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Synthesis of General Chebyshev Characteristic Function for Dual (Single) Bandpass Filters

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Abstract— A new synthesis method for the generation of the generalized Chebyshev characteristic polynomials has been presented. The general characteristic function is generated by linear combination of elementary Chebyshev characteristic functions. The characteristic function is suitable for synthesis of dual bandpass filters as well as direct synthesis for single bandpass filters.

Index Terms—Dual-band Filters, Generalised Chebyshev, Minimum Phase Networks,

I. INTRODUCTION

There has been increasing interest in the past decade in the area of multi-passband filters. In particular dual band filters offers flexibility as well as efficiency in the utilization of communication resources. Many important contributions have been made to the methods of designing of dual band filters [1-3].

The previous methods outlined in [2, 3] involve some form of frequency transformations to generate a lowpass transfer function suitable for dual band filters. Similar lowpass filters for dual band filters may be designed using change of variable based on classical work in [4]. In this paper, however, methods of direct generation of the general Chebyshev lowpass transfer function for dual passband filters will be explained. The outlined method offers a simple and intuitive approach to synthesis of dual (or direct synthesis of single) band filter networks by linearly combining simple elementary characteristic functions.

In section II the generation of the basic prototype characteristic functions is described followed by the method of computing the characteristic polynomials in section III. Finally, a design example is given in section IV.

II. CHARACTERISTIC FUNCTION

For any given filter network, the power transfer function may be defined as [5],

$$|S_{21}(\omega)|^2 = \frac{1}{1+\left(\frac{kF(\omega)}{P(\omega)}\right)^2} \tag{1}$$

where k is a constant, the monic polynomials, $F(\omega)$ and $P(\omega)$ are the reflection and transmission (containing the transfer function's transmission zeros) characteristic polynomial respectively, all dependent on the frequency variable ω rad/s. Let the characteristic function be defined as

$$T_N(\omega) = k\frac{F(\omega)}{P(\omega)} \tag{2}$$

where, N is the degree of the filter network. It may be shown then that for Chebyshev characteristic functions the characteristic function can be found from the linear combination of m number of low degree basic characteristic functions $X_r(\omega)$ based on the following equation,

$$T_N(\omega) = cosh\left\{\sum_{r=1}^{m}[\alpha_r\, acosh\{X_r(\omega)\}]\right\} \tag{3}$$

where α_r (integer) is the corresponding weighting number to the basic characteristic function $X_r(\omega)$. Thus the problem of determining the higher degree rational polynomial $T_N(\omega)$, is reduced to determining some unique lower degree basic characteristic functions $X_r(\omega)$ which act as basic building blocks for higher degree polynomials. Each of the basic prototype is defined by the number and positions of transmission zeros. The overall characteristic function may be obtained by further expansion of (3) as presented below after a bit of mathematical manipulations,

$$T_N(\omega) = \frac{1}{2}\left\{\prod_{r=1}^{m}\left[X_r(\omega)+\sqrt{X_r^{\,2}(\omega)-1}\right]^{\alpha_r} + \prod_{r=1}^{m}\left[X_r(\omega)-\sqrt{X_r^{\,2}(\omega)-1}\right]^{\alpha_r}\right\}. \tag{4}$$

Now the term in (4) is conveniently re-written as

$$X_r(\omega) \pm \sqrt{X_r^{\,2}(\omega)-1} = \frac{U_r \pm W_r\sqrt{V}}{P_r(\omega)} \tag{5}$$

where U_r is simply the numerator of $X_r(\omega)$ and

$$V = (\omega^2-1)(\omega-\alpha)(\omega-\beta) \tag{6}$$

is the polynomial containing the critical (cut-off) points. W_r is the polynomial that results from the factorisation (5) and $P_r(\omega)$ is simply the denominator of $X_r(\omega)$. The next sections show how the basic prototypes may be determined.

