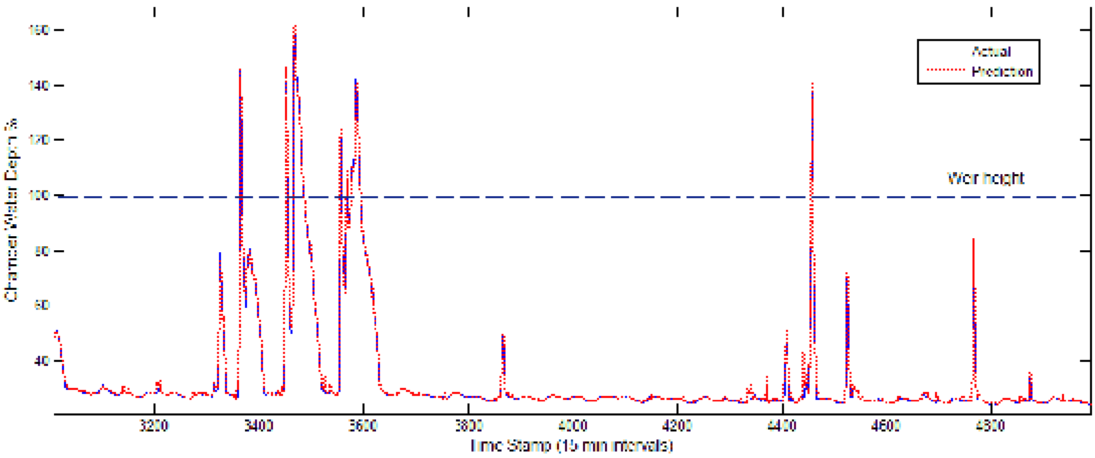

Figure 3: Model prediction and measured flow depth.

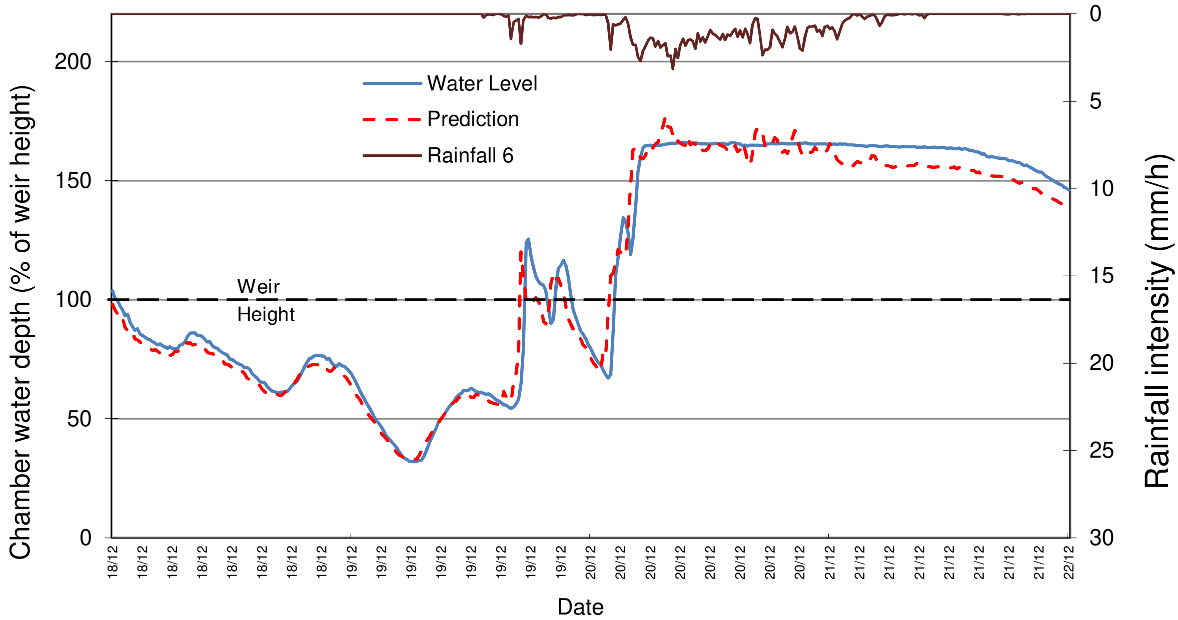

Figure 4: Model prediction (one hour ahead) for selected period

When considering Figure 4, it is also of interest to compare overflow volumes calculated from measured and simulated depths using the standard thin plate weir formula (based on a weir crest at 164%). For the totality of the test data (nearly three month's data), the measured cumulative overflow volume was 3862m³. The predicted cumulative overflow volume was 9288m³. Hence, there was a large over-prediction of overflow volume due to relatively small errors in the depth prediction. We can conclude the architecture is not optimised for this prediction, which was not the intention of the methodology.

