



Deposited via The University of Sheffield.

White Rose Research Online URL for this paper:

<https://eprints.whiterose.ac.uk/id/eprint/83163/>

---

**Monograph:**

Coca, D., Balikhin, M.A. and Billings, S.A. (2001) Comparison of the Dynamic Processes in Plasma Turbulence Observed in the High and Low-B Regions of the Terrestrial Foreshock. Research Report. ACSE Research Report 792 . Department of Automatic Control and Systems Engineering

---

**Reuse**

Items deposited in White Rose Research Online are protected by copyright, with all rights reserved unless indicated otherwise. They may be downloaded and/or printed for private study, or other acts as permitted by national copyright laws. The publisher or other rights holders may allow further reproduction and re-use of the full text version. This is indicated by the licence information on the White Rose Research Online record for the item.

**Takedown**

If you consider content in White Rose Research Online to be in breach of UK law, please notify us by emailing [eprints@whiterose.ac.uk](mailto:eprints@whiterose.ac.uk) including the URL of the record and the reason for the withdrawal request.

COMPARISON OF THE DYNAMICAL PROCESSES IN PLASMA TURBULENCE OBSERVED  
IN THE HIGH- AND LOW- $\beta$  REGIONS OF THE TERRESTRIAL FORESHOCK

D. COCA, M. A. BALIKHIN, S.A. BILLINGS



Department of Automatic Control and Systems Engineering,  
University of Sheffield  
Sheffield, S1 3JD,  
UK

Research Report No.792

June 2001

200704609



# COMPARISON OF THE DYNAMICAL PROCESSES IN PLASMA TURBULENCE OBSERVED IN THE HIGH- AND LOW- $\beta$ REGIONS OF THE TERRESTRIAL FORESHOCK

D. Coca, M. A. Balikhin and S. A. Billings

Dept. of Automatic Control & Systems Engineering,  
University of Sheffield, Mappin Street, Sheffield S1 3JD, UK  
tel: +44 (0)114 222 5250/fax: +44 (0)114 222 5661  
e-mail: coca@acse.shef.ac.uk / balikhin@acse.shef.ac.uk

## ABSTRACT

This paper highlights the fact that the dynamical processes that characterise plasma turbulence observed in the high- $\beta$  region of the terrestrial foreshock are significantly different from the dynamical processes identified in the low- $\beta$  region. The study is based on a time-domain model identified from measurements taken by AMPTE-UKS and AMPTE-IRM satellites.

## INTRODUCTION

Nonlinear system identification based on the NARMAX model (Leontaritis and Billings, 1987) has been previously applied to analyse developed turbulence observed upstream of the quasi-parallel part of the terrestrial bow-shock. The data used in the study (Schwartz and Burgess, 1991; Schwartz *et al.*, 1992) was recorded by the magnetometer instruments aboard AMPTE UKS (PI D. Southwood) and AMPTE IRM (PI H. Lühr) satellites (Southwood *et al.*, 1985; Lühr *et al.*, 1985). One thousand pairs of measurements of  $B_y$  taken by AMPTE UKS and AMPTE IRM, considered to be the input and the output respectively of a nonlinear dynamical system, were used to estimate a time domain model of the turbulence (Coca *et al.*, 1999). The plasma  $\beta$  (the ratio of thermal pressure to magnetic pressure) over the time interval considered for identification (500-750 seconds past 10:50:00 UT) was low compared with a previous interval of observation (0-150 second past 10:50:00 UT).

This study compares the model predictions over the time intervals characterised by high and low plasma  $\beta$ , using data not used in estimating the model. The aim is to investigate if the dynamics in the high- $\beta$  region can also be described accurately by the model identified from observations in the low- $\beta$  region.