A. Basic prototypes for minimum phase lowpass filters

For minimum phase filter networks, all the transmission zeros of the transfer function (1) are either at the origin or infinite in the complex plane. For this class of lowpass filter, the characteristic function satisfies the general differential equation of the form,

$$\frac{dT_N(\omega)}{d\omega} = \frac{C_n(\omega^2+\omega_m^{\,2})\sqrt{T_N^{\,2}(\omega)-1}}{\omega\sqrt{\omega^4-(1+\omega_c^{\,2})\omega^2+\omega_c^{\,2}}}. \tag{7}$$

The term $\omega^2 + \omega_m{}^2$ accounts for a pair of imaginary turning points. The other turning points are provided by the term $\sqrt{T_N{}^2(\omega) - 1}$ and the extra zeros provided by this expression are just the cut-off points at $\omega = -1, -\omega_c, \omega_c$, and 1, which are cancelled out by the denominator term $\sqrt{\omega^4 - (1+\omega_c{}^2)\omega^2 + \omega_c{}^2}$. C_n is a constant and ω_c is the inner cutoff in the normalized domain. By solving this differential equation, the general solution of the characteristic function for minimum phase filter networks is obtained. It may be shown that the solution to (7) may be written as,

$$T_N(\omega) = cosh\begin{Bmatrix} \alpha_1\, acosh\{X_{2-0-0}(\omega)\} \\ +\alpha_2\, acosh\{X_{2-0-1}(\omega)\} \\ +\alpha_3\, acosh\{X_{2-0-2}(\omega)\} \end{Bmatrix} \quad (8)$$

proving the synthesis equation (3) where the basic prototypes are

$$X_{2-0-0}(\omega) = \frac{2\omega^2 - (1+\omega_c{}^2)}{(1-\omega_c{}^2)}$$
$$X_{2-0-1}(\omega) = \frac{\omega^2 - \omega_c}{(1-\omega_c)\omega} \quad (9)$$
$$X_{2-0-2}(\omega) = \frac{(1+\omega_c{}^2)\omega^2 - 2\omega_c{}^2}{(1-\omega_c{}^2)\omega^2}.$$

From (8) for $T_N(\omega)$ to be an even N^{th} degree rational polynomial in ω, $2\alpha_1 + 2\alpha_2 + 2\alpha_3 = N$ i.e.

$$\alpha_1 + \alpha_2 + \alpha_3 = N/2 \quad (10)$$

Thus α_1, α_2 and α_3, must be either zero or positive integers. Also from (8), the number of transmission zeros at the origin for $T_N(\omega)$ is

$$N_{TZ} = N_{OTZ} = \alpha_2 + 2\alpha_3 \quad (11)$$

By assigning different integer values including zero to α_1, α_2 and α_3, different linear combinations of functions in (8) may be obtained as unique solutions to the differential equation (7). $T_N(\omega)$ given by (8) is thus the general solution to the differential equation defined by (7). There are only two equations in α_1, α_2 and α_3 i.e. (10) and (11). Thus one of the three scalars may be chosen and the other two may be determined from (10) and (11). One suitable choice is as follows: For $N_{OTZ} \leq N/2$, choose $\alpha_3 = 0$, then solving (10) and (11) simultaneously yields,

$$\alpha_1 = N/2 - N_{OTZ}$$
$$\alpha_2 = N_{OTZ} \quad (12)$$

For $N_{OTZ} \geq N/2$, choose $\alpha_1 = 0$, then solving (10) and (11) simultaneously yields,

$$\alpha_2 = N - N_{OTZ}$$
$$\alpha_3 = N_{OTZ} - N/2 \quad (13)$$

TABLE I POSSIBLE VALUES FOR SCALARS α_1, α_2 AND α_3

Condition	α_1	α_2	α_3
$N_{OTZ} \leq N/2$	$N/2 - N_{OTZ}$	N_{OTZ}	0
$N_{OTZ} \geq N/2$	0	$N - N_{OTZ}$	$N_{OTZ} - N/2$

The different scalars values are summarized in table I. In this work, the nomenclature $N - N_{FTZ} - N_{OTZ}$ is adopted to depict an N^{th} degree characteristic function with N_{FTZ} transmission zeros at some general complex frequencies, including purely real and imaginary (real frequency), and N_{OTZ} number of transmission zeros at the origin. This yields family of solutions based on the number of transmission zeros at the origin (N_{OTZ}). It is interesting to note that the first characteristic function in (9) is simply the even degree Achieser-Zolotarev characteristic function [6]. The well-known Chebyshev even degree characteristic functions may be obtained from this class by simply setting the parameters $\omega_c = 0$ and $N_{FTZ} = N_{OTZ} = 0$. Additionally, the second prototype in (9) is a well-known function in the design of filters e.g. used in [2] and also used as a conventional normalised bandpass transformation [7]. Therefore, Table I gives all possible transmission zeros at the origin (N_{OTZ}) for any given minimum phase lowpass filter of degree N(N even).