For reasons of brevity, full details of the model sensitivity tests, including derivation of training set length and other validation conducted are not presented but the interested reader is referred to Guo (2011). However, Table 2 provides some sensitivity tests conducted as regards the training and testing sets using different input data along with a range of error metrics. Note that m=11, n=8 and rainfall radar square is 6 (when rainfall *u* is incorporated as an input). We see that including the rainfall in the input (columns 1 to 3) versus the equivalent columns where depth only is used (indicated as *y* only) generally only improves prediction accuracy marginally for training data, and sometimes not at all for testing. However, only using rainfall data and no depth in the input (final column) results in very large errors and, in this case, actually higher error on the test set.

Table 2. Sensitivity tests

Type	p=1, *y* and *u*		p=4, *y* and *u*		p=12, *y* and *u*		p=1, *y* only		p=4, *y* only		p=12, *y* only		p=1, *u* only	
	Train	Test	Train	Test	Train	Test	Train	Test	Train	Test	Train	Test	Train	Test
% error	2.7	2.0	8.2	4.0	21.4	7.8	2.9	1.7	10.9	3.6	23.5	7.9	58.8	75.8
R^2	0.987	0.993	0.929	0.985	0.781	0.968	0.984	0.993	0.907	0.990	0.763	0.973	0.223	-0.075
MAE	1.53	1.48	4.80	3.40	10.5	6.12	1.44	1.39	5.20	2.59	10.96	5.52	30.80	40.15
RMSE	5.19	3.97	11.97	6.11	21.03	8.8	5.64	4.14	13.72	5.03	21.85	8.04	39.60	50.92
MASE	1.04	1.02	3.26	2.34	7.14	4.22	0.98	0.96	3.54	1.78	7.45	3.80	20.95	27.65

Due to an artefact of the data set division the performance is actually better for the test set for the networks featured in Table 1 (and in most cases in Table 2) contrary to normal ANN applications. The training period covered mid-June to late September, while the test data period was for the months October to December. Evidently these two periods will possess quite different rainfall patterns. There was 6.4% more rainfall depth in the training data and it tended to be more intense. However, a more significant factor was that when the characteristics of the data sets were examined in detail, it was discovered the training set had a particular period with CSO depth values significantly above 170%, whereas there is a clear cut-off in values for the test data around 170%. This appears to skew the results at the highest depths, which affects the training set with higher errors compared to the test set. It is suspected that either the monitor or the CSO had a minor issue during this first 3 months i.e. during the training period. The training and test sets were switched around and, using the parameters of ANN-1, we get 1.9% error in training and 2.6% error in testing i.e. a more normal performance on test data. So the performance of the CSO during the first 3 months is less predictable with the architecture than in the later period.

Clearly there are many potential applications for the model in respect of identifying unexpected performance or in a gradual change (e.g. silt build-up). Development of the model is on-going in order to provide this type of interpretation capability. In particular, online processing of data could allow the prediction of CSO performance failures (such as spill events) much earlier and potentially in real time. A full decision support system will also necessitate further classification of model outputs perhaps using fuzzy / Bayesian inference systems or a binomial event discriminator. While the work described has only used measured rainfall, there is the potential to use predicted rainfall (using nowcasting) as an input to the model. However, this type of forecasting would naturally lead to a greater degree of uncertainty and potentially larger errors. There is also significant potential for applying these techniques to other sewerage asset types such as Detention Tanks and Sewer Pumping Stations with a view to enabling wider network performance visibility.

CONCLUSIONS

Rainfall radar data offers a data solution for near real time operational strategies. This work has demonstrated the potential of a data driven approach in capturing the underlying relationship between contributing local rainfall (using radar data) and the water level within a CSO structure downstream. A case study example has shown how rainfall radar data correlates with certain time delays to CSO chamber depth. An ANN model trained with the pseudo-inverse rule was shown to be capable of providing prediction of CSO depth with less than 5% error for predictions 5 time steps ahead (75 minutes) for unseen data. This shows improvement on previous studies using tipping bucket rain gauge measurements. In theory, the longer the range of the rainfall prediction available, the further the water depth can be predicted into the future, however in practice rainfall prediction errors will limit the forecast time of the technique. This tool offers the potential benefit of early detection of unexpected performance behaviour and the identification of various failure modes in both dry and wet weather conditions thus enabling pollution incidents to be managed more proactively. The approach is a very useful alternative to developing a full physical based model of a catchment, removing manual modelling overheads and the data requirements of calibration.

ACKNOWLEDGEMENTS

This work was part supported by the Pennine Water Group Platform Grant, funded by the U.K. Engineering and Physical Sciences Research Council (EP/I029346/1). The authors would like to gratefully thank Yorkshire Water Services for data provision.

REFERENCES

Bracewell, R. 1965 Pentagram Notation for Cross Correlation. *The Fourier Transform and Its Applications*. New York: McGraw-Hill, pp. 46 and 243.