## NONLINEAR MODEL IDENTIFICATION

A Volterra polynomial model, which is just a particular type of NARMAX model

$$y(t) = f(u(t-1), \dots, u(t-n_u), e(t-1), \dots, e(t-n_e)) + e(t)$$

where  $u(k)$ ,  $y(k)$ ,  $e(k)$  denote the input, output and prediction error respectively,  $n_u$  and  $n_e$  are the maximum input and output lags, and  $f(\cdot)$  is a cubic

multivariable polynomial, was identified from the data recorded over the estimation interval (500-750 seconds past 10:50:00 UT). This interval was chosen because of the high amplitude range and frequency content of the input measurements (AMPTE UKS) over this interval. The identified model included deterministic and stochastic terms. The model validation tests performed on the resulting model have shown that there were no unmodelled nonlinear processes.

## MODEL PREDICTIONS IN THE LOW- $\beta$ REGION

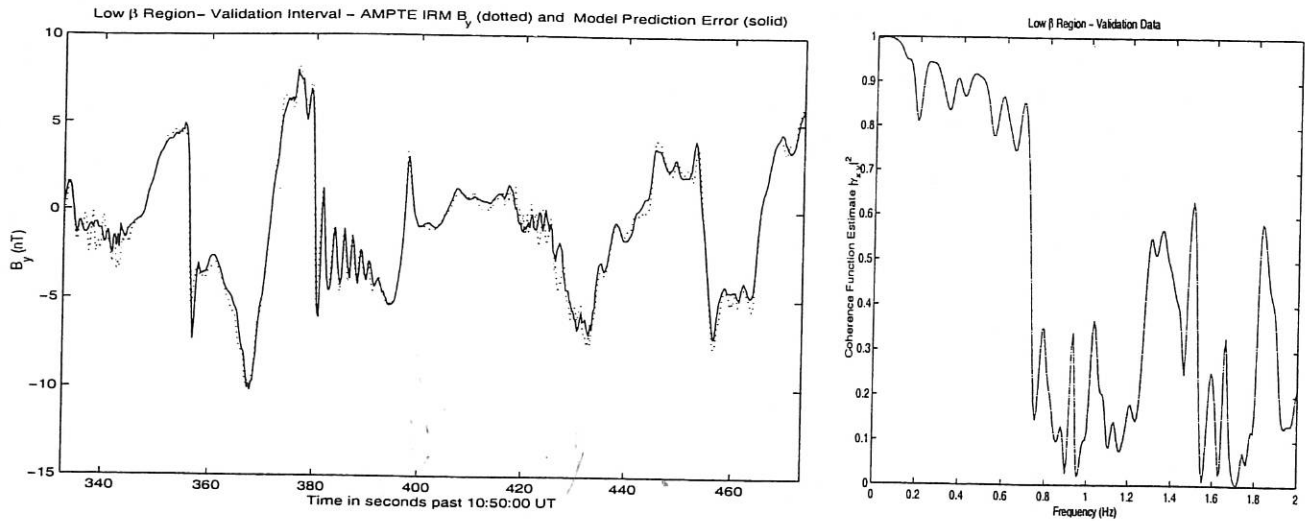
The deterministic part of the identified model was simulated using the AMPTE UKS data as the input. The resulting model predicted output was compared with the original AMPTE IRM measurements in the low- $\beta$  interval.

Figure (1.a) shows the measurements of  $B_y$  taken by AMPTE IRM (dotted) superimposed with the model predicted output (solid) over the estimation interval. To investigate how the model performs over different frequencies the coherence function  $|\gamma(\omega)|^2$  calculated for the real measurements and the model predicted output over the same interval are shown in Figure (1.b). Figure (2.a) shows the measurements of  $B_y$  taken by AMPTE IRM (dotted) superimposed with the model predicted output (solid) over a different interval not used in estimation. The coherence function  $|\gamma(\omega)|^2$  calculated for the real measurements and the model predicted output over the same interval are shown in Figure (2.b).