B. Basic prototypes for symmetrical lowpass filters

The three basic prototypes derived in II (A) are used in design of general Chebyshev characteristic function for symmetrical networks. In general an N^{th} basic prototype characteristic function may be determined analytically by solving a set of N non-linear simultaneous equations based on the behavior of the function and its known values at the critical points, (i.e. α, β, ± 1 as depicted in Fig. 1) using,

$$X_r{}^2(\omega) - 1 = 0\,. \quad (14)$$

For symmetrical networks, the inner cutoff points are,

$$\alpha = \beta = \omega_c \quad (15)$$

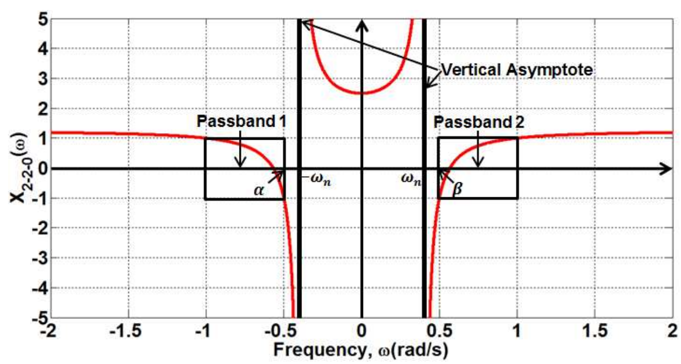

Fig. 1 Example of a plot of the basic characteristic function in II (B)

For example consider a second degree basic prototype for direct synthesis of bandpass filters or synthesis of symmetrical dual band filters shown in Fig. 1 given by,

$$X_{2-2-0}(\omega) = \frac{U_r(\omega)}{P_r(\omega)} = \frac{\omega^2 + p}{d(\omega^2 - \omega_n{}^2)} \quad (16)$$

Since from Fig. 1, $X_{2-2-0}(\pm 1) = 1$ and $X_{2-2-0}(\pm\omega_c) = -1$, then two simultaneous equations may be formed and solved for unknown coefficients p and ε as

$$\frac{1+p}{\varepsilon(1-\omega_n{}^2)} = 1 \text{ and } \frac{\omega_c{}^2 + p}{\varepsilon(\omega_c{}^2 - \omega_n{}^2)} = 1 \text{ where}$$
$$p = \left(2\omega_c{}^2 - \omega_n{}^2(1+\omega_c{}^2)\right)/(2\omega_n{}^2 - \omega_c{}^2 - 1) \quad (17)$$
$$\varepsilon = (\omega_c{}^2 - 1)/(2\omega_n{}^2 - \omega_c{}^2 - 1)$$

Hence the basic symmetrical characteristic function with the transmission zeros prescribed at ${\omega_n}^2$ is given by,

$$X_{2-2-0}(\omega) = \frac{(2{\omega_n}^2 - {\omega_c}^2 - 1)\omega^2 + 2{\omega_c}^2 - {\omega_n}^2(1 + {\omega_c}^2)}{({\omega_c}^2 - 1)(\omega^2 - {\omega_n}^2)}. \tag{18}$$

In fact all the basic prototypes may be found in this way and for symmetrical networks are summarized in Table II. Note that for the $X_{2-2-0}(\omega)$ prototype, $W_r > 0$ for $|\omega_n| > 1$ and $W_r < 0$ for $\omega_c < \omega_n < \omega_c$.

TABLE II: SYMMETRICAL NETWORK BASIC PROTOTYPES

$V = (\omega^2 - 1)(\omega^2 - {\omega_c}^2)$

$N - N_{FTZ} - N_{OTZ}$	$U_r(\omega)$	$W_r(\omega)$	$P_r(\omega)$
$2-0-0$	$2\omega^2 - (1 + {\omega_c}^2)$	2	$1 - {\omega_c}^2$
$2-0-1$	$\omega^2 - \omega_c$	1	$(1 - \omega_c)\omega$
$2-0-2$	$(1 + {\omega_c}^2)\omega^2 - 2{\omega_c}^2$	$2\omega_c$	$(1 - {\omega_c}^2)\omega^2$
$2-2-0$	$(2{\omega_n}^2 - {\omega_c}^2 - 1)\omega^2 +$ $2{\omega_c}^2 - {\omega_n}^2(1 + {\omega_c}^2)$	$\pm 2\sqrt{({\omega_n}^2 - 1)({\omega_n}^2 - {\omega_c}^2)}$	$({\omega_c}^2 - 1)(\omega^2 - {\omega_n}^2)$

C. Basic prototypes for asymmetrical lowpass filters

Similar to the method used for symmetrical prototypes, asymmetrical basic prototypes may be determined and are tabulated in Table III in terms of the required polynomials.