Branisavljevic´, N., Prodanovic´, D. and Pavlovic´, D. 2010 Automatic, semi-automatic and manual validation of urban drainage data. *Water Science and Technology*, **62**(5), 1013-1021.

Einfalt, T., Arnbjerg-Nielsen, K., Golza, C., Jensen, N., Quirmbach, M., Vaes, G., and Vieux, B. 2004 Towards a roadmap for use of radar rainfall data in urban drainage. *Journal of Hydrology*, **299**, 186-202.

Dawson, C. W.; Wilby, R. L. 1999 A comparison of artificial neural networks used for river flow forecasting, Hydrology & earth System Sciences, **3**(4), 529-540.

Duncan, A. P., Chen, A. S., Keedwell, E. C., Djordjević, S. and Savić, D. A. 2013 RAPIDS: Early Warning System for Urban Flooding and Water Quality Hazards. MaLWaS Sympsium, AISB-IACA conference, University of Exeter, April 2013. ISBN: 978-1-908187-33-8.

Evora, N.D and Coulibaly, P. 2009 Recent advances in data-driven modeling of remote sensing applications in hydrology. *Journal of Hydroinformatics*, **11**(3–4), 194–201.
Fernando, A. K., Zhang, X. and Kinley, P. F. 2006 Combined Sewer Overflow Forecasting with Feed-forward, Back-propagation Artificial Neural Network. *Transactions on Engineering, Computing and Technology* V12, ISSN 1305-5313, 58-64.
Guo, N. 2011 Improving the operation and maintenance of CSO structures. PhD thesis, University of Sheffield.
Guo, N. and Saul, A.J. 2011 Improving the operation and maintenance of CSO structures. In *Proceedings of 12th International Conference on Urban Drainage*, Porto Alegre, Brazil.
Kramer, S., Verworn, H.-R., and Ziegler, J. 2005 Radar rainfall time series for the performance assessment of sewer systems. In: *Proceedings of 10th International Conference on Urban Drainage*, Copenhagen, Denmark, 21-26 August 2005.
Kurth, A., Saul, A., Mounce, S.R., Shepherd, W and Hanson, D. 2008 Application of Artificial Neural Networks (ANNs) for the prediction of CSO discharges. *11th International Conference on Urban Drainage*, Edinburgh, Scotland 31 August-5 September.
Li, X., Zhou, F. and Lodewyk, S. 2010 Applications of Artificial Neural Networks in Urban Water System. *Proceedings of Watershed Management 2010 : Innovations in Watershed Management Under Land Use and Climate Change*, ASCE, Madison, Wisconsin, US.
Liukkonen, M., Laakso, I. and Hiltunen, Y. 2013 Advanced monitoring platform for industrial wastewater treatment: Multivariable approach using the self-organizing map. *Environmental Modelling & Software*, **48**, 193-201.
MATLAB® R2012a (The MathWorks Inc., Massachusetts).
Mayoraz, E. 1990 Benchmark of Some Learning Algorithms for Single Layer and Hopfield Networks. *Complex Systems*, **4**, 477-490.
Met. Office, 2009. Fact Sheet 15, Weather Radar. Available from: http://www.metoffice.gov.uk/media/pdf/o/c/fact_sheet_No._15.pdf [Accessed 19/06/2013].
Ostojin, S., Mounce, S. R. and Boxall, J. B. 2011 An artificial intelligence approach for optimising pumping in sewer systems. *Journal of HydroInformatics*, **13**(3), 295-306.
Penrose, R. 1955 A generalized inverse for matrices. *Proceedings of the Cambridge Philosophical Society,* **51**, 406–413. doi:10.1017/S0305004100030401.
Schellart, A.N.A, Shepherd, W. and Saul, A.J. 2012 Influence of rainfall estimation error and spatial variability on sewer flow prediction at a small urban scale. *Advances in Water Resources*, **45**, 65-75. DOI 10.1016/j.advwatres.2011.10.012.
Shepherd, W., Saul, A.J., and Hanson, D. 2010 Case Study of Long Term Sewer Hydraulic Monitoring. *Proceedings of 6th International Conference on Sewer Processes and Networks*, Surfers Paradise, Australia.
Solomatine, D P. and Dulal, K N. 2003 Model trees as an alternative to neural networks in rainfall-runoff modelling, Hydrological Sciences–Journal–des Sciences Hydrologiques, **48**(3), 399-411.
Widrow, B. 1962 *Generalization and information storage in networks of ADALINE Neurons in Self Organizing system*, (M.C. Yovitz, G.T. Jacobi, and G.D. Goldstein, eds) pp. 435-461, Washington, DC: Spartan Books.
Yu, Y., Kojima, K., An, K. and Furumai, H. 2013 Cluster analysis for characterization of rainfalls and CSO behaviours in an urban drainage area of Tokyo. *Water Science and Technology*, **68**(3), 544-551.