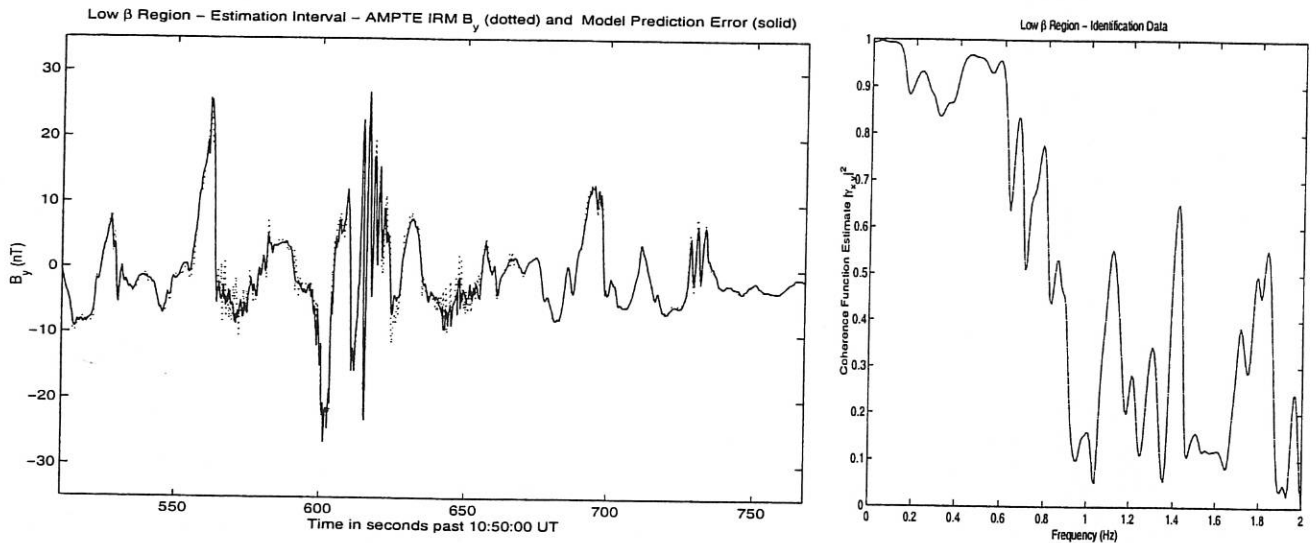
In both cases the model predictions are very accurate over the frequency range which contains most energy of the turbulence.

## MODEL PREDICTIONS IN THE HIGH- $\beta$ REGION

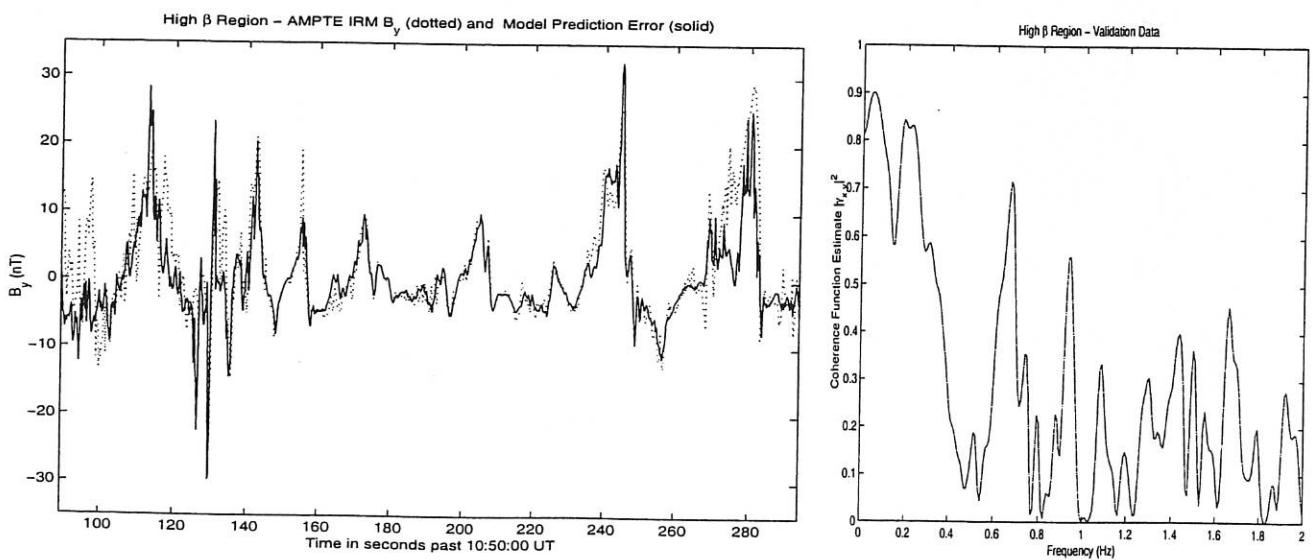
Figure (3.a) shows the model prediction output (solid) is superimposed on the real measurements (dotted) taken over an interval characterised by a higher value of  $\beta$ . The coherence function for this interval is shown in Figure (3.b). In this case the model predictions are quite poor compared to the previous intervals analysed. In particular the coherence functions show that the model fails to predict accurately in the 0.2-0.6 Hz frequency range where the whistler waves are usually observed.