TABLE III: ASYMMETRICAL NETWORK BASIC PROTOTYPES

$V = (\omega^2 - 1)(\omega - \alpha)(\omega - \beta)$

Prototype ($N - N_{FTZ} - N_{OTZ}$) and Position of Dependent Transmission Zero (ω_z) ISB = Inner Stopband, LSB = Lower Stopband, USB = Upper Stopband
$2-2-0$ (ω_z $LSB-ISB/ISB-USB$)
$\omega_z = [(2\alpha\beta + \beta - \alpha)/\omega_n - \alpha - \beta]/[(\alpha + \beta)/\omega_n - \alpha + \beta - 2]$ $\varepsilon = (\alpha + 1)/[(\omega_z - \alpha)/\omega_n + \omega_z + 1]$ $U_r(\omega) = \omega^2 - \varepsilon(\omega_z + 1/\omega_n)\omega + \varepsilon(1 + \omega_z/\omega_n) - 1$ $P_r(\omega) = \varepsilon\big((\omega_z/\omega_n)\omega^2 - (\omega_z + 1/\omega_n)\omega - \omega_z\big)$ $W_r(\omega) = 1$ if ω_n is infinite $W_r(\omega) = \sqrt{1 - \varepsilon^2}$ if ω_n is finite
$2-2-0$ (ω_z $LSB-USB/ISB-ISB$)
$\varepsilon = [\alpha\beta + 1 - (\alpha + \beta)\omega_n]/[1 - \alpha\beta + (\alpha + b - 2\omega_n)\omega_n]$ $\omega_z = (\alpha\beta + 1 + (\alpha\beta - 1)\varepsilon)/(2\varepsilon\omega_n)$ $U_r(\omega) = \omega^2 - (1 + \varepsilon)(\alpha + \beta)\omega/2 + (\alpha\beta - 1)/2 + \varepsilon(\alpha\beta + 1)/2$ $P_r(\omega) = \varepsilon(\omega^2 - (\omega_z + \omega_n)\omega + \omega_z\omega_n)$ $W_r(\omega) = \sqrt{1 - \varepsilon^2}$
$3-1-0$ ($1 - \beta > \alpha + 1$) (ω_z ISB)
$\omega_m = (\alpha + \beta + 2)/2$ $\varepsilon = ({\omega_m}^2 - 2\omega_m - \alpha - \beta - \alpha\beta)/2$ $\omega_z = -({\omega_m}^2 + \alpha\beta)/(2\varepsilon)$ $U_r(\omega) = \omega^3 - (2\omega_m + \alpha + \beta)\omega^2/2$ $+ ({\omega_m}^2 - 2\omega_m + \alpha + \beta + \alpha\beta)\omega/2$ $+ ({\omega_m}^2 - \alpha\beta)/2$ $P_r(\omega) = \varepsilon(\omega - \omega_z)$ $W_r(\omega) = \omega - \omega_m$
$3-1-0$ ($1 - \beta > \alpha + 1$) (ω_z LSB/USB)
$\omega_m = (\beta - \alpha)/2$ $\varepsilon = ({\omega_m}^2 + 2\alpha\omega_m + 1)/2$ $\omega_z = (\alpha{\omega_m}^2 + \beta)/(2\varepsilon)$ $U_r(\omega) = \omega^3 - (2\omega_m + \alpha + \beta)\omega^2/2 + ({\omega_m}^2 + 2\alpha\omega_m - 1)\omega/2$ $+ (-\alpha{\omega_m}^2 + \beta)/2$ $P_r(\omega) = \varepsilon(\omega - \omega_z)$ $W_r(\omega) = \omega - \omega_m$
$3-1-0$ ($1 - \beta < \alpha + 1$) (ω_z ISB)
$\omega_m = (\alpha + \beta - 2)/2$ $\varepsilon = -({\omega_m}^2 + 2\omega_m + \alpha + \beta - \alpha\beta)/2$ $\omega_z = -({\omega_m}^2 + \alpha\beta)/(2\varepsilon)$ $U_r(\omega) = \omega^3 - (2\omega_m + \alpha + \beta)\omega^2/2$ $+ ({\omega_m}^2 + 2\omega_m - \alpha - \beta + \alpha\beta)\omega/2$ $+ (-{\omega_m}^2 + \alpha\beta)/2$ $P_r(\omega) = \varepsilon(\omega - \omega_z)$ $W_r(\omega) = \omega - \omega_m$
$3-1-0$ ($1 - \beta < \alpha + 1$) (ω_z LSB/USB)
$\omega_m = (\alpha - \beta)/2$ $\varepsilon = ({\omega_m}^2 + 2\beta\omega_m + 1)/2$ $\omega_z = \big(\beta{\omega_m}^2 + \alpha\big)/(2\varepsilon)$ $U_r(\omega) = \omega^3 - (2\omega_m + \alpha + \beta)\omega^2/2 + ({\omega_m}^2 + 2\beta\omega_m - 1)\omega/2$ $+ (-\beta{\omega_m}^2 + \alpha)/2$ $P_r(\omega) = \varepsilon(\omega - \omega_z)$ $W_r(\omega) = \omega - \omega_m$
$4-1-0$ (ω_z ISB)
$p = (-\alpha + \beta + 2)/2$ $\omega_{m1} = [\alpha + \beta - p^2 - 2p(\alpha - 1)]/[2(\alpha - \beta + p - 2)]$ $\omega_{m2} = \omega_{m1} + p$ $\varepsilon = (2\beta\omega_{m1} + 2\alpha\omega_{m2} + (\beta + 1){\omega_{m1}}^2 - (\alpha - 1){\omega_{m2}}^2)/2$ $\omega_z = \big(\beta{\omega_{m1}}^2 + \alpha{\omega_{m2}}^2\big)/(2\varepsilon)$ $U_r(\omega) = \omega^4 - (2\omega_{m1} + 2\omega_{m2} + \alpha + \beta)\omega^3/2 +$ $({\omega_{m1}}^2 + {\omega_{m2}}^2 + 2(\beta + 1)\omega_{m1} + 2(\alpha - 1)\omega_{m2} - \alpha + \beta)\omega^2/2 -$ $\big((\beta + 1){\omega_{m1}}^2 + (\alpha - 1){\omega_{m2}}^2 + 2\beta\omega_{m1} - 2\alpha\omega_{m2}\big)\omega/2 +$ $(\beta{\omega_{m1}}^2 - \alpha{\omega_{m2}}^2)/2$ $P_r(\omega) = \varepsilon(\omega - \omega_z)$ $W_r(\omega) = \omega^2 - (\omega_{m1} + \omega_{m2})\omega + \omega_{m1}\omega_{m2}$