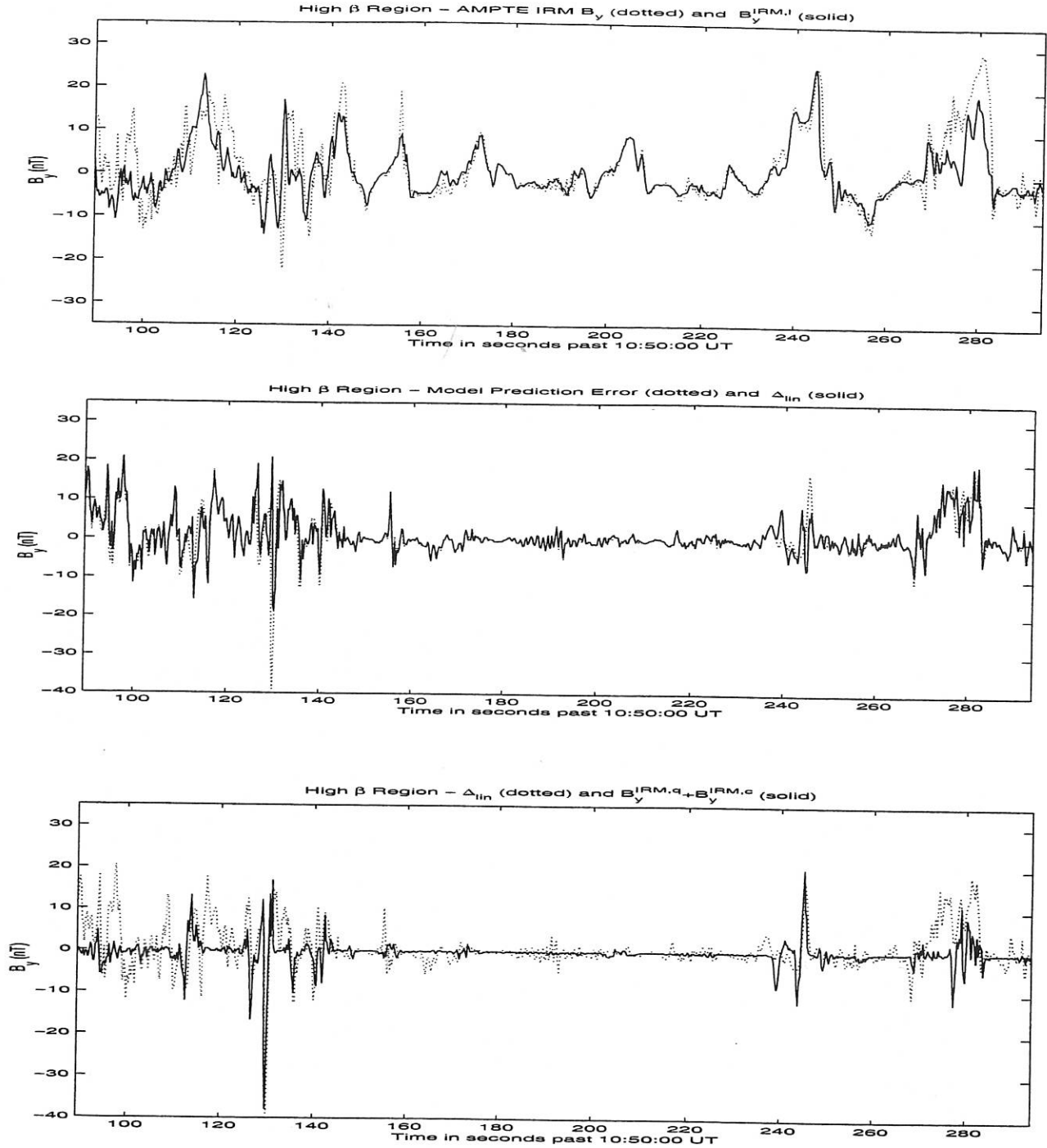
Figures 1: (a) Model predictions over the low  $\beta$  estimation interval (b) Coherence



Figures 2: (a) Model predictions over the low  $\beta$  validation interval (b) Coherence



Figures 3: (a) Model predictions over the high  $\beta$  validation interval (b) Coherence



Figures 4: (a) AMPTE IRM and  $B_y^{IRM,l}$  (b) Model error and  $\Delta_{lin}$  (c) Model error and  $B_y^{IRM,q} + B_y^{IRM,c}$

The model predicted output has been decomposed into its linear, quadratic and cubic contributions  $B_y^{IRM,l}$ ,  $B_y^{IRM,q}$  and  $B_y^{IRM,c}$ .

Figure (4.a) shows the measurements of  $B_y$  (dotted) superimposed by the linear model predictions  $B_y^{IRM,l}$  (solid). Figure (4.b) shows the prediction error (dotted) superimposed with the linear prediction error  $\Delta_{lin}(k)$  (solid) while Figure (4.c) shows the linear prediction error  $\Delta_{lin}(k)$  (dotted) super-imposed by the combined quadratic and cubic components  $B_y^{IRM,q} + B_y^{IRM,c}$  (solid).

## CONCLUSIONS

The results clearly indicate that the dynamics of the turbulence in the region characterised by higher  $\beta$  values is significantly different from the dynamics in the lower  $\beta$  part. In particular, Figures (4.a,b,c) indicate that while the low amplitude structures are still described quite well by the linear part of the model, the quadratic and cubic nonlinearities in the model fail to compensate the linear prediction errors that occur for

the higher amplitude structures. This suggests that the main changes in the dynamics that occur in the high- $\beta$  region involve mainly the nonlinear, three- and four-wave interaction processes. A more comprehensive study is under way.

#### ACKNOWLEDGEMENTS

DC and SAB gratefully acknowledge that part of this work was supported by the UK Engineering and Physical Sciences Research Council.

#### REFERENCES

- Lühr H., Klöckler N., Oelschägel, W., Häusler B., and M. Acuna (1985): The IRM magnetometer, *IEEE Trans. Geosci. And Remote Sensing*, vol. GE-23, 259, 1985.
- Leontaritis, I. J. and Billings S. A. (1987): Input-output parametric models for nonlinear systems – Part 1: Deterministic non-linear systems; Part 2: Stochastic nonlinear systems, *International Journal of Control*, Vol. 41, 303, 1987.
- Schwartz, S. J. and Burgess D. (1991): Quasi-parallel shocks: A patch-work of three-dimensionless structures, *Geophys. Res. Lett.*, Vol. 18, 373, 1991.
- Southwood, D. J., Mier-Jedrzejowicz, W. A. C. and Russell C. T. (1985): The fluxgate magnetometer for the AMPTE UKS subsatellite, *IEEE Trans. Geosci. And Remote Sensing*, Vol. GE-23,301,1985.
- Coca D., Balikhin, M., Billings S. A., Alleyne, H. St. C. K., Dunlop, M., Lühr, H. (1999): Time-Domain Identification of Nonlinear Processes in Space-Plasma Turbulence Using Multi-Point Measurements, *ESA/Imperial College Cluster-II Workshop on Multiscale/Multipoint Plasma Measurements*, Imperial College, London, ESA SP-449, 5, 1999.