III. CHARACTERISTIC POLYNOMIALS

Once the basic prototypes are determined, the computation of the overall characteristic function is fairly straight forward from (4). The weighting integer numbers α_r is the designer's choice depending on the number and positions of transmission zeros required from a given elementary function of table II. By substituting (5) in (4), the reflection polynomial is obtained as follows:

$$F(\omega) = \frac{1}{2}\left\{\prod_{r=1}^{m}\left[U_r + W_r\sqrt{V}\right]^{\alpha_r} + \prod_{r=1}^{m}\left[U_r - W_r\sqrt{V}\right]^{\alpha_r}\right\}. \tag{19}$$

Thus the reflection polynomial is computed by successive application of the general recursive technique [5] with initial conditions $X_0 = 1$ and $Y_0 = 0$ and defined by

$$X_N = U_r X_{N-1} + (W_r V) Y_{N-1}$$
$$Y_N = W_r X_{N-1} + U_r Y_{N-1} \tag{20}$$

where all the parameters are as defined above. (20) is used for m basic prototypes and repeated α_r times for each prototype, each time using the previous results to compute the N^{th} polynomials in (20). Finally

$$F(\omega) = X_N. \tag{21}$$

The monic polynomial $P(\omega)$ is obtained from the prescribed transmission zeros as

$$P(\omega) = \prod_{r=1}^{m} P_r(\omega). \tag{22}$$

For monic polynomials $F(\omega)$ and $P(\omega)$, their normalizing parameter μ and ε respectively are computed at points in ω - plane where both s-parameters $S_{11}(\omega)$ and $S_{21}(\omega)$ are known (e.g. $\omega = 1$) so that the unitary condition [7] and the prescribed return loss level are satisfied. Once the characteristic polynomials are determined, the rest of the synthesis process follows from the standard filter theory in [8].

IV. DESIGN EXAMPLE

A symmetrical dual passbands with cutoffs at 1710-1785 & 1920-1995MHz and 20 dB passband return loss were designed. Using the lowpass to bandpass transformation, the inner cutoff is $\omega_c = 0.5025$. The prescribed transmission zeros were at $\omega = \pm 0.25, 0, \pm 1.75$. Since the dual band is symmetrical, the following lowpass prototypes were linearly combined according to (3) based on the basic prototypes of table II: $2-0-0$, $2-0-1$ and $2-2-0$ with weighting numbers $\alpha_1 = 2$, $\alpha_2 = 1$ and $\alpha_3 = 2$ respectively. The first prototype only provides transmission zeros at infinite, the second provides the required single transmission zero at the origin and the last prototype provides the two pairs of symmetrical transmission zeros - applied iteratively depending on α_r.

TABLE IV $10-4-1$ DUAL BAND FILTER CHARACTERISTIC POLYNOMIALS ($\mu = -1$AND $\varepsilon = 197.6872$)

$$P(p) = p^5 + 3.1250p^3 + 0.1914p$$

$$F(p) = p^{10} + 2.9564p^8 + 3.3175p^6 + 1.7564p^4 + 0.4373p^2 + 0.0410$$

$$E(p) = p^{10} + 1.0152p^9 + 3.4717p^8 + 2.5759p^7 + 4.2763p^6 + 2.2206p^5 + 2.2900p^4 + 0.7535p^3 + 0.5238p^2 + 0.0842p + 0.0410$$

Coupling Matrix for $10-4-1$ Dual Band Filter

	S	1	2	3	4	5	6	7	8	9	10	L
S	0	0.7124	0	0	0	0	0	0	0	0	0	0
1	0.7124	0	0.6601	0	-0.4907	0	0	0	0	0	0	0
2	0	0.6601	0	0.0643	0	0	0	0	0	0	0	0
3	0	0	0.0643	0	0.6752	0	0	0	0	0	0	0
4	0	-0.4907	0	0.6752	0	0.3150	0	0	0	0	0	0
5	0	0	0	0	0.3150	0	0.6816	0.3276	0	0	0	0
6	0	0	0	0	0	0.6816	0	0	0	0	0	0
7	0	0	0	0	0	0.3276	0	0	0.7038	0	-0.0984	0
8	0	0	0	0	0	0	0	0.7038	0	0.4841	0	0
9	0	0	0	0	0	0	0	0	0.4841	0	0.8167	0
10	0	0	0	0	0	0	0	-0.0984	0	0.8167	0	0.7124
L	0	0	0	0	0	0	0	0	0	0	0.7124	0

This gives an overall $10-4-1$ lowpass characteristic function. Using the general recursive formulae (20) the characteristic polynomials were determined as in Table IV in complex variable $p(j\omega)$. Then cascaded synthesis was used to extract the element values and the coupling matrix generated as shown below. The bandpass simulation and topology are shown in Fig. 2.

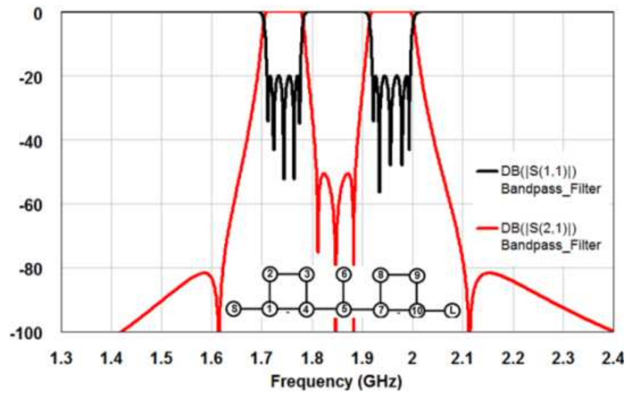

Fig. 2 Bandpass Simulation of the dual band filter in IV with its topology.

V. CONCLUSION

The method of generating the general Chebyshev characteristic function used in the design of relatively close-spaced dual passband filters has been outlined. Linear combinations of the elementary characteristic functions (based on transmission zeros placement) allow high order characteristic functions to be determined.

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